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# Minimizing energy consumption for real-time tasks on heterogeneous platforms under deadline and reliability constraints

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**Abstract:** Low energy consumption and high reliability are widely identified as increasingly relevant issues in real-time systems on heterogeneous platforms. In this paper, we propose a multi-criteria optimization strategy to minimize the expected energy consumption while enforcing the reliability threshold and meeting all task deadlines. The tasks arrive periodically. Each instance of a task is replicated to ensure a prescribed reliability threshold. The platform is composed of processors with different (and possibly unrelated) characteristics, including speed profile, energy cost and failure rate. We provide several mapping and scheduling heuristics to solve this challenging optimization problem. Specifically, a novel approach is designed to control (i) how many replicas to use for each task, (ii) on which processor to map each replica and (iii) when to schedule each replica for each task instance on its assigned processor. Different mappings achieve different levels of reliability and consume different amounts of energy. Scheduling matters because once a task replica is successful, the other replicas of that task instance are canceled, which calls for minimizing the amount of temporal overlap between any replica pair. The experiments are conducted for a comprehensive set of execution scenarios, with a wide range of processor speed profiles and failure rates. The comparison results reveal that our strategies perform better than the random baseline, with a gain in energy consumption of more than 40% for nearly all cases. The absolute performance of the heuristics is assessed by a comparison with a lower-bound; the best heuristics achieve an excellent performance. It saves only 2% less energy than the lower-bound.

**Key-words:** real-time systems, energy-aware systems, reliability, scheduling, heterogeneous platforms

# Minimiser la consommation d'énergie pour des tâches en temps réel sur des plateformes hétérogènes avec contraintes de délais et de fiabilité

**Résumé :** La faible consommation d'énergie et la haute fiabilité sont identifiées comme des problèmes de plus en plus pertinents dans les systèmes en temps réel sur des plateformes hétérogènes. Dans ce rapport, nous proposons une stratégie d'optimisation multi-critère pour minimiser l'espérance de la consommation d'énergie tout en respectant le seuil de fiabilité et toutes les échéances des tâches. Les tâches arrivent périodiquement. Chaque instance d'une tâche est répliquée pour garantir un seuil de fiabilité prescrit. La plateforme est composée de processeurs avec des caractéristiques différentes (et éventuellement sans corrélation), y compris la vitesse, le coût énergétique et le taux de panne. Nous fournissons plusieurs heuristiques de placement et d'ordonnancement pour ce problème d'optimisation difficile. Plus précisément, une nouvelle solution est conçue pour contrôler (i) le nombre de répliques à utiliser pour chaque tâche, (ii) sur quel processeur doit-on placer chaque réplique et (iii) comment ordonnancer chaque réplique de chaque instance de tâche sur le processeur qui lui est affecté. Différents placements atteignent différents niveaux de fiabilité et consomment différentes quantités d'énergie. L'ordonnancement est important car une fois qu'une réplique de tâche réussit, les autres répliques de cette instance sont annulées, ce qui demande de minimiser le recouvrement en temps entre toute paire de répliques. Les expériences sont exécutées pour un grand ensemble de scénarios, avec une large gamme de vitesses et de taux d'échec pour les processeurs. Les résultats montrent que nos stratégies fonctionnent mieux que la référence de base aléatoire, avec un gain de 40 % en consommation d'énergie, dans presque tous les cas. La performance absolue de l'heuristique est évaluée en la comparant avec une borne inférieure. La meilleure heuristique atteint une excellente performance, avec une valeur moyenne supérieure de seulement 2 % à la borne inférieure.

**Mots-clés :** systèmes en temps réel, systèmes sensibles à l'énergie, fiabilité, ordonnancement, plates-formes hétérogènes

## 1 Introduction

Real-time systems are composed of periodic tasks that are regularly input to a parallel computing platform and must complete execution before their deadlines. In many applications, another requirement is reliability: the execution of each task is prone to transient faults, so that several replicas of the same task must be executed in order to guarantee a prescribed level of reliability [1, 2]. Recently, several strategies have been introduced with the objective to minimize the expected energy consumption of the system while matching all deadlines and reliability constraints [3, 4].

This work aims at extending these energy-aware strategies in the context of heterogeneous platforms. Heterogeneous platforms have been used for safety-critical real-time systems for many years [5]. With the advent of multiple hardware resources such as multi-cores, GPUs, and FPGAs, modern computing platforms exhibit a high level of heterogeneity, and the trend is increasing. The multiplicity of hardware resources with very different characteristics in terms of speed profile, reliability level and energy cost, raises an interesting but challenging problem: given several device types, which ones should we keep and which ones should we discard, in order to achieve the best possible tri-criteria trade-off (time, energy, reliability)? Needless to say, this optimization problem is NP-hard, even with two identical error-free processors, simply because of matching deadlines.

This work provides several mapping and scheduling heuristics to solve the tri-criteria problem on heterogeneous platforms. The design of these heuristics is much more technical than in the case of identical processors. Intuitively, this is because the reliability of a replica of one task depends upon the processor which executes it, and is different for each task instance (which we define in Section 2.1 below). More precisely, the reliability of a replica of  $j$ -th instance of  $i$ -th task mapped on processor  $m_k$  can be estimated by  $R(\tau_{i,j}, m_k) = e^{-\lambda_k c_{i,k}}$ , where  $c_{i,k}$  is the worst case execution time (WCET, which is common for all instances of the same task) of task  $\tau_i$  on  $m_k$ , and  $\lambda_k$  the failure rate of  $m_k$ . The total reliability of a task instance can be estimated by a function of the reliability of all its replicas (which we explicit in Equation 1 below); hence, it is not known until the end of the mapping process, unless we pre-compute an exponential number of reliability values. Then there are many processors to choose from, and those providing a high reliability, thereby minimizing the number of replicas needed to match the reliability threshold, may also require a high energy cost per replica: in the end, it might be better to use less reliable but also less energy-intensive processors. Furthermore, the reliability is not enough to decide for the mapping: if two processors offer similar reliabilities for a task, it might be better to select the one with smaller execution time, in order to increase the possibility of mapping other tasks without exceeding any deadline. Altogether, we face a complicated decision, and we provide several criteria to guide the

mapping process.

Overall, the objective is to minimize the expected energy consumption while matching all deadlines and reliability constraints. The expected energy consumption is the average energy consumed over all failure scenarios. Consider a sample execution: whenever the execution of a task replica succeeds, all the other replicas are instantaneously deleted; therefore, the actual amount of energy consumed depends both upon the error scenario (which replica is the first successful) and upon the overlap between replicas (some replicas are partially executed and interrupted when the successful one completes). Given a mapping, the scheduling phase aims at reducing overlap between any two replicas of the same task. Note that having an overlap-free scheduling is not always possible, because of utilization constraints. Also, deciding whether an overlap-free scheduling exists for a given mapping is NP-hard [6], even for deterministic tasks.

Finally, in actual real-time systems, tasks often complete before their worst-case execution time, or WCET, so that execution times are routinely modeled as stochastic. For instance, one typically assumes that the execution time of every instance of task  $\tau_i$  on  $m_k$  follows a common uniform probability distribution in the range  $[\beta_{b/w}c_{i,k}, c_{i,k}]$  for some constant  $\beta_{b/w} < 1$  (ratio of best case over worst case).

In the end, the expected energy consumption must also be averaged over all possible values for execution times in addition to over all failure scenarios. To assess the performance of our heuristics, we use a comprehensive set of execution scenarios, with a wide range of processor speed profiles and failure rates. When the failure rate is low, most heuristics are equivalent, but when the failure rate is higher, only a few heuristics achieve good performance. Because we have no guarantee on the performance of the global mapping and scheduling process, we analytically derive a lower-bound for the expected energy consumption of any mapping. This bound cannot always be met. Nevertheless, we show that the performance of our best heuristics remains quite close to this bound in the vast majority of simulation scenarios.

The main contributions of the paper are the following:

1. The formulation of the tri-criteria optimization problem;
2. The design of several mapping and scheduling heuristics;
3. The characterization of a lower-bound for energy consumption;
4. An experimental evaluation based on a comprehensive set of simulations scenarios, showing that our best heuristics achieve a significant gain over the whole spectrum of application and platform parameters.

The rest of the paper is organized as follows. Section 2 provides a detailed description of the optimization problem under study, including a few notes on its complexity. The mapping and scheduling heuristics are described in



Table 1: Key Notations

Notation	Explanation
$N$ and $M$	number of tasks and of processors
$\tau_i$	the $i$ -th task
$\tau_{i,j}$	the $j$ -th instance of the $i$ -th task
$m_k$	the $k$ -th processor
$p_i$	period (deadline) for each instance of task $\tau_i$
$L = \text{lcm}_{1 \leq i \leq n} p_i$	hyperperiod of the system
$w_{i,j,k}$	actual execution time for task instance $\tau_{i,j}$ on processor $m_k$
$c_{i,k}$	WCET for task $\tau_i$ on processor $m_k$
$u_{i,k} = \frac{c_{i,k}}{p_i}$	utilization of task $\tau_i$ executing on processor $m_k$
$u_k$	utilization of $m_k$ (sum of utilization of replicas assigned to $m_k$ )
$\mathcal{R}_i$	target reliability threshold for task $\tau_i$
$\lambda_k$	failure rate of processor $m_k$
$R(\tau_{i,j}, m_k)$	reliability of task $\tau_{i,j}$ on processor $m_k$
$P_{k,d}$	dynamic power consumed per time unit on processor $m_k$
$P_{k,s}$	static power consumed per time unit on processor $m_k$
$E_s$	total static energy consumption of the system
$E_d$	total dynamic energy consumption of the system
$E_d(\tau_{i,j}, m_k)$	dynamic energy cost of task instance $\tau_{i,j}$ on processor $m_k$

Section 3 and 4 respectively. The lower-bound of energy consumption is introduced in Section 5. Section 6 is devoted to a comprehensive experimental comparison of the heuristics. Section 7 presents related work. Finally, Section 8 gives concluding remarks and hints for future work.

## 2 Model

The inputs to the optimization problem are a set of real-time independent tasks, a set of non-identical processors and a reliability target. Key notations are summarized in Table 1.

### 2.1 Platform and tasks

The platform consists of  $M$  heterogeneous processors  $m_1, m_2, \dots, m_M$  and a set of  $N$  periodic atomic tasks  $\tau_1, \tau_2, \dots, \tau_N$ . Each task  $\tau_i$  has WCET  $c_{i,k}$  on the processor  $m_k$ . The WCETs among different processors are not necessarily related. In the experiments, we generate the  $c_{i,k}$  values with the method proposed in [7], where we have two parameters to control the correlation among task execution times and processors (see Section 6.1 for details). Each periodic task  $\tau_i$  generates a sequence of *instances* with period  $p_i$ , which is equal to its deadline. The  $j$ -th instance of task  $\tau_i$  is released at date  $(j-1)p_i$  and its deadline is  $jp_i$ . The whole input pattern repeats every hyperperiod of length  $L = \text{lcm}_{1 \leq i \leq n} p_i$ . Each task  $\tau_i$  has  $\frac{L}{p_i}$  instances within the hyperperiod.

As already mentioned, real-time tasks usually complete execution earlier

than their estimated WCET: actual execution times are assumed to be data-dependent and non-deterministic, randomly sampled from some probability distribution whose support is upper bounded by the WCET. See Section 6.1 for details on the generation of actual execution times from WCET values. The *utilization*  $u_{i,k}$  of task  $\tau_i$  executing on processor  $m_k$  is defined as  $u_{i,k} = \frac{c_{i,k}}{p_i}$ . The utilization of a processor is the sum of the utilizations of all task instances that are assigned to it.

The execution of any task can be preempted, that is, to be temporarily stopped (for instance to allow for the execution of another task) and later resumed without any induced penalty.

## 2.2 Reliability

We consider transient faults, modeled by an exponential probability distribution of rate  $\lambda_k$  on processor  $m_k$ . Thus, fault rates differ from one processor to another. This is a very natural assumption for a heterogeneous platform made of different-type processors. At the end of the execution of each task, there is an *acceptance test* to check the occurrence of soft errors induced by the transient faults. It is assumed that acceptance tests are 100% accurate, and that the duration of the test is included within the task WCET [3].

The *reliability* of a task instance is the probability of executing it successfully, in the absence of software faults. It is related to its execution time. But in our problem, only the tasks WCETs are known, but not the actual execution time. Thus, in our heuristics, we use the WCETs to estimate the reliability of task instances and to make mapping decisions. The reliability of task instance  $\tau_{i,j}$  on processor  $m_k$  with WCET  $c_{i,k}$  can be estimated by  $R(\tau_{i,j}, m_k) = e^{-\lambda_k \times c_{i,k}}$ . Two instances  $\tau_{i,j}$  and  $\tau_{i,j'}$  of the same task  $\tau_i$  have the same estimated reliability if they are executed on the same processor, because their WCETs are the same. But the actual reliability during execution is not the same ( $e^{-\lambda_k \times w_{i,j,k}}$  and  $e^{-\lambda_k \times w_{i,j',k}}$  respectively) because of their different actual execution times.

In the final schedule, task instance  $\tau_{i,j}$  may have several replicas executing on different processors, in order to match its reliability threshold. Let  $alloc(i, j)$  denote the index set of the processors executing a replica of  $\tau_{i,j}$ . The mapping achieves the following reliability  $R(\tau_{i,j})$  for task instance  $\tau_{i,j}$ :

$$R(\tau_{i,j}) = 1 - \prod_{k \in alloc(i,j)} (1 - R(\tau_{i,j}, m_k)) \quad (1)$$

Indeed, the task will succeed if at least one of its replicas does. The success probability is thus equal to 1 minus the probability of all replicas failing, which is the expression given in Equation (1). It also means that all other replicas (executing or not started) can be canceled when the first one is successfully finished.

Each instance  $\tau_{i,j}$  of task  $\tau_i$  has a reliability threshold  $\mathcal{R}_i$  which is an input of the problem and that must be met by the mapping. In other words,

the constraint writes  $R(\tau_{i,j}) \geq \mathcal{R}_i$  for  $1 \leq i \leq N$  and  $1 \leq j \leq \frac{L}{p_i}$ . This threshold is always satisfied during the actual execution, although we use the estimated reliability for mapping; this is because the actual execution time can never be greater than the WCET and, thus, the actual reliability is never smaller than the estimated one.

Because the tasks are independent, it is natural to assume that they might have different reliability thresholds: a higher threshold means that more resources should be assigned for the task to complete successfully with a higher probability. In the experiments we use  $\mathcal{R}_i = \mathcal{R}$  for all tasks, but our heuristics are designed to accommodate different thresholds per task.

### 2.3 Power and energy

The power consumed per time unit on processor  $m_k$  is composed of two parts, static power ( $P_{k,s}$ ) and dynamic power ( $P_{k,d}$ ). The static power is consumed permanently if the processor  $m_k$  is used in the schedule. In contrast, the dynamic power is consumed only if the processor is actually executing a task. To summarize, we have  $2M$  input values,  $\{P_{1,s}, P_{2,s} \dots P_{M,s}\}$  for static powers and  $\{P_{1,d}, P_{2,d} \dots P_{M,d}\}$  for dynamic powers.

The total static energy consumption of the system during one hyperperiod is simply given by

$$E_s = \sum_{k \in Used} P_{k,s} \times L \quad (2)$$

where  $Used$  denotes the index set of the processors used by the schedule.

Similarly to Section 2.2, the dynamic energy consumption of one replica of task instance  $\tau_{i,j}$  on processor  $m_k$  is estimated using the WCET  $c_{i,k}$  of task  $\tau_i$ :

$$E_d(\tau_{i,j}, m_k)^{estimated} = P_{k,d} \times c_{i,k} \quad (3)$$

in which we use the same value  $c_{i,k}$  for all instances  $\tau_{i,j}$  of the same task  $\tau_i$  on  $m_k$  because their WCET is the same. Thus, the estimated dynamic energy consumption during one hyperperiod, which is used to compute the schedule, is expressed as

$$E_d^{estimated} = \sum_{(i,j,k) | k \in alloc(i,j)} E_d(\tau_{i,j}, m_k)^{estimated} \quad (4)$$

But the actual energy consumption of  $\tau_{i,j}$  will likely be lower than stated in Equation (3). First the actual execution time  $w_{i,j,k}$  is typically smaller than the WCET  $c_{i,k}$ . In addition, recall that we cancel all other replicas of a task instance as soon as one of its replicas has successfully completed. Thus, the execution time of a replica can be reduced or even zeroed out by the success of another replica of the same task instance.

We define a scenario  $s = \{W, F\}$  as the random drawing of the execution times  $w_{i,j,k}$  and of the failure booleans  $f_{i,j,k}$  for all  $(i, j, k)$  where

$k \in \text{alloc}(i, j)$ .  $f_{i,j,k} = \text{True}$  if the replica of task instance  $\tau_{i,j}$  executed on processor  $m_k$  can be successfully completed, and  $f_{i,j,k} = \text{False}$  if it will be victim of a failure. Again, some replicas will be interrupted or never launched, depending upon the scenario. Given a scenario, we let  $r_{i,j,k}$  denote the actual execution times of each replica of each task instance. These values are dynamically computed as the execution of the schedule progresses. We compute the actual dynamic energy as

$$E_d(\tau_{i,j}, m_k)_s^{\text{actual}} = P_{k,d} \times r_{i,j,k} \quad (5)$$

$$E_{d,s}^{\text{actual}} = \sum_{(i,j,k) | k \in \text{alloc}(i,j)} E_d(\tau_{i,j}, m_k)_s^{\text{actual}} \quad (6)$$

Finally, we weight each scenario  $s \in S$  by its probability  $p_s$ , where  $S$  is the set of all scenarios, and compute the expected dynamic energy consumption during one hyperperiod:

$$E_d^{\text{expected}} = \sum_{s \in S} (p_s * E_{d,s}^{\text{actual}}) \quad (7)$$

Altogether, the expected energy consumption of the schedule during one hyperperiod becomes:

$$E^{\text{expected}} = E_s + E_d^{\text{expected}} \quad (8)$$

The optimization objective is to find the schedule which minimizes this value. Clearly, there is no closed-form formula for the expected energy consumption, whose value is approximated by Monte-Carlo simulations.

## 2.4 Optimization Objective

The objective is to determine a set of replicas for each task, a set of processors to execute them, and to build a schedule of length at most  $L$ , so that the expected energy consumption is minimized, while matching the reliability threshold  $\mathcal{R}_i$  for each task  $\tau_i$  and the deadline  $j \times p_i$  of each task instance  $\tau_{i,j}$ . As already mentioned in Section 1, the expected energy consumption is an average made over all possible execution times randomly drawn from their distributions, and over all failure scenarios (with every component weighted by its probability to occur). An analytical formula is out of reach, and we use Monte-Carlo sampling in the experiments. However, we stress the following two points:

- To guide the design of the heuristics, we use a simplified objective function; more precisely, we use WCETs instead of (yet unknown) actual execution times, and we conservatively estimate the dynamic energy of a task as the sum of the dynamic energy of all its replicas. Because mapping decisions are based upon WCETs, the number of enrolled

processors does not depend upon actual execution times and the static energy is always the same for all scenarios, namely the length of the period times the sum of the static powers of the enrolled processors (see Equation (2)).

- To assess the absolute performance of the heuristics, we derive a lower-bound for the dynamic energy. This bound is based upon actual execution times but neglects scheduling constraints and assumes no overlap between any two task replicas, hence it is not reachable in general. However, we show that our best heuristics achieve performance close to this bound.

## 2.5 Complexity

The global optimization problem is obviously NP-hard, since it is a generalization of the makespan minimization problem with a fixed number of parallel processors [8]. The optimization of the sole scheduling phase is also NP-hard for identical processors: if the number of replicas has already been decided for each task, and if the assigned processor of each replica has also been decided, the scheduling phase aims at minimizing the expected energy consumption by avoiding overlap between the replicas of a same task [6]. In fact, the heterogeneity of the processors, both in terms of power cost and fault rate, dramatically complicates the problem. Proposition 1 shows that the problem remains NP-complete when mapping identical and deterministic tasks with same period onto processors with different fault-rates, even without any energy constraint; in other words, choosing the optimal subset of processors to execute each task and its replicas to match their reliability threshold is an intrinsically combinatorial problem! We formally state the corresponding decision problem:

**Definition 1** (RELIABILITY). Consider  $n$  identical and deterministic tasks  $\tau_i$  with same period and deadline  $p = 1$ , and same reliability threshold  $\mathcal{R}$ , to be executed on a platform composed of  $M$  heterogeneous processors. On processor  $m_k$ ,  $1 \leq k \leq M$ , each task  $\tau_i$  has execution time  $c_{i,k} = 1$  and reliability  $R(\tau_i, m_k) = R_k$ . Is it possible to map all tasks so as to match the reliability threshold for each task?

Because the execution time on each processor is always equal to the period/deadline, the hyperperiod of length  $L = 1$  is composed of a single instance of each task, and all replicas will necessarily be executed in parallel in the time interval  $[0, 1]$ ; hence each processor executes at most one replica.

**Proposition 1.** RELIABILITY is NP-Complete.

*Proof.* The problem is obviously in NP. For the completeness, we use a reduction from 3-PARTITION [8]. Consider an arbitrary instance  $\mathcal{I}_1$  of 3-PARTITION: given  $3m$  positive integers  $a_i$ ,  $1 \leq i \leq 3m$  with  $\frac{B}{4} < a_i < \frac{B}{2}$

and  $\sum_{i=1}^{3m} a_i = mB$ , can we find a partition of  $\{1, 2, \dots, 3m\}$  into  $m$  subsets  $I_j$  such that  $\sum_{i \in I_j} a_i = B$  for  $1 \leq j \leq m$ ? We build the following instance  $\mathcal{I}_2$  of RELIABILITY: we have  $M = 3m$  processors and  $n = m$  tasks. A replica on processor  $m_k$ ,  $1 \leq k \leq M$ , has reliability  $R_k = \frac{a_k}{A}$  where  $A = 6B^2 + 2B^3$ . Finally, the target reliability threshold for each task is  $\mathcal{R} = \frac{B}{A}(1 - 3\frac{B}{A})$ . The size of  $\mathcal{I}_2$  is clearly polynomial (and even linear) in the size of  $\mathcal{I}_1$ .

We now show that  $\mathcal{I}_2$  has a solution if and only if  $\mathcal{I}_1$  does. Assume first that  $\mathcal{I}_1$  has a solution and let  $I_j$  denote the  $m$  subsets of the partition. Note that each subset has exactly three elements because on the condition on the size of the  $a_i$ 's. For  $\mathcal{I}_2$ , we map task  $\tau_i$ ,  $1 \leq i \leq n = m$ , on the three processors whose index belongs to  $I_i$ . This is a valid mapping onto the  $M = 3m$  processors because the  $m$  subsets form a partition of  $\{1, 2, \dots, M\}$ . Each task has three replicas. Writing  $I_i = \{i_1, i_2, i_3\}$ , the reliability of  $\tau_i$  writes

$$\begin{aligned} \mathcal{R}_i &= 1 - \prod_{q=1}^3 (1 - R_{i_q}) \\ &= \frac{\sum_{q=1}^3 a_{i_q}}{A} - \frac{\sum_{1 \leq q < r \leq 3} a_{i_q} a_{i_r}}{A^2} + \frac{a_{i_1} a_{i_2} a_{i_3}}{A^3} \\ &\geq \frac{\sum_{q=1}^3 a_{i_q}}{A} - \frac{\sum_{1 \leq q < r \leq 3} a_{i_q} a_{i_r}}{A^2} \end{aligned} \quad (9)$$

Since  $\sum_{q=1}^3 a_{i_q} = B$  and  $a_i < \frac{B}{2}$  for all  $1 \leq i \leq 3m$ , we obtain

$$\mathcal{R}_i \geq \frac{B}{A} - \frac{3B^2}{4A^2} \geq \frac{B}{A} - \frac{3B^2}{A^2} = \mathcal{R}$$

hence  $\mathcal{I}_2$  has a solution.

