

A low-rank tensor method for parameterized linear fluid-structure interaction problems

Sébastien Riffaud, Miguel Angel Fernández, Damiano Lombardi

▶ **To cite this version:**

Sébastien Riffaud, Miguel Angel Fernández, Damiano Lombardi. A low-rank tensor method for parameterized linear fluid-structure interaction problems. CMBE 2022 - 7th International Conference on Computational and Mathematical Biomedical Engineering, Jun 2022, Milan, Italy. hal-04010551

HAL Id: hal-04010551
https://inria.hal.science/hal-04010551

Submitted on 1 Mar 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

7th International Conference on Computational and Mathematical Biomedical Engineering - CMBE2021
27–29 June 2022, Italy
P. Nithiarasu and C. Vergara (Eds.)

A LOW-RANK TENSOR METHOD FOR PARAMETERIZED LINEAR FLUID-STRUCTURE INTERACTION PROBLEMS

Sébastien Riffaud[1], Miguel Angel Fernández[1,2], and Damiano Lombardi[1,2]
[1]Inria Paris, Commedia, 75012 Paris,
{sebastien.riffaud,miguel.fernandez,damiano.lombardi}@inria.fr
[2]Sorbonne Université & CNRS UMR 7598 LJLL, 75005 Paris

SUMMARY

This talk will present a low-rank tensor method for parameterized linear fluid-structure interaction problems. In this work, the fluid is governed by the incompressible Stokes equations, and we assume a thick-walled solid model. In order to reduce the computational cost associated with the coupled problem, we develop a low-rank tensor method based on the low-rank Generalized Minimal Residual method. This one is then employed in an uncertainty quantification task to perform a large number of simulations. The preliminary results show the significant reduction of the computational cost provided by the proposed method.

Key words: *fluid-structure interaction problem, low-rank method, cardiovascular mathematics*

1 INTRODUCTION

In cardiovascular modeling, the patient-specific data needed to simulate blood flow may be uncertain or even unavailable. The choice of appropriate parameters is therefore crucial since a small variation can strongly affect the hemodynamic indicators of clinical interest. However, uncertainty quantification tasks require efficient models due to the large number of simulations to perform for the different queried parameters. For theses reasons, we develop a low-rank tensor method to significantly reduce the computational cost associated with cardiovascular simulations. This low-rank tensor method is then employed to evaluate the hemodynamic indicators for a large number of parameters in order to assimilate patient-specific data.

2 METHODOLOGY

Let Ω^f be an open set representing the fluid domain, Ω^s be an open set representing the solid domain, and Σ be the interface between Ω^f and Ω^s. In this work, the fluid is governed by the incompressible Stokes equations

$$\begin{cases} \rho^f \partial_t \boldsymbol{u} - \mathrm{div}(\boldsymbol{\sigma}_f) = 0 \quad \text{in} \quad \Omega^f \times \mathbb{R}^+, \\ \mathrm{div}(\boldsymbol{u}) = 0 \quad \text{in} \quad \Omega^f \times \mathbb{R}^+, \end{cases} \tag{1}$$

where ρ^f denotes the fluid density, $\boldsymbol{u}$ denotes the fluid velocity, p denotes the fluid pressure, $\boldsymbol{\sigma}_f$ denotes the fluid Cauchy stress tensors, given by $\boldsymbol{\sigma}_f = 2\mu\boldsymbol{\epsilon}(\boldsymbol{u}) - p\boldsymbol{I}$ with $\boldsymbol{\epsilon}(\boldsymbol{u}) = \frac{1}{2}(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T)$ and μ the fluid viscosity. In the solid domain, we assume a thick-walled model

$$\begin{cases} \rho^s \partial_t \dot{\boldsymbol{d}} - \mathrm{div}(\boldsymbol{\sigma}_s) = 0 \quad \text{in} \quad \Omega^s \times \mathbb{R}^+, \\ \dot{\boldsymbol{d}} = \partial_t \boldsymbol{d} \quad \text{in} \quad \Omega^s \times \mathbb{R}^+, \end{cases} \tag{2}$$

