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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Heuristics for Network Coding in Wireless Networks

Song Yean Cho, Cédric Adjih, Philippe Jacquet

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Heuristics for Network Coding in Wireless Networks

Song Yean Cho, Cédric Adjih, Philippe Jacquet

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Abstract:

Multicast is a central challenge for emerging multi-hop wireless architectures such as wireless mesh networks, because of its substantial cost in terms of bandwidth.

In this report, we study one specific case of multicast: broadcasting, sending data from one source to all nodes, in a multi-hop wireless network. The broadcast we focus on is based on network coding, a promising avenue for reducing cost; previous work of ours showed that the performance of network coding with simple heuristics is asymptotically optimal: each transmission is beneficial to nearly every receiver. This is for homogenous and large networks of the plan.

But for small, sparse or for inhomogeneous networks, some additional heuristics are required. This report proposes such additional new heuristics (for selecting rates) for broadcasting with network coding. Our heuristics are intended to use only simple local topology information. We detail the logic of the heuristics, and with experimental results, we illustrate the behavior of the heuristics, and demonstrate their excellent performance.

Key-words: wireless, broadcast, network coding, hypergraph, min-cut, heuristics, rate selection, subgraph selection

Heuristiques pour le codage de réseau dans les réseaux sans fil

Résumé : Le multicast est un enjeu central pour les nouvelles architectures de réseaux sans fil, tels que les réseaux maillés “mesh”, à cause de son coût substantiel en termes de bande passante.

Dans ce rapport, nous étudions une méthode de multicast spécifique: la diffusion à tout le réseau, dans un réseau sans fil multi-sauts. Plus précisément, la diffusion que nous considérons est fondée sur l’utilisation du codage de réseau, une méthode prometteuse pour réduire le coût. Nos précédents travaux ont montré que la performance du codage de réseaux avec une heuristique simple était asymptotiquement optimale : chaque transmission est utile à presque tous les destinataires. C’est vrai pour de grands réseaux homogènes du plan Euclidien.

Mais pour des réseaux plus petits, ou inhomogènes, des heuristiques supplémentaires sont requises. Ce rapport propose de telles heuristiques (de choix de débit) pour la diffusion par codage de réseau. Nos heuristiques ont pour objectif d’utiliser seulement des informations simples concernant la topologie locale. Nous détaillons la logique des heuristiques, et par des résultats expérimentaux, nous illustrons le comportement de ces heuristiques, et mettons en évidence leur excellente performance.

Mots-clés : sans fil, diffusion, codage de réseau, hypergraph, coupe minimale, heuristique, sélection de débit

1 Introduction

The confirmed success of wireless networks has made wireless communication ubiquitous. One of the predicted use of wireless networks is multicast, which could be used for multimedia content diffusion, video conference, software distribution, and a number of other applications. However, in multi-hop networks such as wireless mesh networks, multicast transmissions incur a substantial cost by simple virtue of requiring relaying over several forwarders, in order to cover every destination. For this reason efficient techniques for multicasting are of prime interest. One of them is the recently proposed method, network coding. Network coding was introduced by the seminal work of [1] as a new paradigm where intermediate nodes mix information from different flows (different bits or different packets). We use network coding specifically for broadcasting rather than general multicasting in wireless multi-hop networks.

The problem that we are addressing is *efficient broadcast*:

- Broadcast packets from one source to all nodes, with the minimum number of transmissions.

Without network coding, finding the optimal broadcasting is an NP complete problem [2], but a number of heuristics exist for efficient broadcasting such as MPR-flooding [3] or techniques based on connected dominating sets [4,5]. But with network coding, the optimal broadcasting can be found in polynomial time. Finding an optimal solution (subgraph selection) consists in finding the coding nodes and their optimal rates [6, 7, 8]. This problem can be formulated as a linear program, which can be solved in polynomial time [11, 10], and possibly in a distributed fashion [10].