Assume now that  $\mathcal{I}_2$  has a solution, and let  $I_i$  denote the indices of the processors executing replicas of task  $\tau_i$  for  $1 \leq i \leq n$ . The subsets  $I_i$  are pairwise disjoint, but we do not know whether they form a partition yet (some processors may have not been enrolled in the mapping). But assume that some  $I_i$  has only 1 element  $i_1$ : we get  $\mathcal{R}_i = \frac{a_{i_1}}{A} \geq \mathcal{R}$ . This leads to

$$a_{i_1} \geq B \left(1 - 3\frac{B}{A}\right) = B - \frac{3B^2}{A} \geq B - \frac{1}{2}$$

The last inequality comes from the definition of  $A$  which has been chosen large enough. But  $a_{i_1} < \frac{B}{2} < B$  implies  $a_{i_1} \leq B - 1$  since we have integers. Hence a contradiction. Similarly, if some  $I_i$  has only 2 elements  $i_1$  and  $i_2$ : we get

$$\mathcal{R}_i = \frac{a_{i_1} + a_{i_2}}{A} - \frac{a_{i_1} a_{i_2}}{A^2} \geq \mathcal{R}$$

hence  $\frac{a_{i_1} + a_{i_2}}{A} \geq \mathcal{R}$ . We conclude as before, because it implies that  $a_{i_1} + a_{i_2} \geq B - \frac{1}{2}$ , while  $a_{i_1} + a_{i_2} < 2\frac{B}{2} = B$  implies  $a_{i_1} + a_{i_2} \leq B - 1$ . As a consequence, each subset  $I_i$  has at least 3 elements. We do have a partition of  $\{1, 2, \dots, 3m\}$ , and each subset  $I_i$  has exactly 3 elements. Then, writing

$I_i = \{i_1, i_2, i_3\}$  as before, we have  $\mathcal{R}_i \geq \mathcal{R} = B(1 - 3\frac{B}{A})$  for all  $1 \leq i \leq m$ , where  $\mathcal{R}_i$  is given by Equation (9). We derive that

$$\begin{aligned} \frac{\sum_{q=1}^3 a_{i_q}}{A} &\geq \frac{B}{A} \left(1 - 3\frac{B}{A}\right) + \frac{\sum_{1 \leq q < r \leq 3} a_{i_q} a_{i_r}}{A^2} - \frac{a_{i_1} a_{i_2} a_{i_3}}{A^3} \\ &\geq \frac{B}{A} \left(1 - 3\frac{B}{A}\right) - \frac{a_{i_1} a_{i_2} a_{i_3}}{A^3} \end{aligned}$$

As  $a_i < \frac{B}{2}$  for  $1 \leq i \leq 3m$ , we can deduce:

$$\frac{\sum_{q=1}^3 a_{i_q}}{A} \geq \frac{B}{A} \left(1 - 3\frac{B}{A}\right) - \frac{B^3}{8A^3}$$

Recall that  $A = 6B^2 + 2B^3$  and  $B$  is positive integer. Hence,

$$\begin{aligned} \sum_{q=1}^3 a_{i_q} &\geq B \left(1 - 3\frac{B}{A}\right) - \frac{B^3}{8A^2} \geq B \left(1 - 3\frac{B}{A}\right) - \frac{B^3}{A} \\ &\geq B - \frac{3B^2 + B^3}{A} \geq B - \frac{1}{2} \end{aligned}$$

Then  $\sum_{q=1}^3 a_{i_q} \geq B$  because we have integers. But the sum of the  $3m$  integers  $a_i$  is  $mB$ , hence all the sums are indeed equal to  $B$ , and we have a solution to  $\mathcal{I}_1$ . This concludes the proof.  $\square$

### 3 Mapping

In the mapping phase, we need to define the number of replicas for each task instance, as well as the execution processor for each replica, aiming at meeting the reliability target and deadline, while minimizing the energy cost. As hyperperiods are repetitive, we aim at finding a feasible mapping within one hyperperiod (time length  $L$ ). One difficulty introduced by platform heterogeneity is that we do not know the number of replicas needed for each task instance to reach its reliability threshold, before completing the mapping process, because different processors have different failure rates and speeds and, hence, they provide different reliabilities for each replica. Therefore, the simpler three-step method of [3, 6] cannot be applied.

Since the estimation of dynamic energy consumption ( $E_d(\tau_{i,j}, m_k)^{estimated}$ ) and reliability ( $R(\tau_{i,j}, m_k)$ ) of task instance  $\tau_{i,j}$  on processor  $m_k$  do not depend on the index of instance  $j$ , these two values will be the same for all instances of the same task. Thus, in the following description, instead of making decision for each task instance  $\tau_{i,j}$ , we will use  $E_d(\tau_i, m_k)$  and  $R(\tau_i, m_k)$  as the estimated dynamic energy consumption and reliability on processor  $m_k$ , for all instances of task  $\tau_i$ , and we will make the same mapping decision for all instances of the same task.

Table 2: Example

$m_k$	$E(\tau_i, m_k)$	$R(\tau_i, m_k)$	$\frac{R(\tau_i, m_k)}{E(\tau_i, m_k)}$	$-\frac{\log_{10}(1-R(\tau_i, m_k))}{E(\tau_i, m_k)}$
1	1	0.9	0.9	1
2	1	0.9	0.9	1
3	2	0.99	0.495	1
4	1	0.99	0.99	2
5	2	0.9	0.45	0.5

When mapping all the  $\frac{L}{P_i}$  instances of a given task on a processor, we use the standard *Earliest Deadline First (EDF)* scheduling heuristic [9]. EDF tells us that a given processor is a fit for that replica if and only if the utilization of that processor does not exceed 1. Recall that the utilization of a processor is the sum of the utilizations of all task instances assigned to it.

As shown in Algorithm 1, given a set of tasks with their periods and reliability targets and a set of heterogeneous processors, we first order the tasks according to TASKMAPCRITERION, which can be either:

- *deW (inW)*: decreasing (increasing) average work size  $\bar{c}_i = \frac{c_{i,1} + c_{i,2} + \dots + c_{i,M}}{M}$ ;
- *deMinW (inMinW)*: decreasing (increasing) minimum work size  $\bar{c}_i = \min_{1 \leq k \leq M} c_{i,k}$ ;
- *deMaxW (inMaxW)*: decreasing (increasing) maximum work size  $\bar{c}_i = \max_{1 \leq k \leq M} c_{i,k}$ ;
- *random*: random ordering.

Then, for each task in the ordered list, we order the processors for mapping its replicas according to PROCMAPCRITERION, which can be either:

- *inE*: increasing energy cost;
- *deR*: decreasing reliability;
- *deP*: decreasing ratio of  $-\frac{\log_{10}(1-R(\tau_i, m_k))}{E(\tau_i, m_k)}$  (explained below);
- *random*: random ordering.

We use the example shown in Table 2 to explain how to design a better criterion in PROCMAPCRITERION. Assume there are five processors with different energy and reliability configurations (the first two processors are identical). Considering only the reliability, we cannot distinguish between the third and fourth processors. Apparently, the fourth processor is better since it consumes less energy and provide the same level of reliability. The problem is the same when ordering processors only according to energy cost.



This gives us a hint that we need to consider energy and reliability interactively. A first idea would be to use the ratio  $\frac{R(\tau_i, m_k)}{E(\tau_i, m_k)}$ , which expresses the reliability per energy unit of every instance of task  $\tau_i$  executing on processor  $m_k$ . But consider a task instance with a reliability target  $\mathcal{R}_i = 0.98$ : it requires either the third processor alone or the first two processors together. Both solutions match the reliability goal with the same energy cost 4. We aim at a formula that would give the same weight to both solutions. The ratio  $-\frac{\log_{10}(1-R(\tau_i, m_k))}{E(\tau_i, m_k)}$  is a good candidate, because the total energy cost is the sum of all processors while the reliability is a product. This discussion explains how we have derived the third criterion *deP* in PROCMAPCRITERION, namely to order processors by decreasing ratio of  $-\frac{\log_{10}(1-R(\tau_i, m_k))}{E(\tau_i, m_k)}$ .

For the mapping phase, for task  $\tau_i$ , we add replicas for all its instances  $\tau_{i,j}$  in the order of the processor list until the reliability target  $\mathcal{R}_i$  of each instance is reached. The algorithm uses the probability of failure  $PoF = 1 - R(\tau_i) = \prod_{k \in alloc(i)} (1 - R(\tau_i, m_k))$  (Equation (1)). The mapping process always ensures that: (i) no two replicas of the same task are assigned to the same processor; (ii) the utilization  $u_k$  of each processor does not exceed 1.

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**Algorithm 1:** Replication setting and mapping
 

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**Input:** A set of tasks  $\tau_i$  with reliability targets  $\mathcal{R}_i$  and periods  $p_i$ ; a set of heterogeneous processors  $m_k$

**Output:** An allocation  $\sigma_m$  of all replicas on the processors

```

1 begin
2   order all the tasks with TASKMAPCRITERION and renumber them  $\tau_1, \dots, \tau_N$ 
3   /* initialize the utilization of all processors to zero */
4    $u \leftarrow [0, \dots, 0]$ 
5   /* iterate through the ordered list of tasks */
6   for  $i \in [1, \dots, N]$  do
7     /* order processors for each task */
8     order all processors for task  $\tau_i$  with PROCMAPCRITERION and renumber
9     them  $proc_1, \dots, proc_M$ 
10    /* this ordered list may differ from task to task */
11     $k = 1$ 
12     $PoF = 1$ 
13    while  $1 - PoF < \mathcal{R}_i$  do
14      /*  $u_{temp}$  is the utilization of processor  $m_k$  after adding a
15      replica of task  $\tau_i$  */
16       $u_{temp} = u_k + u_{i,k}$ 
17      if  $u_{temp} \leq 1$  then
18         $u_k = u_{temp}$ 
19         $PoF = PoF \times (1 - R(\tau_i, m_k))$ 
20        for all instances of  $\tau_i$ , add a replica on  $proc_k$ 
21       $k++$ 
22      if  $k > m$  then
23        return not feasible
24  return  $\sigma_m$ 

```

---

## 4 Scheduling

In the scheduling phase, we aim at ordering the tasks mapped on each processor, with the objective to further minimize the energy consumption during execution. Recall that the success of any replica leads to the immediate cancellation of all the remaining replicas, a crucial source of energy saving. Thus, our approaches identify a primary replica for each task instance, then all other replicas for that instance become secondaries. The goal of the proposed scheduling heuristics is to avoid overlaps between the execution of the primary and secondary replicas of each task instance: the primary must be terminated as soon as possible, while the secondaries must be delayed as much as possible.

After the mapping phase, we have a static schedule on each processor, also called the *canonical* schedule, which is based upon the WCET of each task and EDF. The EDF schedule can use preemption, so that a given task instance may be split into several chunks. As a result, from the canonical schedule, each processor has an ordered list made up with all task instance chunks that it has to execute during the hyperperiod, together with their starting and finish times. As the actual execution time of a real-time task instance will be shorter than its WCET, the canonical schedule will never be executed exactly as such. Still, it can be used as the baseline to compute the maximum delay for secondary replicas without missing any deadline.

To determine the primary replica for each task instance, we could make the decision offline or online. The offline strategy means that we use pre-knowledge before real execution, where we consider the following criteria:

- EDF\_WCET: choose the processor that can execute the replica the fastest;
- EDF\_ENERGY: choose the processor that can execute the replica with the minimum dynamic energy consumption;
- EDF\_RELIABILITY: choose the processor that can execute the replica the most reliably.

Note that for offline strategies, the primary replicas for different task instances of the same task is on the same processor. After determining the primary replicas, we deploy the following techniques based on the canonical schedule to differentiate the primary replica and the secondary replicas:

- Scale the WCETs of all tasks by  $\frac{1}{\alpha}$ , where  $\alpha$  is the utilization, which also gives a valid canonical schedule, but with longer worst case expected execution times for all tasks, which we call the scaled canonical schedule as shown in Figure 1. This gives a better reference to further delay the start time of secondary replicas. Here is why: at the mapping phase, as long as the total utilization of replicas that are mapped onto

the processor is less than or equal to one, then we are able to find a valid scheduling using EDF. Assume we have mapped three tasks  $t_i, t_j, t_k$  onto processor  $p$ , and that the utilization is  $\alpha = \frac{c_{i,p}}{p_i} + \frac{c_{j,p}}{p_j} + \frac{c_{k,p}}{p_k}$ . Either we keep the mapping and have a fraction  $1 - \alpha$  of the interval where  $p$  is idle, or we artificially slow down the execution times of all three tasks by a factor  $\alpha$ , then we will have a new utilization  $\beta = \frac{c_{i,p}}{p_i\alpha} + \frac{c_{j,p}}{p_j\alpha} + \frac{c_{k,p}}{p_k\alpha} = 1$ , which also gives us a feasible mapping without any idle time. Then we could delay the start time of secondary replicas without conflict timeliness as long as they will finish executions not later than their finish times in the scaled canonical schedule (Figure 2).

- Consider a *scheduling interval* defined by two consecutive deadlines in the canonical schedule. Inside the interval, task chunks to be executed are ordered by the EDF policy. We observe that we can freely reorder the chunks without missing any deadline, by definition of an interval. It means that in each interval, we could reorder to execute all primary replicas first, and then secondary replicas (Figure 3).
- Since we have delayed the start time of secondary replicas, there are some idle slots in the schedule. We could take advantage of these idle slots by pre-fetching other primary replica chunks in the availability list: those primaries that have been released but which were scheduled later because they have lower EDF priority than the current secondary replicas.

Based on the above ideas, considering one interval in the improved static schedule, it: 1) starts with the primary replica chunks; 2) fills in the idle slot by inserting other primary replica chunks that are available; 3) ends by the secondary replica chunks. Then, during the real execution, according to the failure case, the scheduler will dynamically cancel the execution of secondaries if the primary replica finished successfully. If the replica selected as primary is the least expensive energy-wise (like the one selected by EDF\_ENERGY), and if no secondary replica has started executing yet, then the energy consumption will be minimal for that task. However, because the choice of the primary replica is highly dependent on the characteristics of processors, different tasks may have their primary replicas on the same processor, which would lead to load imbalance and heavy overlaps. To avoid this situation, we design another type of method to choose primary replicas on-the-fly:

- EDF\_START\_TIME: choose the processor that can start the execution of the replica the earliest (given already made scheduling decisions).

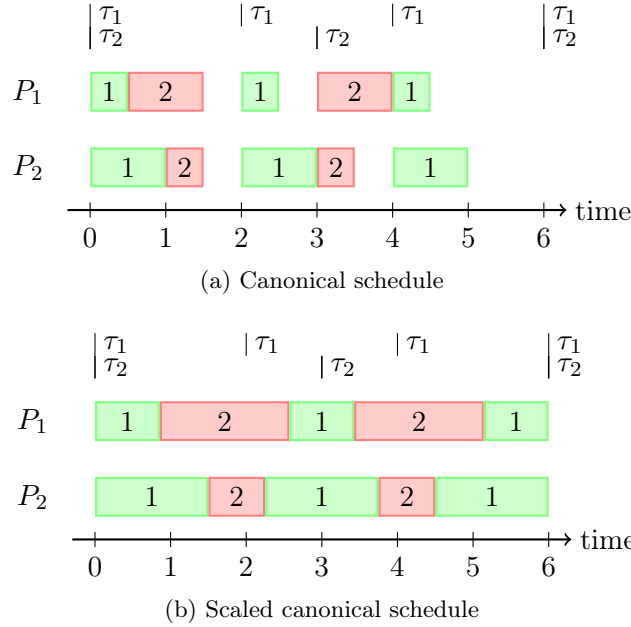


Figure 1: Example of scaling WCETs with two tasks  $\tau_1$  and  $\tau_2$  ( $c_{1,1} = c_{2,2} = 0.5$ ,  $c_{1,2} = c_{2,1} = 1$ ,  $p_1 = 2$  and  $p_2 = 3$ ) on each processor with total utilization  $u_1 = \frac{7}{12}$  and  $u_2 = \frac{4}{6}$ .

This online strategy dynamically chooses the replica starting the earliest as the primary replica for each task instance. This tends to spread primaries onto different processors, which gives more slack time on each processor to reduce overlaps. As a side note, it will possibly be the case that two different instances of the same task have not the same primary processor. Then all other replicas of the same task become secondaries and are delayed according to the scaled canonical schedule, as shown in Figure 2. We could not implement the “reordering inside intervals” shown in Figure 3 for EDF\_START\_TIME, which means that the end time of secondaries will not exceed the one planned in the scaled canonical schedule, as this is an offline improvement that requires the primary replica known in advance. But the online strategy has two major advantages compared to the offline strategies: (1) as long as we finish one replica successfully, we can safely cancel other replicas of the same task instance earlier than in offline schedules, which gives more flexibility to adjust the schedule afterwards; (2) the static schedules have to reserve time slots for the secondaries that correspond to their WCET (it is impossible to know their actual execution times before execution). Since their actual execution times are usually shorter, this dynamically frees some time slots that the online schedule uses to prefetch available primary replica chunks.

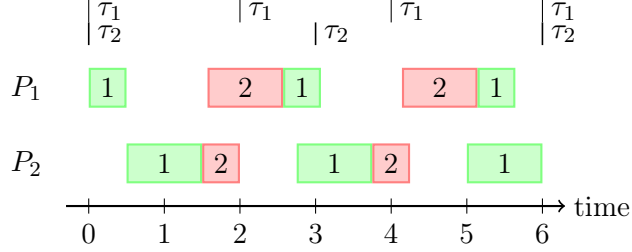


Figure 2: Static schedule when prioritizing primaries while delaying secondaries according to their finish times in the Scaled canonical schedule.

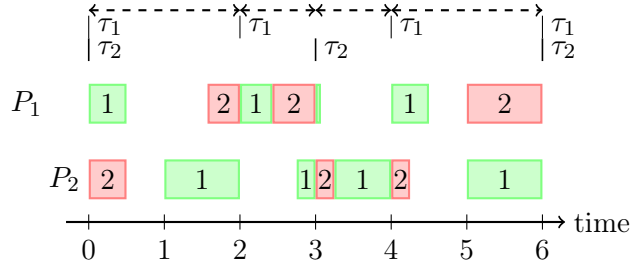


Figure 3: Reordering chunks freely inside intervals.

## 5 Lower bound

In this section, we explain how to derive a lower-bound for the expected energy consumption of a solution to the optimization problem, namely a mapping/scheduling heuristic that uses some of the selection criteria outlined in Sections 3 and 4.

For each problem input, namely  $N$  tasks  $\tau_i$  with reliability thresholds  $\mathcal{R}_i$ ,  $M$  processors  $m_k$  with failure rates  $\lambda_k$ , and with all WCET  $c_{i,k}$ , we compute a solution, i.e., a mapping and scheduling of all replicas. We first use Monte-Carlo simulations (see Section 6) and generate several sets of values for the actual execution time  $w_{i,j,k}$  of task instance  $\tau_{i,j}$  on processor  $m_k$ . The values  $w_{i,j,k}$  are drawn *uniformly across processors* as some fraction of their WCET  $c_{i,k}$  (refer to Section 6.1 for details).

Now, for each set of values  $w_{i,j,k}$ , we generate a set of failure scenarios, compute the actual energy consumed for each scenario, and report the average of all these values as the expected energy consumption. A failure scenario operates as follows. We call an event the end of the execution of a replica on some processor. At each event, we flip a biased coin (weighted with the probability of success of the replica on that processor) to decide whether the replica is successful or not. If it is, we delete all other replicas of the same task instance. At the end of the execution, we record all the dynamic energy that has been actually spent, accounting for all complete

and partial executions of replicas, and we add the static energy given by Equation (2). This leads to the energy consumption of the failure scenario. We average the values over all failure scenarios and obtain the expectation, denoted as  $E(\{w_{i,j,k}\})$ .

In addition, we also compute a lower-bound  $LB(\{w_{i,j,k}\})$  as follows. Our goal is to accurately estimate the energy consumption of an optimal solution. Since the static energy depends upon the subset of processors that are used in the solution (see Equation (2)), we need to try all possible subsets. Given a processor subset  $\mathcal{S}$ , we consider each task instance  $\tau_{i,j}$  independently, and try all possible mappings of replicas of  $\tau_{i,j}$  using only processors in  $\mathcal{S}$ . Thus we explore all subsets  $\mathcal{T}$  of  $\mathcal{S}$ . A subset  $\mathcal{T}$  is *safe* if mapping a replica of  $\tau_{i,j}$  on each processor of  $\mathcal{T}$  meets the reliability criterion  $\mathcal{R}_i$ , and if no strict subset of  $\mathcal{T}$  is safe. Note that safe sets are determined using the WCETs  $c_{i,k}$ , and not using the  $w_{i,j,k}$ , because of the problem specification. Now for each safe subset  $\mathcal{T}$ , we try all possible orderings (there are  $k!$  of them if  $|\mathcal{T}| = k$ ); for each ordering, we compute the expected value of the dynamic energy consumption as follows: if, say,  $\mathcal{T} = \{m_1, m_3, m_4\}$  and the ordering is  $m_3, m_4, m_1$ , then we compute

$$P_{3,d}w_{i,j,3} + (1 - e^{-\lambda_3 w_{i,j,3}})P_{4,d}w_{i,j,4} \\ + (1 - e^{-\lambda_3 w_{i,j,3}})(1 - e^{-\lambda_4 w_{i,j,4}})P_{1,d}w_{i,j,1}.$$

We see that we optimistically assume no overlap between the three replicas, and compute the dynamic energy cost as the energy of the first replica (always spent) plus the energy of the second replica (paid only if the first replica has failed) plus the energy of the third replica (paid only if both the first and second replicas have failed), and so on. Note that here we use execution times and failure probabilities based upon the actual execution times  $w_{i,j,k}$  and not upon the WCETs  $c_{i,k}$ . The value of the sum depends upon the ordering of the processors in  $\mathcal{T}$ , hence we check the 6 orderings and retain the minimal value. We do this for all safe subsets and retain the minimal value. Finally we sum the results obtained for each task instance and get the lower-bound for the original processor subset  $\mathcal{S}$ . We stress that this bound is not necessarily tight, because our computation assumes no overlap for any replica pair, and does not check the utilization of each processor (which may exceed 1). The final lower-bound  $LB(\{w_{i,j,k}\})$  is the minimum over all processor subsets. Although the computation has exponential cost, due to the exploration of all processor subsets  $\mathcal{S}$ , the computation of the expected energy for a given ordering in a subset  $\mathcal{T}$  of  $\mathcal{S}$  obeys a closed-form formula.

## 6 Performance evaluation

This section assesses the performance of our different strategies to map and schedule real-time tasks onto heterogeneous platforms. In Section 6.1, we

describe the parameters and settings used during the experimental campaign. We present the results in Section 6.2. The algorithms are implemented in C++ and in R. The related computing code and experimental data are publicly available in [10]. The comprehensive set of results can be found in the appendix of the report.

## 6.1 Experimental methodology

In the experiments, we have  $M = 10$  processors and  $N = 20$  tasks. The hyperperiod  $L$  of the system is fixed at 300. The task instances  $\tau_{i,j}$  are arrived every  $p_i$ , chosen between one of the following values: 20, 30, 50, 60, 100, 150. The set of WCETs is generated by the method proposed in [7], as mentioned in Section 2.1. The WCET values are controlled by the correlation factor between the different tasks ( $cor_{\text{task}}$ ) and between the different processors ( $cor_{\text{proc}}$ ). For example,  $cor_{\text{task}} = 0$  (resp.  $cor_{\text{proc}} = 0$ ) means that the WCET values between different tasks on one processor (resp. between different processors for one task) are completely randomly generated. Inversely,  $cor_{\text{task}} = 1$  (resp.  $cor_{\text{proc}} = 1$ ) means that the WCET values between different tasks on one processor (resp. between different processors for one task) are all the same. We vary these two parameters between 0.25 and 0.75 to visualize the result under different correlation conditions, while guaranteeing a certain degree of randomness. We also define a parameter *basicWork* as the estimated total utilization of the system with a single replica per task instance, in order to study the impact of system workload pressure:

$$basicWork = \frac{\sum_{i,k} u_{i,k}}{M^2} \quad (10)$$

In Equation (10), we use the average utilization on the  $M$  processors ( $\frac{\sum_k u_{i,k}}{M}$ ) to estimate the pressure that one replica of task  $\tau_i$  can give on the system. We sum up the average utilization of all  $N$  tasks, and we divide this value by  $M$  because  $M$  processors are available. Hence, *basicWork* represents an estimation of the fraction of time that processors are used if each task has a single replica. In the experiments, we vary *basicWork* from 0.1 to 0.3.

To generate the actual execution times for each task instance from the task WCETs, we use two parameters. The first one,  $\beta_{b/w}$ , is global to all tasks:  $\beta_{b/w}$  is the ratio between the best-case execution time and the worst-case execution time. It is the smallest possible ratio between the actual execution time of a task instance and the WCET of that task. Therefore, the actual execution time of all instances of task  $\tau_i$  on processor  $m_k$  belongs to  $[\beta_{b/w}c_{i,k}, c_{i,k}]$ . We consider five possible values of  $\beta_{b/w}$ : 0.2, 0.4, 0.6, 0.8, and 1. The second parameter,  $\beta_{i,j}$ , is task instance dependent:  $\beta_{i,j}$  describes whether the task instance  $\tau_{i,j}$  is a small one or a large one.  $\beta_{i,j}$  is randomly drawn in  $[0, 1]$ .  $\beta_{i,j} = 0$  means that task instance  $\tau_{i,j}$  has the shortest possible execution time, and  $\beta_{i,j} = 1$  means that the actual execution is equal to its

worst case execution time. Overall, the actual execution time of task instance  $\tau_{i,j}$  on processor  $m_k$  is thus defined as:  $w_{i,j,k} = (\beta_{b/w} + (1 - \beta_{b/w})\beta_{i,j})c_{i,k}$ .

For a processor  $m_k$  in the platform, we fix its static power  $P_{k,s}$  at 0.001 as in the literature [11, 12, 13]. For the dynamic power and the failure rate, we have two sets of parameters. The first parameter set also follows values from previous work [11, 12, 13]. For this set, we have a relatively high dynamic power and very low failure rate. Therefore, the replicas using this first set of parameters succeed in almost all cases. Then, to evaluate our heuristics in the context when failures occur more frequently, we introduce a second set of parameters where the replicas have lower dynamic power and relatively high failure rate. For the first set, we choose randomly the dynamic power  $P_{k,d}$  between 0.8 and 1.2, and the failure rate  $\lambda_k$  between 0.0001 and 0.00023. And for the second set, we have  $P_{k,d}$  10 times smaller (between 0.08 and 0.12), and  $\lambda_k$  100 times larger (between 0.01 and 0.023). With the second set of parameters, the actual reliability of one replica ranges from 0.1 to 0.99. To be more realistic, in our experiments processors with a larger dynamic power  $P_{k,d}$  have a smaller failure rate  $\lambda_k$ . It means that a more reliable processor costs more energy than a less reliable one. We guarantee this by ordering inversely the  $P_{k,d}$ 's and the  $\lambda_k$ 's after generating the random values.

Due to space limitations, and because the general trends about the relative performance of heuristics is the same in both sets, we only include here figures about the large failure rate set; figures for the small failure rate set can be found in the appendix.

We vary the local reliability target  $\mathcal{R}_i$  between 0.8 and 0.95. for the big failure rate set and between 0.9 and 0.98 for the small failure rate set. This is to give the system a reasonable freedom while mapping and scheduling. The reliability target is relatively high, implying that tasks need several replicas to reach it. But it is chosen low enough so that feasible mappings can be found in the vast majority of scenarios.

## 6.2 Results

In this section, we compare the performance of the different criteria presented in Sections 3 and 4, and we choose the criterion which performs the best. Next, we analyze the impact of the different parameters on the performance of the chosen criterion.

