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A Bayesian Framework for Poisson Process Characterization of Extremes with Objective Prior

Théo Moins¹, Julyan Arbel¹, Anne Dutfoy², Stéphane Girard¹

¹Statify, Inria Grenoble Rhône-Alpes

²EDF R&D dept. Périclès

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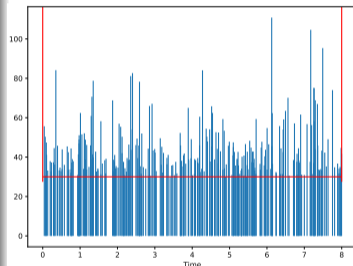
Motivations

Let (X_1, \dots, X_n) be i.i.d. r.v. and the associated point process N_n , evaluated on $I_u = [u, +\infty)$.

Theorem 1 (S. Coles 2001)

Under mild conditions and for a sufficiently large u , N_n can be approximated by a non-homogeneous Poisson process N of intensity measure Λ with parameters $\theta = (\mu, \sigma, \xi) \in \mathbb{R} \times \mathbb{R}^+ \times \mathbb{R}$, such that for all $x > u$,

$$\Lambda(I_x) = \int_x^{+\infty} \lambda(t) dt = \begin{cases} \left\{ 1 + \xi \left(\frac{x-\mu}{\sigma} \right) \right\}_+^{-\frac{1}{\xi}} & \text{if } \xi \neq 0, \\ \exp\left(-\frac{x-\mu}{\sigma}\right) & \text{if } \xi = 0. \end{cases}$$



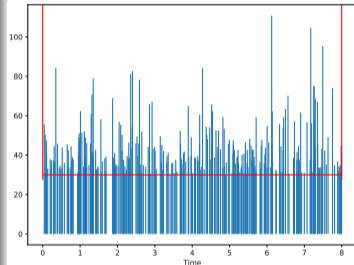
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$$\text{Likelihood: } p_{PPP}(\mathbf{x} \mid \mu, \sigma, \xi) = \exp \left\{ -m \left(1 + \xi \left(\frac{u - \mu}{\sigma} \right) \right)^{-\frac{1}{\xi}} \right\} \sigma^{-n} \prod_{i=1}^n \left(1 + \xi \left(\frac{x_i - \mu}{\sigma} \right) \right)^{-\frac{1}{\xi} - 1}$$

The Bayesian Point of view

Inference on $\theta = (\mu, \sigma, \xi)$ using Bayesian algorithms such as MCMC.

The main advantages are (S. G. Coles et al. 1996):

- Consideration of expert information with informative prior,
- Can be used in any case, even when the likelihood is irregular (for $\xi < -1$),
- Access to the posterior predictive distribution:

$$p(\tilde{x} | \mathbf{x}) = \int \underbrace{p(\tilde{x} | \theta)}_{\text{randomness of future obs.}} \underbrace{p(\theta | \mathbf{x})}_{\text{parameter uncertainty}} d\theta.$$

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Computational Bayesian issues: prior elicitation, reparameterization, etc.

Uninformative prior for extremes

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Theorem 2 (Moins et al.)

The uniform prior on $(\mu, \log \sigma, \xi)$ leads to a proper posterior for the Poisson process model for extremes as soon as $n \geq 3$.

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Theorem 3 (Moins et al.)

Jeffreys prior for a Poisson process for extremes with parameters $\theta = (\mu, \sigma, \xi)$ exists provided $\xi > -1/2$, and can be written as

$$\pi_{J,PP}(\mu, \sigma, \xi) \propto \frac{(1 + \xi (\frac{u-\mu}{\sigma}))^{-\frac{3}{2\xi}-1}}{\sigma^2(1 + \xi)(1 + 2\xi)^{\frac{1}{2}}}.$$

It is improper and leads to a proper posterior as soon as $\xi > -1/2$.

An orthogonal parameterisation

Alternative parameterisation: instead of (μ, σ, ξ) , run MCMC on an orthogonal parameterisation suggested for another purpose by (Chavez-Demoulin et al. 2005):

$$(r, \nu, \xi) = \left(m \left(1 + \xi \left(\frac{u - \mu}{\sigma} \right) \right)^{-1/\xi}, (1 + \xi)(\sigma + \xi(u - \mu)), \xi \right).$$

r : intensity of PP, expected number of exceedances.

This ensures full orthogonality, which means that $I(r, \nu, \xi)$ is diagonal.

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$$\pi_{J,PP}(r, \nu, \xi) \propto \frac{r^{1/2}}{\nu(1 + \xi)(1 + 2\xi)^{1/2}}.$$

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Our experiments show better convergence with orthogonal parameters.

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