However, we adopt a different, even simpler, approach: previous work [12, 13] has shown that a simple heuristic for selecting rates could achieve asymptotically the optimal efficiency for homogeneous large and dense wireless networks of the plane — and also that, noticeably, it would outperform methods without network coding. This is asymptotically true for homogeneous networks, but the heuristic needs adjustments for less homogeneous, smaller or sparser networks. The adjustment is the topic of this report. Our key contributions are the following:

- We propose an improved heuristic for rate selection, inspired by [14]. It requires only local topology information: knowledge of two-hop neighbors.
- We empirically study its performance on representative graphs with different densities and different sizes. We investigate and explain the variation of the performance, and also compare it to other techniques (including without using network coding).

The rest of the paper is organized as follows: section 2 details the network model and definitions, section 2.6 describes the heuristics, section 3 analyzes performance with experimental results and section 4 concludes.

2 Definitions

2.1 Network Model

In this report, we study the problem of broadcasting from one source to all nodes. We will focus on getting a preliminary idea of the performance of our heuristics for wireless networks. Hence, we will assume an ideal wireless model in this report (and realistic models would be subject of future work): lossless wireless transmissions without collisions or interferences. We also assume that each node of the network is operating below its maximum transmission capacity.

In an idealized model, multi-hop wireless networks can be modeled as *unit disk graphs* of the plane, where two nodes are neighbors if their distance is lower than a fixed radio range as seen in Fig. 1. In addition, in wireless networks, the *wireless broadcast advantage* is used: each transmission is overheard by several nodes. As a result the graph is in reality a (*unit disk*) *hypergraph*. Precisely, we consider the following networks: unit disk [hyper]graphs where nodes are either distributed randomly (Fig. 1.1) or more regularly organized in a lattice (Fig. 1.2). In addition, in both cases, we also consider their variants where the network is a torus, with wrap-around connections in both the x and y directions as on Fig. 1.3.

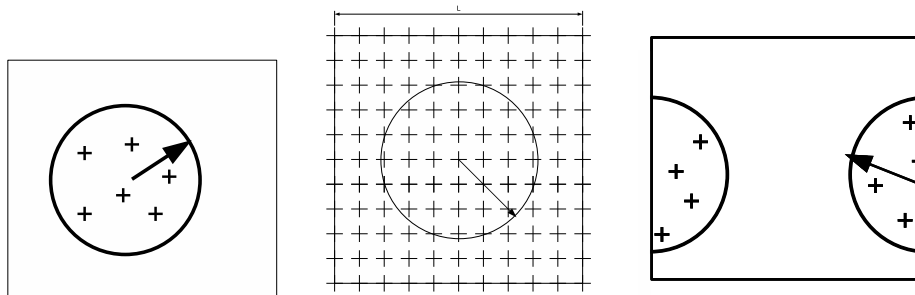


Fig. 1.1: random unit disk graph

Fig. 1.2: lattice graph

Fig. 1.3: on a torus

Figure 1: Network Models

2.2 Notation

We will consider the following items in each instance of the several types of graphs in section 2, and use the following notation in the rest of the report:

- **Nodes:** \mathcal{V} , set of vertices (nodes) of the graph
- **Number of nodes:** N

- **Expected/average number of neighbors:** M
- **Hyperedge :** $H_v, H_v \subset \mathcal{V}$ is the subset of nodes which are reached by one transmission of node v (the neighbors of v)
- **Rate:** Each node v retransmits coded packets with a fixed rate C_v .
- **Cost per broadcast:** E_{cost} ; defined in section 2.3
- **Optimal cost per broadcast :** E_{optimal} (section 2.3)
- **Min-cut of the source s , for broadcast to the entire network:** $C_{\text{min}}(s)$; see section 2.4

2.3 Performance

Because we focus on broadcasting, our approach uses a simplified subgraph selection, where every node is a coding node. The solution for the simplified subgraph consists of defining the rate of each node – by the heuristic.

To measure the performance of the heuristics, we use the number of transmissions per broadcast as the metric for the cost. We consider:

- the number of retransmissions from every node per unit time (directly given by selected rate).
- the number of packets successfully broadcasted from the source to the entire network per unit time; it is the achievable broadcast rate.

By dividing the number of retransmissions by the number of packets successfully broadcasted, we obtain our metric for the cost per broadcast and denote it E_{cost} . For reference, we will also use the cost of the optimal solution, E_{optimal} , obtained by solving the linear program as presented in [10].

The number of packets successfully broadcasted per the unit time, i.e. the maximum achievable broadcast rate is computed as the *min-cut* from the source to every destination in the network considered as a hypergraph as defined in section 2.4.