Due to lack of space, we choose only to present a representative subset of the results. The comprehensive set of all results is available in the appendix. We choose as default values  $\beta_{b/w} = 1$ ,  $basicWork = 0.3$ ,  $\mathcal{R}_i = 0.95$ . This set of parameters is chosen to observe results when the platform is under maximum pressure. We fix  $cor_{task}$  and  $cor_{proc}$  both at 0.5 as default values.

Each experiment is an average of 25 WCETs sets generated as follows. We first generate 5 sets of periods. For each of these sets of periods, 5 WCET matrices are generated. For each WCET matrix, we generate 10 sets



of random  $P_{k,d}$  and  $\lambda_k$  values. For each  $P_{k,d}$  and  $\lambda_k$  generated, the final result is the average over 10 executions. Overall, we run 2,500 randomly generated experiments for each set of  $\beta_{b/w}$ , *basicWork*,  $\mathcal{R}_i$ ,  $cor_{task}$  and  $cor_{proc}$  values. The total number of experiments ran is 2,700,000 for each heuristic.

Each result point is represented as the percentage of energy saved over the random baseline method (defined below). Hence, on all the figures and in all the tables, the higher, the better. The random baseline method is defined as follows: for each task, we add replicas randomly on available processors until reaching the task reliability target during the mapping phase; for scheduling, we randomly order replicas mapped on each processor and execute them in sequence and as soon as possible. We compare our different strategies to the lower-bound proposed in Section 5. During the scheduling phase, we add several other references called EDF\_PLAIN, EDF\_REF\_PAPER, and SMALLEST. They all use the same mapping as our heuristics, but have different scheduling strategies: EDF\_PLAIN is simply the plain EDF heuristic. EDF\_REF\_PAPER is the heuristic proposed in [3]. SMALLEST considers a single replica for each task instance, the replica which costs the least energy (hence, SMALLEST is not always a valid heuristic as it may not satisfy the reliability threshold). In fact, SMALLEST shows an ideal case for the scheduling phase: every instance of every task successfully executes one replica, the replica which costs the least energy, while all other replicas are not yet started and are canceled after the first replica is completed. In contrast, LOWERBOUND calculates the lowest expected energy consumption without considering the load of processors. Thus, we can find in the experimental results that SMALLEST can have a better result than LOWERBOUND in some cases.

For the 2,500 random trials for each setting, in the figures, we report the average number of replicas needed in total for the 20 tasks (on the left side), the average number of failures that occur per time unit (in the middle), and the average percentage of random trials in which we find feasible mapping (on the right side). These numbers are reported in black above the horizontal axis on each figure.

### 6.2.1 Task ordering criteria for the mapping phase

In Table 3, we summarize the performance of different task ordering criteria during the mapping phase for different  $cor_{task}$  and  $cor_{proc}$  parameter sets. We see that, for each  $(cor_{task}, cor_{proc})$  group, the difference between the different task ordering criteria, including random, is not obvious. The difference is at most 0.7% between the best and the worst criteria for any given set of  $(cor_{task}, cor_{proc})$  values. We conclude that these criteria do not critically influence energy consumption. Therefore, in the following, for ordering tasks, we will only consider *deMinW*, which performs globally slightly better than others. We now focus on how to select processors during the mapping phase,

	<i>random</i>	<i>deW</i>	<i>inW</i>	<i>deMinW</i>	<i>deMaxW</i>	<i>inMinW</i>	<i>inMaxW</i>
$cor_{task} = 0.25, cor_{proc} = 0.25$	54.8%	54.7%	55.1%	55.0%	54.7%	54.8%	55.1%
$cor_{task} = 0.25, cor_{proc} = 0.50$	47.6%	47.5%	48.0%	47.6%	47.5%	48.0%	48.0%
$cor_{task} = 0.25, cor_{proc} = 0.75$	33.5%	33.8%	33.7%	33.8%	33.8%	33.7%	33.7%
$cor_{task} = 0.50, cor_{proc} = 0.25$	55.1%	55.4%	55.1%	55.4%	55.4%	55.1%	55.1%
$cor_{task} = 0.50, cor_{proc} = 0.50$	48.6%	48.7%	48.8%	48.7%	48.7%	48.8%	48.8%
$cor_{task} = 0.50, cor_{proc} = 0.75$	33.4%	33.6%	33.6%	33.6%	33.6%	33.6%	33.6%
$cor_{task} = 0.75, cor_{proc} = 0.25$	54.6%	54.9%	54.4%	54.9%	54.8%	54.4%	54.4%
$cor_{task} = 0.75, cor_{proc} = 0.50$	44.2%	44.4%	44.3%	44.3%	44.4%	44.3%	44.3%
$cor_{task} = 0.75, cor_{proc} = 0.75$	32.9%	33.2%	33.2%	33.2%	33.1%	33.2%	33.2%

Table 3: Percentage of energy saved over the baseline when using different task ordering heuristics during the mapping phase under big failure rate, with  $basicWork = 0.3$ ,  $\beta_{b/w} = 1$ , and  $\mathcal{R}_i = 0.95$ .

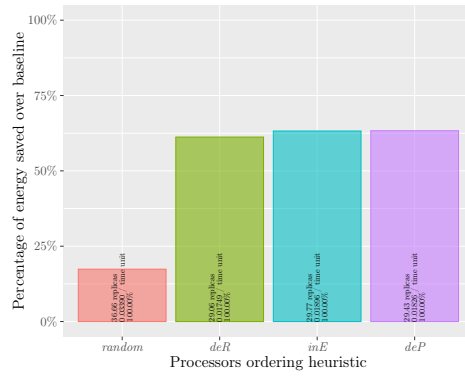


Figure 4: Percentage of energy saved over the baseline when using  $deMinW$  and different processor ordering heuristics during the mapping phase under big failure rate, with  $cor_{proc} = 0.5$ ,  $cor_{task} = 0.5$ ,  $basicWork = 0.3$ ,  $\beta_{b/w} = 1$ , and  $\mathcal{R}_i = 0.95$ .

and on the scheduling strategies.

### 6.2.2 Processor ordering criteria for the mapping phase

Figure 4 shows the performance of different processor ordering criteria. We see from this figure that our criteria perform globally well. Our heuristics, random excepted, can save around 60% of energy compared to the baseline scheme. Among the different criteria,  $inE$  and  $deP$  have similar performance, and this performance is better than that of  $deR$ . In the following, we will consider  $deP$  as the default criterion for ordering processors.

### 6.2.3 Scheduling heuristics

For the scheduling strategies, Figure 5 shows that, after carefully selecting the mapping criterion, our heuristics can save up to 66.9% of the energy

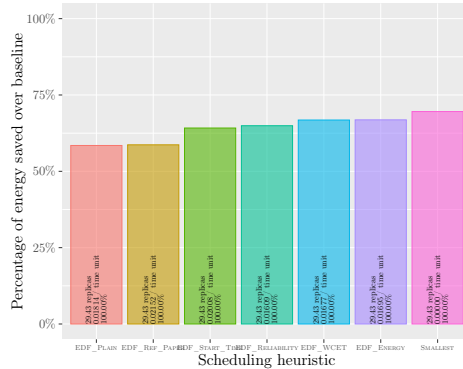


Figure 5: Percentage of energy saved over the baseline when using the *deMinW* and *deP* heuristics during the mapping phase and different criteria during the scheduling phase under big failure rate, with  $cor_{proc} = 0.5$ ,  $cor_{task} = 0.5$ ,  $basicWork = 0.3$ ,  $\beta_{b/w} = 1$ , and  $\mathcal{R}_i = 0.95$ .

when compared to the random baseline. Among the different heuristics, we find that the EDF\_START\_TIME heuristic, which chooses the earliest replica as primary, and which has an energy saving performance of 64.2%, is the worst of our original heuristics. On the contrary, EDF\_ENERGY performs the best (with a performance of 66.9%). It saves 8.4% more energy than EDF\_PLAIN, and the performance is only 2.7% worse than SMALLEST. Recall that SMALLEST sums up, for each task instance, only one replica which costs the least energy, without considering failures or replica overlapping. It is very likely that this bound is unreachable, especially when the system is under high pressure. EDF\_WCET performs similarly to EDF\_ENERGY, and EDF\_RELIABILITY performs slightly worse. In fact, as the energy consumption of a replica depends a lot on its WCET, it is reasonable that EDF\_WCET and EDF\_ENERGY tend to choose the same primary replica and have similar performance. This result shows that it is better to choose a replica which costs less energy as primary. In contrast, the earliest starting replica may not lead to minimal energy consumption, although we could have expected that it would induce less overlap among replicas.

In the following paragraphs, we only consider EDF\_ENERGY and we focus on the variation of its performance for the different parameters. We add EDF\_PLAIN, EDF\_START\_TIME, and SMALLEST as references.

#### 6.2.4 Task and processor correlation

Figures 6 and 7 show results when task correlation or processor correlation varies. We see that, when  $cor_{task}$  and  $cor_{proc}$  are not high ( $< 0.75$ ), the performance remains relatively stable. The EDF\_ENERGY strategy achieves an energy saving percentage over the random baseline of around 65%. This

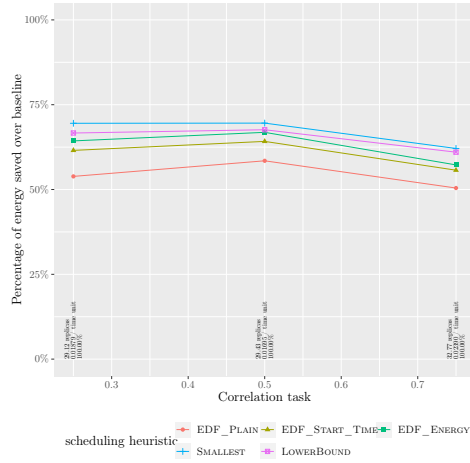


Figure 6: Percentage of energy saved over the baseline when using *deMinW* and *deP* during mapping for different scheduling criteria, under big failure rate, when varying  $cor_{task}$ , with  $cor_{proc} = 0.5$ ,  $basicWork = 0.3$ ,  $\beta_{b/w} = 1$ , and  $\mathcal{R}_i = 0.95$ .

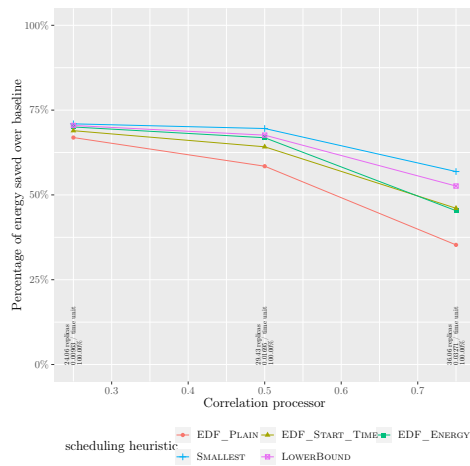


Figure 7: Percentage of energy saved over the baseline when using *deMinW* and *deP* during mapping for different scheduling criteria, under big failure rate, when varying  $cor_{proc}$ , with  $cor_{task} = 0.5$ ,  $basicWork = 0.3$ ,  $\beta_{b/w} = 1$ , and  $\mathcal{R}_i = 0.95$ .

result is close to the SMALLEST and LOWERBOUND references. When  $cor_{task}$  and  $cor_{proc}$  is high, we can observe that the performance decreases. In the case of  $cor_{task} = 0.75$  or  $cor_{proc} = 0.75$ , the performance of EDF\_ENERGY decreases respectively to 57.3% and 45.4%, but the difference with LOWERBOUND remains small (3.8% and 7.2% respectively).

The figures also show that the performance of EDF\_START\_TIME is closer to that of EDF\_ENERGY when  $cor_{task}$  or  $cor_{proc}$  is high. The reason is that, with a higher  $cor_{task}$  or  $cor_{proc}$ , the WCETs of different tasks on the same processor, or the WCETs of the same task on different processors, are more similar. In both cases, the orders of the processors for different tasks become more similar, and are more closely related to the power and/or the reliability parameters ( $P_{k,d}$  and  $\lambda_k$ ). Therefore, after the mapping phase, we have a few fully used processors, while the other processors have a low load. The overlapping increases, and the performance becomes worse. Inversely, when  $cor_{task}$  or  $cor_{proc}$  is not high, if we simply choose the replica which costs the least energy as the primary replica, the primary replicas are more randomly distributed on the different processors because of the high randomness of WCETs, and the overlapping is minimal. Then it is possible that we finish the primary early and with success, and delete all secondary replicas before starting the processing of any of them. This is why, when  $cor_{task}$  or  $cor_{proc}$  is high, the EDF\_ENERGY strategy cannot save as much energy as in other cases.

### 6.2.5 Task variability

Figure 8 presents the results when  $\beta_{b/w}$  varies. We observe that the result of each heuristic is (almost) independent of the value of  $\beta_{b/w}$ . This is because we map and schedule tasks based on their WCETs, so the mapping and scheduling results are independent of the value of  $\beta_{b/w}$ . Furthermore, each task  $\tau_i$  has the same  $\beta_i$  on the different processors. Therefore the energy savings tend to be similar. More precisely, the performance of EDF\_ENERGY varies only for 0.5% when  $\beta_{b/w}$  increases from 0.2 to 1. So we can conclude that the result is independent of  $\beta_{b/w}$ .

When  $\beta_{b/w}$  is small, actual execution times can greatly differ from the WCETs which are used for mapping and scheduling. However, in this case, the heuristics perform as well as in the case  $\beta_{b/w} = 1$ . This shows that the heuristics are very robust.

### 6.2.6 Utilization and reliability threshold

On Figure 9, we observe the performance of the different heuristics when varying both *basicWork* and  $\mathcal{R}_i$ . We see that, when *basicWork* and  $\mathcal{R}_i$  increase, the differences between heuristics become larger. EDF\_PLAIN performs worse, while the performance of EDF\_ENERGY remains stable. The

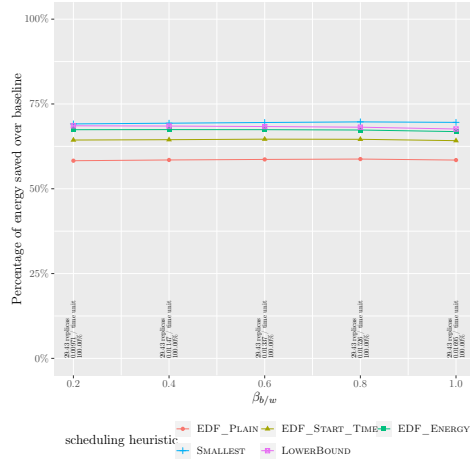


Figure 8: Percentage of energy saved over the baseline when using *deMinW* and *deP* during mapping for different scheduling criteria, under big failure rate, when varying  $\beta_{b/w}$ , with  $cor_{task} = 0.5$ ,  $cor_{proc} = 0.5$ ,  $basicWork = 0.3$ , and  $\mathcal{R}_i = 0.95$ .

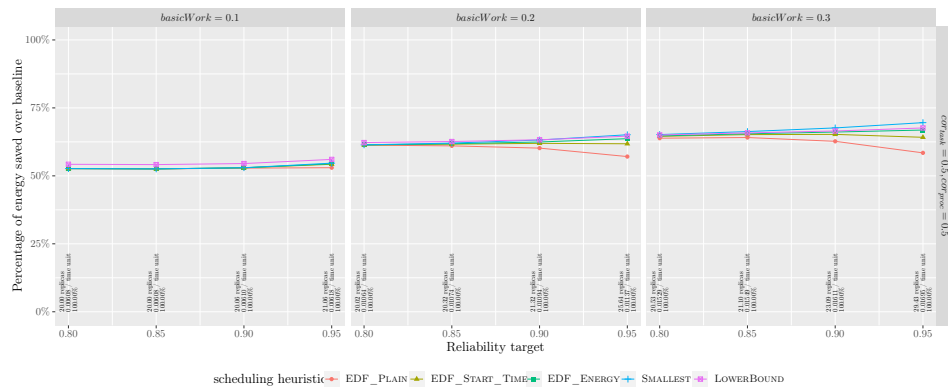


Figure 9: Percentage of energy saved over the baseline when using *deMinW* and *deP* during mapping for different scheduling criteria, under big failure rate, when varying  $basicWork$  and  $\mathcal{R}_i$ , with  $cor_{task} = 0.5$ ,  $cor_{proc} = 0.5$  and  $\beta_{b/w} = 1$ .

difference between the SMALLEST and LOWERBOUND references and our heuristics also increases with *basicWork* and  $\mathcal{R}_i$ . When *basicWork* = 0.3 and  $\mathcal{R}_i$  = 0.95, this difference of percentages is only of 1.5% for SMALLEST and 0.4% for LOWERBOUND. In fact, with the increase of the system load, the WCETs increase, and each replica has a higher chance to be victim of a failure. But LOWERBOUND does not consider the load of processors, and SMALLEST estimates the energy consumption without considering failures or overlapping. This explains the growing gap between these references and the heuristics when *basicWork* and  $\mathcal{R}_i$  increase.

In summary, our strategy maintains a stable performance when *basicWork* and  $\mathcal{R}_i$  increase. In contrast, EDF\_PLAIN performs worse. The difference between EDF\_ENERGY and the LOWERBOUND reference is lower than 1% even in the worst case, which stresses the high performance of the designed strategy.

### 6.2.7 Number of replicas, number of failures and success rate

We counted the number of replicas that failed during the execution in each experiment. In the set with small failure rate, we have on average 0.0002 failed replicas per time unit. On the contrary, in the set with big failure rate, the average number of failed replicas per time unit increases to 0.0129.

In Figure 4, we observe that *random* needs more replicas and is the victim of a larger number of failures. *deR* achieves the lowest total number of replicas and total number of failures encountered because it always uses the processors with highest reliability. *deP* achieves a slightly lower number of replicas and failures than *inE* because it considers both reliability and energy .

In Figure 5, the total number of replicas is the same for all criteria because they all use the same mapping. For the number of failures, the difference is not obvious between strategies. Unsurprisingly, EDF\_RELIABILITY has the lowest failure rate. Then, EDF\_WCET and EDF\_ENERGY have slightly better failure rates than the other heuristics.

Figures 6 and 7 show that the number of replicas and failures remain relatively stable when  $cor_{task}$  or  $cor_{proc}$  is not high. But there is a relative increase when  $cor_{task}$  or  $cor_{proc}$  reaches 0.75.

As for  $\beta_{b/w}$ , we see on Figure 8 that, when  $\beta_{b/w}$  increases, the number of replicas remains the same, while the number of failures increases. This is because we map replicas according to the task WCET which is independent on the value of  $\beta_{b/w}$ . However, when  $\beta_{b/w}$  is larger, we have longer actual execution times and, thus, a higher probability that failures actually happen.

While increasing  $\mathcal{R}_i$  and *basicWork*, we observe in Figure 9 that the number of replicas increases, and also the number of failures. With higher  $\mathcal{R}_i$ , we need more replicas to satisfy the reliability target, and more replicas executed at the same time means a higher probability that failures actually occur. As

	$cor_{\text{task}} = 0.25$	$cor_{\text{task}} = 0.50$	$cor_{\text{task}} = 0.75$
$cor_{\text{proc}} = 0.25$	100%	100%	100%
$cor_{\text{proc}} = 0.50$	99.971%	100%	100%
$cor_{\text{proc}} = 0.75$	99.111%	99.992%	100%

Table 4: Success rate of random trials under big failure rate, when varying  $cor_{\text{task}}$  and  $cor_{\text{proc}}$ .

	$basicWork = 0.1$	$basicWork = 0.2$	$basicWork = 0.3$
$\mathcal{R} = 0.80$	100%	100%	100%
$\mathcal{R} = 0.85$	100%	100%	100%
$\mathcal{R} = 0.90$	100%	100%	99.940%
$\mathcal{R} = 0.95$	100%	100%	98.825%

Table 5: Success rate of random trials under big failure rate, when varying  $cor_{\text{task}}$  and  $cor_{\text{proc}}$ .

for  $basicWork$ , a higher  $basicWork$  means larger average WCETs. Longer replicas lead to lower reliability. Thus failures happen more frequently, and we need also more replicas to reach the reliability threshold.

The success rate can be found in Tables 4 and 5. We observe that the proportion of experiments for which we can find a feasible mapping decreases when the system pressure becomes large. The fraction of instances for which we can build a solution also decreases when  $cor_{\text{task}}$  is low and  $cor_{\text{proc}}$  is high. This is why we used  $\mathcal{R}_i = 0.95$  and  $basicWork = 0.3$  as the highest reliability target and basic work parameter, and we avoided to use extreme correlation values in our experiments. Within all platforms in the experiments, all the tested heuristics are able to find a valid solution in all tested configurations with small failure rate. And in the big failure rate cases, heuristics are able to build valid solutions for more than 99.8% of the instances. The very high success rate of our experiments means that we can be confident in our results and that they do not suffer of any bias.

### 6.2.8 Summary

In conclusion, in the experiments we compared different criteria for the mapping and scheduling phases, and we also studied the influence of the different parameters. Tables 6 and 7 present a synthetic view of the results achieved by different heuristics, or for different correlations, when the system is under the highest pressure ( $\beta_{b/w} = 1$ ,  $basicWork = 0.3$ , and  $\mathcal{R}_i = 0.95$ ).

Among the different criteria used in the heuristics, we observe from Table 6 that the  $deP$  method is the best processor ordering for the mapping phase, and EDF\_ENERGY performs best among the scheduling criteria. In



	<i>random</i>	<i>deR</i>	<i>inE</i>	<i>deP</i>
EDF_PLAIN	6.9%	57.2%	58.2%	58.5%
EDF_REF_PAPER	8.8%	57.3%	58.1%	58.7%
EDF_START_TIME	17.0%	62.1%	64.0%	64.2%
EDF_RELIABILITY	23.6%	62.1%	65.3%	64.9%
EDF_WCET	23.8%	64.2%	66.8%	66.8%
EDF_ENERGY	24.2%	64.4%	66.8%	66.9%
LOWERBOUND	67.6%			
SMALLEST	51.8%	67.4%	69.7%	69.6%

Table 6: Percentage of energy saved over the baseline when using different processor ordering criteria during mapping and using different scheduling criteria under big failure rate, with  $cor_{\text{task}} = 0.5$ ,  $cor_{\text{proc}} = 0.5$ ,  $\beta_{b/w} = 1$ ,  $basicWork = 0.3$ , and  $\mathcal{R}_i = 0.95$ .

	$cor_{\text{task}} = 0.25$	$cor_{\text{task}} = 0.5$	$cor_{\text{task}} = 0.75$
$cor_{\text{proc}} = 0.25$	72.4%	70.0%	67.1%
$cor_{\text{proc}} = 0.50$	64.3%	66.9%	57.3%
$cor_{\text{proc}} = 0.75$	42.6%	45.4%	46.4%

Table 7: Percentage of energy saved over the baseline when using the *deM-inW* task ordering criterion and the *deP* processor ordering criterion during mapping, and using the EDF\_ENERGY scheduling criterion, under big failure rate, when varying  $cor_{\text{task}}$  and  $cor_{\text{proc}}$ , with  $\beta_{b/w} = 1$ ,  $basicWork = 0.3$ , and  $\mathcal{R}_i = 0.95$ .

the case of high system pressure, our best criterion can still achieve 66.9% of energy saving compared to the random baseline. The difference of performance is 42.7% if we use *random* as the processor ordering criterion during the mapping phase, and the difference is only 2.7% compared to the SMALL-EST reference during the scheduling phase. To study the efficiency of our selected heuristic under different  $cor_{\text{task}}$  and  $cor_{\text{proc}}$ , Table 7 presents the performance while using *deMinW*, *deP* and EDF\_ENERGY as criteria. Our strategies can save more than 40% of the energy consumed by the baseline in all cases, and this percentage can be as high as 72.4% in the best case. Furthermore, we report a median energy saving percentage of only 1.6% less than that of the LOWERBOUND; and an average of only 2.0% less.

We can confidently conclude that our best strategies perform remarkably well over the whole experimental settings.

## 7 Related work

### 7.1 Scheduling for heterogeneous platforms

There is a huge literature on scheduling for heterogeneous platforms. Here, we only reference some very recent work closely related to our problem but dealing with non-periodic tasks. [12] maximizes the reliability of an energy-constrained DAG executed on a heterogeneous platform while using DVFS. Conversely, [13] minimizes the energy consumption of a reliability-constrained DAG executed on a heterogeneous platform while using, or not, DVFS. A group of authors published a book [14] and several articles on the problem of DAG scheduling on heterogeneous platforms. In Chapter 2 of book [14] and in [15] these authors consider the energy minimization when scheduling a DAG with or without DVFS. However, these two references do not consider reliability. In [11] they considered the same problem while satisfying some reliability goal.

Overall, these studies do not consider real-time applications. The deadlines which constrain real-time tasks render problems significantly harder to tackle.

### 7.2 Scheduling real-time applications on homogeneous platforms

Liu and Layland first introduced the Earliest Deadline First (EDF) and the Rate Monotonic (RM) scheduling policies for real-time systems and provided the utilization bounds for both policies in 1973 [9]. Since then, the real-time scheduling problem has been extensively studied. There exists a very significant literature on real-time scheduling for multiprocessor systems. However, most work is devoted to homogeneous processor systems, as exemplified by the survey [16] which ignores altogether heterogeneous systems, and by the

more recent survey [17] where only 9 of the 78 references deal with heterogeneous platforms. [3] minimizes the energy when scheduling independent tasks with different deadlines on a homogeneous platform while satisfying some threshold on reliability. The study [6] improved the solution from [3], in particular by carefully avoiding overlaps between primary and secondary replicas. [4] considers the same problem; however, it uses checkpointing to cope with failures when all other works consider replication.

We refer the interested reader to [6, 3, 16, 17] for a comprehensive overview of the related work for homogeneous platforms. Heterogeneous platforms make the problem even harder because processors can have different speeds, energy costs, and failure rates. Therefore, the processor preferred for one task by one of the objectives and constraints—deadline satisfaction, energy minimization, reliability threshold satisfaction— may be the worst processor for another objective or constraint. The heuristics have thus to perform complicated trade-offs in these three-criteria settings.

