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# Social Networking and Population Growth

## A complex mathematical relationship

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### Introduction

Growth and disease transmission rates within human populations are continuously changing [4] and will be of increasing importance over the coming decades. The modelling of social network construction in pair-bonding species is, therefore, necessary for understanding how connection between individuals influence these critical dynamics.

The established and accepted Ecological Constraints model [1] hypothesises that nonlinear and constrained population growth occurs as a function of competition for limited ecological resources. Meanwhile, existing Markovian models of population growth [2] have not yet explored the dynamics of pair formation within such species and, as such, the effect of these dynamics on population growth. These simpler Markovian models, however, have been rigorously analysed [3] and present an appropriate framework for further mathematical exploration.

We propose a novel nonlinear Markovian model to explore the effect of mutual pair-bonding on population growth and embed this model within the novel Dynamic Markovian Ecological Network (DyME-Net) model to explore how subdivision may overcome these nonlinearities.

We explore the **hypothesis** that nonlinear population growth within pair-bonding species could be explained by the probabilistic functions of mate choice, even in the absence of ecological constraints.

### Markov Chain Models

To explore mutual pair-bonding, let there exist three populations  $X(t)$ ,  $Y(t)$ , and  $Z(t)$  which respectively represent males, females, and coupled populations. We assume that males and females uniformly choose a random member of the opposite sex with whom they wish to mate and that they may propose to this individual with rates  $q_m$  and  $q_f$ , respectively. A couple is only formed in the case where the individual proposed to also chooses the proposing individual. Couples may then produce a male or female progeny respectively with rates  $r_m$  and  $r_f$ , whilst individuals may leave the population with rates  $p_m$  and  $p_f$ . This is illustrated in Figure 1.