2.4 Achievable Broadcast Rate: Min-cut

A central result of the performance of network coding in wireless networks gives the maximum broadcast (more generally: multicast) rate for a source. It is given by the *min-cut* from the source to each individual destination of the networks, viewed as a hypergraph [15].

Let us consider the source s , and one of the broadcast destinations $t \in \mathcal{V}$. The definition of a *s-t cut* is: a partition of the set of vertices V in two sets S, T such as $s \in S$ and $t \in T$. Let $Q(s, t)$ be the set of such *s-t cuts*: $(S, T) \in Q(s, t)$.

We denote ΔS , the set of nodes of S which are neighbors of at least one node of T ; the *capacity of the cut* $C(S)$ is defined as the maximum rate between the nodes in S and the nodes in T :

$$C(S) \triangleq \sum_{v \in \Delta S} C_v \quad (1)$$

The *min-cut* between s and t is the cut of $Q(s, t)$ with the minimum capacity. Let us denote $C_{\min}(s, t)$ its capacity. From [9, 16, 15], the maximum broadcast capacity is given by the minimum of capacity of the min-cut of every destination $C_{\min}(s)$, with:

$$C_{\min}(s, t) \triangleq \min_{(S, T) \in Q(s, t)} C(S) \quad \text{and} \quad C_{\min}(s) \triangleq \min_{t \in \mathcal{V} \setminus \{s\}} C_{\min}(s, t)$$

Once computed, the performance given by the global min-cut $C_{\min}(s)$ can be achieved by different coding methods (precisely or asymptotically).

2.5 Previous Heuristic

In [12, 13] we proposed a simple heuristic, where most nodes have the same rate except the source and some nodes near the edge of the network (*exceptional nodes*): IREN/IRON (*Increased Rate for Exceptional Nodes, Identical Rate for Other Nodes*).

The simple heuristic achieved asymptotically near-optimal efficiency for unit disk graphs in large dense homogenous networks. In other words, the simple heuristic enables for every transmission to bring *innovative* information to almost every receiver.

Let us reproduce the logic of the heuristic:

1. Assume that the every node has an identical retransmission rate. Assume it is 1, arbitrarily, e.g. one packet per second.
2. Then every node with M neighbors can receive M coded packets per second. Assume that nearly all of them innovative.
3. Then the source should inject at least M packets per a second.
4. An issue is the nodes of near the border, because they have less neighbors - so in order to be safe, their rate is set to M as well (IREN).

Within this framework, it was proven that the achievable broadcast capacity, the min-cut, would be M for lattice graphs, and asymptotically M for dense random unit-disk graphs. For large lattice or random unit-disk graphs, the cost per broadcast would also converge to the optimal. However the result is asymptotical; for a practical instance of a graph, the cost of the “increased rate” of the border nodes could be considerable, and unnecessary high.

An immediate way to alleviate the cost is to omit step 4 of the reasoning entirely, ie, ignoring the issue of border, thus using the following heuristics:

- IRON only (Identical rate for other nodes): every node retransmits with rate 1, except for the source, which transmits with rate M .

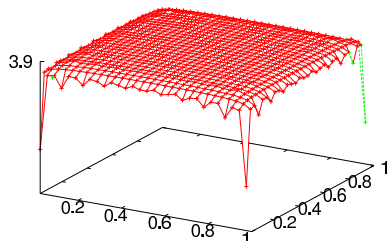


Fig. 2.1: min-cut with IRON only, on a lattice

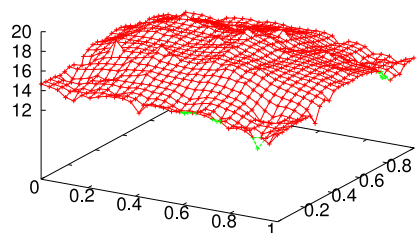


Fig. 2.2: min-cut with IRON only, on a random unit disk

Figure 2: Min-cut with IRON only

For illustrative purposes, we show the maximum achievable broadcast rate, for two instances of the networks with the rates selected by “IRON only” on Fig. 2.