### **7.3 Scheduling real-time applications on heterogeneous platforms**

Some related works target the scheduling of real-time applications on heterogeneous platforms, but without considering fault tolerance. For instance, [18] targets the execution of a DAG, but considering neither energy consumption nor fault-tolerance (when DAGs are scheduled, tasks are always assumed to have the same deadline). [19] targets the execution of independent tasks that access shared resources, the access to resources being exclusive. Their objective is to maximize the number of instances for which a solution is found. [20], [21] and [22] minimize energy consumption by using DVFS, [20] when scheduling independent tasks, [21] a DAG, and [22] a moldable application. [23] considers the scheduling of independent tasks and DAGs under an energy constraint, while [24] considers the scheduling of independent tasks under a thermal constraint. [25] proposes a fully polynomial-time approximation scheme (FPTAS) for minimizing the energy consumption for a set of independent tasks executed on a set of heterogeneous (unrelated) processing elements.

Some related work considers the execution of real-time applications on heterogeneous failure-prone platforms but is limited to coping with a single failure per task or per processor. [26] maximizes the reliability of the considered DAG but does not consider energy consumption and follows the primary/backup technique and, thus, is limited to at most one failure per task of the DAG. [27] attempts to maximize resource utilization (and does not consider energy) when scheduling a set of independent tasks. It assumes that at most one processor can fail, which enables the simultaneous scheduling of several backup tasks on the very same processor, since at most one of them will need to be executed. [28] minimizes the energy consumed for the

execution of a DAG while satisfying a reliability threshold. The proposed solution uses DVFS and Power Mode Management (i.e., the ability to switch off idle processors to low-power inactive state). This solution, however, cannot produce a schedule more reliable than the original one. It also supports at most one fault per processor. [29] minimizes the energy consumed for the execution of a set of independent tasks while satisfying a reliability threshold using DVFS and following a primary-backup approach.

Very few studies consider the execution of real-time applications on heterogeneous failure-prone platforms and can cope with two or more failures per task. [30] minimizes the energy consumed for the execution of a set of independent tasks while satisfying a reliability threshold. The proposed solution uses DVFS. This solution, however, is based on a primary-backup approach that is then extended. This approach, by design, cannot produce a schedule more reliable than the original one with two replicas per task, strongly relies on DVFS, and schedules several replicas of a same task on the same processor (what most other approaches forbid). [5] targets the execution of a DAG on a heterogeneous platform while satisfying a reliability threshold. However, the objective is not the minimization of energy consumption but the maximization of the *utilization of energy consumption*, which can be seen as a yield of reliability improvement with respect to increased energy consumption. As a consequence, [5] produces energy greedy schedules (see subplots (a-1), (b-1), and (c-1) of Figure 1 in [5]). In Chapter 3 of the already mentioned book [14], the authors consider cost minimization (which can be energy minimization) when scheduling a DAG under deadline and reliability constraints. Therefore, we consider the same problem but for a set of independent tasks rather than for a DAG. Because of the dependence between tasks and the chosen as-soon-as-possible scheduling of [14], this solution tends to schedule simultaneously the different replicas of a single task. As already pointed out in the studies [6, 3] this can lead to a significant waste of energy. Therefore, it would have been unfair to compare our solution to that of [14] applied to independent tasks.

From what precedes, we have identified only a single existing solution that enables to schedule real-time tasks on heterogeneous platforms while minimizing energy consumption and satisfying some bound on the overall reliability. However, this solution being dedicated to DAGs lacks the possibility to minimize overlapping between replicas of a same task, which has been previously shown to be crucial [6] and which we have given special care to in this paper (see Section 4).

## 8 Conclusion

In this work, we have studied the problem of executing periodic real-time tasks on an heterogeneous platform, with several objectives: minimizing en-

ergy consumption, guaranteeing reliability thresholds, and meeting all deadlines. For each task, we decide how many replicas should be launched, and on which processors to map them. We tagged one replica per task as “primary” replica and the other ones as “secondary” replicas. To obtain an absolute measure for the evaluation of our heuristics, we have computed a theoretical lower-bound on energy consumption.

Extensive simulations show that our best heuristics always achieve very good performance, very close to the LOWERBOUND (on average the percentage of energy saving of our best heuristic is only 2% less than that of LOWERBOUND). This performance was reached by considering processors in the *deP* order when mapping the replicas of a task (roughly speaking, *deP* is the ratio of a task failure rate by its energy cost), and by tagging as “primary” the replicas which cost the smallest dynamic energy.

Furthermore, while all decisions are taken with worst-case execution times (WCETs) of tasks as only input, the simulations used actual execution times; the best heuristic always achieved excellent performance even when the actual execution times were far smaller than the WCETs, showing the robustness of our approach.

Future work will aim at extending the algorithms to periodic graphs of tasks instead of independent task sets. The dependences between tasks will dramatically complicate the mapping and scheduling problems.

## Acknowledgment

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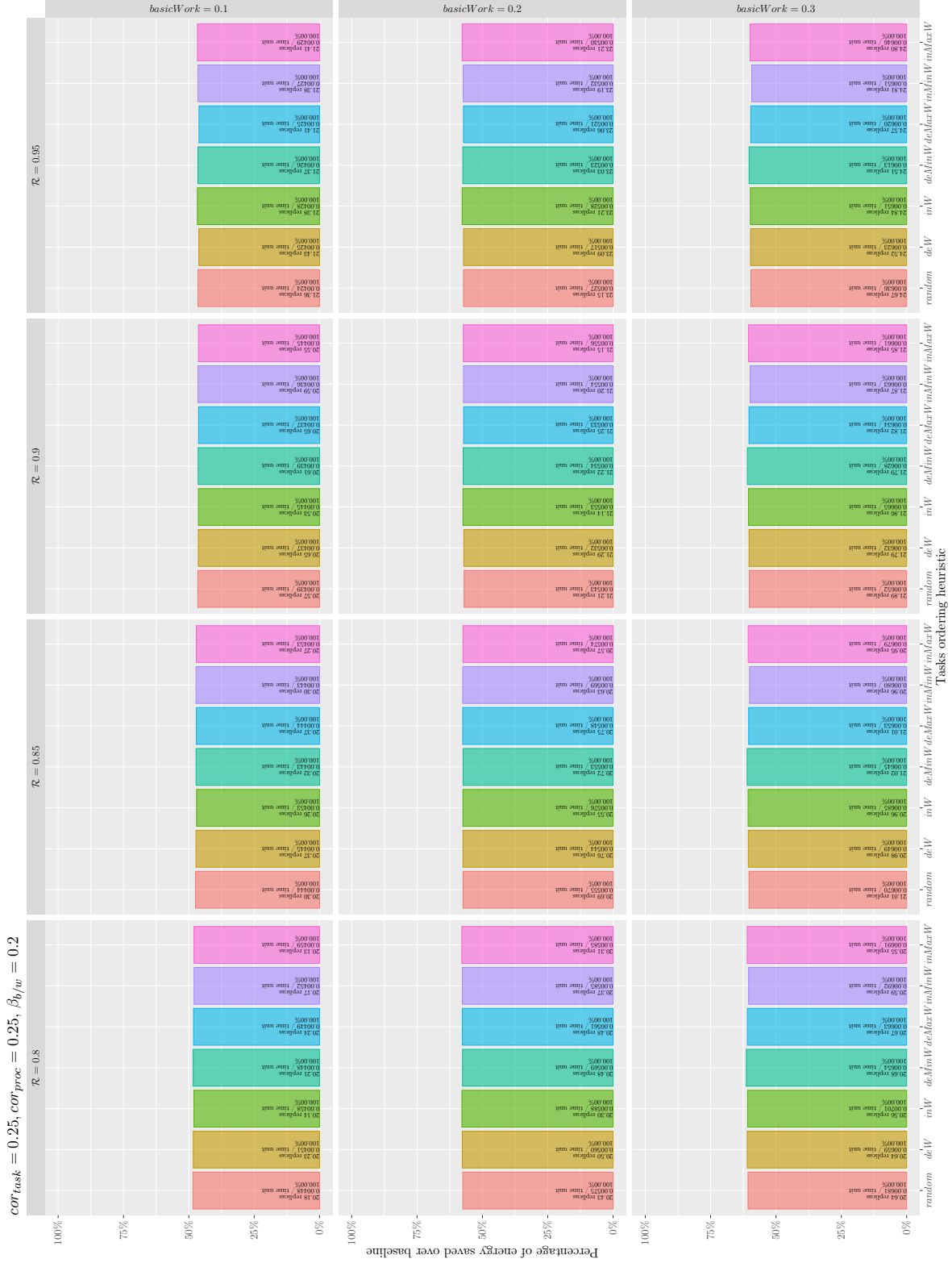
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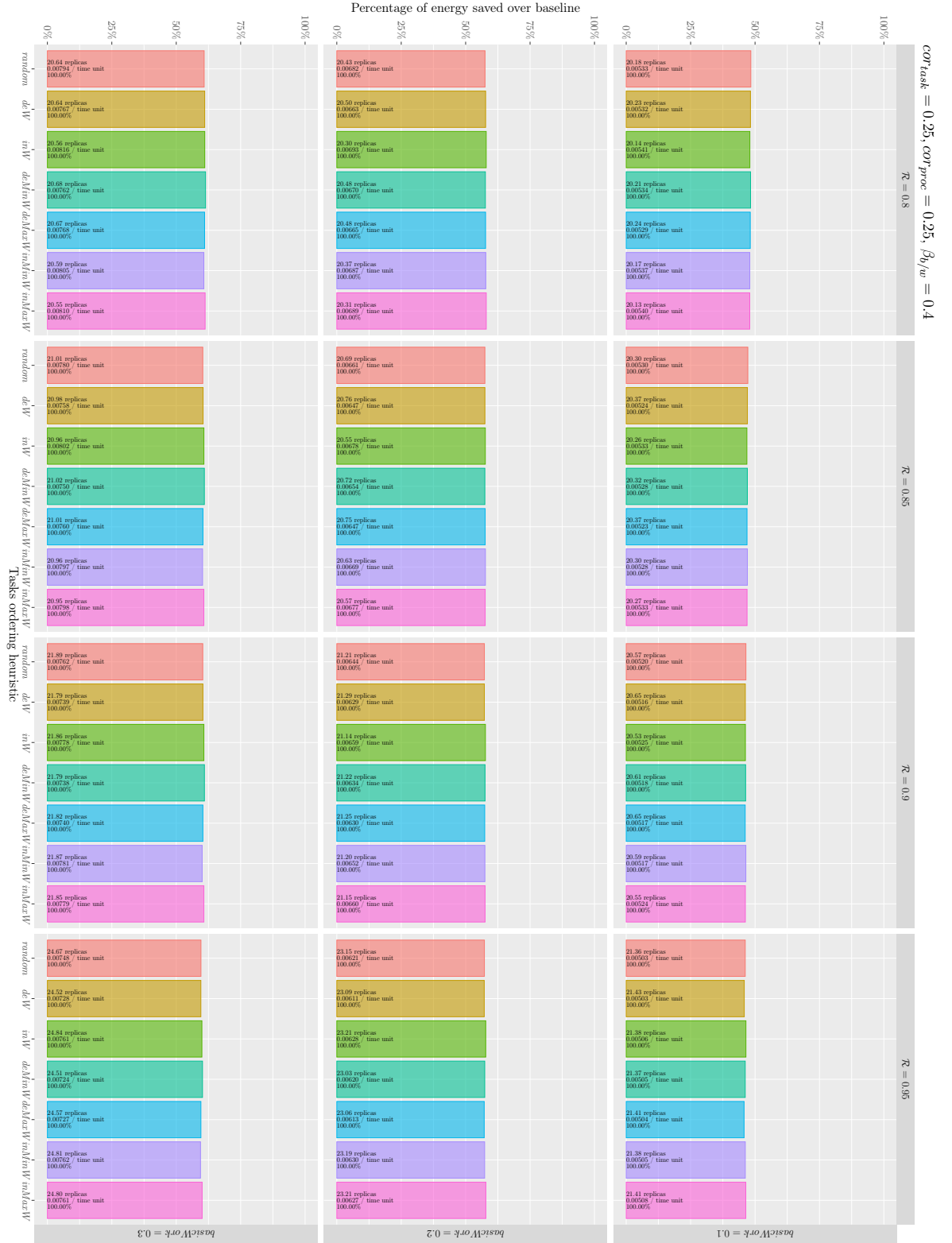


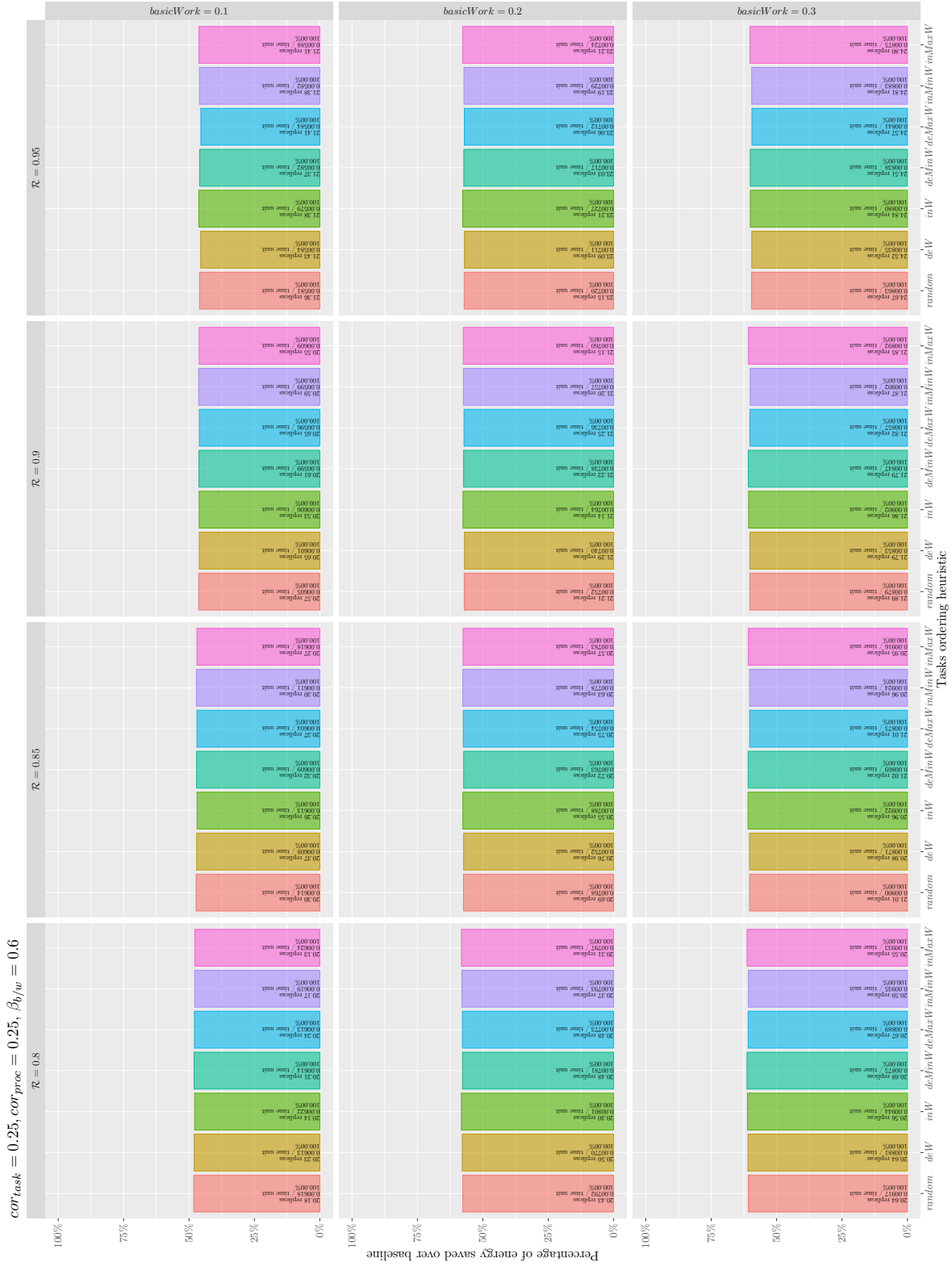
## **Task ordering criteria for mapping phase**

The following section presents percentage of energy saved over baseline using different task ordering criteria under big failure rate.



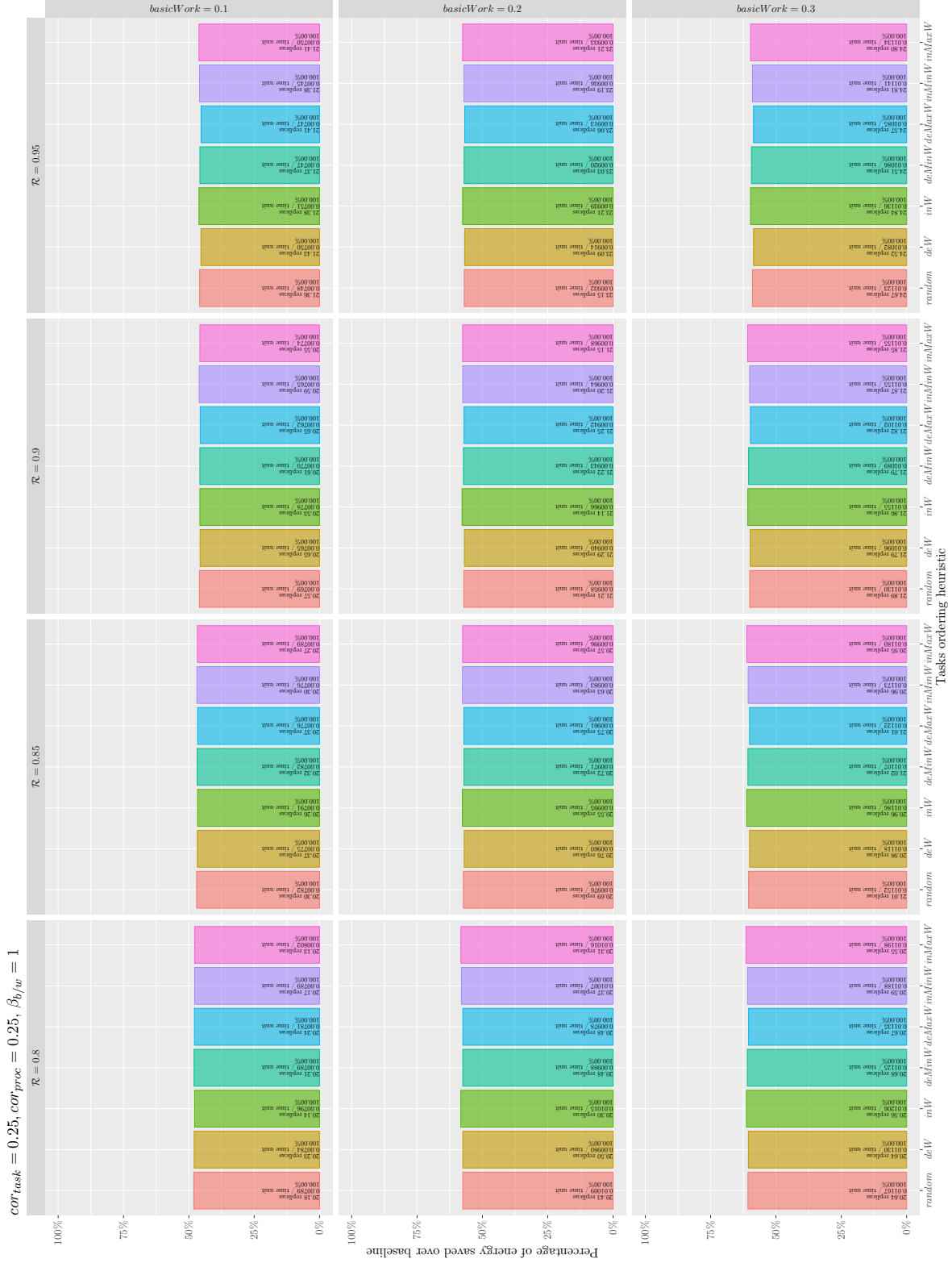
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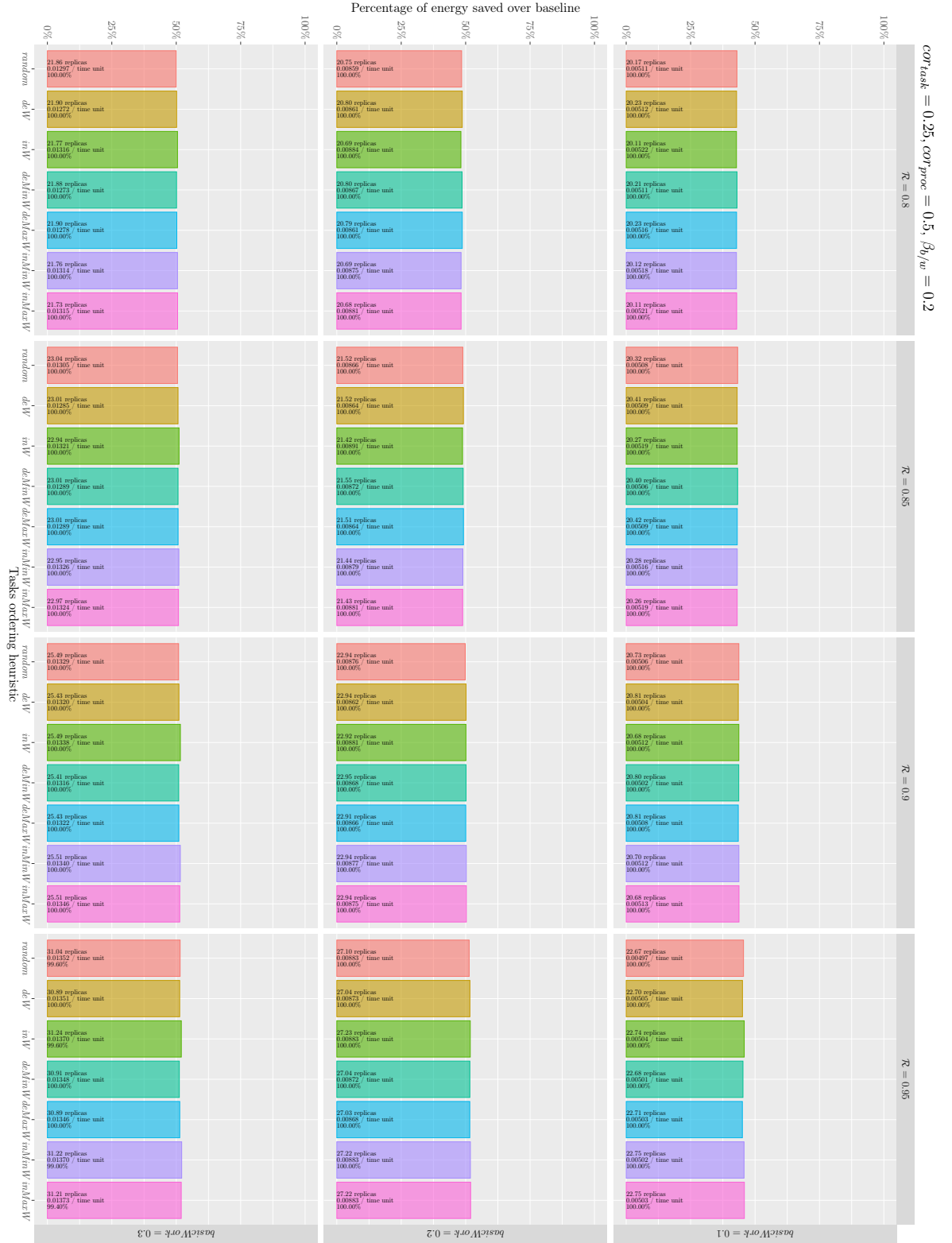


