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An imperative programming language characterizing FBQP

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► **To cite this version:**

Emmanuel Hainry, Romain Péchoux, Mário Silva. An imperative programming language characterizing FBQP. QPL 2022 - Quantum Physics and Logic, Jun 2022, Oxford, United Kingdom. . hal-03895106

HAL Id: hal-03895106

<https://inria.hal.science/hal-03895106v1>

Submitted on 12 Dec 2022

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Introduction

Quantum programming languages are a useful tool, not only for writing complex algorithms, but also for abstracting and reasoning about their properties and to learn more about what can be done efficiently by quantum computers [1].

However, merely being able to describe a quantum program does not inform us on its complexity, meaning that it may not ultimately be efficiently run on a quantum computer. Consequently, there is a need for static analysis tools and techniques for reasoning about and certifying the complexity of these quantum algorithms.

We introduce an imperative programming language called QPT (Quantum Poly-Time) with recursion rules that captures the complexity class FBQP (Functions Bounded-error Quantum Polytime), the class of functions computable in polynomial time by a quantum Turing machine with at most 1/3 probability of error, commonly accepted as the class of feasible problems for quantum computers. Our result takes advantage of a function algebra proposed by Yamakami [2] characterizing FBQP.

Syntax

$$\begin{aligned}
 P &\triangleq D :: S \\
 i, j &\triangleq n \in \mathbb{Z} \mid |\bar{q}| \\
 b &\triangleq i < j \mid i = j \\
 D &\triangleq \varepsilon \mid \mathbf{Proc} \text{ } proc(m, \bar{p})\{S[m]\}, D \\
 \sigma &\triangleq \emptyset \mid \bar{q} \mid \sigma[i] \mid \sigma_1 \oplus \sigma_2 \mid \text{remove}(\sigma, i) \\
 U &\triangleq \mathbf{NOT} \mid \mathbf{ROT}_\theta \mid \mathbf{PHASE}_\theta \\
 S &\triangleq \mathbf{skip} \\
 &\mid \mathbf{if} \ b \ \mathbf{then} \ S \\
 &\mid \sigma[i]* = U \\
 &\mid S_1; S_2 \\
 &\mid \mathbf{Case} \ C(\sigma[i]) = m \ \mathbf{then} \ S_m \\
 &\mid \mathbf{call} \ proc(i, \sigma)
 \end{aligned}$$

Sets σ are sorted sets of qubits together with basic operations. The **Case** structure implements a quantum choice: controlled on the state of qubit $\sigma[i]$, we apply S_0 or S_1 to the remainder of the qubits.

Rule for termination (R1)

Let $proc_i \sim proc_j$ mean that $proc_i$ and $proc_j$ are mutually recursive procedures. The following condition ensures that a program with procedure declarations D will always terminate:

$$\begin{aligned}
 \forall \mathbf{Proc} \ proc_i(\bar{p})\{S_i\} \in D, \\
 \forall \mathbf{call} \ proc_j(\sigma) \in S_i, \\
 \quad proc_i \sim proc_j \Rightarrow \sigma \text{ is a proper subset of } \bar{p},
 \end{aligned}$$

i.e., any call to a mutually recursive function must strictly decrease the amount of qubits available.

Rule for poly-time (R2)

We prevent an exponential number of procedure calls by limiting the number of recursive calls in each branch of a **Case**. Let $\mathbf{RCalls}(\cdot)$ represent the maximum number of recursive calls for any branch, then

$$\forall \mathbf{Proc} \ proc_i(\bar{p})\{S_i\} \in D, \mathbf{RCalls}(proc_i) \leq 1,$$

i.e., multiple recursive calls can be done only on different branches of the **Case** structure.

QPT \sim FBQP

Soundness: For any program P in QPT following rules **R1** and **R2**, there exists a poly-sized uniform family of circuits $(C_n)_{n \in \mathbb{N}}$ for each input size n that simulates P .

Completeness: For any function f in FBQP with size-bounding polynomial p , and any constant $\varepsilon \in [0, 1/2)$, there exists a program P in QPT following rules **R1** and **R2** such that, running P on polynomially extended input state ρ_x^p , for $x \in \{0, 1\}^n$, a measurement of the first $|f(x)|$ of the output qubits will result in $f(x)$ with probability at least $1 - \varepsilon$.