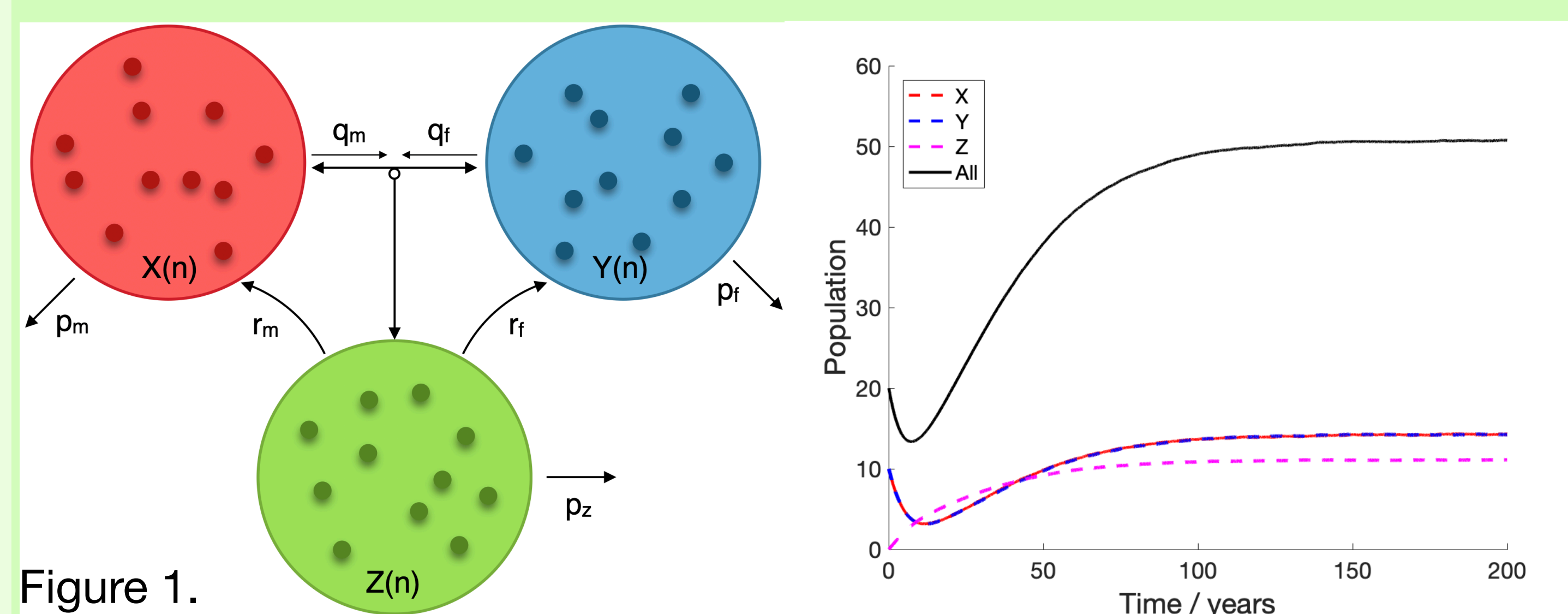


Figure 1. Initialising with  $X(0)=Y(0)=10$  and approximate parameters for the UK population, we find (Figure 1) that the populations of males, females, and couples plateau at approximately 50 individuals. Numerical analysis shows the rate limiting factor to be the probabilistic formation of couples.

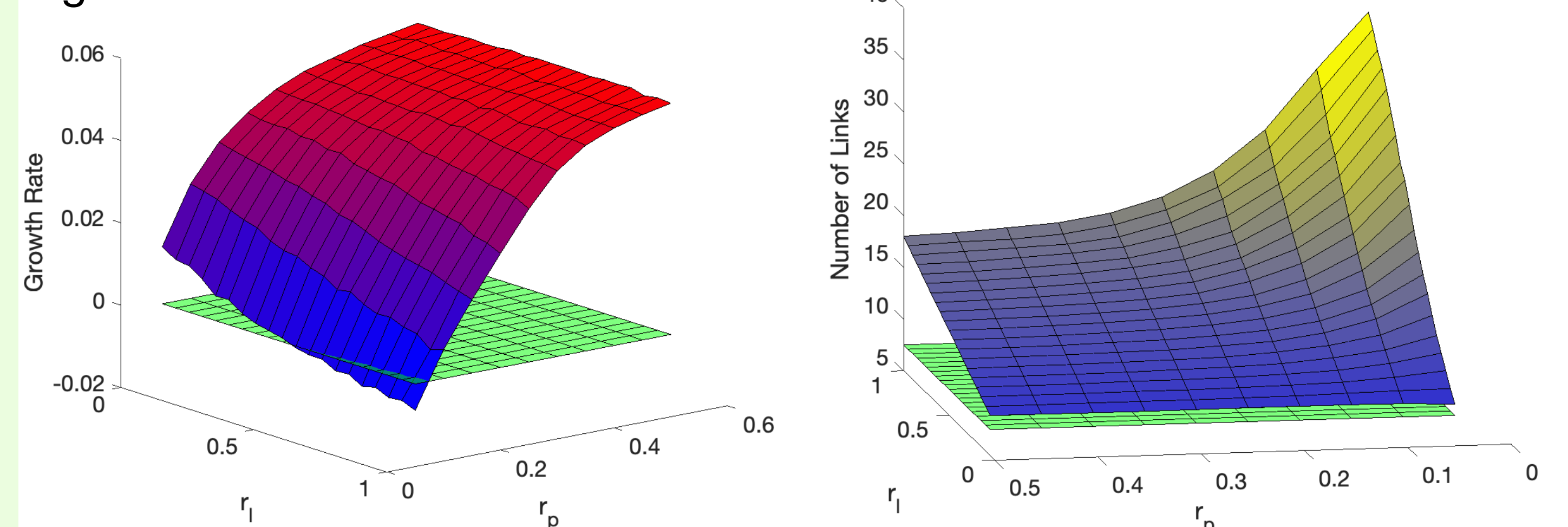
### DyME-Net Model

In order to explore how nonlinearities in population growth, as a result of probabilistic pair formation, may be mitigated by sub-popular decomposition, we embed the assumptions of the mutual pairing Markov model into a network model, where nodes represent individuals (or reproductive pairs) and edges represent links. Nodes may then experience each of the following dynamics with its associated computational action indicated after the colon. Rate parameters are indicated within brackets.

- Social link creation ( $r_\ell$ ): new edge between nodes  $i$  and  $j$
- Reproductive union creation ( $r_p$ ): consolidation of nodes  $i$  and  $j$
- Reproductive union separation ( $r_s$ ): separation of node  $k$
- Reproduction ( $r_b$ ): Creation of a new node  $k$  with  $n_k \in \{5, \dots, 10\}$  edges
- Death ( $r_d$ ): Dissolution of node  $k$  and associated edges

To initialise the model, we begin with  $N_0=100$  nodes each with  $n_i \in \{5, \dots, 10\}$  associated edges. Measured growth rates (exponential trends)  $>0$  indicate population growth whilst those  $<0$  indicate population decline.