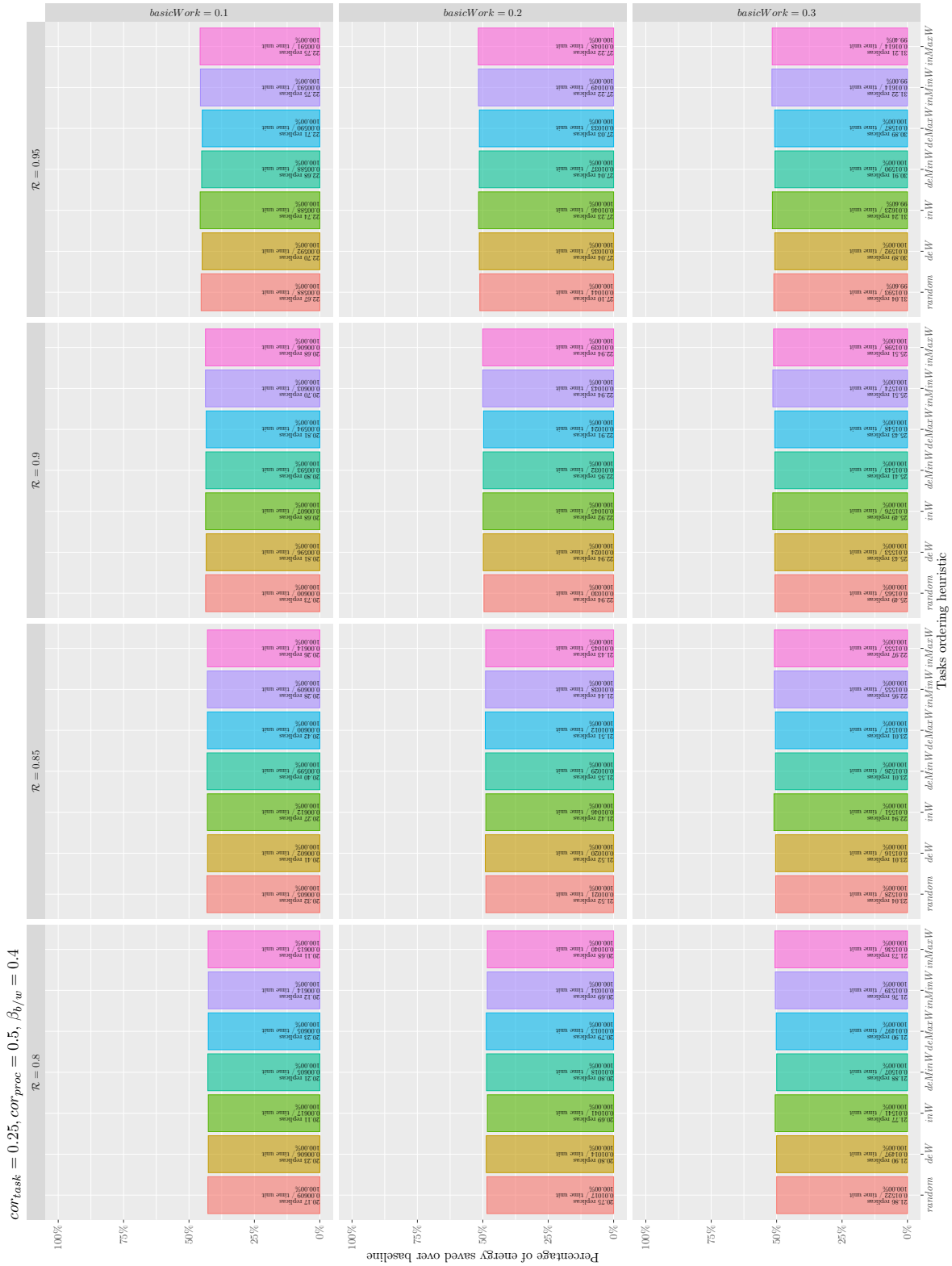


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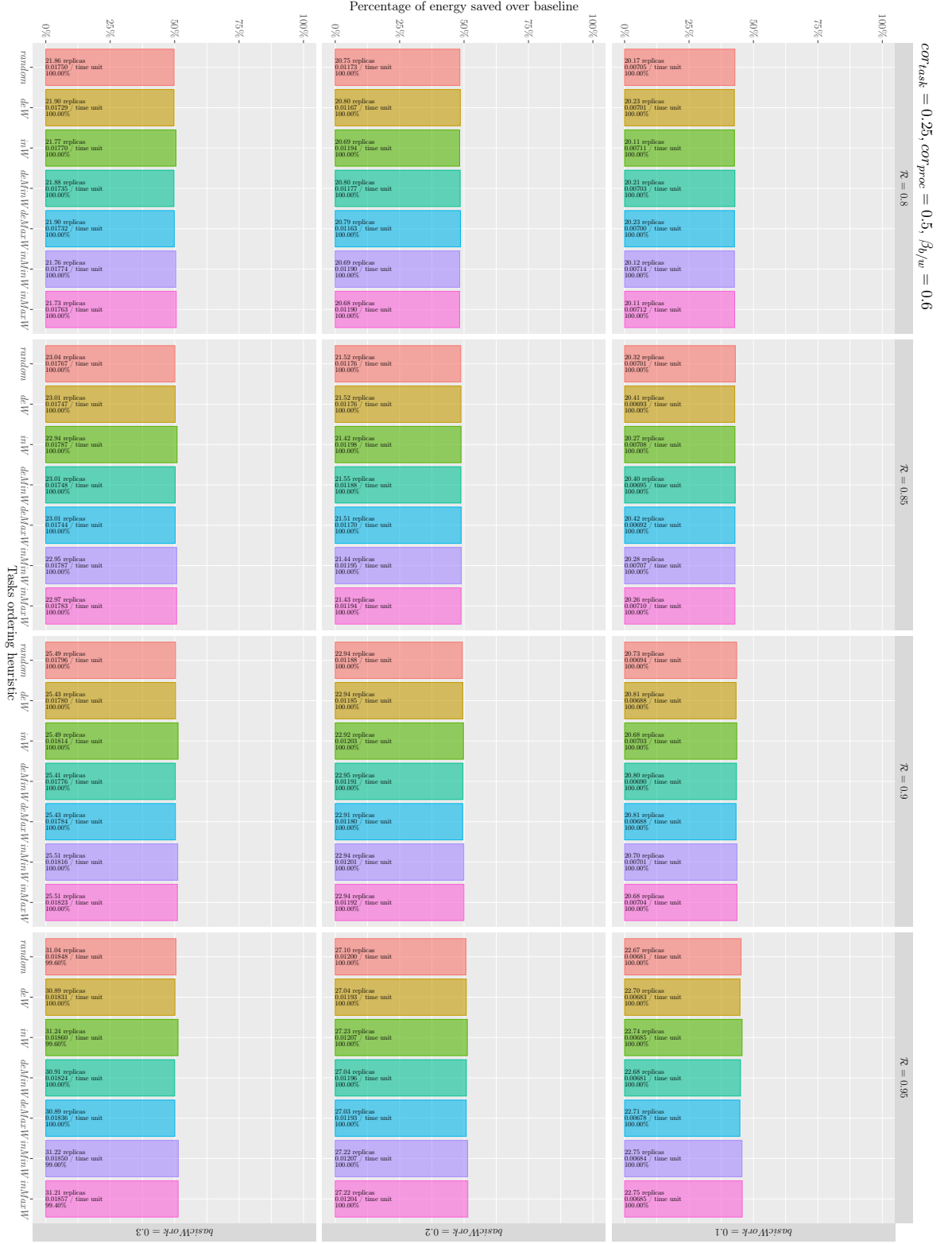


