



**HAL**  
open science

# Sensitivity analysis for two wave breaking models used by the Green-Naghdi equation

Maria Kazolea, Subodh Joshi, Mario Ricchiuto

► **To cite this version:**

Maria Kazolea, Subodh Joshi, Mario Ricchiuto. Sensitivity analysis for two wave breaking models used by the Green-Naghdi equation. WCCM-ECCOMASS 2020 - 14th World Congress in Computational Mechanics and Ecomass Congress 2020, Jan 2021, Paris / Virtual, France. hal-03402971

**HAL Id: hal-03402971**

**<https://inria.hal.science/hal-03402971>**

Submitted on 26 Oct 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Sensitivity analysis for two wave breaking models used by the Green-Naghdi equation

**Maria Kazolea**  
Subodh M. Joshi  
Mario Ricchiuto

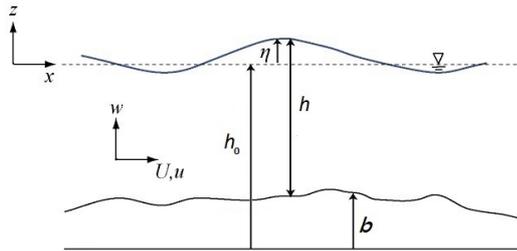
# Context and motivation

Near shore: wave transformation due to interaction with complex bathymetry

- Strong non-linearity, dispersion
- Refraction diffraction
- Shoaling
- **Breaking**
- Induced currents
- Run-up, inundation



# Modeling and discretisation



$$h_t + (hu)_x = 0$$

$$(hu)_t + (hu^2)_x + gh\eta_x = \phi$$

$$(I + \alpha\mathcal{T})\phi = \mathcal{W} - \mathcal{R} + (r_{wb})_x$$

$$\mathcal{W} = g\mathcal{T}(h_0\eta_x)$$

$$(r_{wb})_x = (\nu_t H u_x)_x$$

$$\mathcal{R} = h\mathcal{Q}(u)$$

$$\mathcal{T}(\cdot) = -\frac{1}{3}h^2(\cdot)_{xx} - \frac{1}{3}hh_x(\cdot)_x + \frac{1}{3}[h_x^2 + hh_{xx}](\cdot) + \left[b_x h_x + \frac{1}{2}hb_{xx} + b_x^2\right](\cdot)$$

$$\mathcal{Q}(\cdot) = 2hh_x(\cdot)_x^2 + \frac{4}{3}h^2(\cdot)_x(\cdot)_{xx} + b_x h(\cdot)_x^2 + b_{xx}h(\cdot)(\cdot)_x + \left[b_{xx}h_x + \frac{1}{2}hb_{xxx} + b_x b_{xx}\right](\cdot)^2$$

- Elliptic part: Continuous Galerkin FE method
- Hyperbolic part: Third order MUSCL FV method

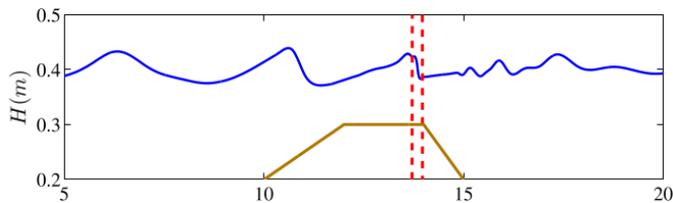
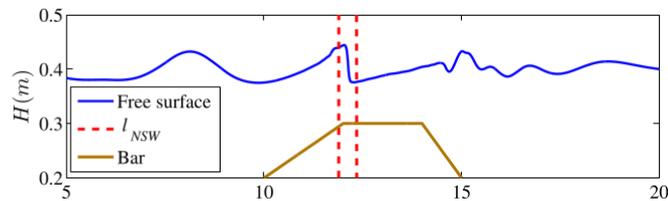
# Breaking closure: shallow water dissipation -Hybrid

1. Detect Breaking Regions
2. Remove dispersive terms
3. Shallow water shock
4. Dissipation



# Breaking closure: Shallow water dissipation -Hybrid

## 1. Detect and flag breaking regions



- Surface variation criterion:

$$|\eta_t| \geq \gamma \sqrt{gH}$$

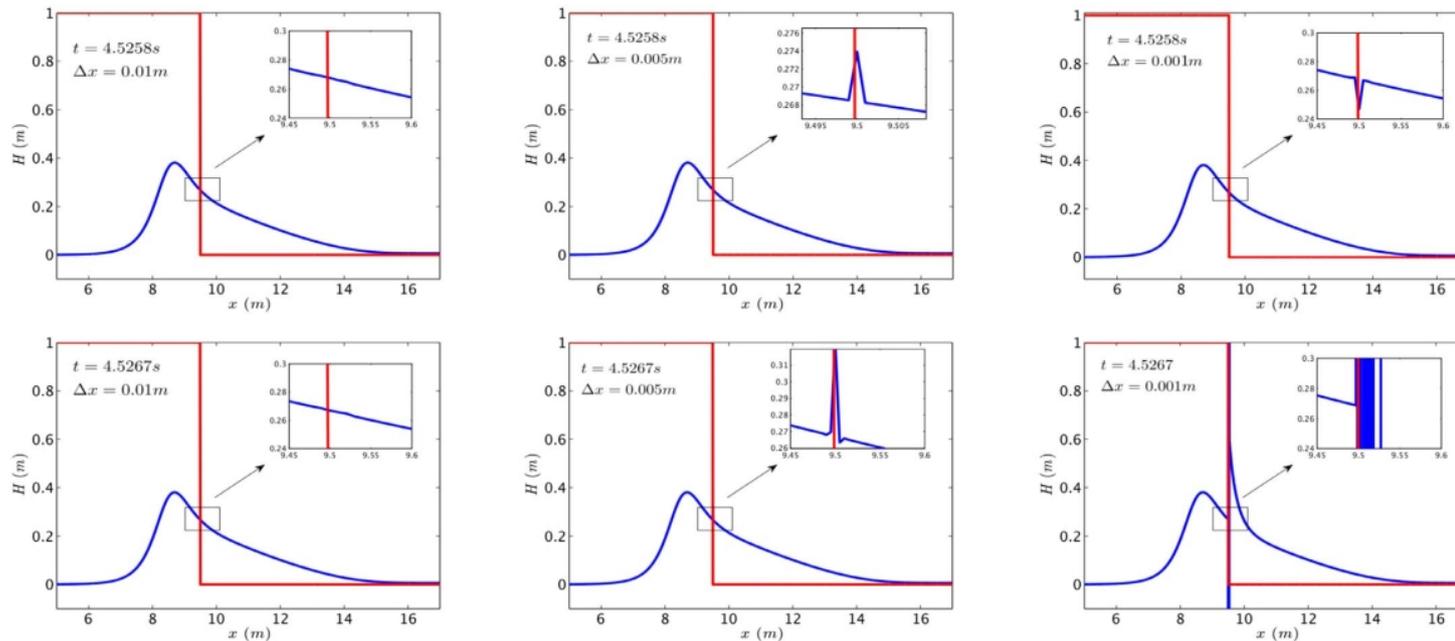
- Local slope angle criterion:

$$\|\nabla\eta\| \geq \tan\varphi_c$$

- The Froude number of each wave is computed and breaking is deactivated if  $Fr < 1.3$

# Breaking closure

- Codes using this closure: Funwave, BOSZ, Coulwave, Mike 21 Tucwave, Slows etc...
- Drawback: can not be used on variable meshes\*



Solitary wave breaking on a slope: hybrid treatment with order reduction at the coupling interface. Wave height at times  $t = 4.5258$  s,  $4.5267$  s (top and bottom rows), on mesh sizes (from left to right)  $\Delta x = 0.01$  m,  $0.005$  m,  $0.001$  m.

\*

- P. Bacigaluppi, M. Ricchiuto, P. Bonneton, *Implementation and Evaluation of Breaking Detection Criteria for a Hybrid Boussinesq model*, *Water Waves*, 2, 2020
- Kazolea, M., Ricchiuto, M.: *On wave breaking for Boussinesq-type models*. *Ocean Modelling* 123, 16–39 (2018)
- Shi, F., Kirby, J.T., Harris, J.C., Geiman, J.D., Grilli, S.T.: *A high-order adaptive time-stepping TVD solver for Boussinesq modeling of breaking waves and coastal inundation*. *Ocean Modelling* 43, 36–51 (2012)

