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Discrete models of complex systems: Recent trends and analytical challenges



Andreas Deutsch^a, Nazim Fatès^{b,*}, Danuta Makowiec^c

^a Center for Information Services and High Performance Computing, Technische Universität Dresden, Dresden, Germany

^b Université de Lorraine, CNRS, Inria, LORIA, F-54000 Nancy, France

^c University of Gdansk, Department of Theoretical Physics and Astrophysics, Gdańsk, Poland

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ABSTRACT

The editors present a special issue of *Physica D* dedicated to discrete models of complex systems. Three main axes structure this issue: simple cellular automata, networks and models of natural or artificial complex systems.

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Context

In 1984, a special issue in *Physica D* presented an exceptional panel of articles on complex systems with a special emphasis on cellular automata [1]. In one of those articles, Stephen Wolfram proposed a rule-based classification of cellular automata into four classes: information-erasing rules, periodic rules, chaotic rules and complex rules. Although these classes were not formally defined, Wolfram's classification triggered a wide interest in cellular automata and in other discrete models where a global complex behaviour would emerge from simple local interactions. The need for discrete models is further supported particularly by applications in the life sciences where discrete elements such as cells and organisms are key for driving complex dynamics such as development and evolution [2]. Almost forty years after this landmark *Physica D* publication, we wished to measure the distance that has been travelled since then.

This special issue follows the SOLSTICE conference organised in Dresden, 15–17 July, 2019 (see: <https://solstice2019.loria.fr>).

In our special issue, three main axes are represented:

- studies on cellular automata,
- studies on networks,
- models of natural or artificial complex systems.

Studies on binary cellular automata and beyond

Binary rules possess a minimal number of states; nevertheless they allow us to gain insights that are also relevant for more complicated automata. Regarding the question of cellular automata classification, the survey presented by Vispoel et al. offers an excellent state of the art [3]. The authors start with Wolfram's classification and then discuss various attempts of formalisations of this classification, in particular the well-known classification of Li & Packard, Culik & Yu, or Kůrka. They also compare various parameters used to quantify the behaviour of the CA rules: Lyapunov exponents, estimates of the entropy, Kolmogorov complexity, etc. They demonstrate that “there is no single all-purpose CA classification scheme. Each approach has its merits and suits specific needs, depending on the family of CA under consideration and the research context”. They call for bridging the gap between the phenomenological approaches, which focus on statistical measures obtained with numerical simulations, and analytical methods, which obtain their results based on the sure path of proving the properties they study.

Cellular automata can be synchronous or asynchronous. Among the questions raised in the *Physica D 10* issue, an important point concerned the role of synchronous transitions of a cellular automaton in the emergence of a particular behaviour [4]. Interestingly enough, the study of asynchronous cellular automata still represents a challenging field (see e.g. Ref. [5] for a survey). Here, Ruivo et al. give us an interesting perspective: revisiting the global synchronisation problem, they show that even though it is

* Corresponding author.

E-mail addresses: andreas.deutsch@tu-dresden.de (A. Deutsch), nazim.fates@loria.fr (N. Fatès), danuta.makowiec@ug.edu.pl (D. Makowiec).

impossible to have a perfect synchronous solution, in an asynchronous framework where the problem is redefined, solutions do exist [6].

In the article of Fei et al. we quit the world of binary rules and discover a universal asynchronous cellular automaton with ternary states and with a local rule described by a set of only ten elementary instructions [7]. The rule is, to date, the simplest known and the intelligent way the authors take advantage of the random fluctuations of signals opens promising paths for the design of future original computing devices.

Another interesting and long-lasting question is the characterisation of cellular automata with the number-conservation property. Indeed, cellular automata represent flows of information but it is delicate to identify those automata ensuring that a global mass is conserved. The number of rules which possess this property is small and their characterisation is a challenge. Here, Wolnik et al. present their last progresses in the analysis of two-dimensional number-conserving rules [8]. In a second article, Wolnik et al. tackle the problem of combining three aspects: (a) the number-conservation property, (b) the reversibility property and, (c) going beyond binary rules and exploring k -ary states [9].

To deepen these questions, Roth, who is an artist motivated by “scientific material”, takes us on a journey of exploring the entropy of the space–time diagrams of cellular automata and making a bridge with the fundamental concept of robustness [10].

Networks

As expressed by S. Kaufman in the *Physica D* 10 issue, networks present another fundamental model of discrete complex systems [11]. Here, Karimi et al. “square” this concept and present a survey on networks of networks [12]. A network of networks is a collection of networks which are themselves interrelated by a network topology. Their bibliographical analysis shows that this concept has been used with different names and in various contexts such as physics (especially nanotechnology), biology, neural networks, social networks, etc.

Boolean networks have been made popular by the work of S. Kauffman who suggested them as models for genetic networks and cell differentiation. These models have been mainly studied based on a synchronous update. Here, Fabre-Monplaisir et al. present a study with combinatorial explorations of the asynchronous dynamics of such a system [13]. Once again, we see that asynchronous updating opens up new challenges, here driven by the various possibilities of defining an asynchronous updating scheme.

As a class of models, cellular automata have always been a means to test simple ideas in social sciences. With “How to be influential being weakly connected”, Krawczyk and Kułakowski are not afraid of tackling a major threat of today’s world: the formation of oligarchies [14]. We wish to the authors to be indeed influential, and, at the same time, to stay away from oligarchies!

Discrete models of complex systems

Not far away from the previous study, we find the concept of Heider’s balance [15]: this concept aims at capturing how equilibrium states may form in a network of actors which classify their neighbours with binary states, as enemies of friends. Here, the authors, Malarz et al. model the problem on a triangular lattice and study a simple deterministic rule. They exhibit a rich behaviour where the friendly and hostile groups evolve with some rich dynamics which tend to minimise the energy of the system, interpreted here as the quantity of unbalanced triads, that is, patterns where three individuals are in a state of dissonance.

It is also a pleasure to see that connections with other practical domains of computer science have been made. In their article, Maiti et al. explore a promising path by considering error-correcting algorithms which are realised with cellular automata [16].

Last, but not least, Makowiec et al. develop a model to explain various aspects of the heart beat dynamics, among which sinus arrhythmia [17]. The authors put a two-dimensional stochastic cellular automaton under their “numerical microscope” and propose various scenarios on how we can understand variations of the rhythm of heart contractions as a result of the way spiral waves develop and propagate. We can remark that the numerical simulations are carried out on a model of excitable cellular automata, which sends us back to the origins and the models of reaction–diffusion mechanisms invented by Turing [18].

This special issue reveals the richness of the field of discrete complex systems and shows that there is a wide range of problems still to be explored. With this panel of articles, we see on the one hand how far our knowledge on such intriguing systems has come since the *Physica D* 10 edition. On the other hand, it seems that there is still so much to learn. Now, the greatest challenge might be to build bridges between the different domains: mathematics, computer science, natural sciences and, maybe even more important, social sciences.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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