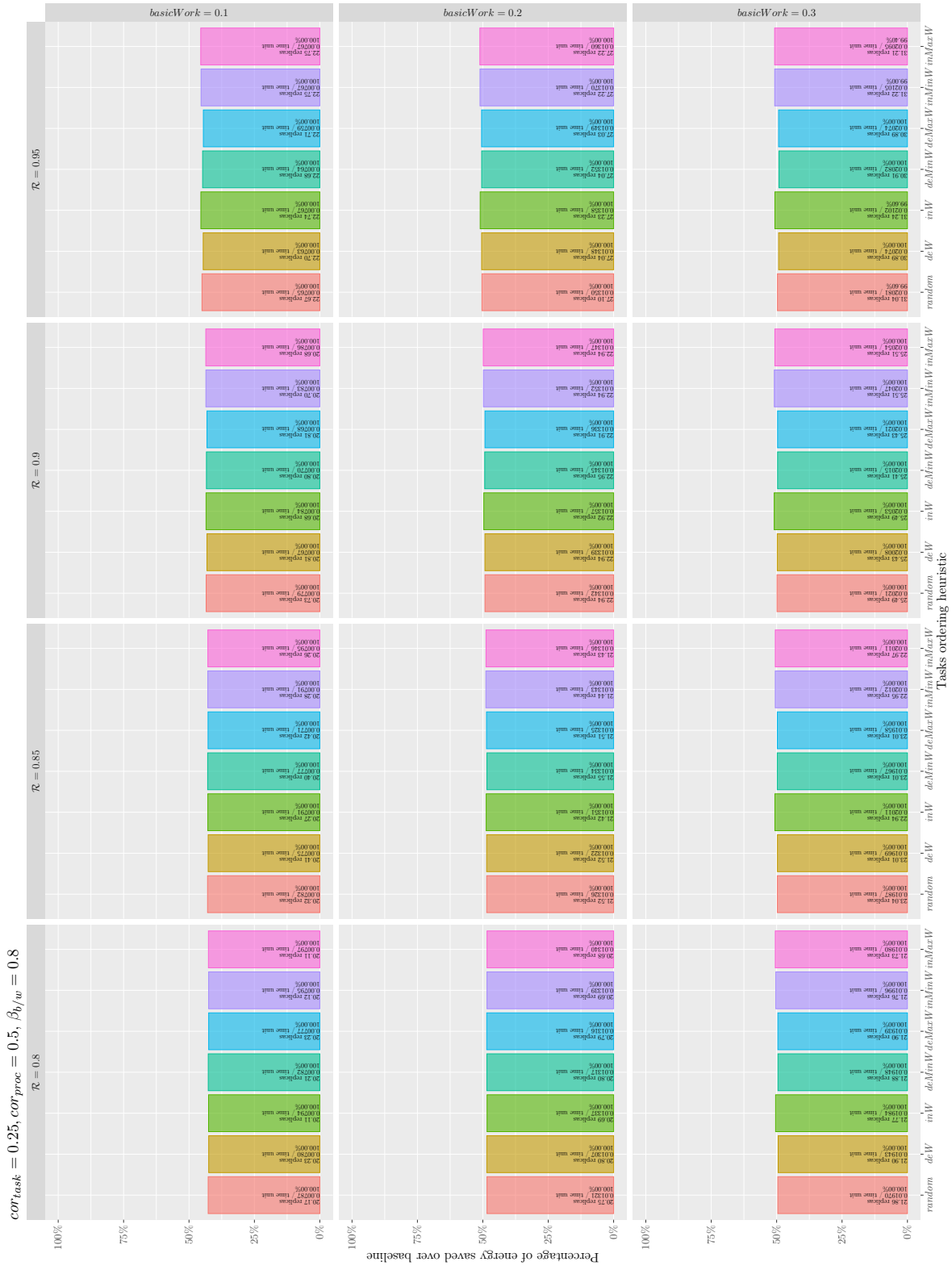


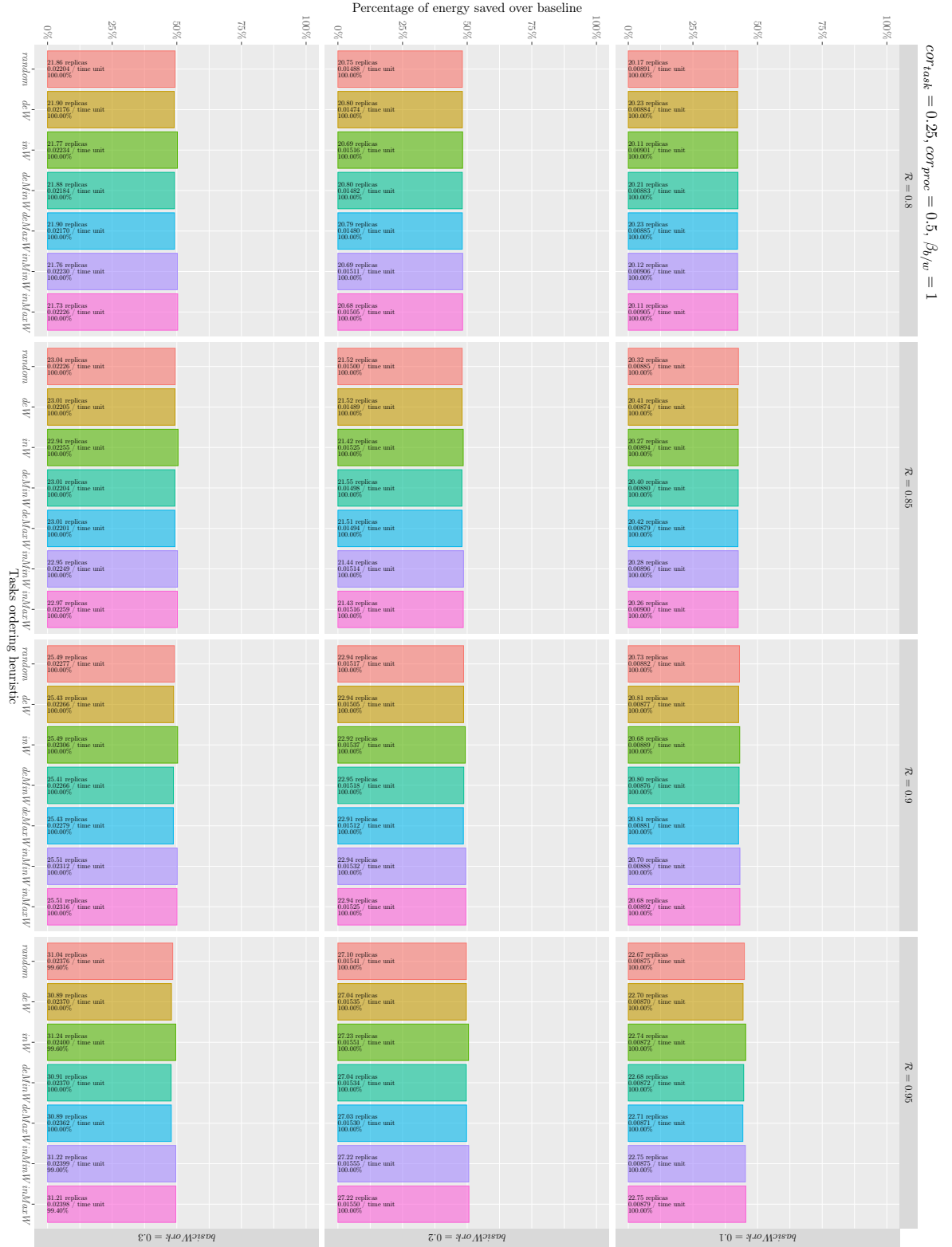






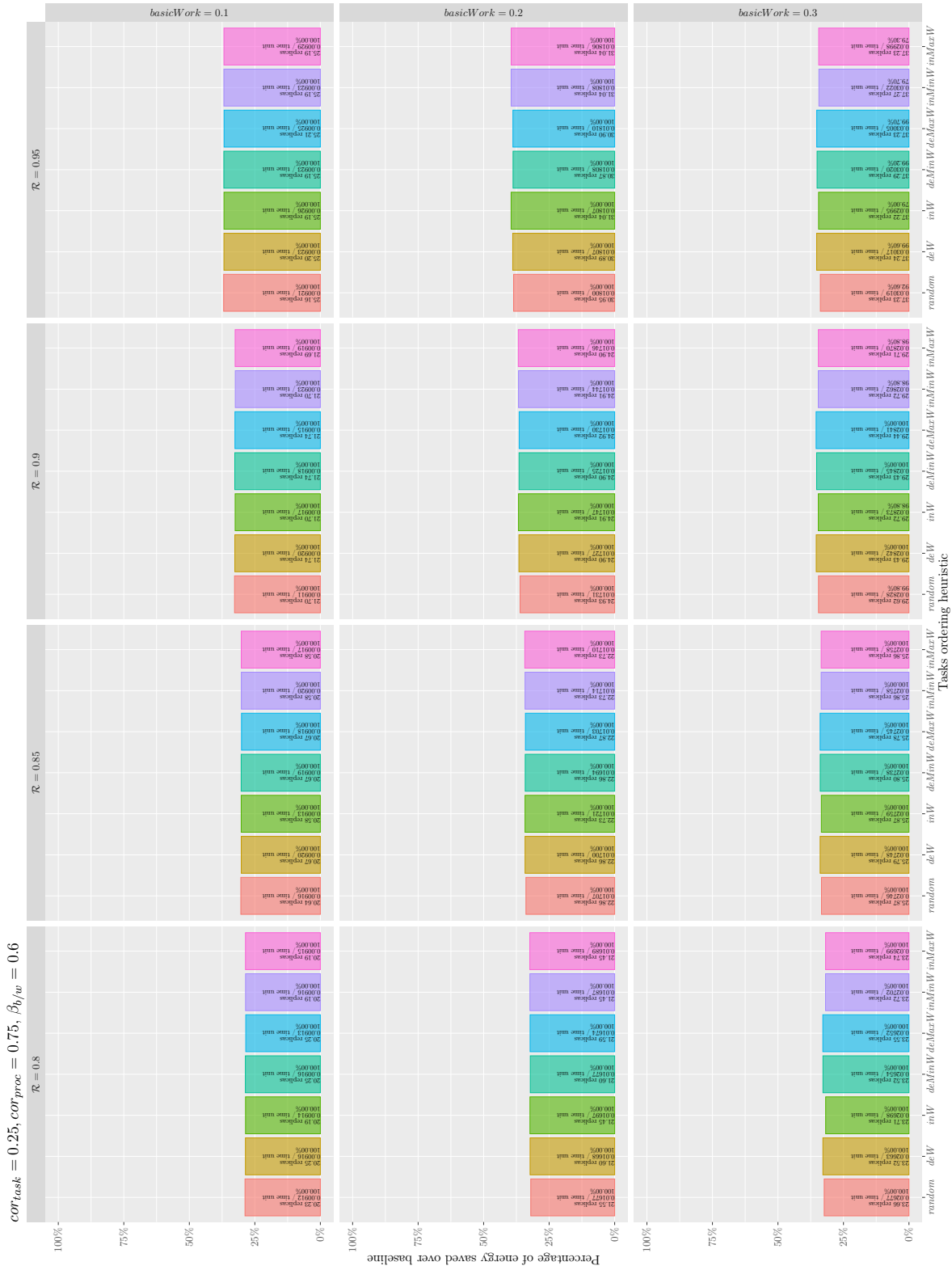






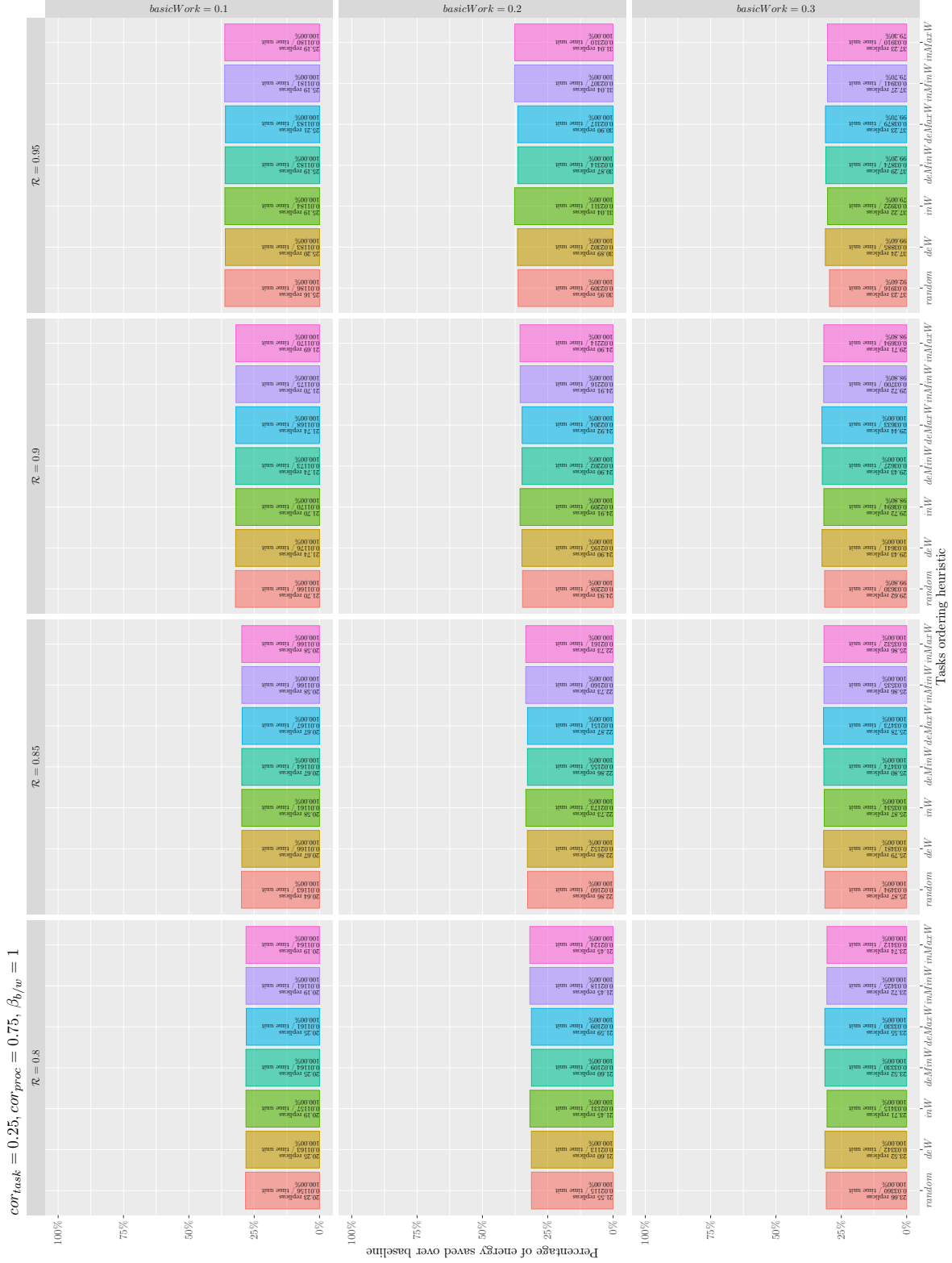




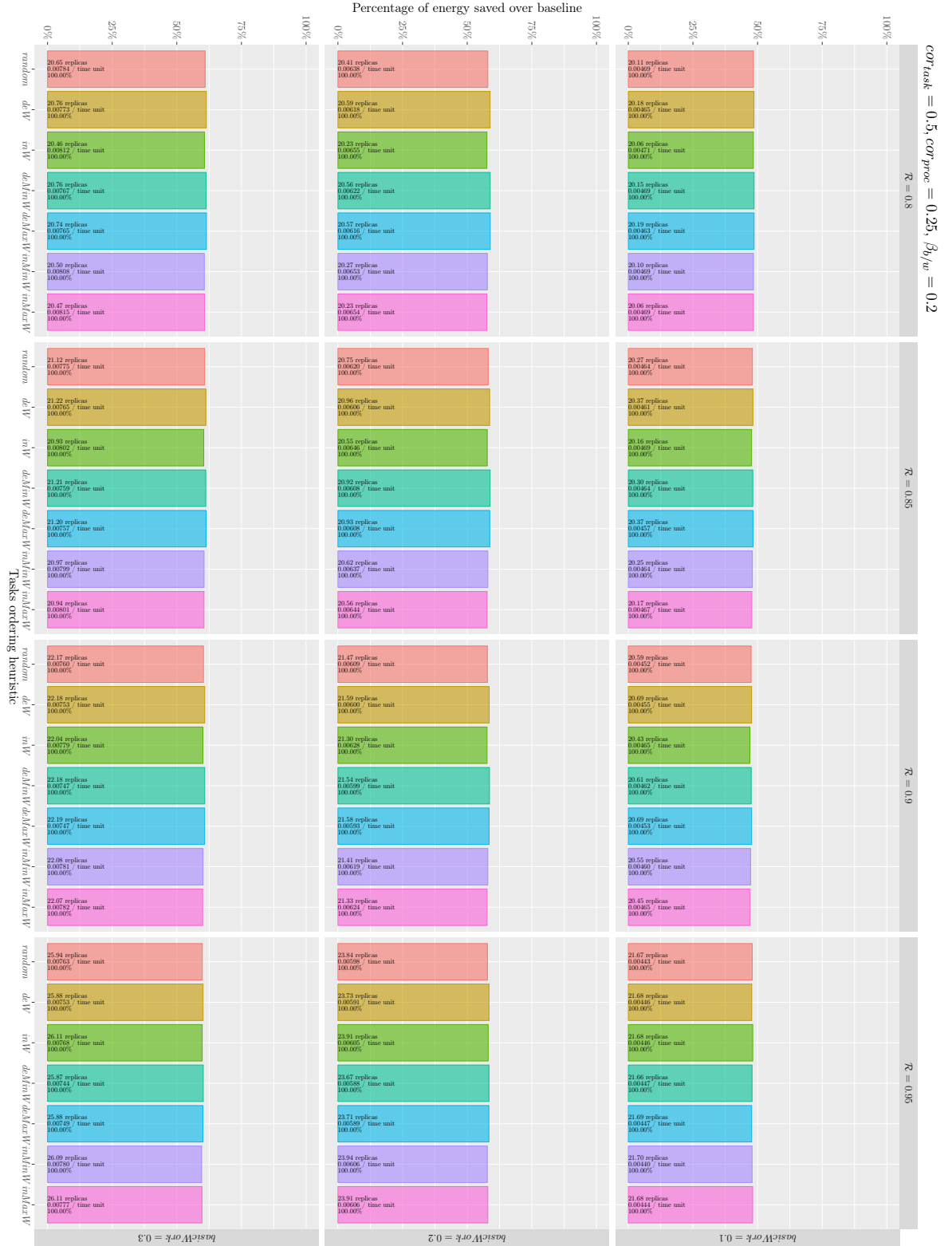


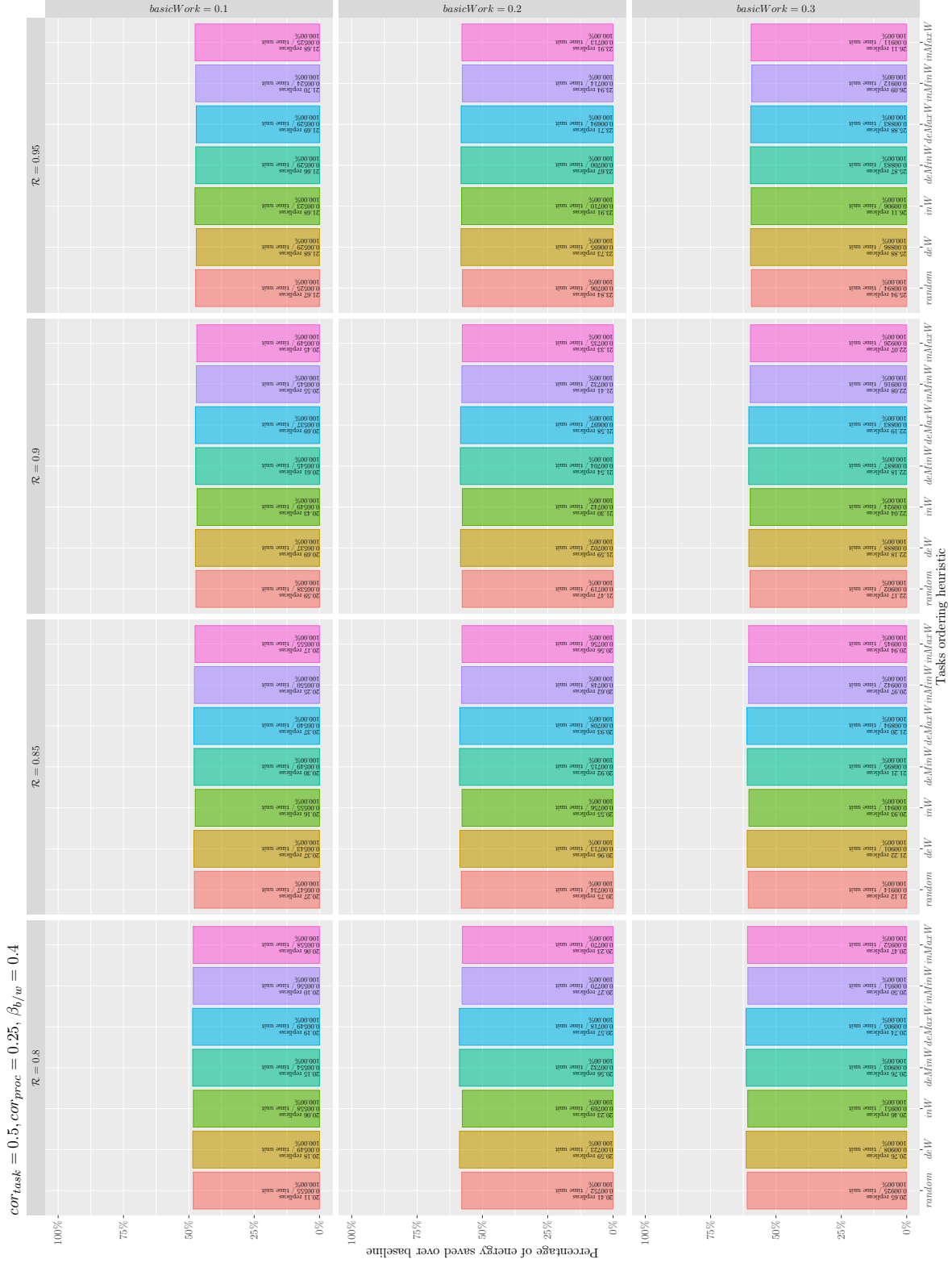
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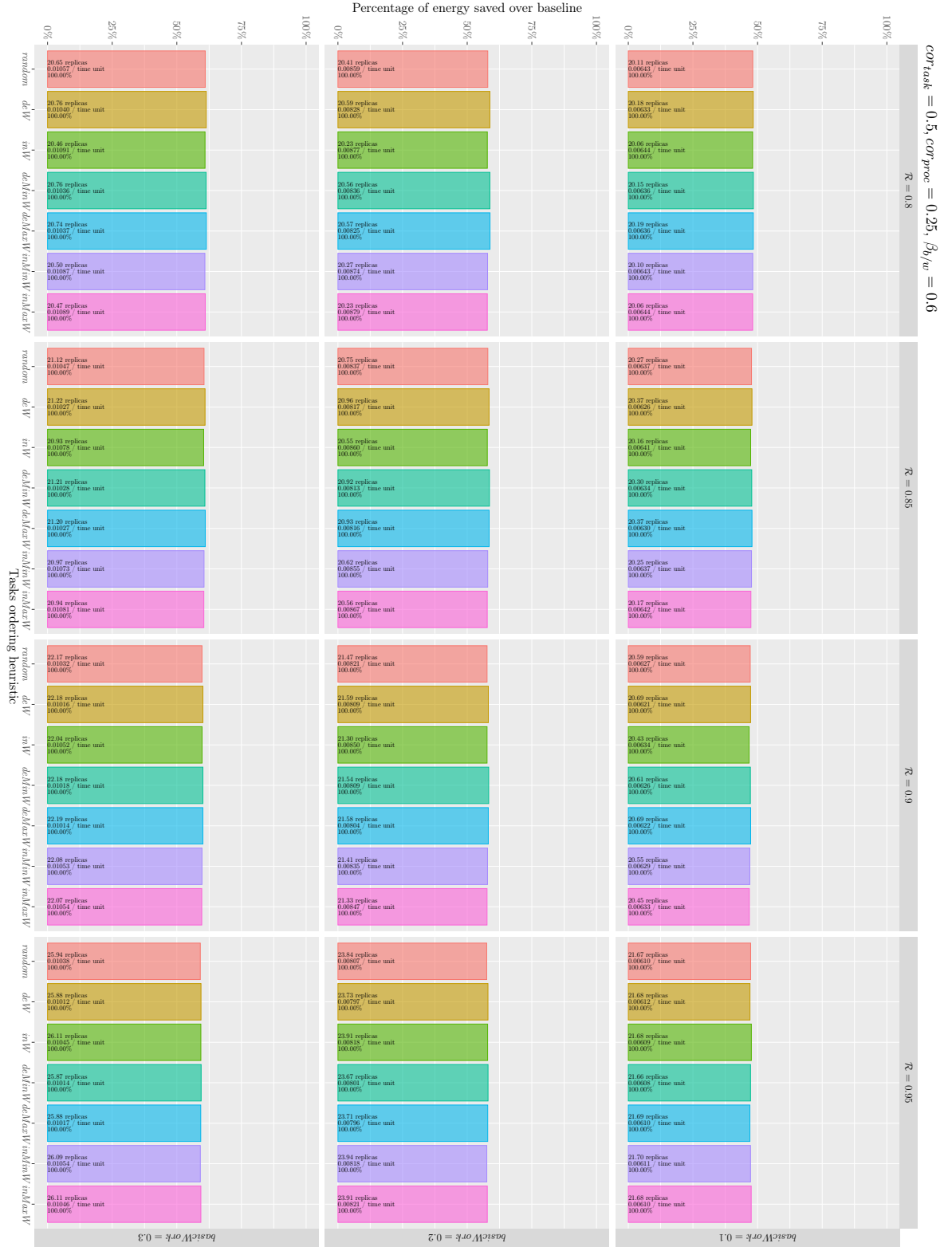


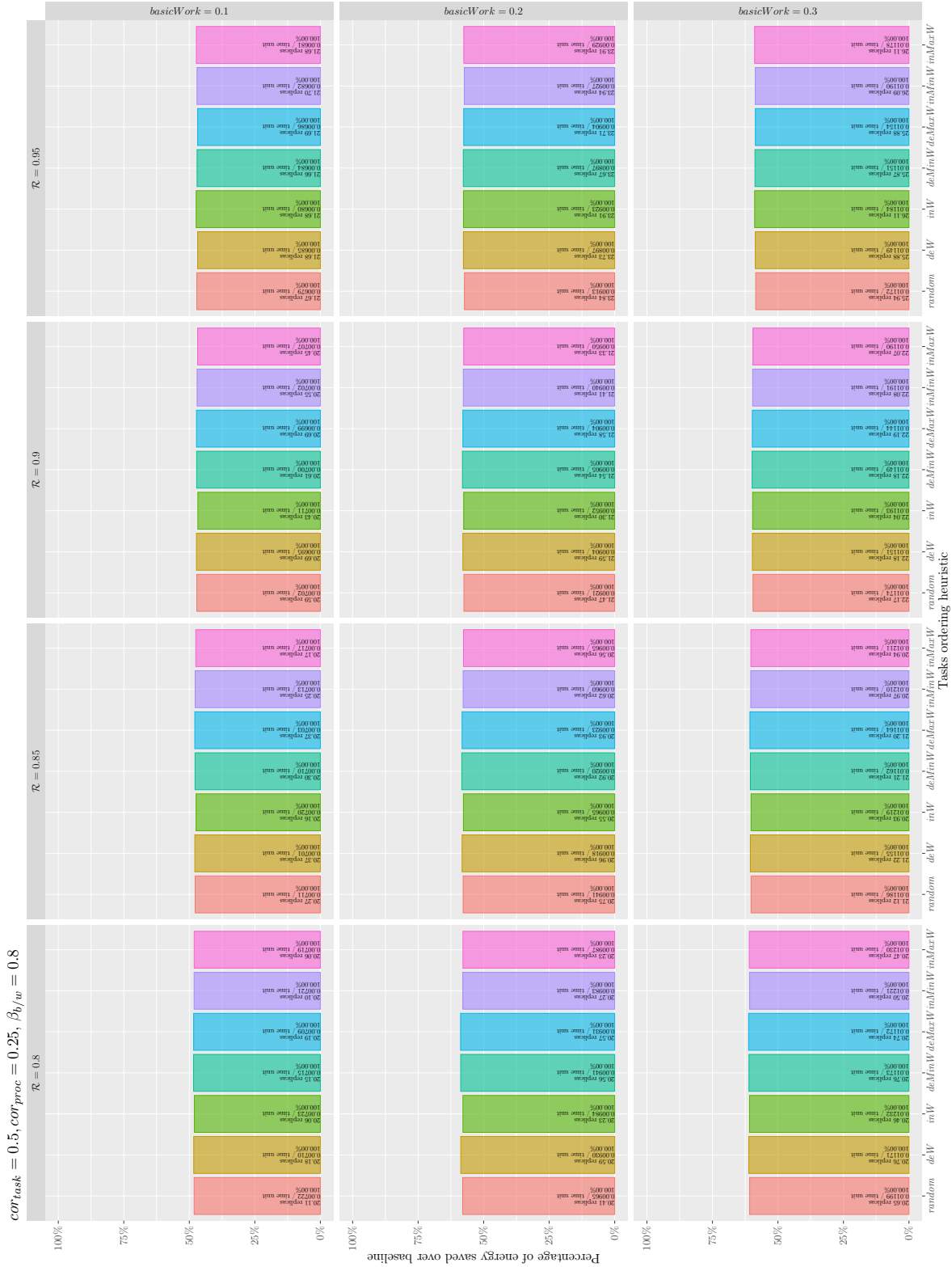


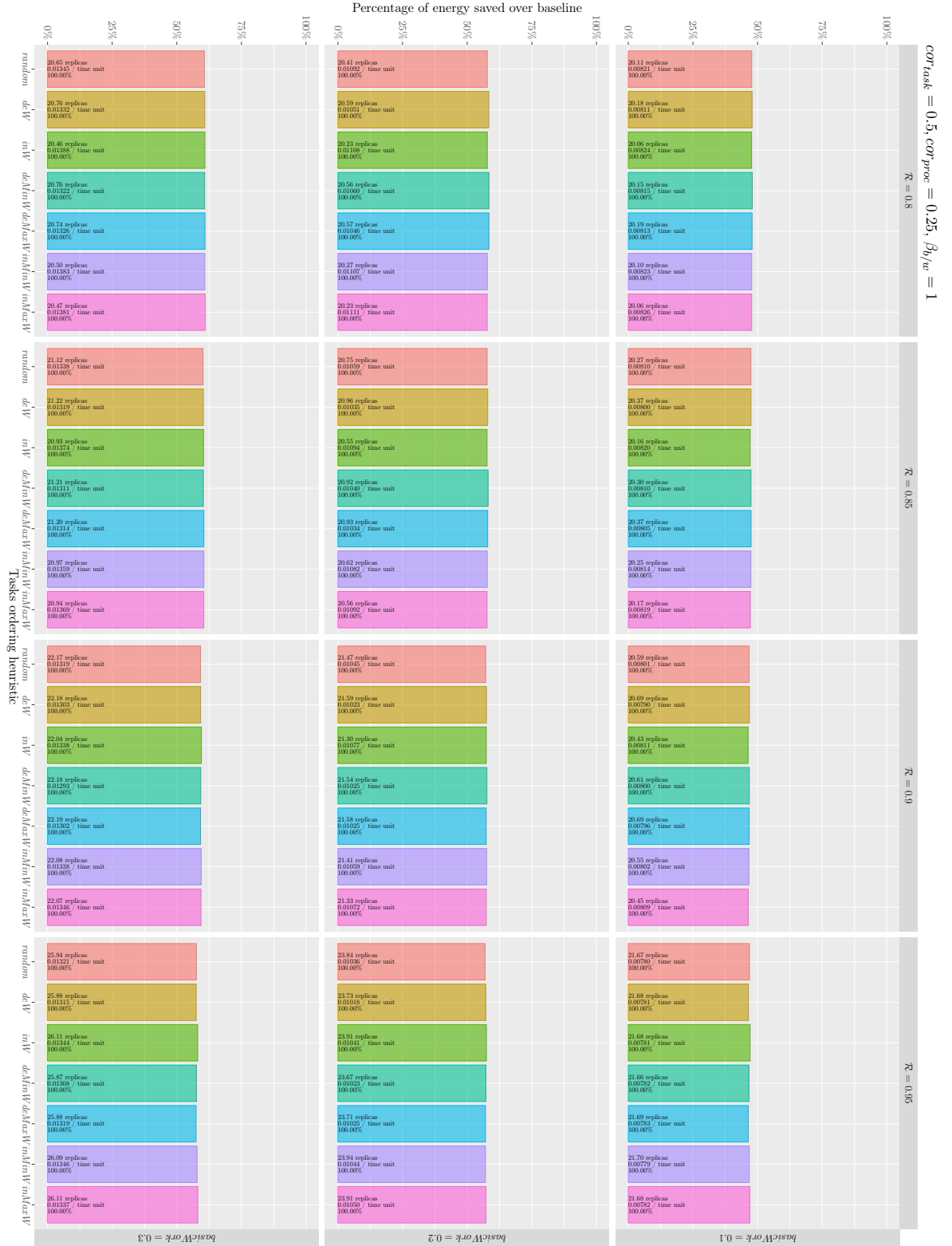


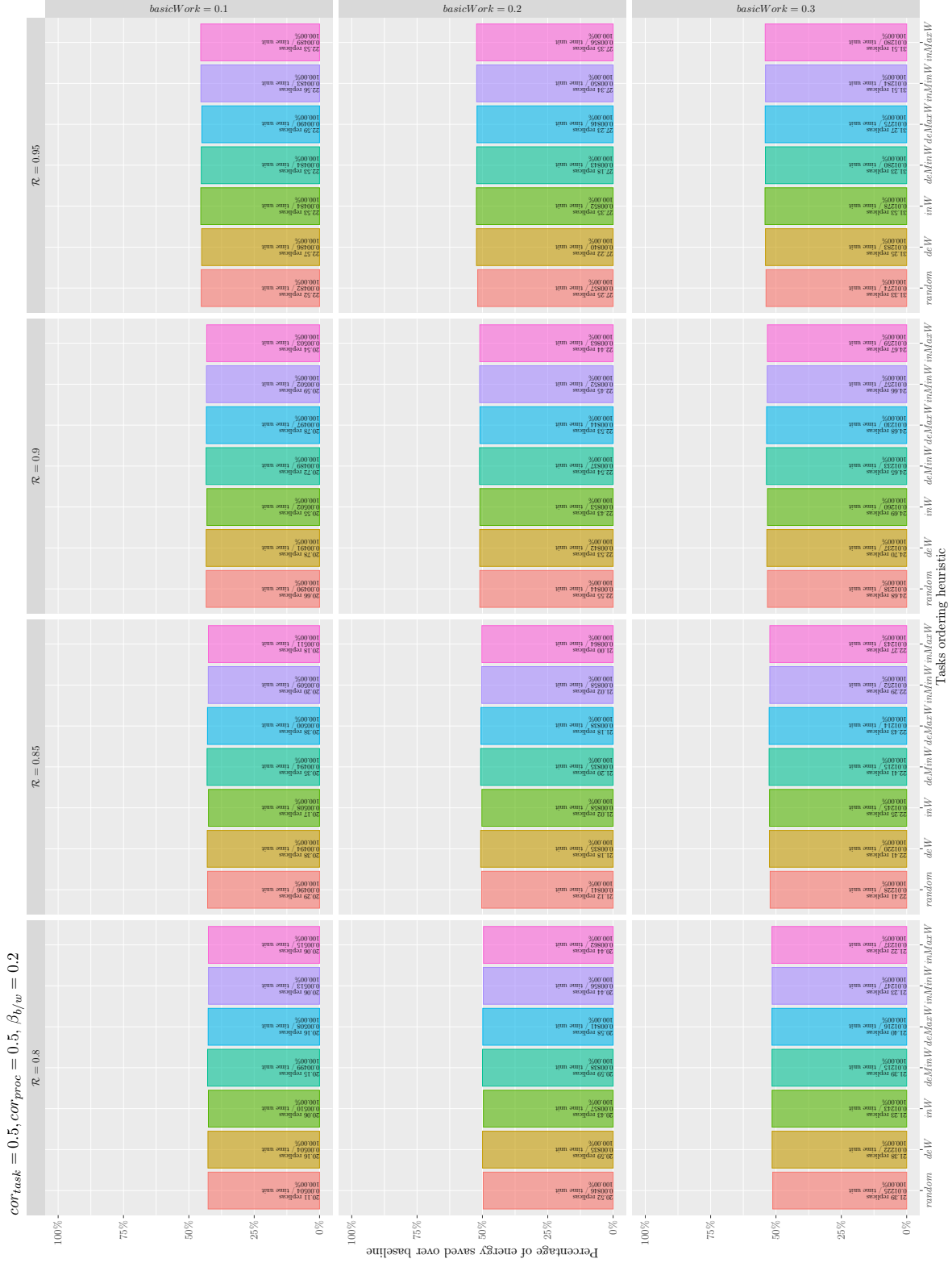


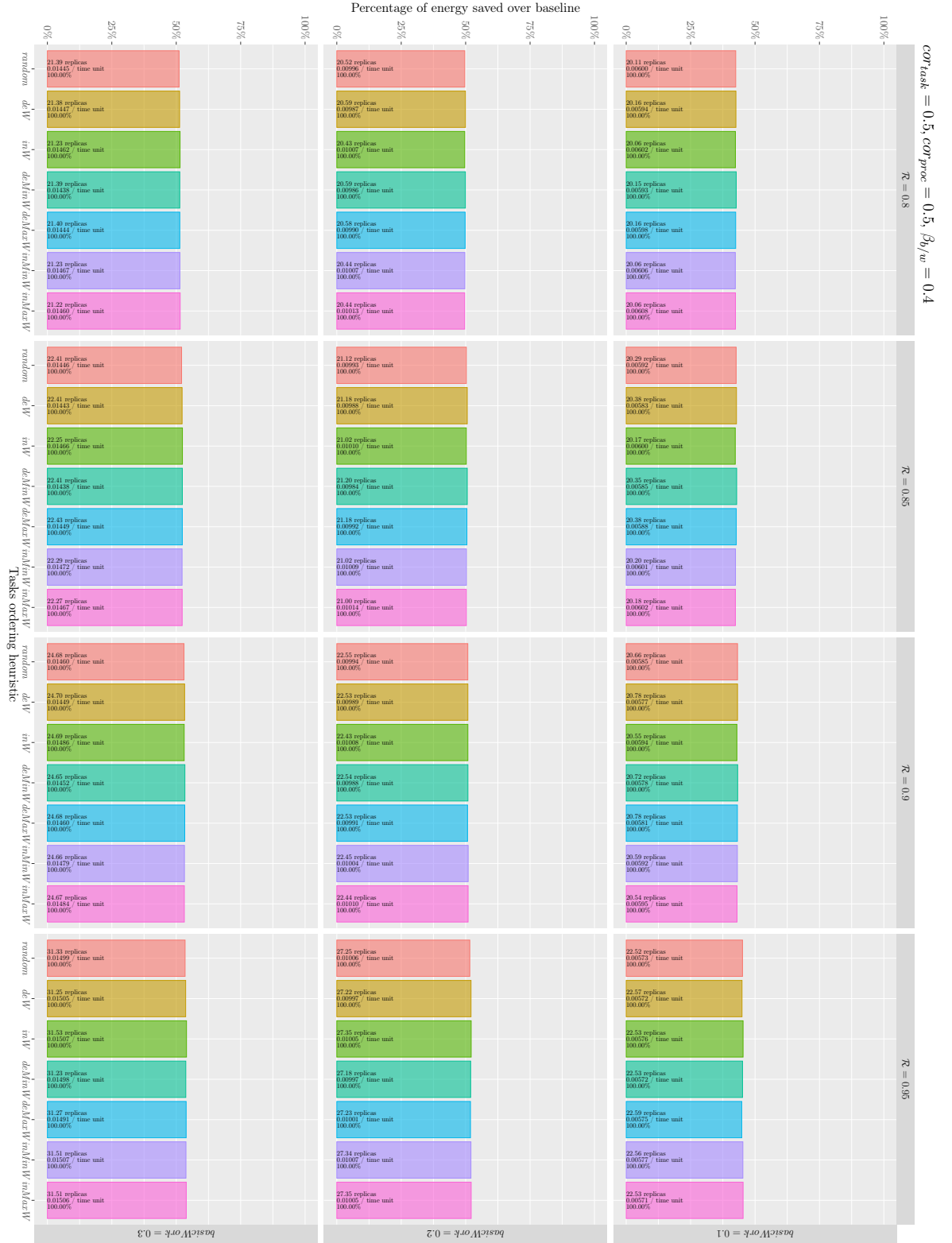
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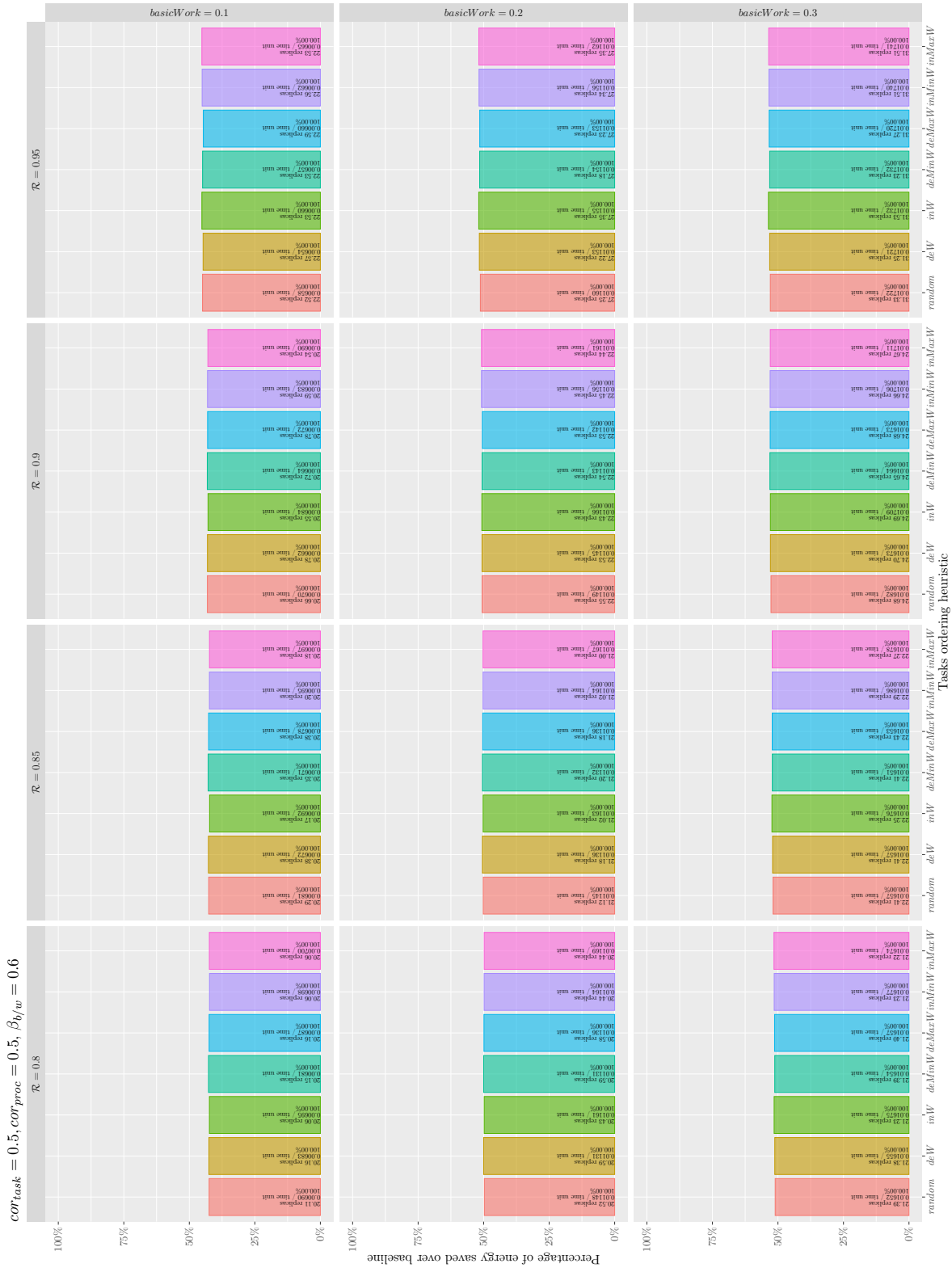