# Breaking closure: Eddy viscosity model

via a PDE based TKE model

1. Detect breaking Regions (same as before)  $\gamma$ ,  $\varphi_c$
2. Dissipation required through the definition of artificial viscosity:

$$v_t = C_\nu \sqrt{\kappa_b \kappa_b + \kappa_j \kappa_j} A$$

$$k_t + uk_x = \mathcal{D} + \mathcal{P} - \mathcal{E}$$

$$\mathcal{D} = \sigma v_t k_{xx} \quad \mathcal{E} = -C_\nu^3 \frac{k^{3/2}}{l_t} \quad \mathcal{P} = B(t, x) \frac{l_t}{\sqrt{C_\nu^3}} (u^s)^3$$

$$l_t = \kappa H$$

$$\kappa, \sigma, C_\nu = \kappa_b, \sigma_b, C_{\nu_b} \text{ Bore}$$

$$\kappa, \sigma, C_\nu = \kappa_j, \sigma_j, C_{\nu_j} \text{ Hydraulic jump}$$

# Sensitivity analysis model

- Surrogate model – Kriging model
- ANOVA
  - Sobol decomposition

$$Y = F(\bar{\mathbf{x}}) = F_0 + \sum_{i=1}^M F_i(x_i) + \sum F_{ij}(x_i x_j) + \sum_{1 \leq i < j \leq M} F_{ij}(x_i, x_j) + \cdots + F_{1,2,\dots,M}(\bar{\mathbf{x}}) \quad \bar{\mathbf{x}} = [x_1, x_2, \dots, x_M]$$

$$F_0 = E(Y)$$

$$F_i(x_i) = E(Y | x_i) - F_0$$

$$F_{ij}(x_i, x_j) = E(Y | x_i, x_j) - F_0 - F_i - F_j$$

# Sensitivity analysis model

- First order indices

$$S_i = \frac{\text{Var}_{x_i}(P(Y|x_i))}{\text{Var}(Y)}$$

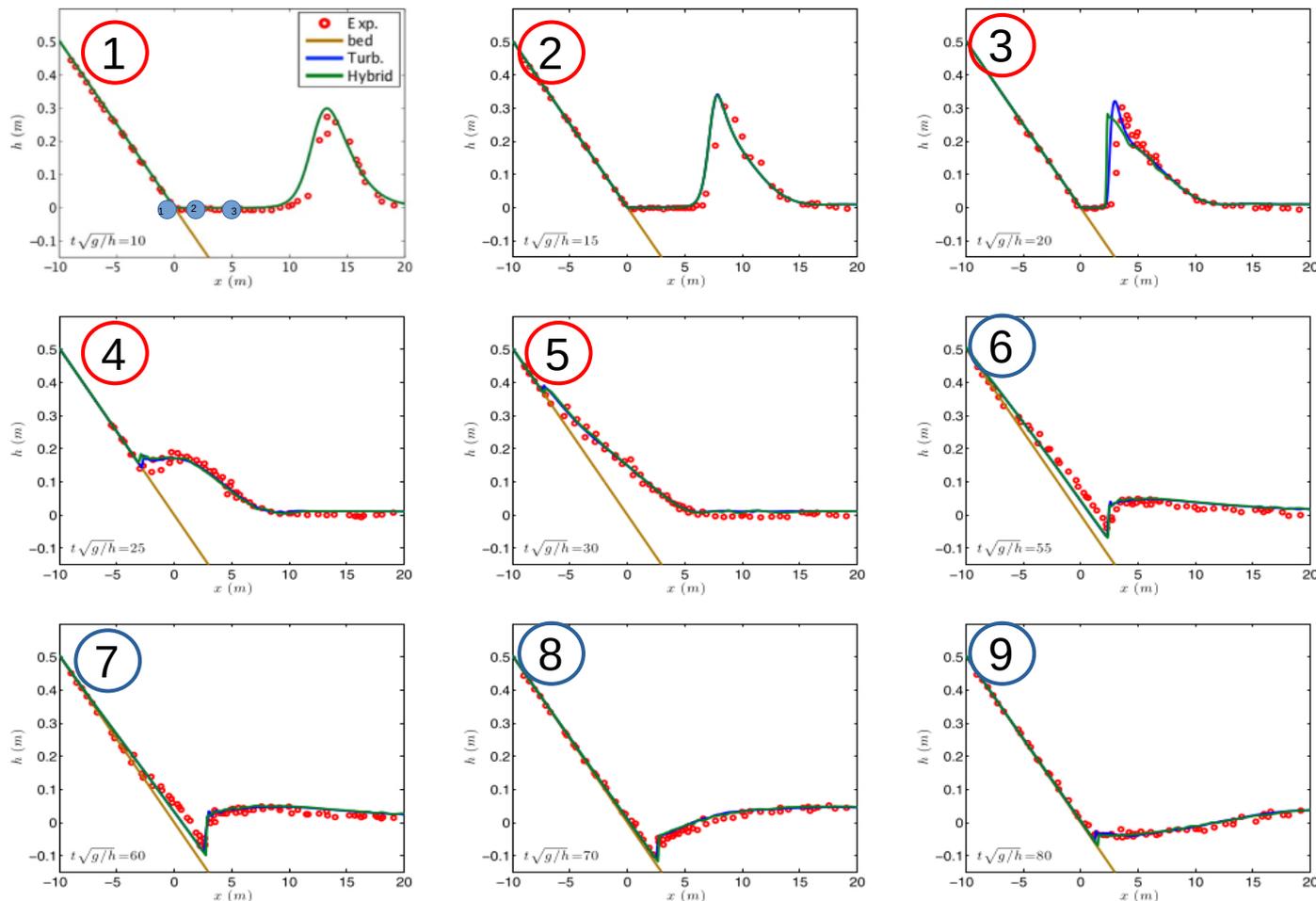
- Second order indices

$$S_{i,j} = \frac{\text{Var}_{x_i,x_j}(P(Y|x_i,x_j))}{\text{Var}(Y)}$$

- Total Sobol index of an input variable == sum of all Sobol indices involving this variable.

# Solitary wave on a plane beach

- Set up:  $A/d = 0.28\text{m}$ ,  $d=1\text{m}$ ,  $x=[-20, 80]\text{m}$ ,  $Nm=0.01$ ,  $WGs: 0,2,5\text{m}$
- 1-5 : First phase of the flow (shoaling breaking and runup)
- 6-9 : Second phase of the flow (run-down and breaking )



Free surface elevation of solitary wave run-up on a plane beach for the GN model.

# Solitary wave on a plane beach: Input parameters

- Surface variation criterion:

$$|\eta_t| \geq \gamma \sqrt{gH} \quad \gamma \in [0.2, 0.6]$$

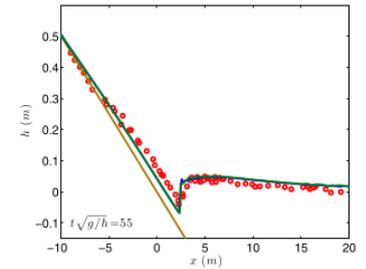
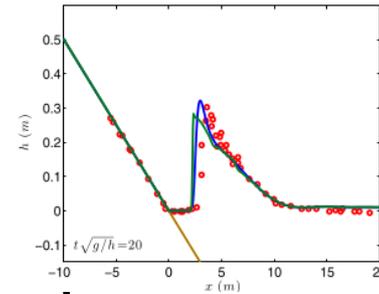
- Local slope angle criterion:

$$\|\nabla\eta\| \geq \tan\varphi_c \quad \varphi_c \in [0.3, 0.53]$$

- Constant parameters:  $\Delta x = 0.05$ ,  $Cfl = 0.01$ ,  $Fr < 1.3$

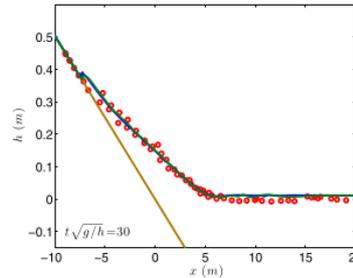
# Solitary wave on a plane beach: Output parameters

× Wave's breaking point location.

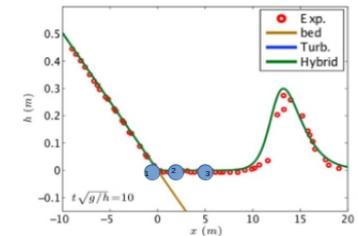


× Wave's height at the breaking point.

× Maximum run up.

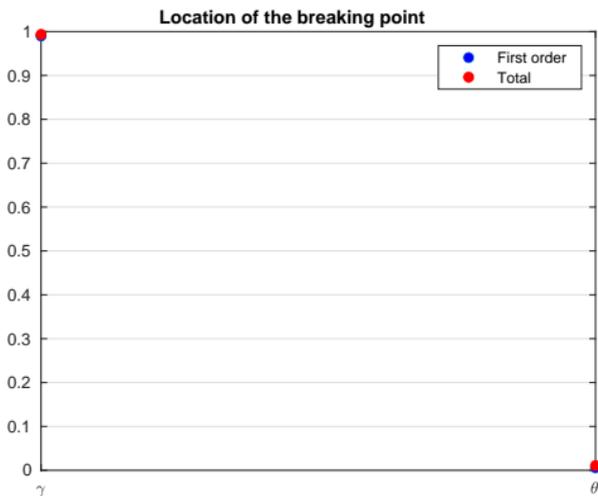
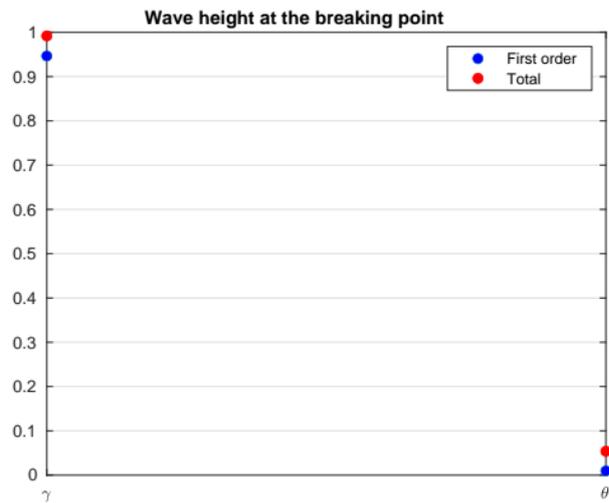


× Maximum wave height at wave gauges.