Building poly-sized circuits

Dealing with procedures that include recursive branching, a straightforward approach to building the circuit will easily require an exponential number of gates, e.g. in the following procedure:

$$\begin{aligned}
 \mathbf{Proc} \ f(\bar{p})\{ \\
 \quad \mathbf{if} \ |\bar{p}| > 1 : \\
 \quad \quad \mathbf{Case} \ C(\bar{p}[1]) = 0 \ \mathbf{then} \\
 \quad \quad \quad \mathbf{call} \ f(\text{remove}(\bar{p}, 1)) \\
 \quad \quad \mathbf{else} \ \mathbf{Case} \ C(\bar{p}[2]) = 1 \ \mathbf{then} \\
 \quad \quad \quad \mathbf{call} \ f(\text{remove}(\bar{p}, \{1, 2\})) \\
 \quad \quad \mathbf{else} \ \bar{p}[1]* = U \} :: \\
 \mathbf{call} \ f(\bar{q})
 \end{aligned}$$

As a proof of **Soundness**, we provide an algorithm to build all programs following rule **R2** with a polynomially large set of gates and wires, such as in the example of Figure 1 for the above function applied on an input \bar{q} of size $n = 5$.

For function f the number of gates and wires needed is $O(n^2)$.

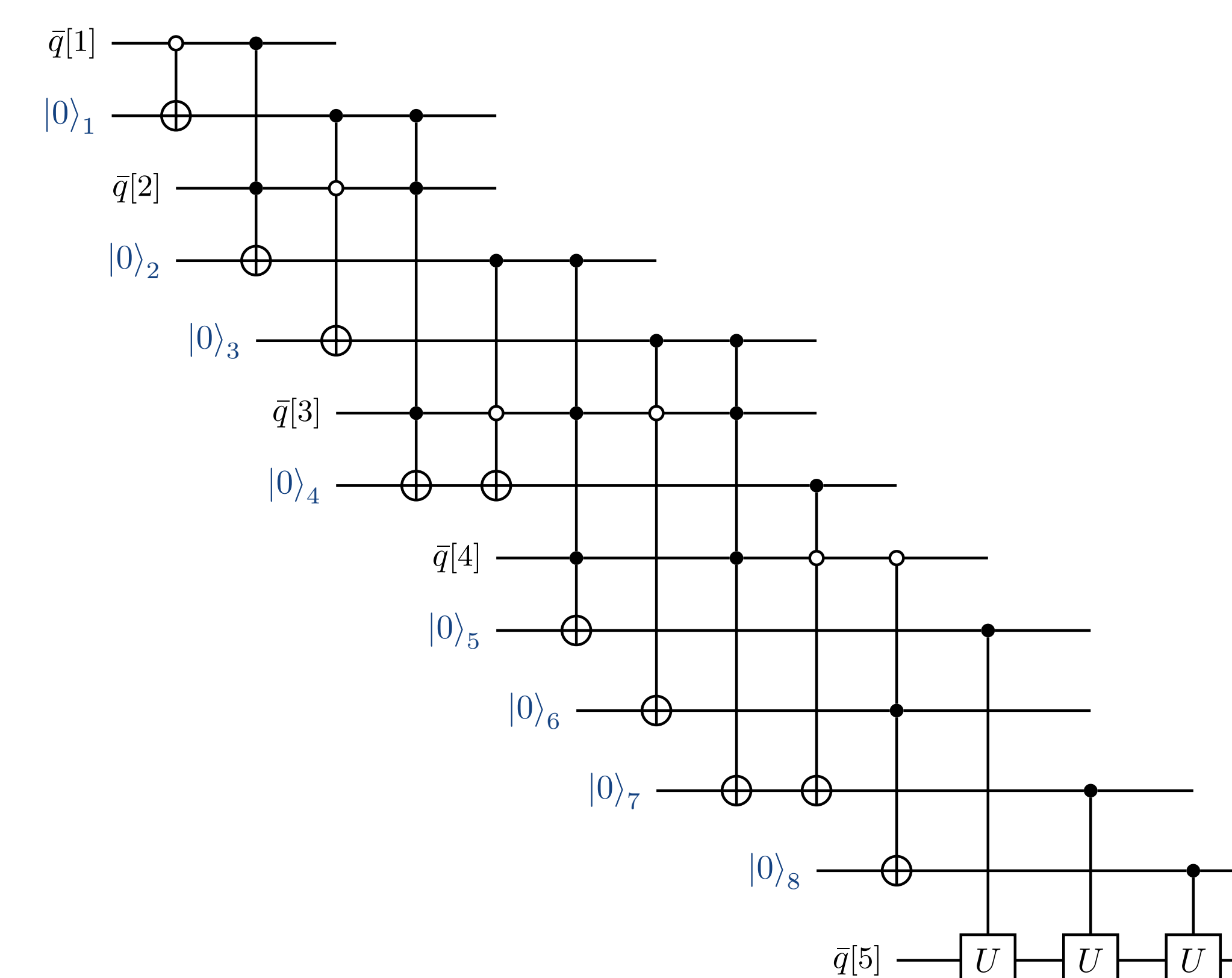


Figure 1: Example circuit of a recursive quantum function f applied on a set of 5 qubits.