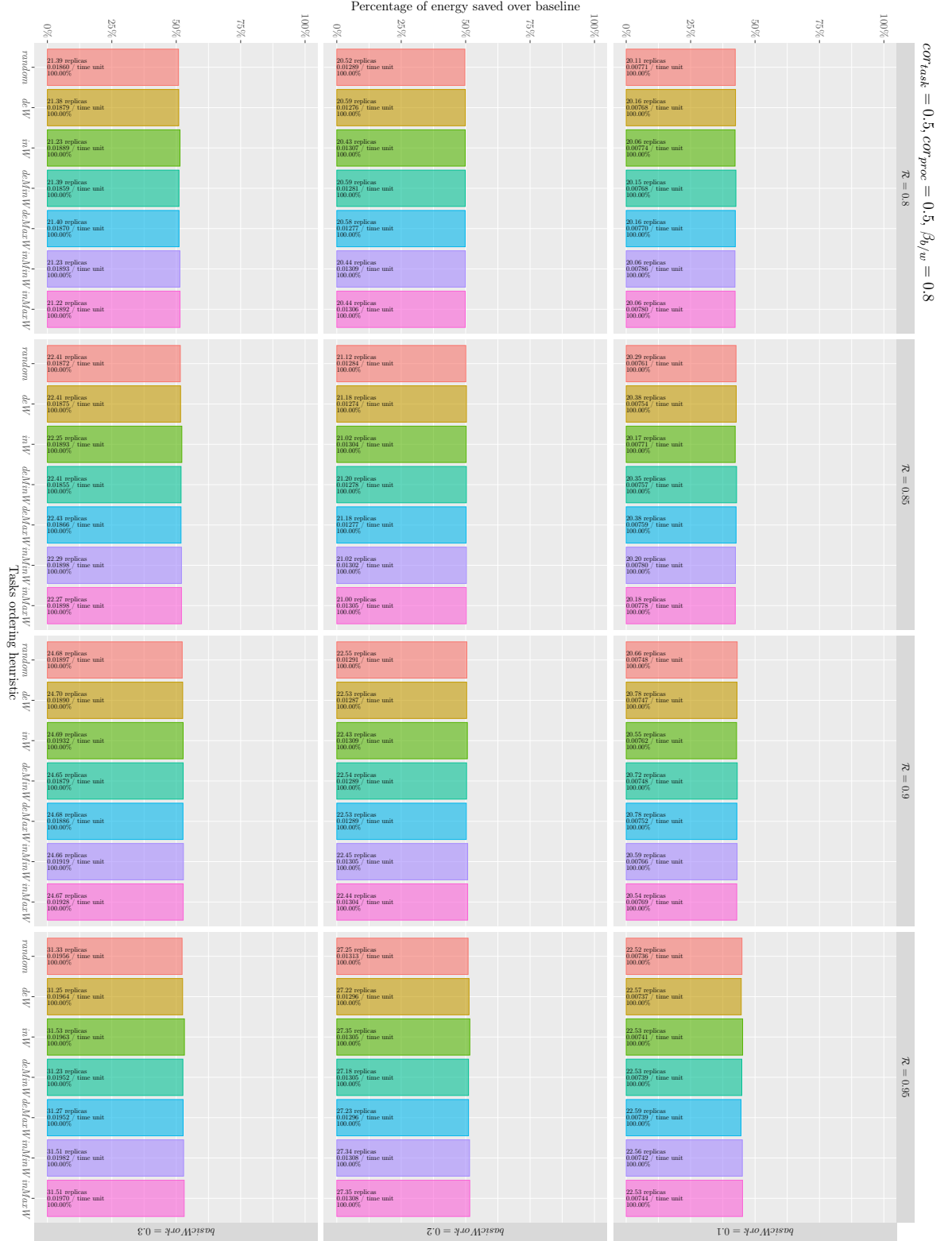


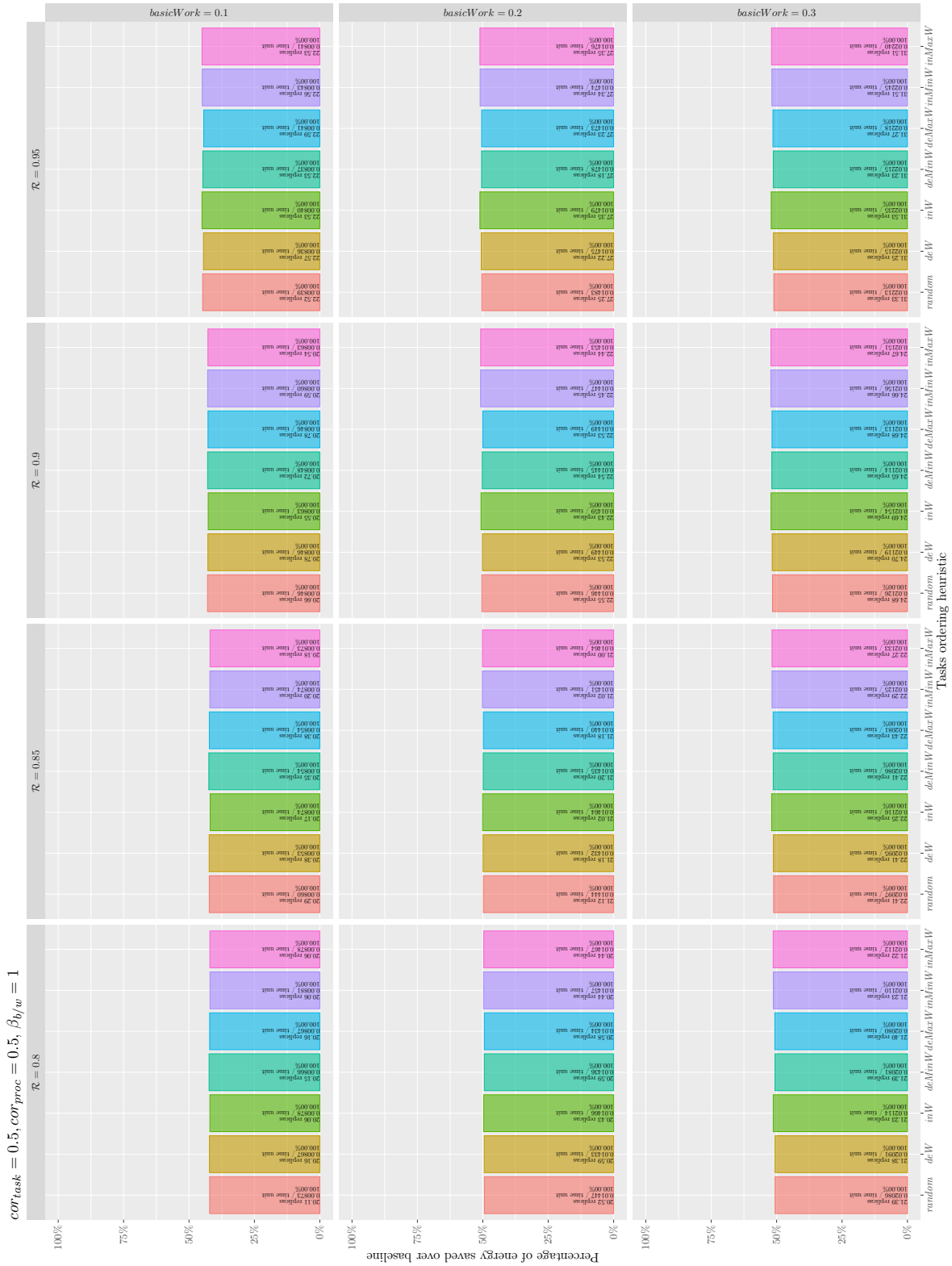






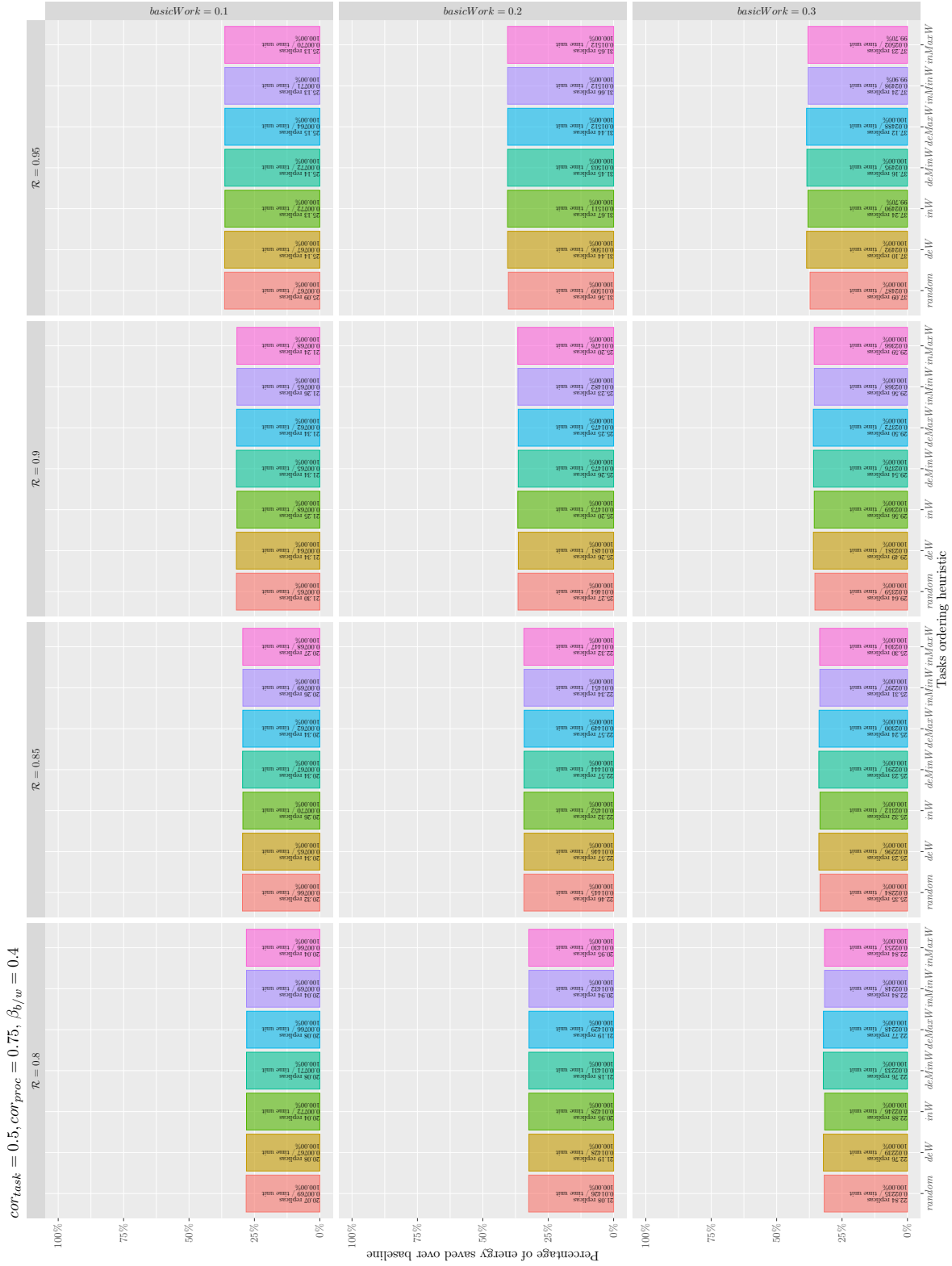






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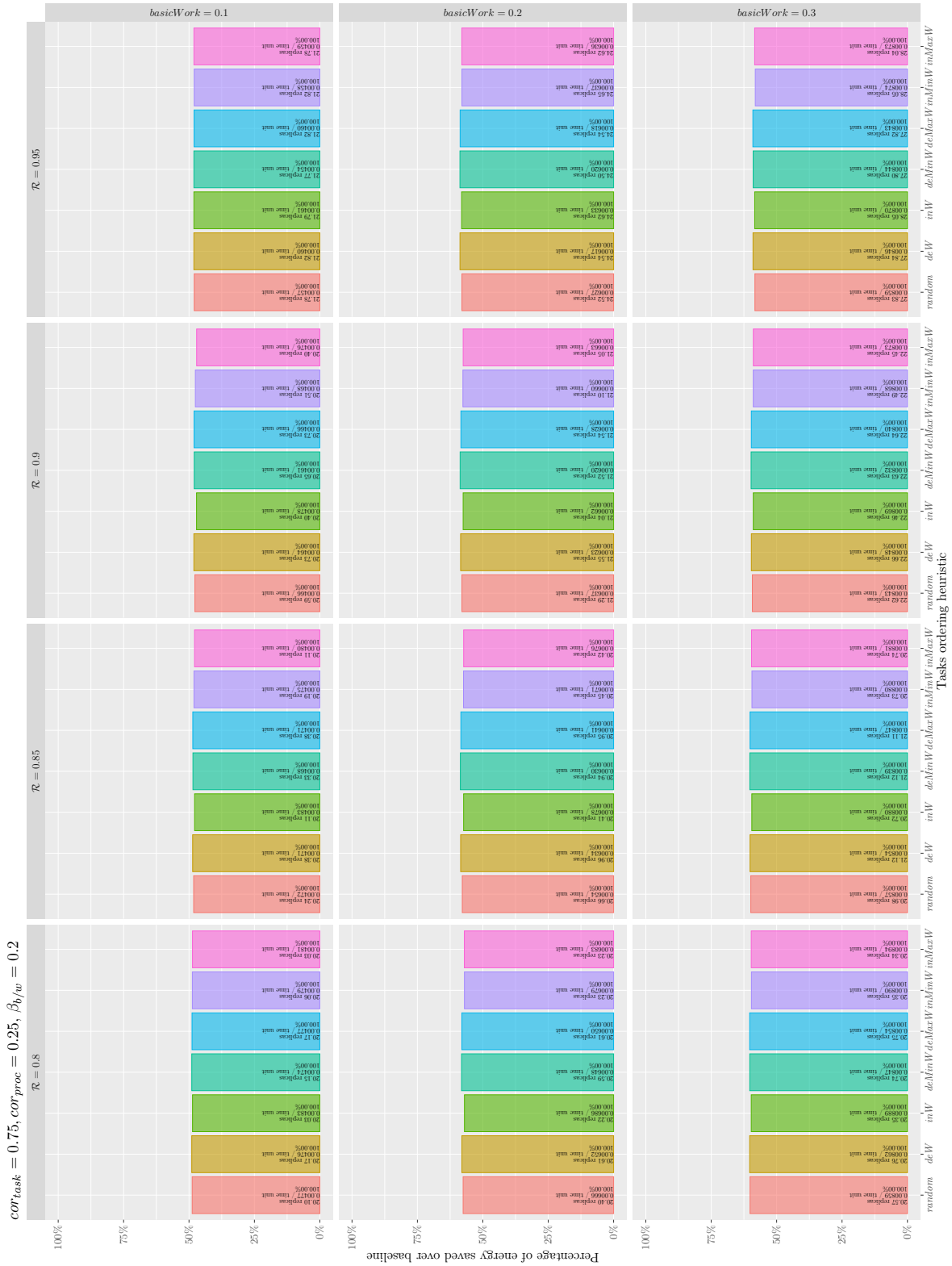