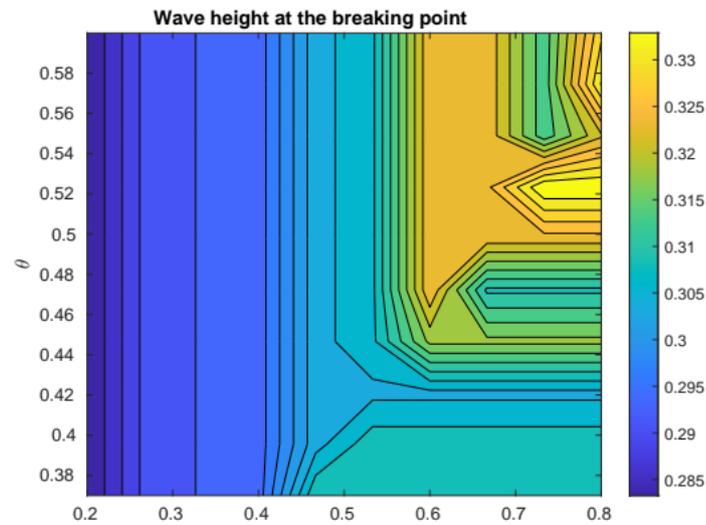


# Solitary wave on a plane beach (Hybrid)

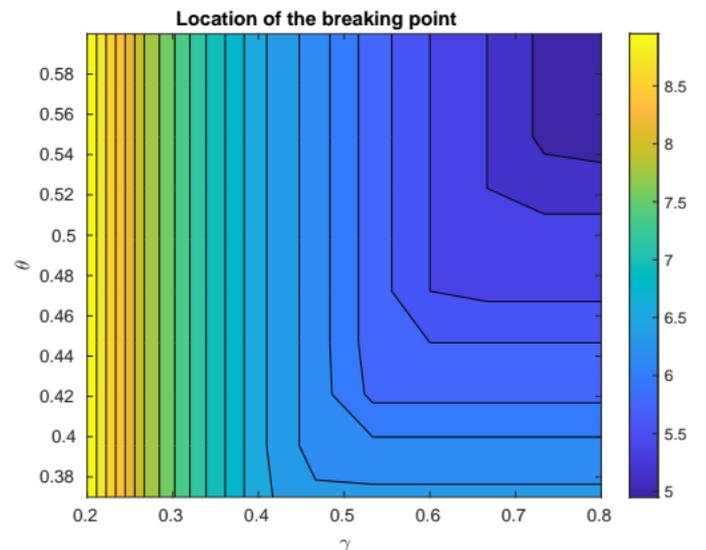
## Sobol's indices



## Contours



Range: 0.0521  
Std(standard deviation) :0.0139



Range: 4.2  
Std: 1.175

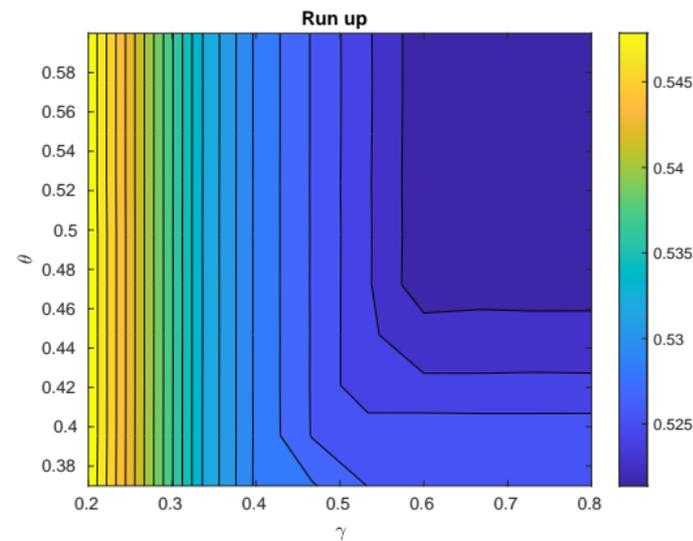
First phase of  
the flow 1-5

# Solitary wave on a plane beach (Hybrid)

Sobol's indices



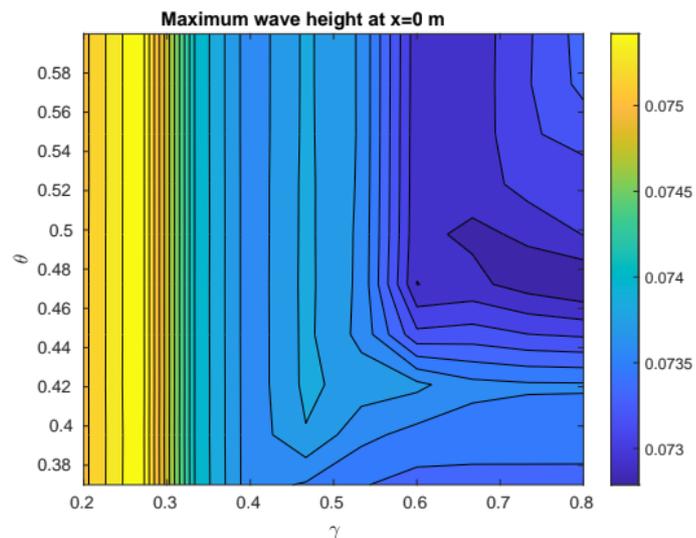
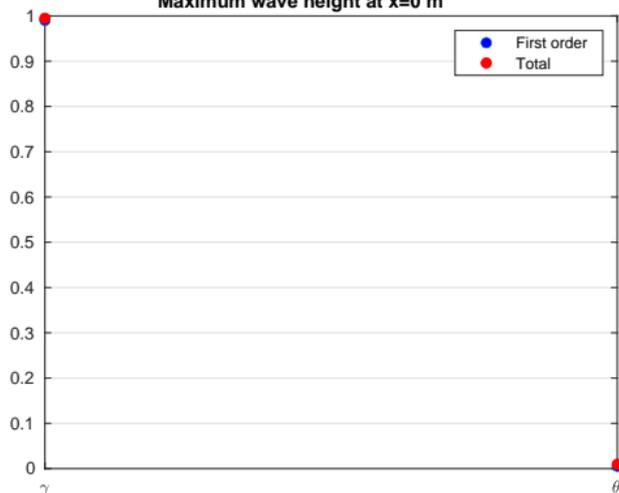
Contours



First phase of  
the flow 1-5

Range: 0.0278  
Std: 0.0087

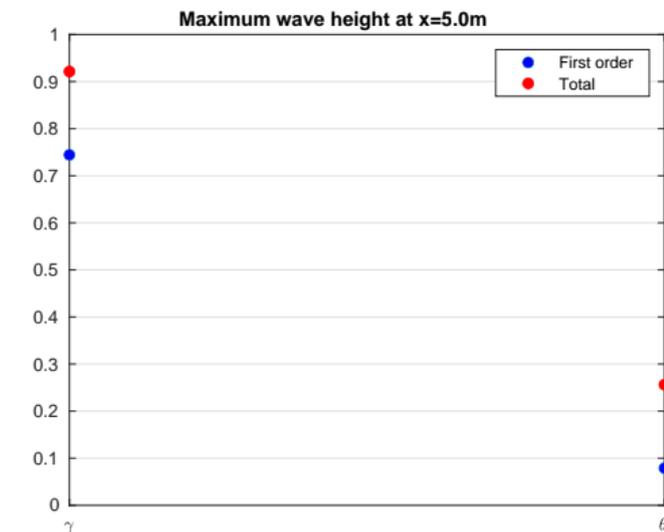
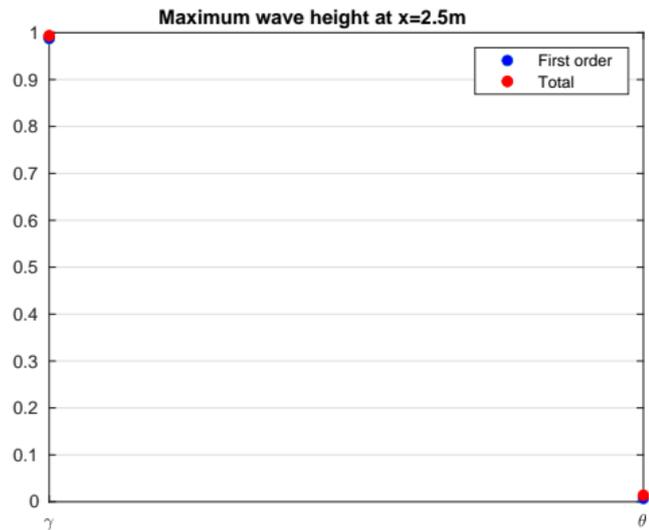
Maximum wave height at x=0 m