where ρ^s denotes the solid density, $\boldsymbol{d}$ denotes the solid displacement, $\boldsymbol{\sigma}_s$ denotes the solid Cauchy stress tensors, given by $\boldsymbol{\sigma}_s = 2\lambda_1\boldsymbol{\epsilon}(\boldsymbol{d}) + \lambda_2 \mathrm{div}(\boldsymbol{d})\boldsymbol{I}$ with λ_1 and λ_2 the Lamé parameters. The coupling conditions between the fluid and solid domains read

$$\begin{cases} \boldsymbol{u} = \dot{\boldsymbol{d}} \quad \text{on} \quad \Sigma \times \mathbb{R}^+, \\ \boldsymbol{\sigma}_f \cdot \boldsymbol{n} = \boldsymbol{\sigma}_s \cdot \boldsymbol{n} \quad \text{on} \quad \partial\Sigma \times \mathbb{R}^+, \end{cases} \tag{3}$$

where $\boldsymbol{n}$ is the outward unit normal. Completed with appropriate initial and boundary conditions, the coupled problem (1, 2, 3) is discretized by an affine finite-element method using the Brezzi-Pitkäranta pressure stabilization in the fluid in order to avoid the *inf-sup* compatibility issues and a Nitsche treatment of the surface coupling. The resulting semi-discrete system is solved in time by the implicit Euler scheme, leading at each time step t_n to the resolution of the linear system

$$\boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{y}(t_n,\boldsymbol{\theta}) = \boldsymbol{b}(t_n,\boldsymbol{\theta}), \tag{4}$$

where $\boldsymbol{\theta} = [\rho^f, \mu, \rho^s, \lambda_1, \lambda_2] \in \mathbb{R}^5$ denotes the input parameter and $\boldsymbol{y} = [\boldsymbol{u}, p, \boldsymbol{d}] \in \mathbb{R}^{N_{\boldsymbol{x}}}$ denotes the solution.

The first part of this work concerns the evaluation of the solution $\boldsymbol{y}$ corresponding to a possibly large number of input parameters $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_{N_{\boldsymbol{\theta}}}$. For this purpose, since

$$\boldsymbol{A}(\boldsymbol{\theta}) = \rho^f \boldsymbol{A}^{\rho^f} + \mu \boldsymbol{A}^{\mu} + \rho^s \boldsymbol{A}^{\rho^s} + \lambda_1 \boldsymbol{A}^{\lambda_1} + \lambda_2 \boldsymbol{A}^{\lambda_2} + \frac{1}{\mu}\boldsymbol{A}^{\frac{1}{\mu}} + \boldsymbol{A}^{c},$$

system (4) is first written in matrix notation as follows

$$\begin{aligned}\boldsymbol{A}^{\rho^f}\boldsymbol{Y}(t_n)\boldsymbol{D}^{\rho^f} + \boldsymbol{A}^{\mu}\boldsymbol{Y}(t_n)\boldsymbol{D}^{\mu} + \boldsymbol{A}^{\rho^s}\boldsymbol{Y}(t_n)\boldsymbol{D}^{\rho^s} + \boldsymbol{A}^{\lambda_1}\boldsymbol{Y}(t_n)\boldsymbol{D}^{\lambda_1} \\ + \boldsymbol{A}^{\lambda_2}\boldsymbol{Y}(t_n)\boldsymbol{D}^{\lambda_2} + \boldsymbol{A}^{\frac{1}{\mu}}\boldsymbol{Y}(t_n)\boldsymbol{D}^{\frac{1}{\mu}} + \boldsymbol{A}^{c}\boldsymbol{Y}(t_n) = \boldsymbol{B}(t_n),\end{aligned}$$