The Fig. 2 represents the min-cut $C_{\min}(s, t)$ of each node t with respect to its position when the total number of nodes is $N = 400$. Both x and y axis represent the position of each node on the plane; the value on the vertical z axis represents min-cut¹. On a lattice, Fig. 2.1, every node has $M = 4$ neighbors, and the source is in the middle of the network. In the random unit disk graph in Fig. 2.2 we select the node with the most number of neighbors as the source, and generate graphs with density (the average number of nodes in a range) equal to $M = 20$.

The min-cut was computed with the software library implementing the maxflow computation algorithm from [17] (and one additional layer to model a directed hypergraph as a directed graph). The optimizations for tree reuse from [18] were also used.

As seen in Fig. 2, the nodes near the border have a min-cut much lower than nodes in the middle of the network (4 compared to 2 for Fig. 2.1). From section 2.4, recall that maximum achievable broadcast rate from the source, $C_{\min}(s)$, is the minimum of the min-cuts $C_{\min}(s, t)$ to each destination t . Hence these nodes near the border become the bottleneck and dictate a bound for the source rate much lower than what other nodes could have achieved. Also in random unit-disk graph Fig. 2.1, notice the same phenomenon occurs on the borders but

¹for the randomly generated unit disk graph, there is interpolation

also that the irregularity of number of neighbors results in irregularity of the min-cut (and hence potential lower minimum min-cut).

These are reasons of inefficiency of IRON.

2.6 The Proposed Heuristic: IR-MS

As illustrated with the simple heuristics in section 2.5, the lack of neighbors lets border nodes and some other nodes receive less than M packets per second and these nodes cause a decrease of the performance. We name these nodes *starving nodes*

To alleviate the bottleneck from the starving nodes, their neighbors compensate their starving by increasing the transmission rates with the following heuristic, inspired by [14]:

- IR-MS (Increased Rate for the Most Starving node): the rate of a node v is set to C_v , with:

$$C_v = \frac{M}{\min_{u \in H_v} (|H_u|)},$$
 where H_w is the set of neighbors of w , and M is the source rate.

This rate selection adjusts the step 2 of the reasoning presented in section 2.5, to the fact that some nodes have less than M neighbors. When IR-MS, if a node v has less than $|H_v| < M$ neighbors, it would still receive a packet rate $\geq \frac{M}{|H_v|}$ from each of its $|H_v|$ neighbors, and hence an overall rate at least equal to M . Notice that this does not necessarily result in a min-cut $C_{\min}(s, v) \geq M$.

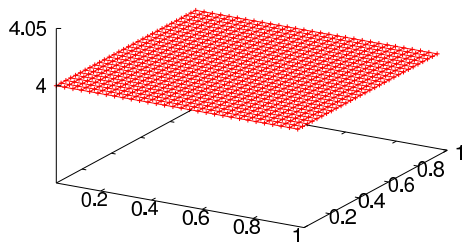


Fig. 3.1: min-cut with IR-MS, on a lattice

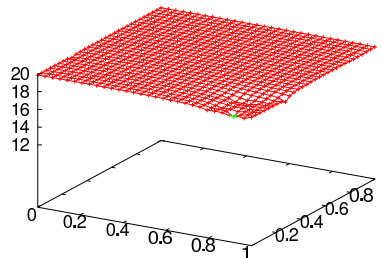


Fig. 3.2: min-cut with IR-MS, on a random unit disk

Figure 3: Min-cut with IR-MS

The Fig. 3 represents the value of the min-cut with the rate selection of IR-MS, with the same topologies and same parameters as for IRON only, in previous Fig. 2. It appears that for these topologies, the min-cut has the value which is much closer to the targeted value M . The increased rates of IR-MS at neighbors of bottleneck may bring additional costs. However this additional cost is relatively much lower than the increase of min-cut, thus the performance is overall improved. We will present more systematic experiments on the performance of the heuristic IR-MS in the next section.