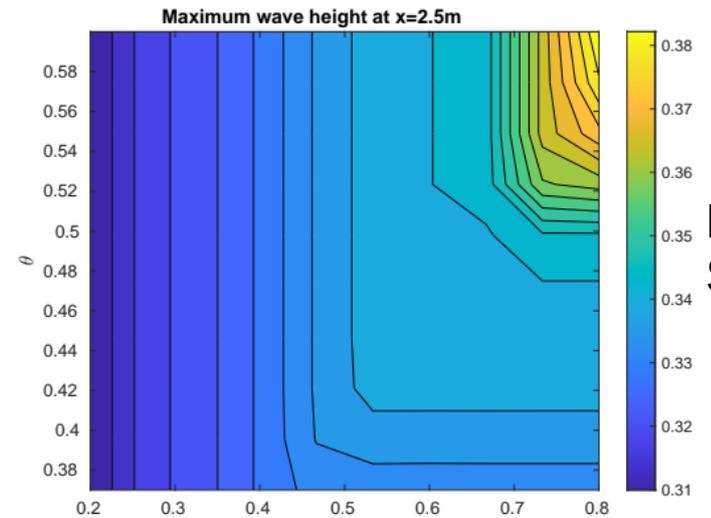
Range: 0.0028  
Std:  $8.1753 \cdot 10^{-4}$

# Solitary wave on a plane beach (Hybrid)

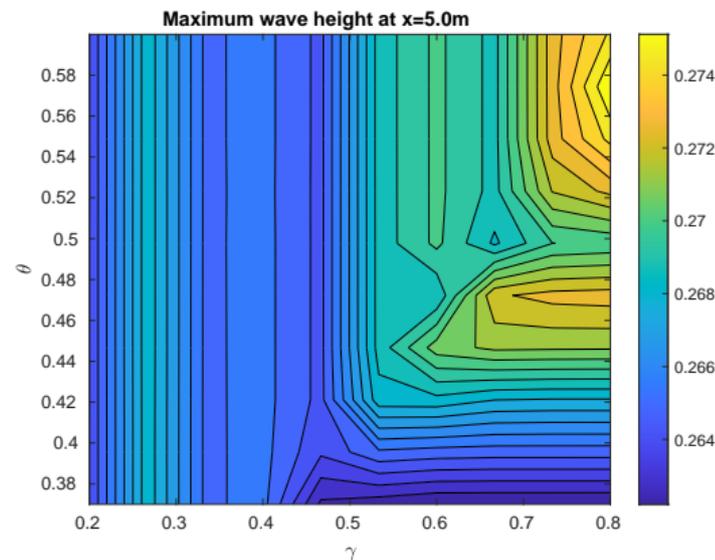
## Sobol's indices



## Contours



Range: 0.0758  
Std: 0.0151

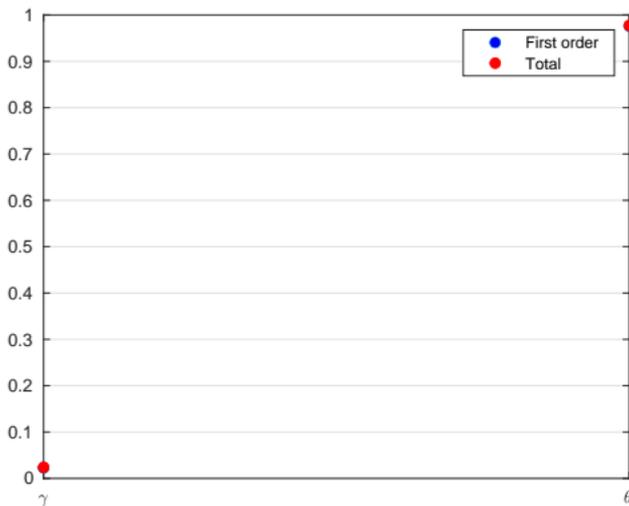
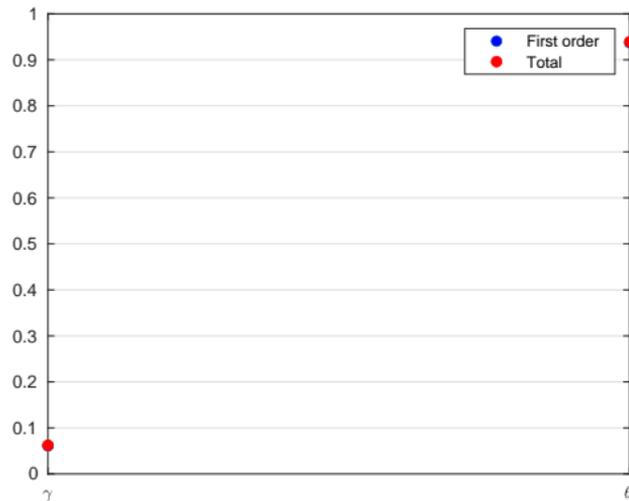


Range: 0.0135  
Std: 0.0032

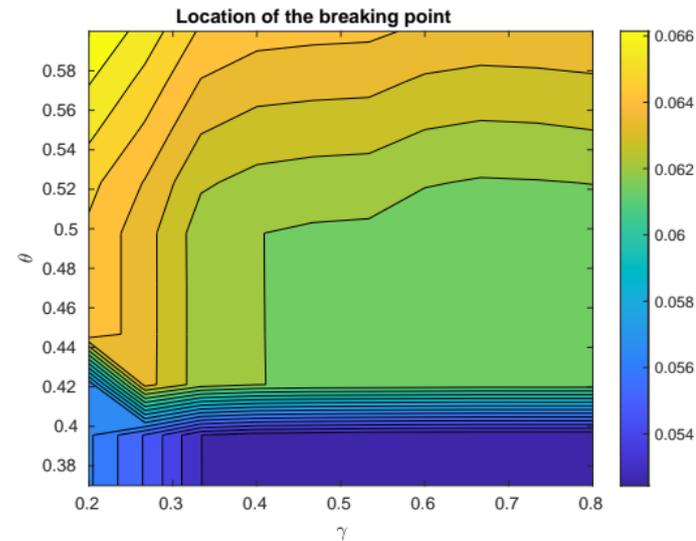
First phase of  
the flow 1-5

# Solitary wave on a plane beach (Hybrid)

Sobol's indices

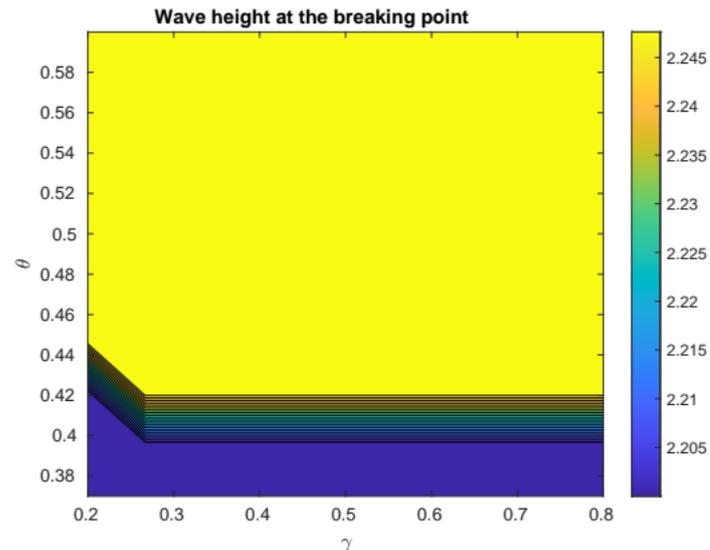


Contours



Second phase  
of the flow 6-9

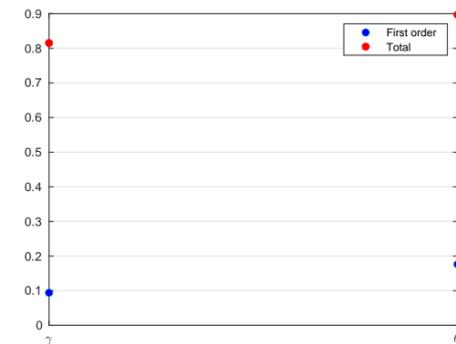
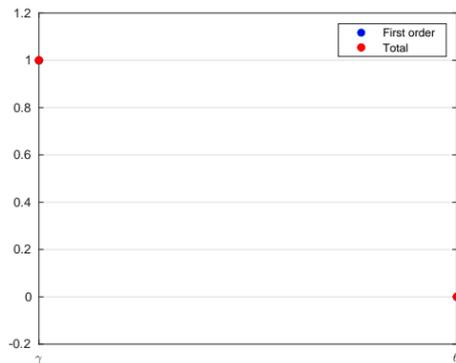
Range: 0.0144  
Std: 0.0041