Example: QFT

The quantum Fourier transform (QFT) is in FBQP, appearing as a subroutine of Shor's algorithm. It contains two recursive patterns, following rules **R1** and **R2**, highlighted in the circuit of Figure 2.

$$\begin{aligned}
 \mathbf{Proc} \ QFT(\bar{p})\{ \\
 \quad \bar{p}[1]* = H; \\
 \quad \mathbf{call} \ Chain(2, \bar{p}); \\
 \quad \mathbf{call} \ QFT(\text{remove}(\bar{p}, 1))\},
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{Proc} \ Chain(m, \bar{p})\{ \\
 \quad \mathbf{Case} \ C(\bar{p}[2]) = 1 \ \mathbf{then} \ \bar{p}[1]* = R_m; \\
 \quad \mathbf{call} \ Chain(m + 1, \text{remove}(\bar{p}, 2))\} ::
 \end{aligned}$$

call $QFT(\bar{q})$

The circuit can be implemented with size $O(n^2)$ gates, for an input with n qubits.

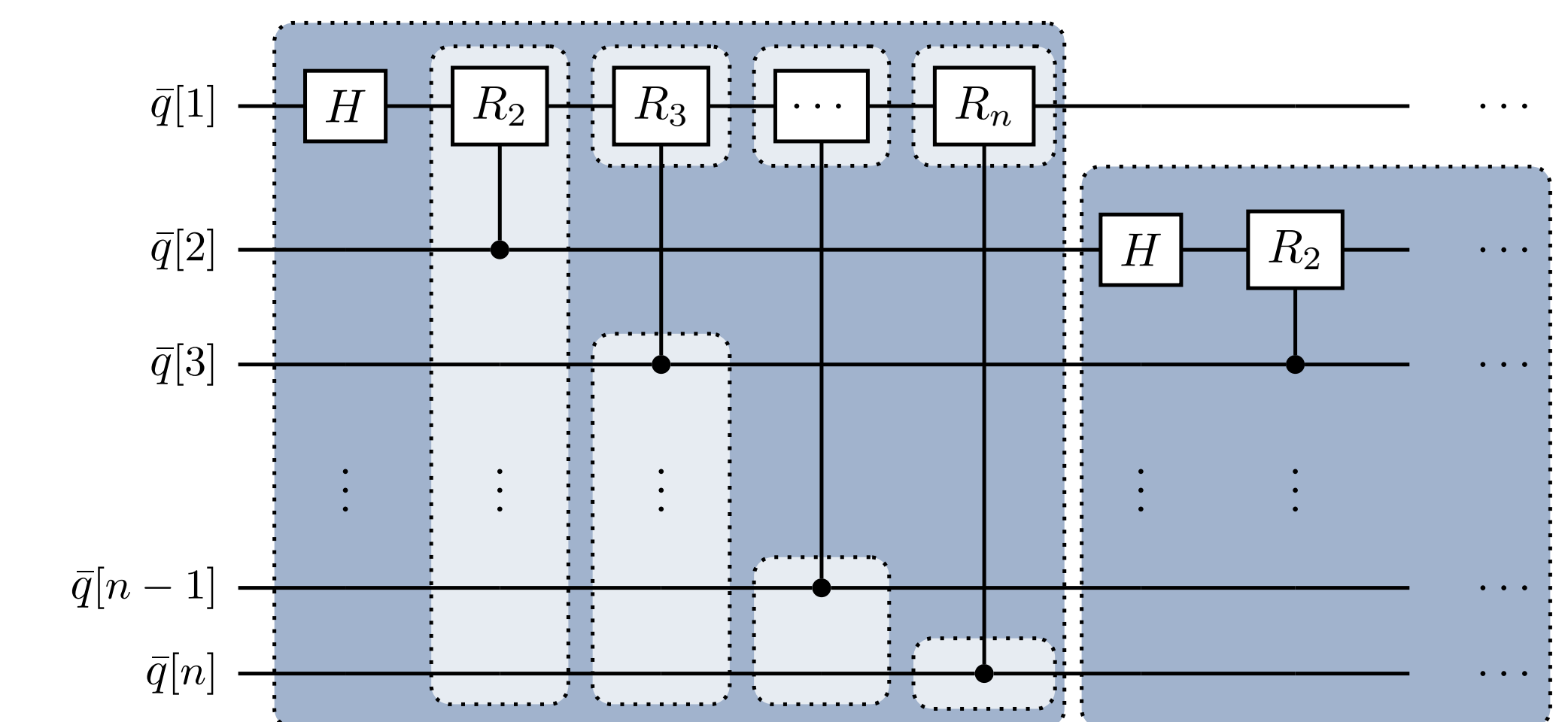


Figure 2: Representation of two recursive patterns with decreasing qubit set in the QFT.

References

- [1] Peter Selinger. Towards a quantum programming language. *Mathematical Structures in Computer Science*, 14(4):527–586, 2004.
- [2] Tomoyuki Yamakami. A schematic definition of quantum polynomial time computability. *J. Symb. Log.*, 85(4):1546–1587, 2020.