3 Experimental Results

3.1 Efficiency on Different Types of Networks

We first evaluate the efficiency of the heuristic IR-MS, with the relative cost w.r.t. the optimal: $E_{\text{rel-eff}} \triangleq \frac{E_{\text{optimal}}}{E_{\text{cost}}}$

One reference point is the approximative upper bound in [13], the achievable performance without network coding, which translates into: $E_{\text{bound-rel-eff}}^{(\text{no-coding})} \approx 0.609 \dots$

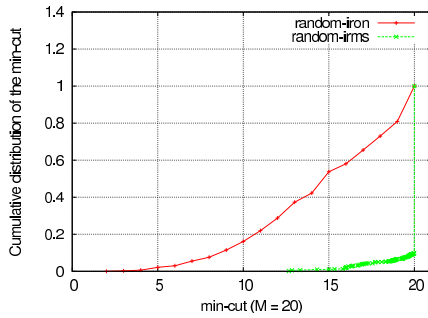


Fig. 4.1: micut: cumulative distribution

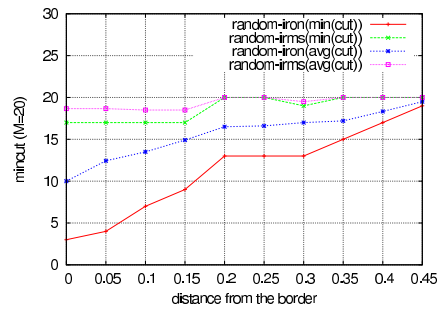


Fig. 4.2: mincut vs distance from the border

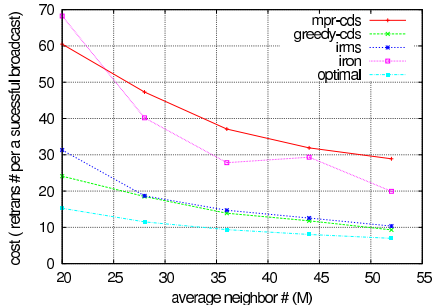


Fig. 4.3: cost vs avg neighbor num on square $N=200$

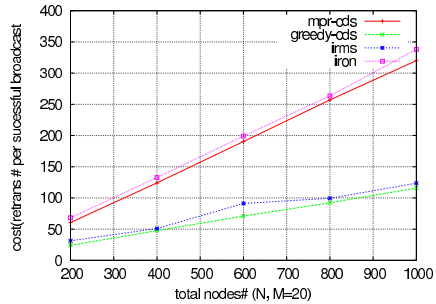


Fig. 4.4: cost vs total node num on square $M=20$

For comparison purposes, we evaluated the IR-MS heuristic on instances of lattice unit disk graphs and random unit disk graphs, both with and without torus effect — four variants in total. Our parameters are the following: number of nodes $N = 196$, the avg. number of neighbors is successively 4, 12, 28, 48, 80.

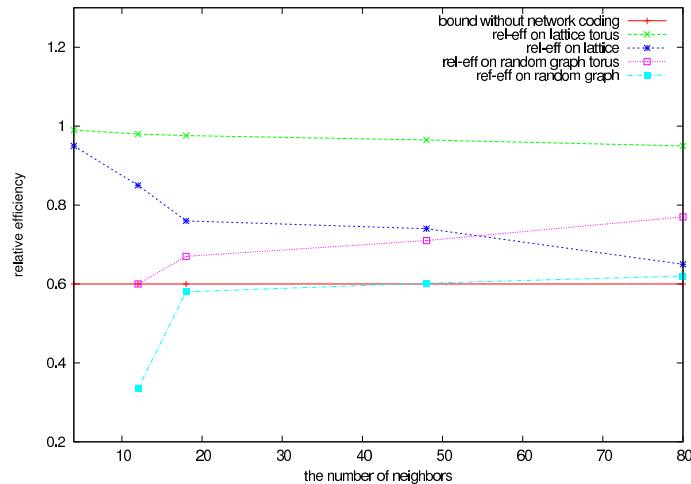


Figure 5: Relative cost (efficiency compared to optimal)

Fig. 5 represents the efficiency obtained for different cases (average of 10 results). The central result is the appreciable performance of IR-MS compared to the achievable performance without network coding: it mostly outperforms the upper bound without network coding (0.609).