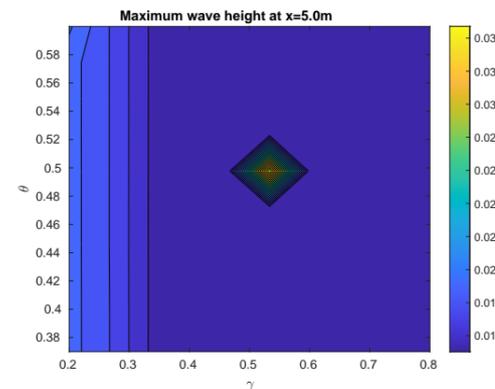
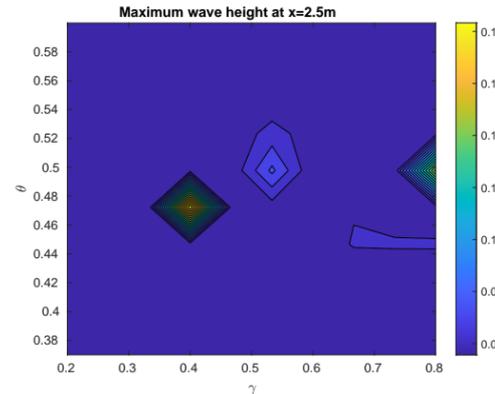
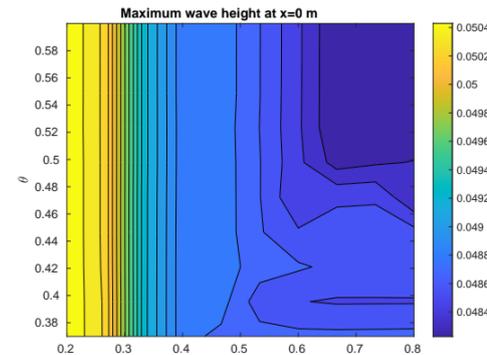
Range: 0.05  
Std: 0.0205

# Solitary wave on a plane beach (Hybrid)

Sobol's indices



Contours



Second phase  
of the flow 6-9

Range: 0.023  
Std:  $7.48 \times 10^{-4}$

Range: 0.01340  
Std: 0.0161

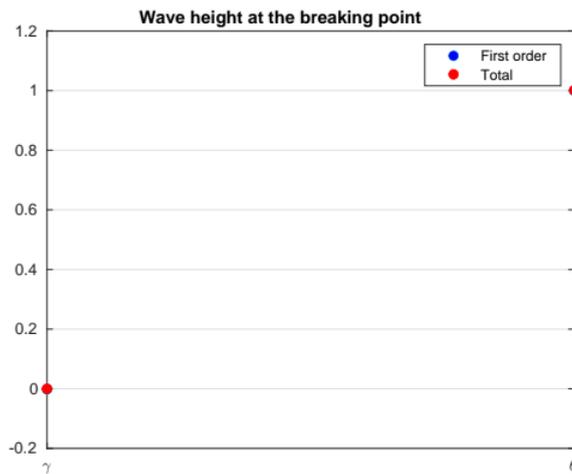
Range: 0.0207  
Std: 0.0024

# Solitary wave on a plane beach (Hybrid)

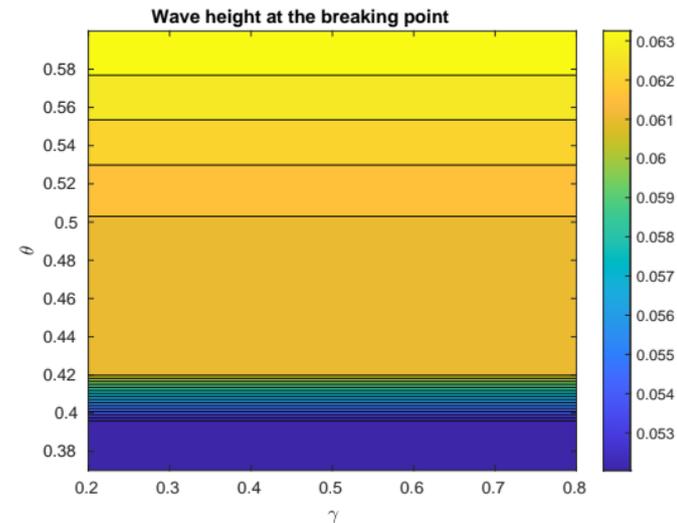
Second phase  
of the flow 6-9

Same initial  
condition

Sobol's indices

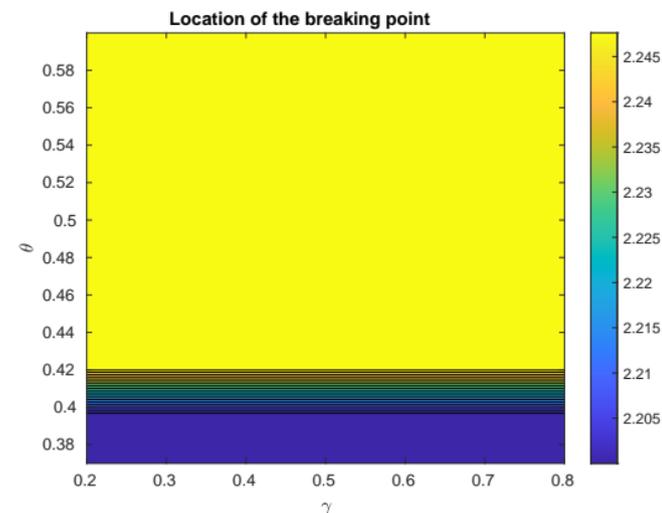
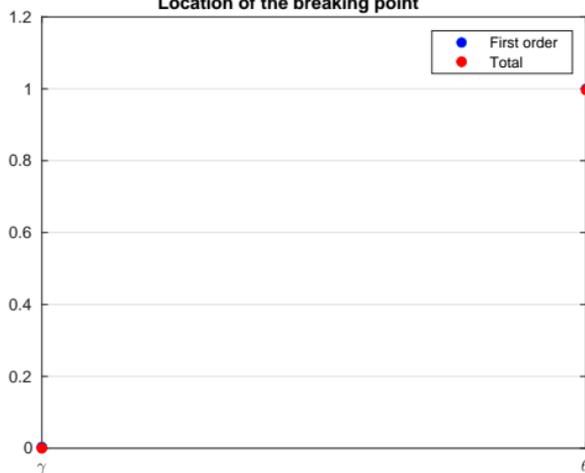