where

$$\begin{aligned}
&\boldsymbol{Y}(t_n) = [\boldsymbol{y}(t_n,\boldsymbol{\theta}_1), \ldots, \boldsymbol{y}(t_n,\boldsymbol{\theta}_{N_{\boldsymbol{\theta}}})] \in \mathbb{R}^{N_{\boldsymbol{x}} \times N_{\boldsymbol{\theta}}},\\
&\boldsymbol{D}^{\rho^f} = \mathrm{diag}(\rho^f_1, \ldots, \rho^f_{N_{\boldsymbol{\theta}}}) \otimes \boldsymbol{I}_{(5N_{\boldsymbol{\theta}})\times(5N_{\boldsymbol{\theta}})},\\
&\boldsymbol{D}^{\mu} = \boldsymbol{I}_{(N_{\boldsymbol{\theta}})\times(N_{\boldsymbol{\theta}})} \otimes \mathrm{diag}(\mu_1, \ldots, \mu_{N_{\boldsymbol{\theta}}}) \otimes \boldsymbol{I}_{(4N_{\boldsymbol{\theta}})\times(4N_{\boldsymbol{\theta}})},\\
&\boldsymbol{D}^{\rho^s} = \boldsymbol{I}_{(2N_{\boldsymbol{\theta}})\times(2N_{\boldsymbol{\theta}})} \otimes \mathrm{diag}(\rho^s_1, \ldots, \rho^s_{N_{\boldsymbol{\theta}}}) \otimes \boldsymbol{I}_{(3N_{\boldsymbol{\theta}})\times(3N_{\boldsymbol{\theta}})},\\
&\boldsymbol{D}^{\lambda_1} = \boldsymbol{I}_{(3N_{\boldsymbol{\theta}})\times(3N_{\boldsymbol{\theta}})} \otimes \mathrm{diag}(\lambda^1_1, \ldots, \lambda^{N_{\boldsymbol{\theta}}}_1) \otimes \boldsymbol{I}_{(2N_{\boldsymbol{\theta}})\times(2N_{\boldsymbol{\theta}})},\\
&\boldsymbol{D}^{\lambda_2} = \boldsymbol{I}_{(4N_{\boldsymbol{\theta}})\times(4N_{\boldsymbol{\theta}})} \otimes \mathrm{diag}(\lambda^1_2, \ldots, \lambda^{N_{\boldsymbol{\theta}}}_2) \otimes \boldsymbol{I}_{(N_{\boldsymbol{\theta}})\times(N_{\boldsymbol{\theta}})},\\
&\boldsymbol{D}^{\frac{1}{\mu}} = \boldsymbol{I}_{(5N_{\boldsymbol{\theta}})\times(5N_{\boldsymbol{\theta}})} \otimes \mathrm{diag}\left(\frac{1}{\mu_1}, \ldots, \frac{1}{\mu_{N_{\boldsymbol{\theta}}}}\right),\\
&\boldsymbol{B}(t_n) = [\boldsymbol{b}(t_n,\boldsymbol{\theta}_1), \ldots, \boldsymbol{b}(t_n,\boldsymbol{\theta}_{N_{\boldsymbol{\theta}}})] \in \mathbb{R}^{N_{\boldsymbol{x}} \times N_{\boldsymbol{\theta}}}.
\end{aligned}$$

Then, the solution $\boldsymbol{Y}$ is approximated by a low-rank tensor in order to significantly reduce the number of degrees of freedom:

$$\widetilde{\boldsymbol{Y}}(t_n) = \boldsymbol{U}(t_n)\boldsymbol{V}(t_n)^T,$$

where $\boldsymbol{U}(t_n) \in \mathbb{R}^{N_x \times N_r}$ and $\boldsymbol{V}(t_n) \in \mathbb{R}^{N_\theta \times N_r}$ with $N_r \ll N_{\boldsymbol{x}}, N_{\boldsymbol{\theta}}$. The approximate solution is finally determined at low cost by the low-rank Generalized Minimal RESidual (GMREST) method [1].

In the second part of this work, the previous low-rank tensor method is employed in an uncertainty quantification task.

3 RESULTS AND CONCLUSIONS

The proposed method is evaluated on the propagation of a pressure-wave within a straight tube in two-dimensions. The parameter space is defined in accordance with cardiovascular applications as $\boldsymbol{\theta} \in [0.9, 1.1] \times [0.03, 0.04] \times [1, 1.2] \times [1.1 \cdot 10^6, 1.2 \cdot 10^6] \times [1.6 \cdot 10^6, 1.8 \cdot 10^6]$. The preliminary results show the significant reduction of the computational cost provided by the low-rank tensor method.

REFERENCES

[1] Y. Saad and M. Schultz. A generalized minimal residual algorithm for solving nonsymmetric linear systems. *SIAM Journal on scientific and statistical computing*, 1986.