Now consider the four different types of networks:

- First type, the most regular graphs: lattice unit disk graphs on torus. IR-MS closely approaches optimality (> 0.95). This is because every node has the same number of neighbors and the rate adjustment is not needed.
- Second type, lattice unit disk graphs without torus: the nodes near the border have less neighbors than others are the issue. IR-MS successfully solves the border issue, show in next section, approaching the targeted maximum broadcast rate, $\frac{\text{min-cut}}{M} = 1$. However efficiency decreases as density increases, because of an increase of the cost of nodes near the border.
- Third type, random unit disk graph on torus. No border effect here, but IR-MS has to overcome the effects of non-homogeneity, which is done convincingly.
- Fourth type, genuine random unit disk graph. The performance is acceptable, but on low density, performance becomes lower, because IR-MS does not fully achieve the maximum broadcast rate, M .

3.2 Distribution of the Min-cut

In this section, further results for case of the random unit disk graphs are provided: these graphs were the ones with the most problematic performance.

Again, the minimum of the min-cuts $C_{\min}(s, t)$ (for all $t \in \mathcal{V}$) decides the overall maximum broadcast rate of the source. Hence good performance is achieved when the distribution of these min-cuts is tighter. Deeper insight is gained by analyzing the cumulative distribu-

tion of the min-cut of each node, for the random graph $N = 400$ previously studied: it is displayed on Fig. 4.1. As evidenced, without IR-MS, the distribution of the min-cut is wider, but with IR-MS, the distribution is closer to M (with a peak for $M = 20$, the targetted min-cut). Still, there is some room for improvement, because a few nodes have a min-cut around 15.

The Fig. 4.2 provides additional information about the value of the min-cut depending on the position of the nodes, for IR-MS and IRON respectively. The distance of each nodes to the border of the network is computed, and statistics are made for the min-cut of nodes with same distance to the border. The Fig. 4.1 gives the average min-cut and the minimum min-cut for nodes at a given distance from the border.

It evidences that the border effect is key, with a lower min-cut when the node is near the border: again, we see that IR-MS improves the minimum min-cut but does not always achieve the target min-cut = M .

3.3 Random Unit Disk Graphs N, M

The Fig. 4.3 and Fig. 4.4, show different perspectives on the performance. The performance measured on these graphs, is E_{cost} , the number of retransmissions per broadcast packet.

First, different algorithms are compared:

- Network coding with IRON.
- Network coding with IR-MS.
- MPR-based dominating sets from [5] (with performance close to MultiPoint-Relays (MPR)-based flooding of [3]). It is representative of the performance of algorithms, with only local topology information, without network coding.
- Connected dominating set from [4] (efficient variant of a greedy algorithm). It is representative of the performance of centralized connected dominating sets algorithms (and to some extent, representative of what could be achieved without network coding).
- for Fig. 4.3, the optimal solution with network coding.

Fig. 4.3 displays the performance of each algorithm when the density of the network increases.

One result is that the performance of the optimal network coding is about $\frac{1}{3}$ better than the performance of the connected dominating set from [4], without network coding.

Then it appears that IR-MS is also close to this efficient dominating set, which is already an interesting result. The gap between IR-MS and the optimal with network coding however indicates that some improvements are possible.

Last, MPR-based dominating sets, which use only local topology information are have the lowest performance: this indicates that network coding, with IR-MS, would be useful in practice, if a fully distributed solution could be designed (with distributed min-cut computation [19]).

The Fig. 4.4 shows the results for increasing density ; they are essentially similar.

In appendix A, additional results are given on Fig. A, for higher densities ($N = 400$ on Fig. Fig. 7.1, and $N = 800$ on Fig. Fig. 7.2), and for torus (torus with $N = 200$ on

Fig. Fig. 7.3 and torus with $N = 400$ on Fig. Fig. 7.4). As one can see, the results are even better on a torus, because there is no border effect.