Contours



Range: 0.0118  
Std: 0.0041

Location of the breaking point



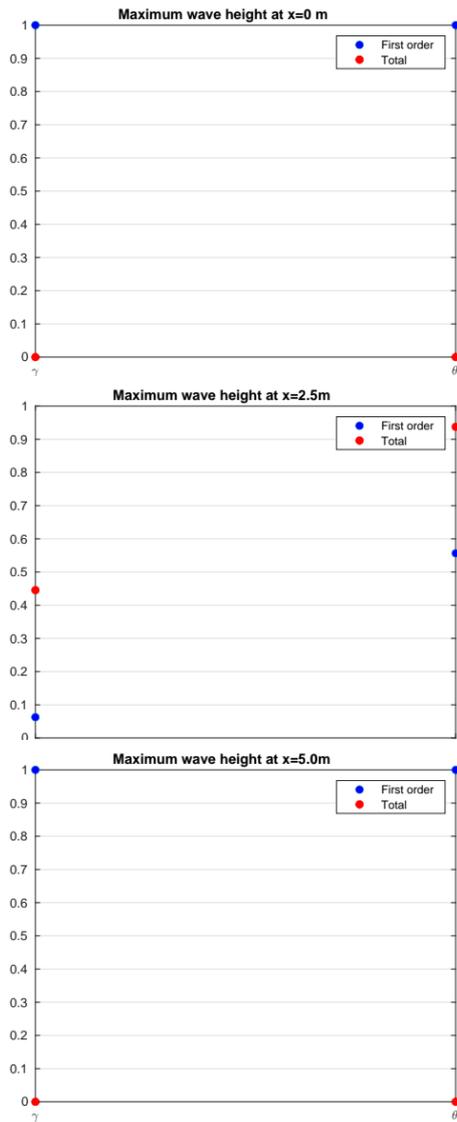
Range: 0.05  
Std: 0.0201

# Solitary wave on a plane beach (Hybrid)

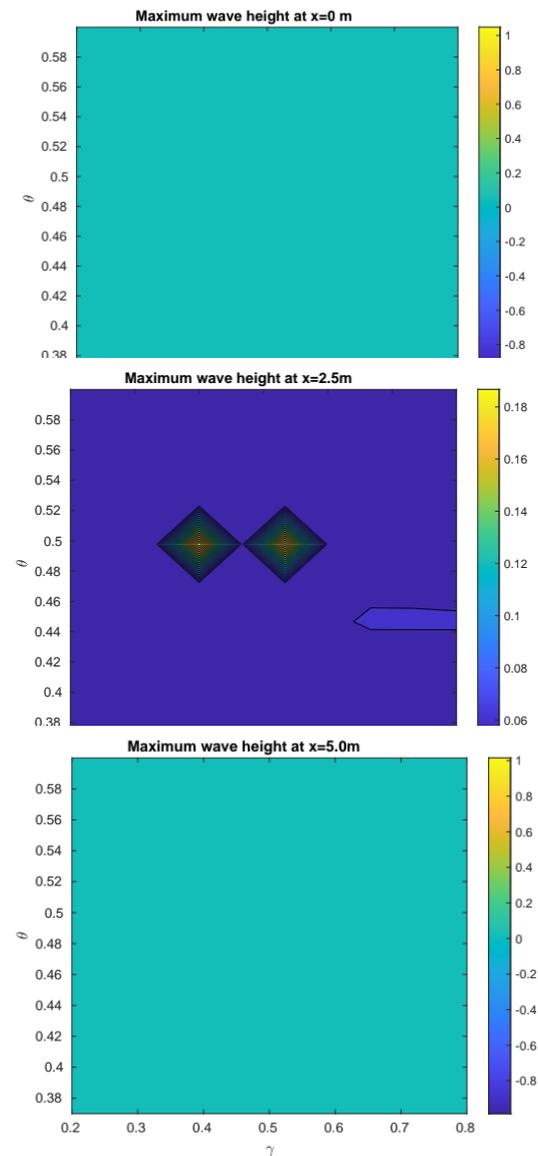
Second phase  
of the flow 6-9

Same initial  
condition

Sobol's indices



Contours

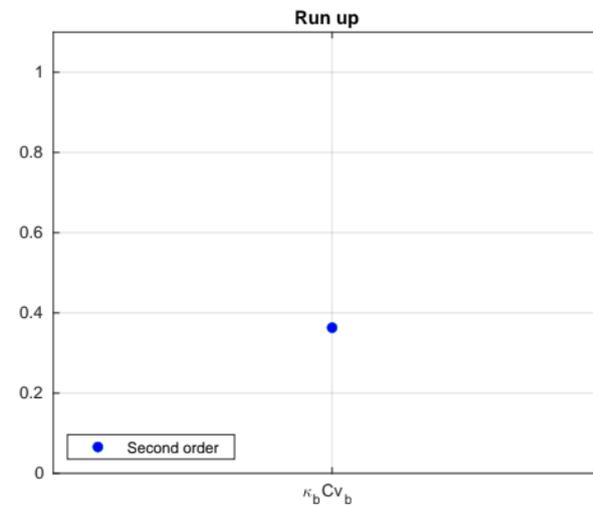
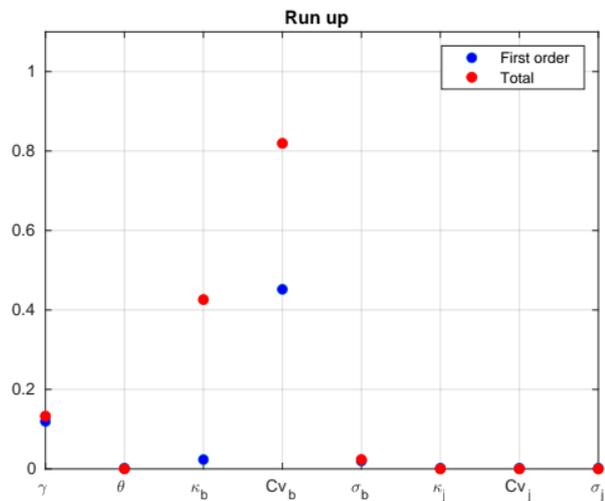
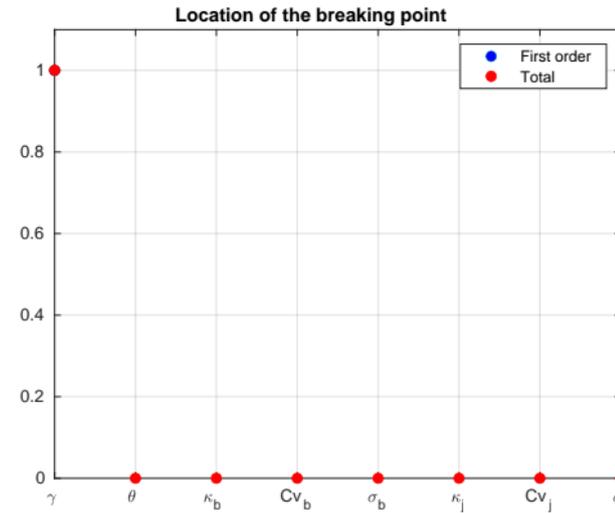
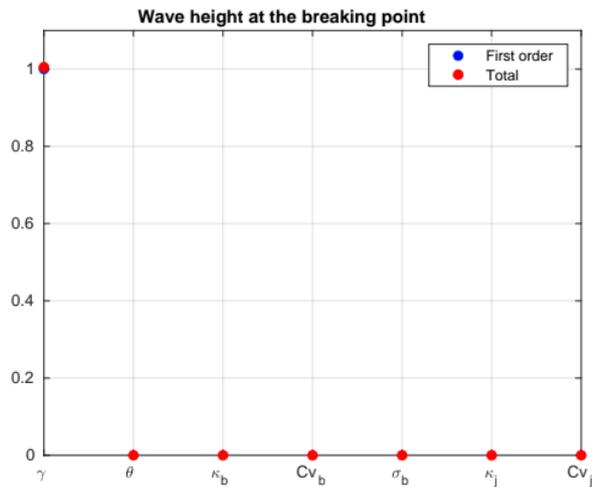


Range: 0.1402  
Std: 0.019

# Solitary wave on a plane beach (TKE)

First phase of the flow 1-5

## Sobol's indices

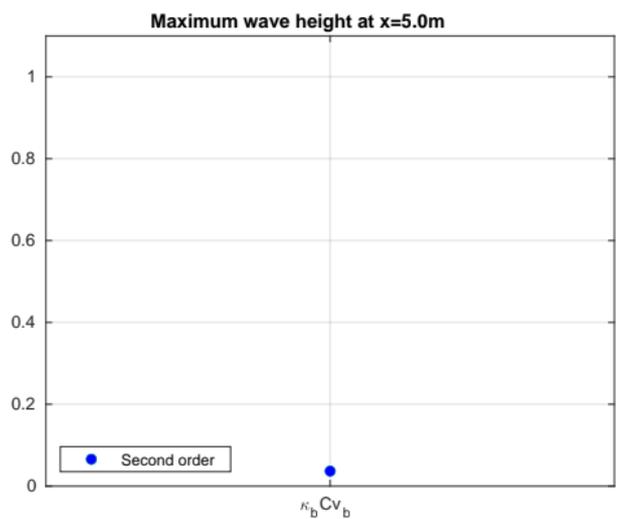
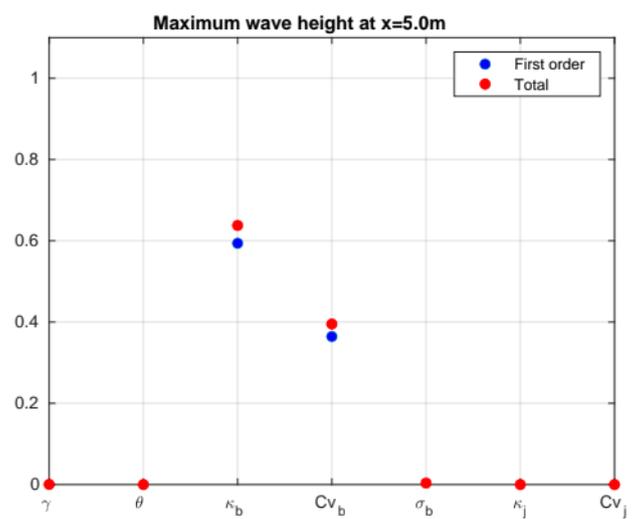
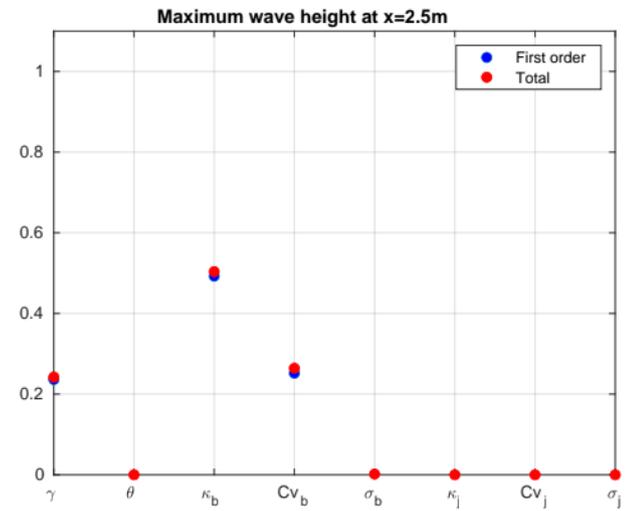
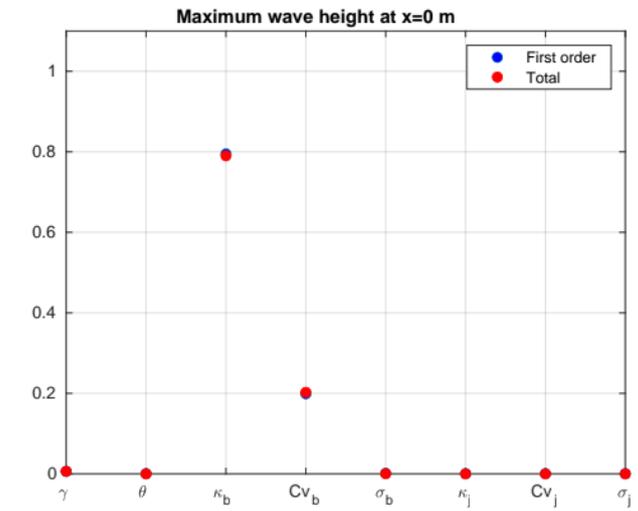