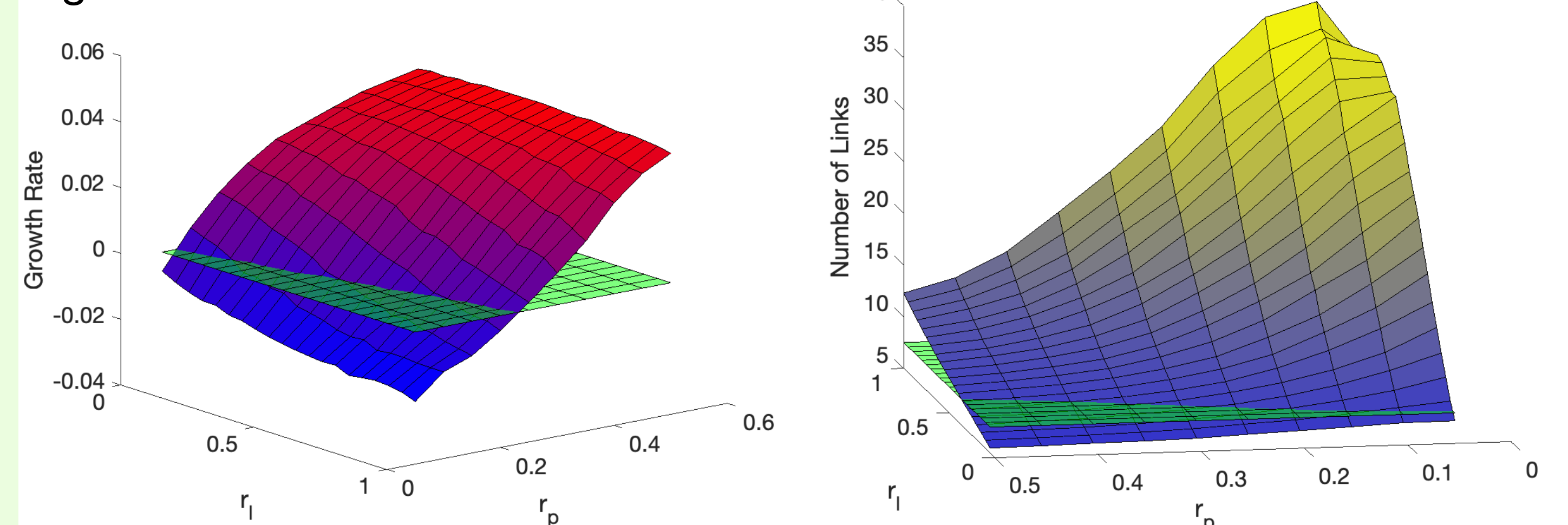
Figure 2.



Results show that increasing the social linking rate, contrary to intuition, decreases the rate of population growth, under DyME-Net assumptions (Figure 2). Increasing the rate of proposal leads to a nonlinear increase in growth rate, possibly as a result of coupling saturation. Simulations also demonstrate an increasing in the average number of social links correlating with a decrease in proposal rate, perhaps owing to the decrease in population growth such that social linking dynamics dominate the system.

Increasing the separation rate ( $r_s=0.1$ ) concurrently increases the significance of social linking,  $r_\ell$ , and proposal,  $r_p$ , rates on population growth (Figure 3). Low proposal rates now guarantee population decline whilst social linking rates now profoundly influence the degree to which proposal rates must increase in order to guarantee population growth.

Figure 3.



### Conclusions

The convergence of DyME-Net to the Markovian mutual-pairing model as  $r_\ell \rightarrow \infty$  (data not shown) demonstrates that complex societies overcome nonlinear and constrained population growth through subpopular decomposition. Moreover, the parsimony of DyME-Net makes it an interesting candidate for data-driven parameterisation and the superposition of disease transmission or social isolation models. Further analysis may elucidate and clarify these dynamics, allowing for population planning and improved disease control in pair-bonding species.

### References

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