3.4 Difficulties for Distributed Rate Selection

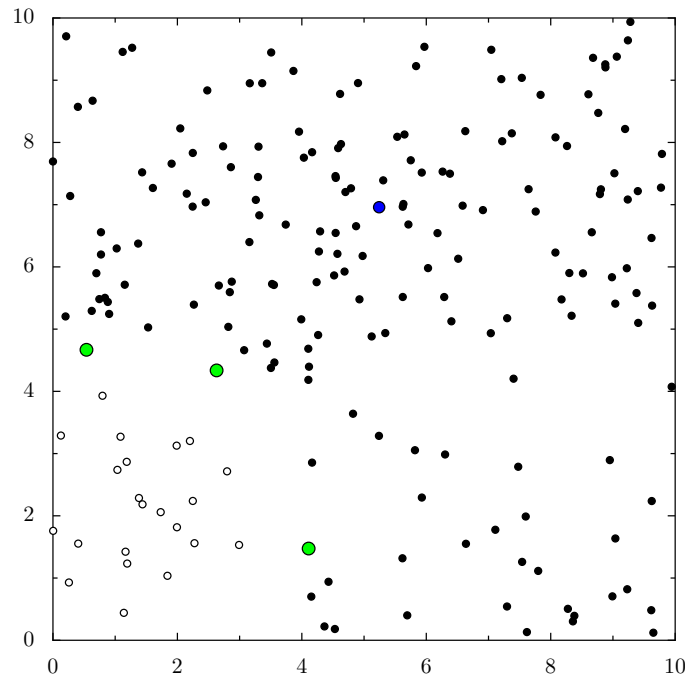


Figure 6: Example of cut

Finally, the Fig. 6 gives an example of an instance of a random unit disk graph, where the performance of IR-MS was found to be low. The cut (S/T corresponding to the min-cut) for a node at the bottom of left corner is represented: the green dots are the only nodes which are connecting the part in the corner to the rest of the network. With IR-MS, the green nodes do not have extremely starving nodes as neighbors, and hence increase their rate little. But data from the source is only transmitted through these green nodes to the sets of white dots, hence for this reason their rate should be greater. This example perfectly illustrates the difficulties found in sparse networks: notice how coordination between the green nodes would require multi-hop communication to detect the issue.

4 Conclusion

We proposed and experimentally studied a heuristic for efficient broadcasting with network coding only using static local information: one hop or two hop neighbors. We showed

excellent performance of this rate selection, and detailed reasons for variations of performance. Future work includes the use of dynamic information for the heuristics in complement of the static local topology information.

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APPENDIX

A Additional Results

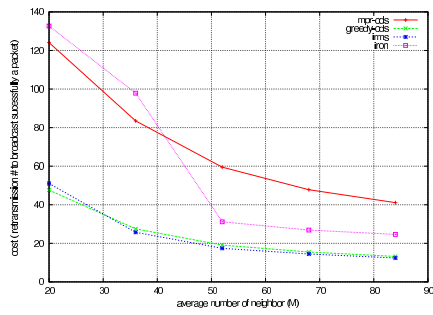


Fig. 7.1: cost vs avg neighbor num on square $N=400$

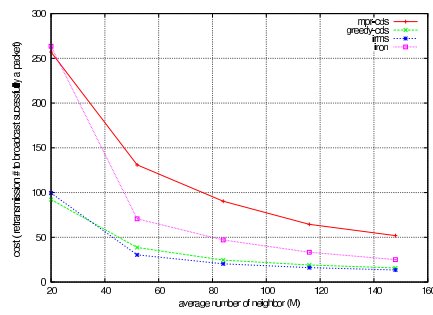


Fig. 7.2: cost vs total node num on square $N=800$

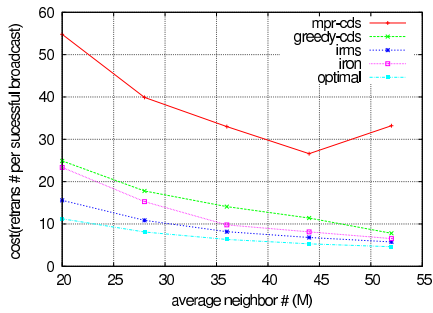


Fig. 7.3: cost vs avg neighbor num on square **torus** $N=200$

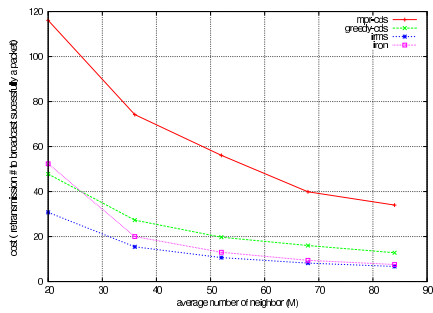


Fig. 7.4: cost vs total node num on square $M=20$

Figure 7: Additional results for higher density and for torus square



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