Range: 0.027  
Std: 0.0076

# Solitary wave on a plane beach (TKE)

First phase of the flow 1-5

## Sobol's indices-wave gauges

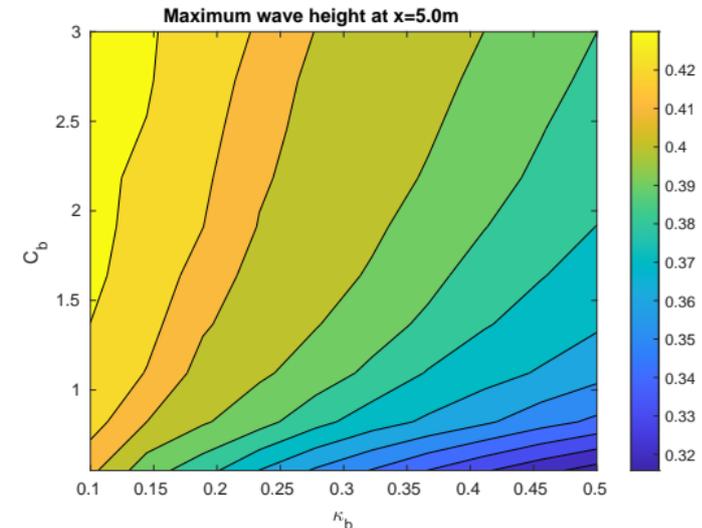
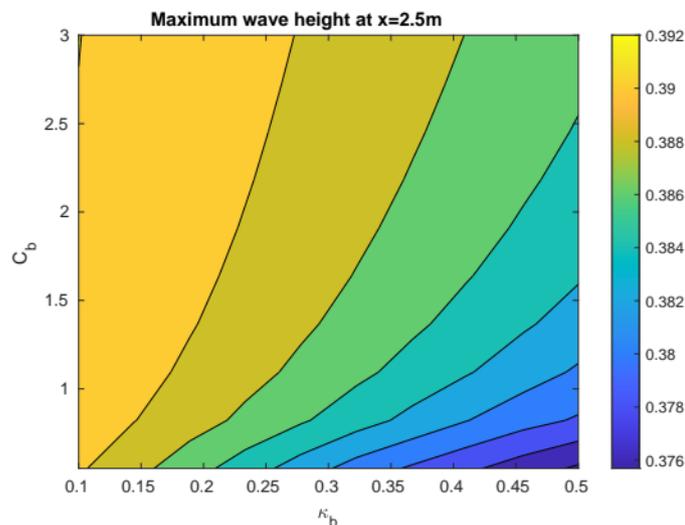
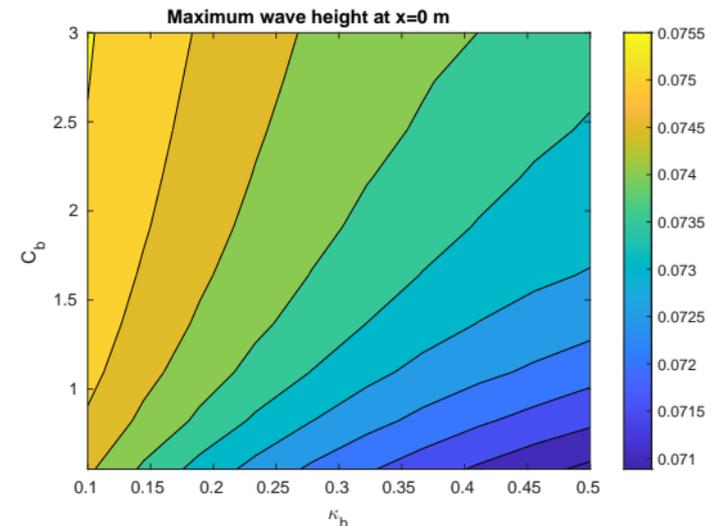
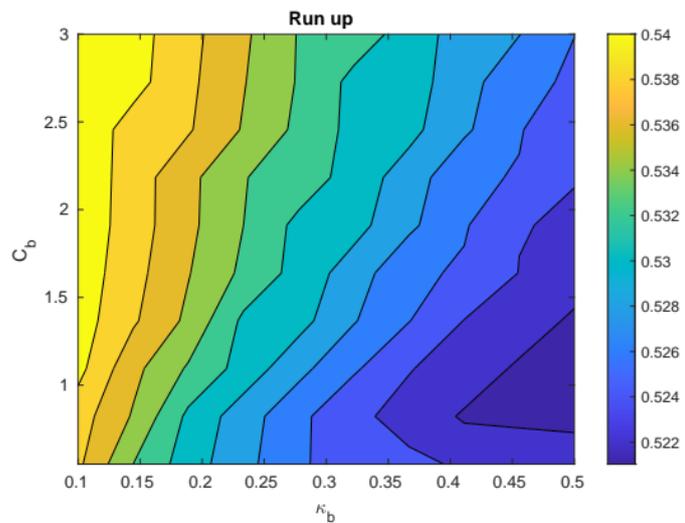


Range: 0.1501  
Std: 0.0515

# Solitary wave on a plane beach (TKE)

First phase of  
the flow 1-5

If we freeze all the input parameters except  $\kappa_b$  and  $c_b$

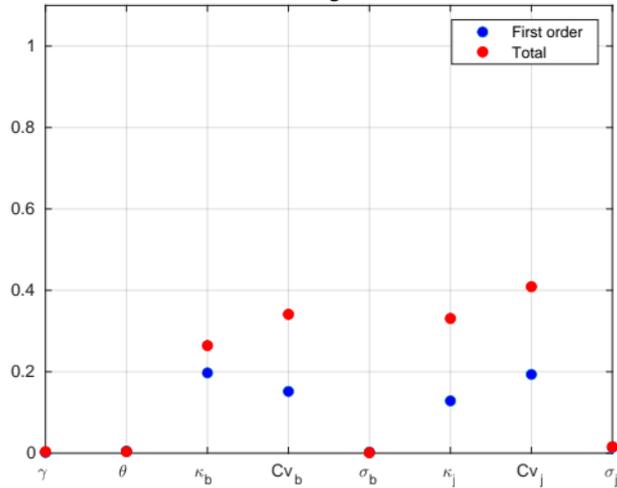


# Solitary wave on a plane beach (TKE)

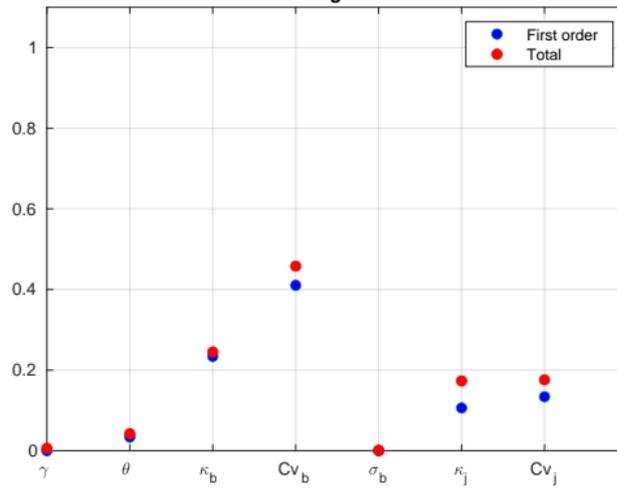
Second phase of the flow 6-9

Same initial condition

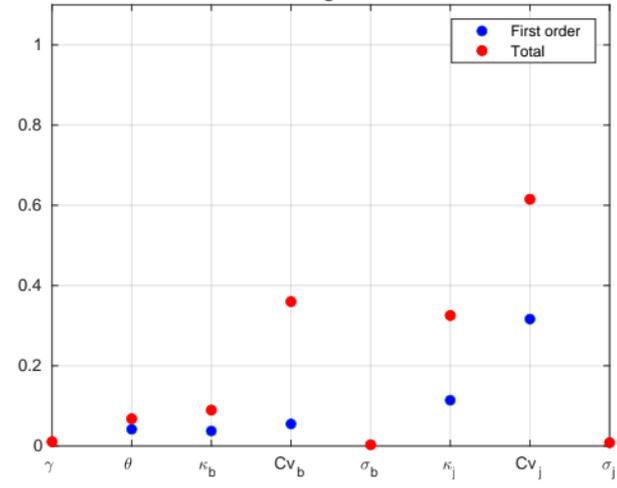
Maximum wave height at x=0 m



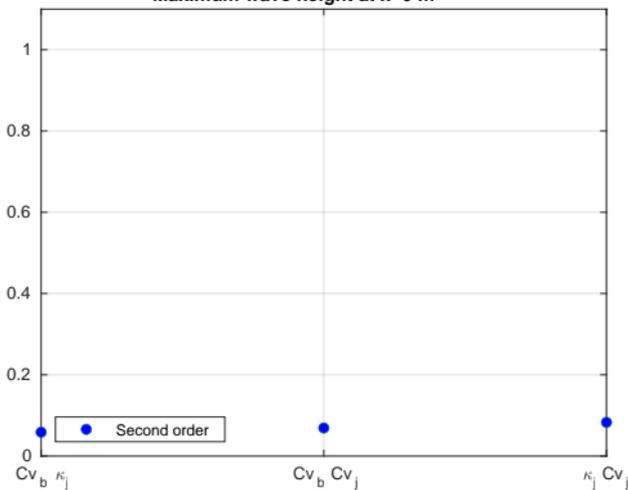
Maximum wave height at x=2.5m



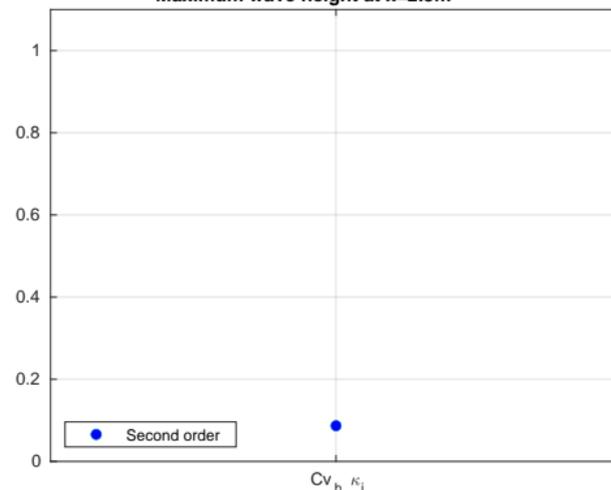
Maximum wave height at x=5.0m



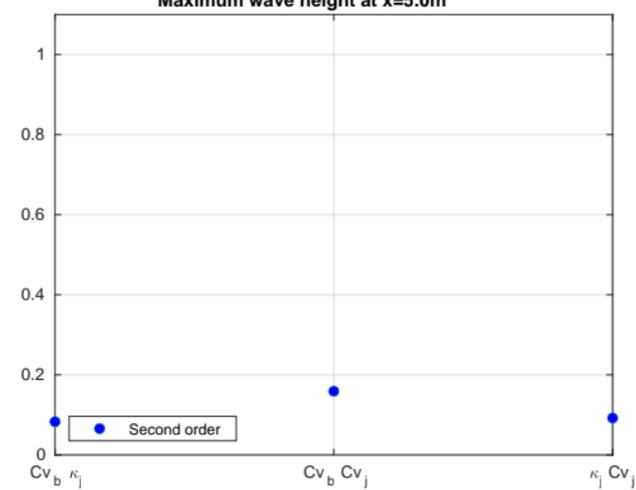
Maximum wave height at x=0 m



Maximum wave height at x=2.5m



Maximum wave height at x=5.0m

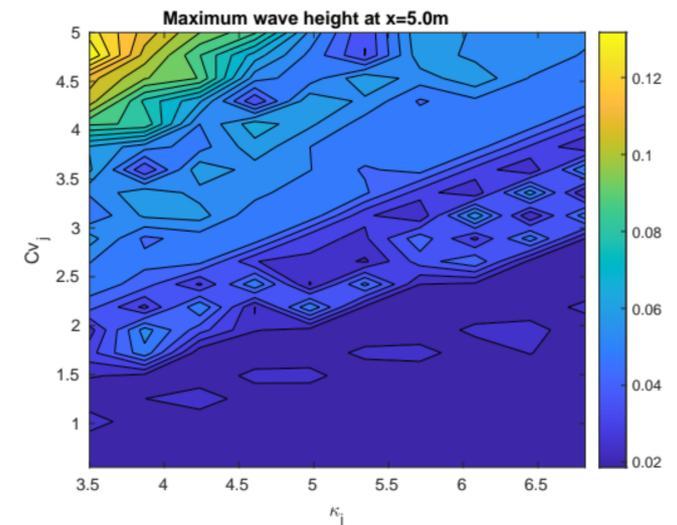
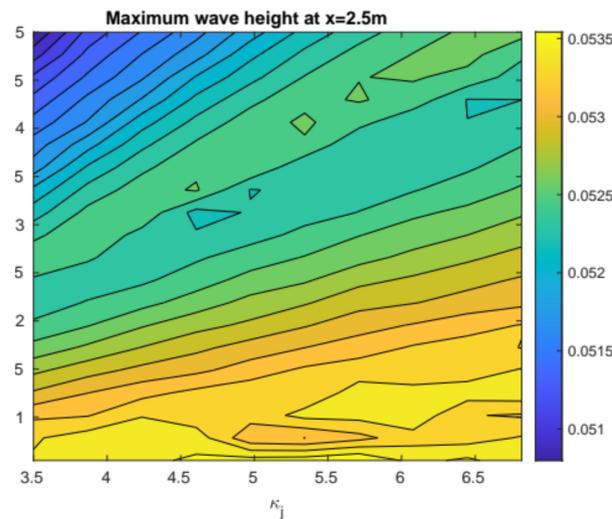
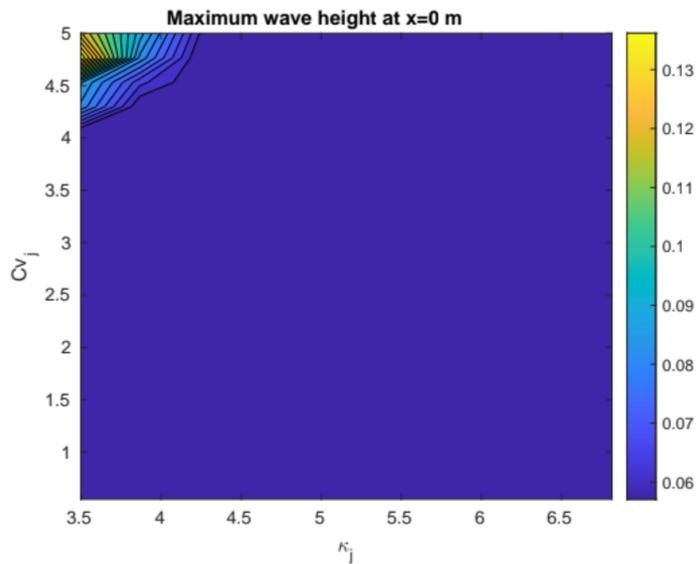
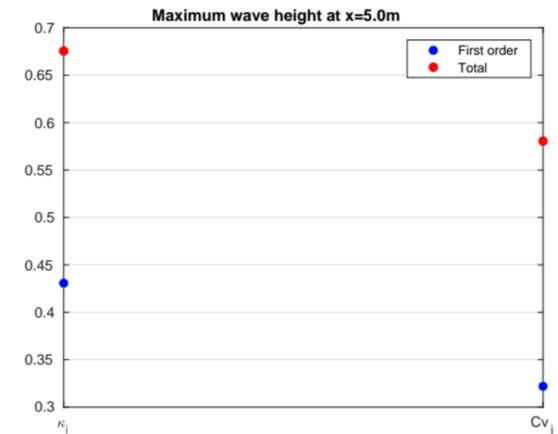
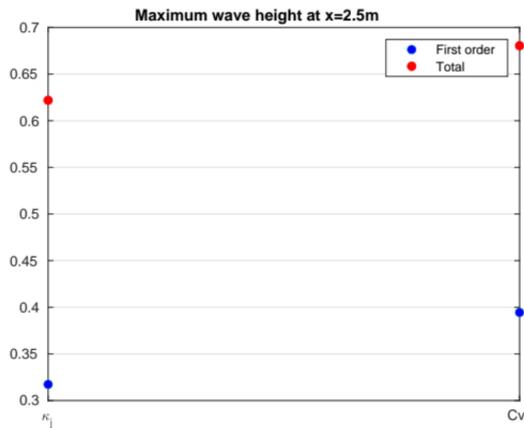
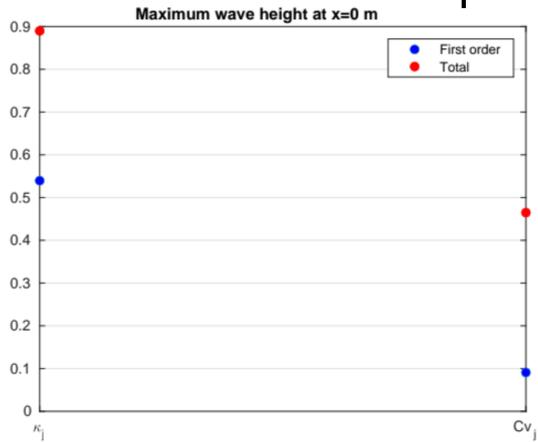


# Solitary wave on a plane beach (TKE)

Second phase of the flow 6-9

Same initial condition

If we freeze all the input parameters except  $\kappa_j$  and  $c_j$



# Conclusions

- Hybrid closure: when works , more robust
- Tke always works but many more parameters to tune
- The variation with TKE quite smooth /no discontinuity in the output parameters

*Thank you!!*