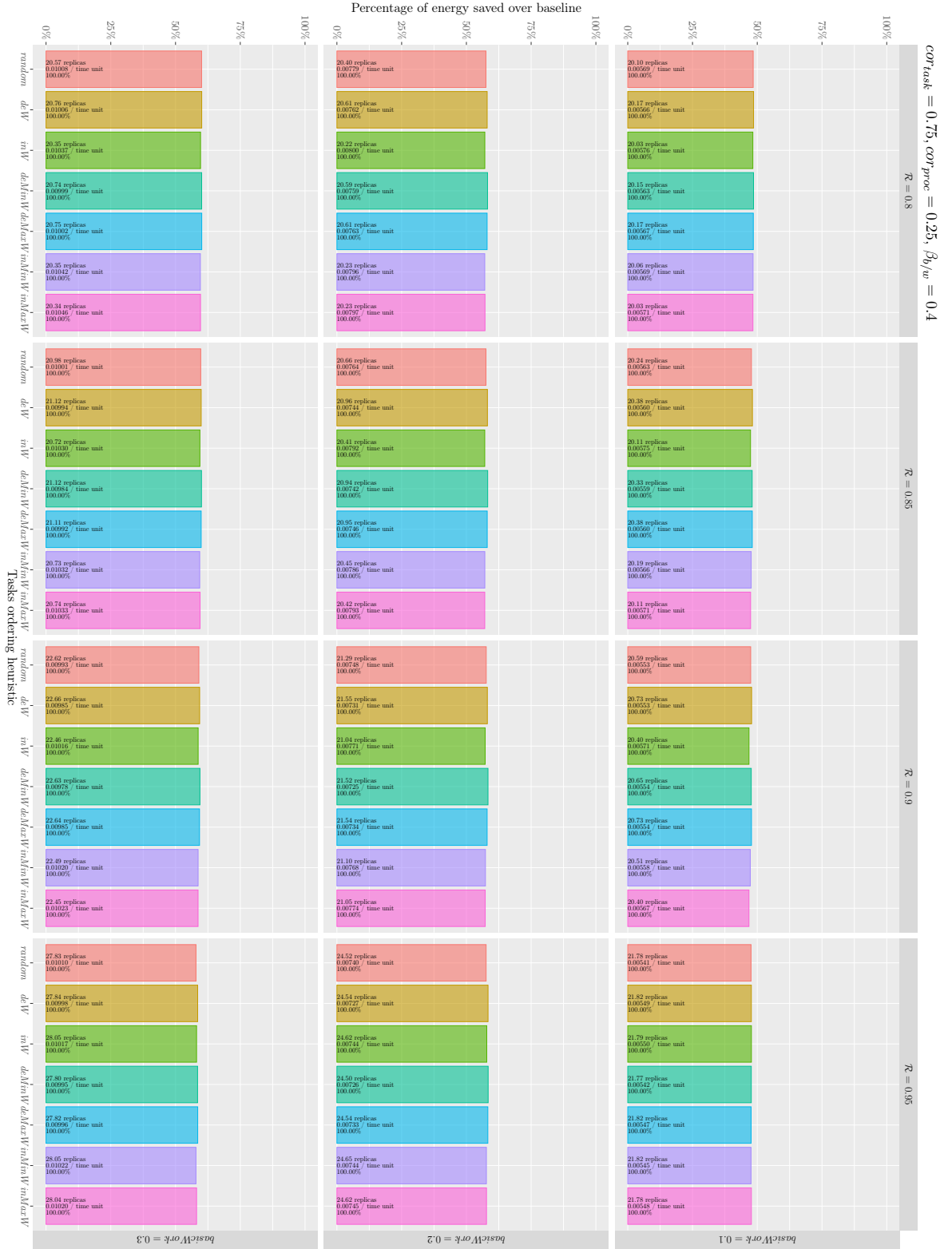


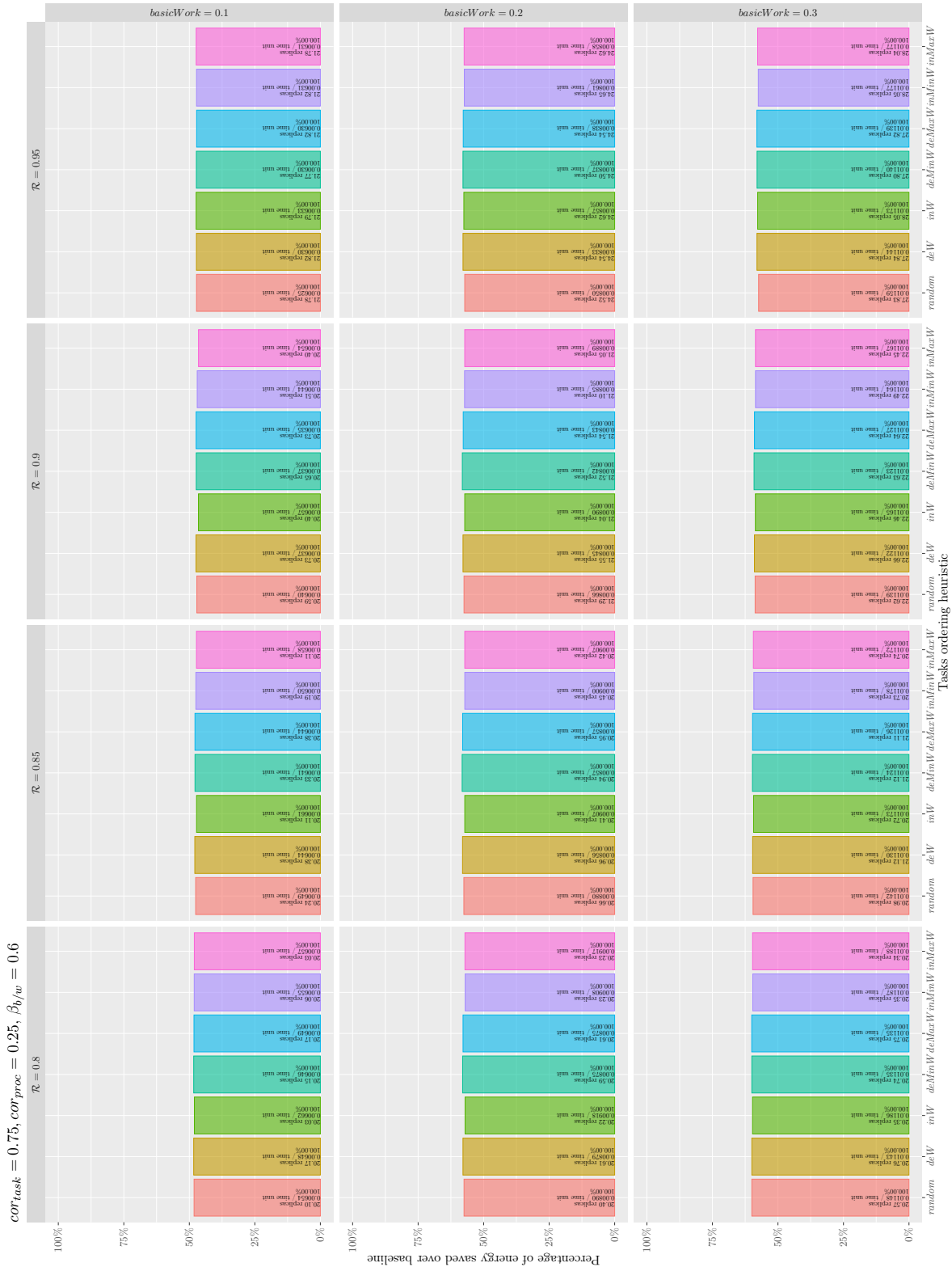


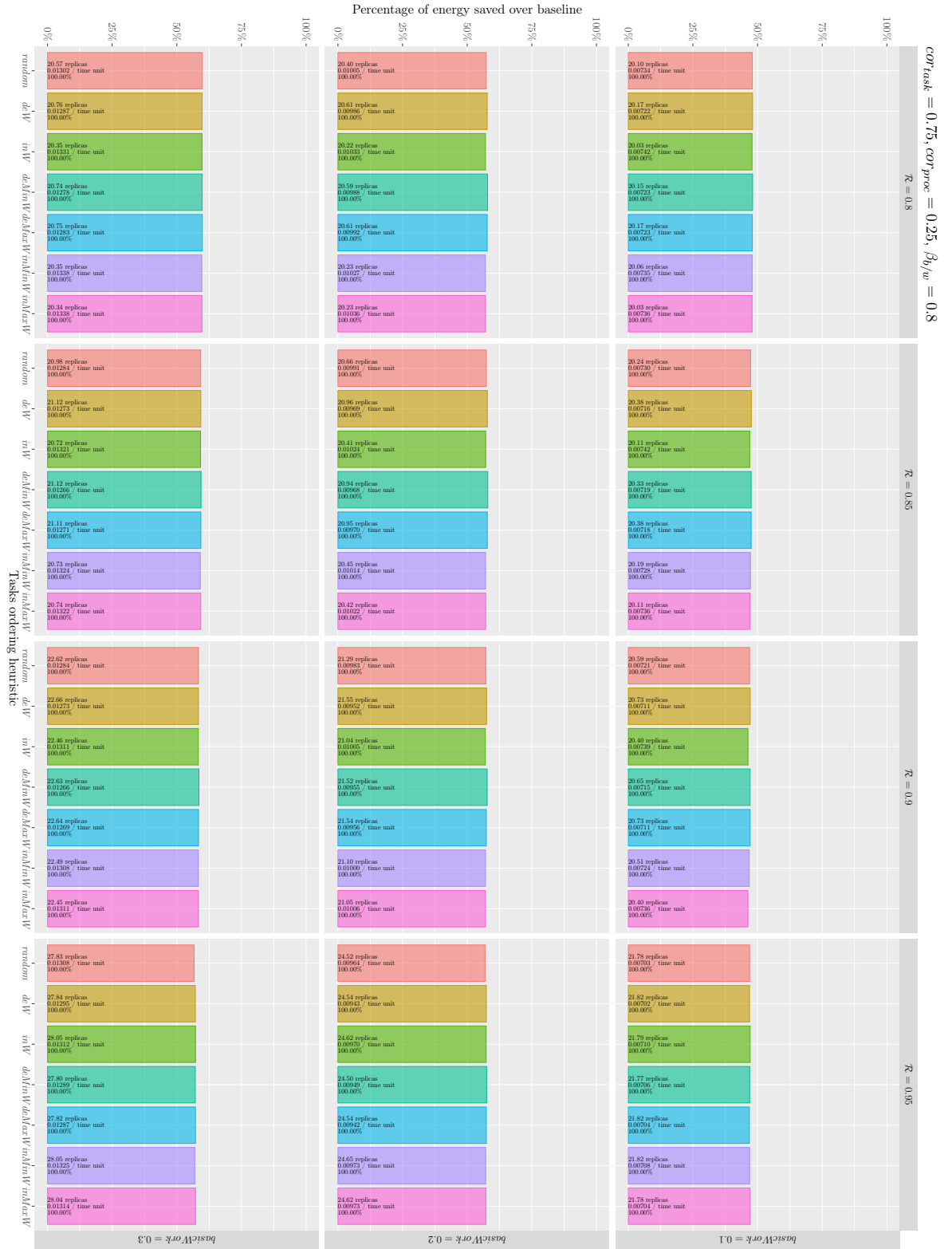


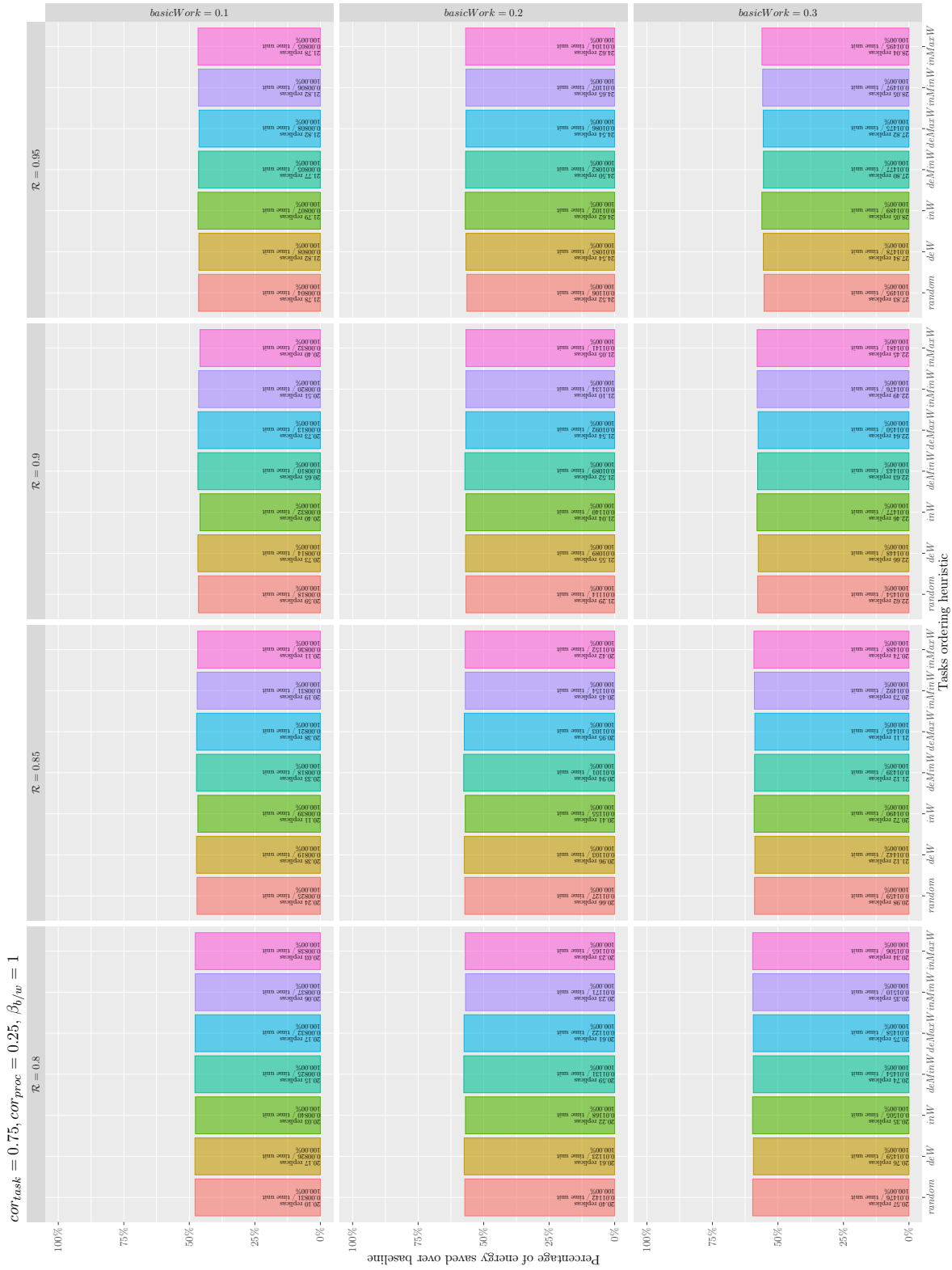




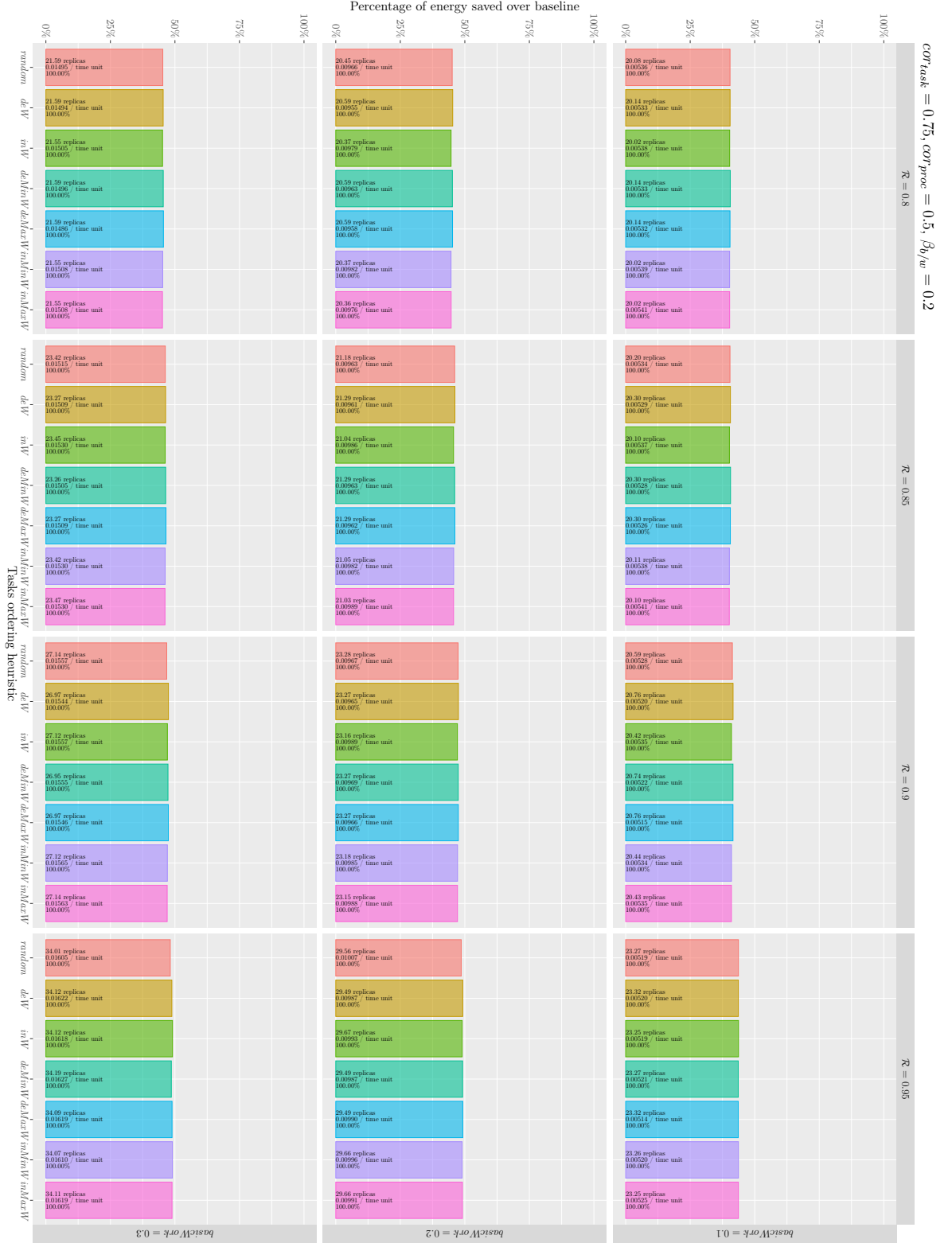


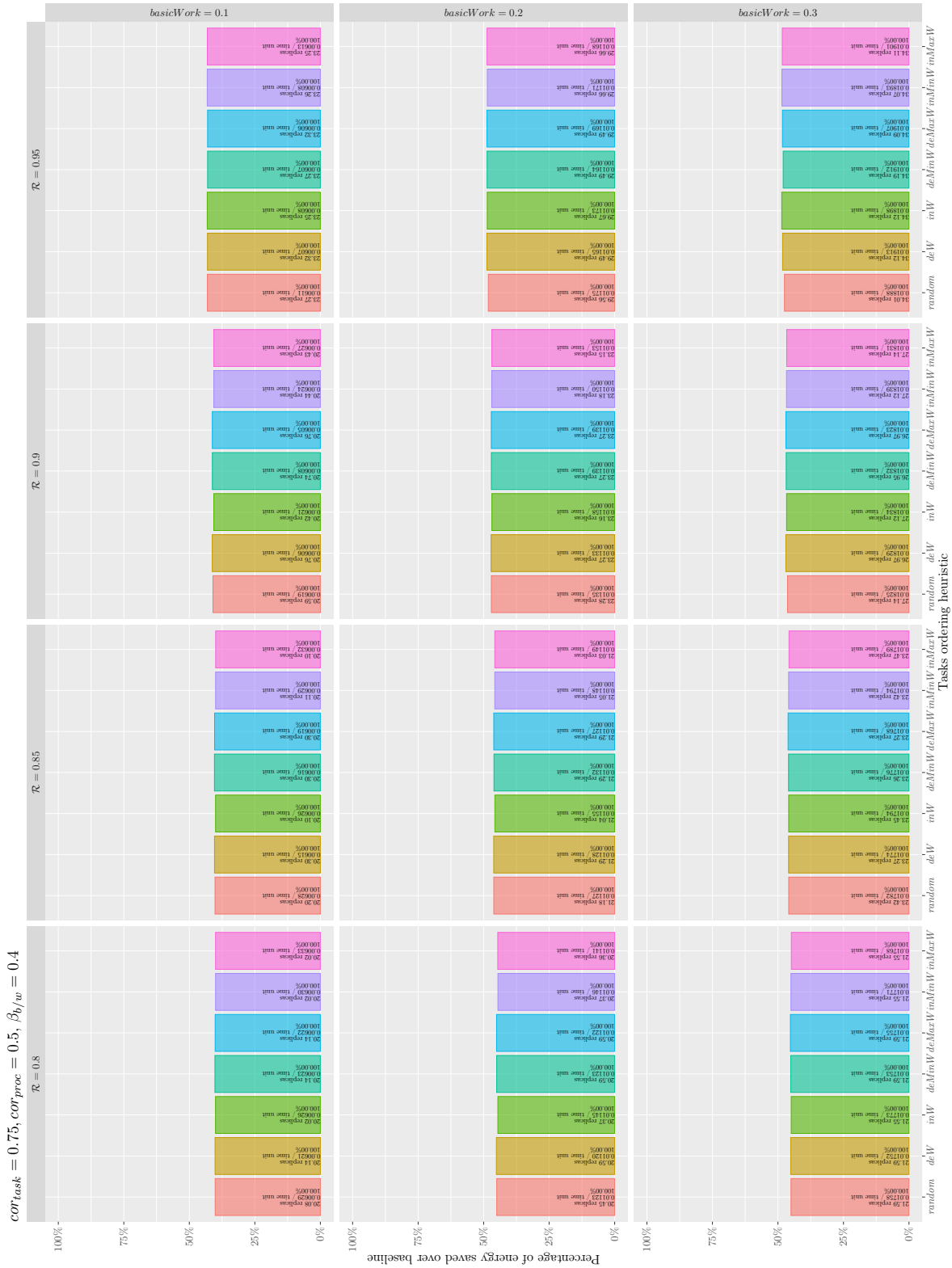


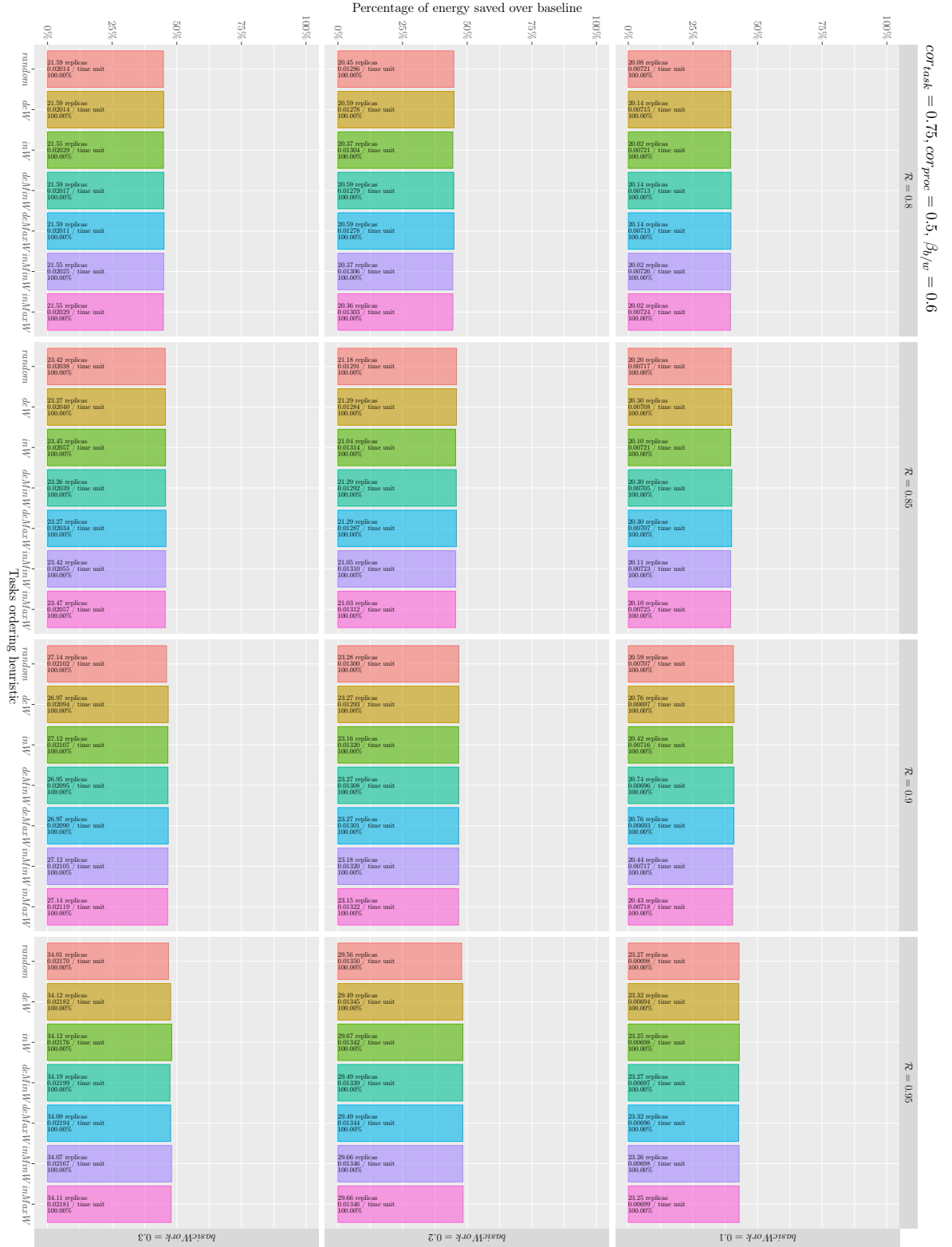


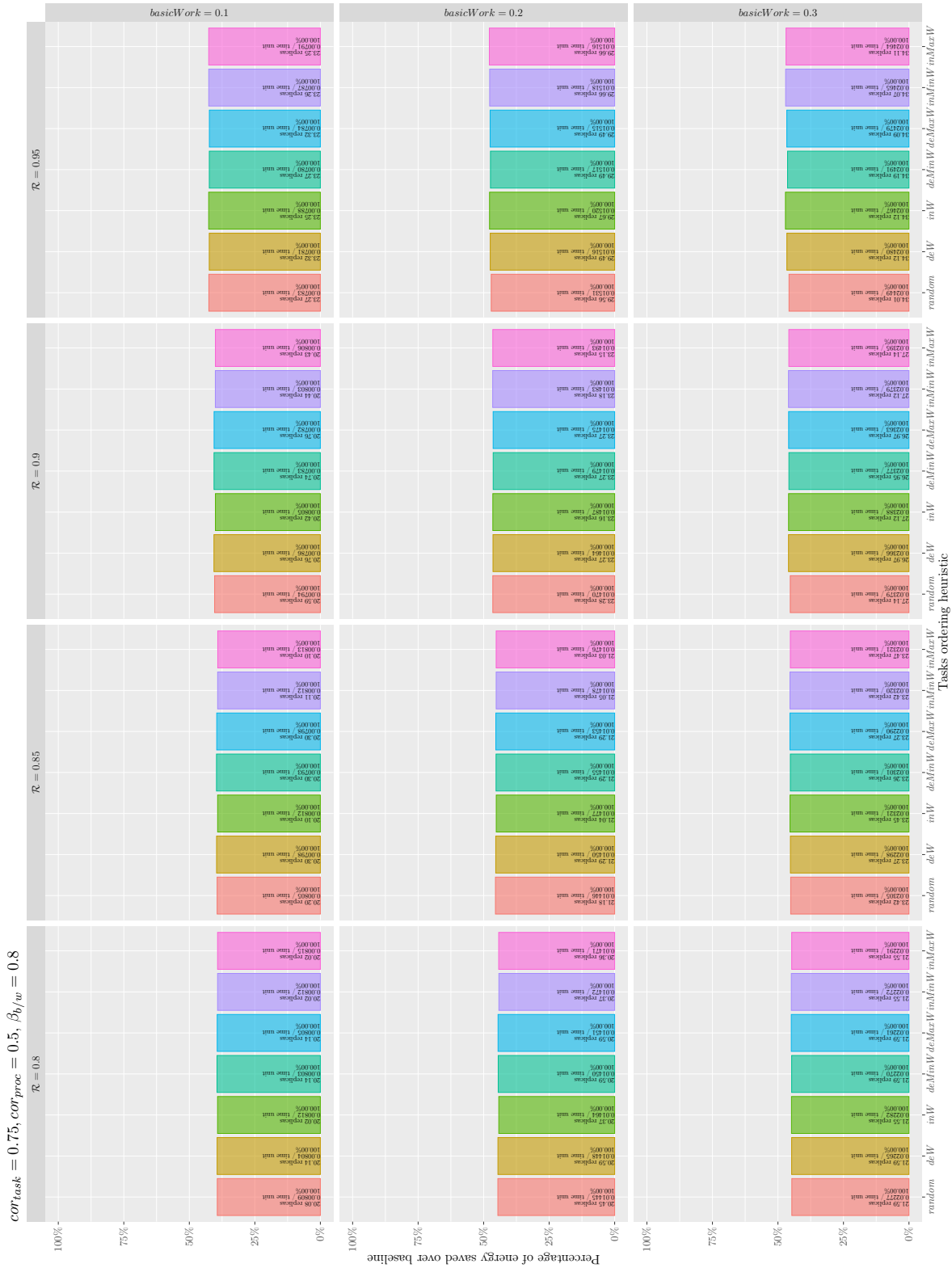


$cot_{task} = 0.75, cot_{proc} = 0.25, \beta_{hw} = 1$



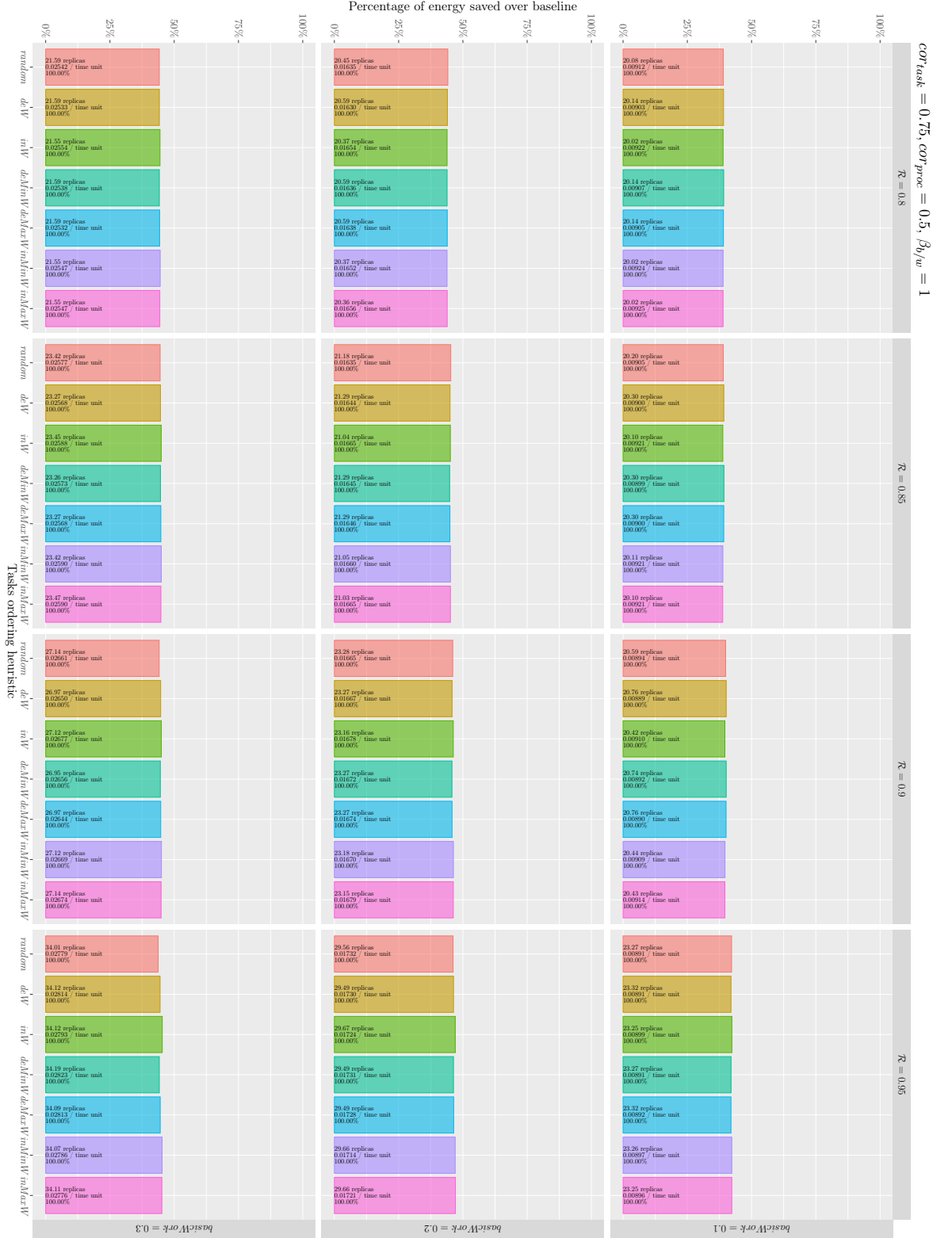


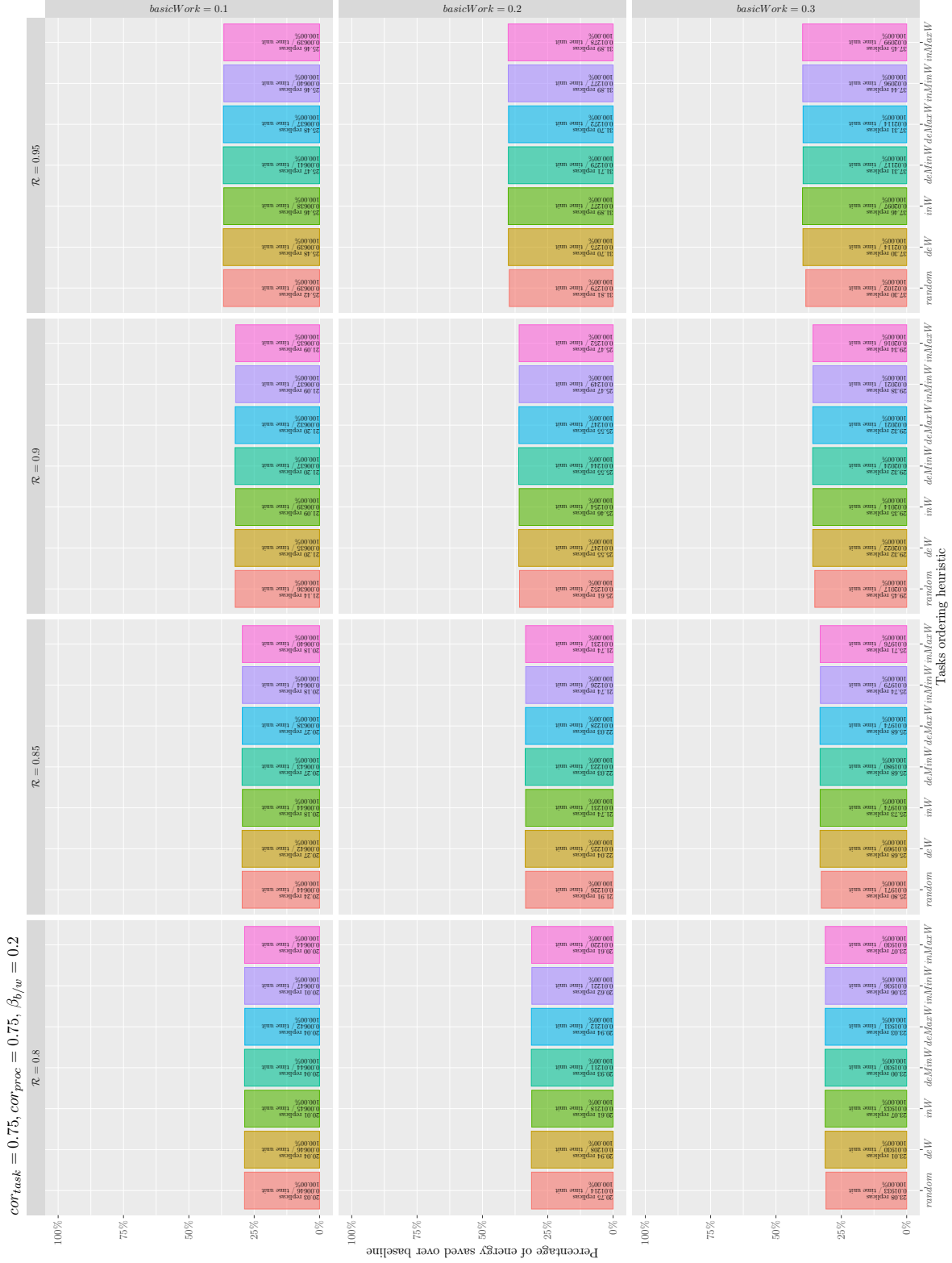




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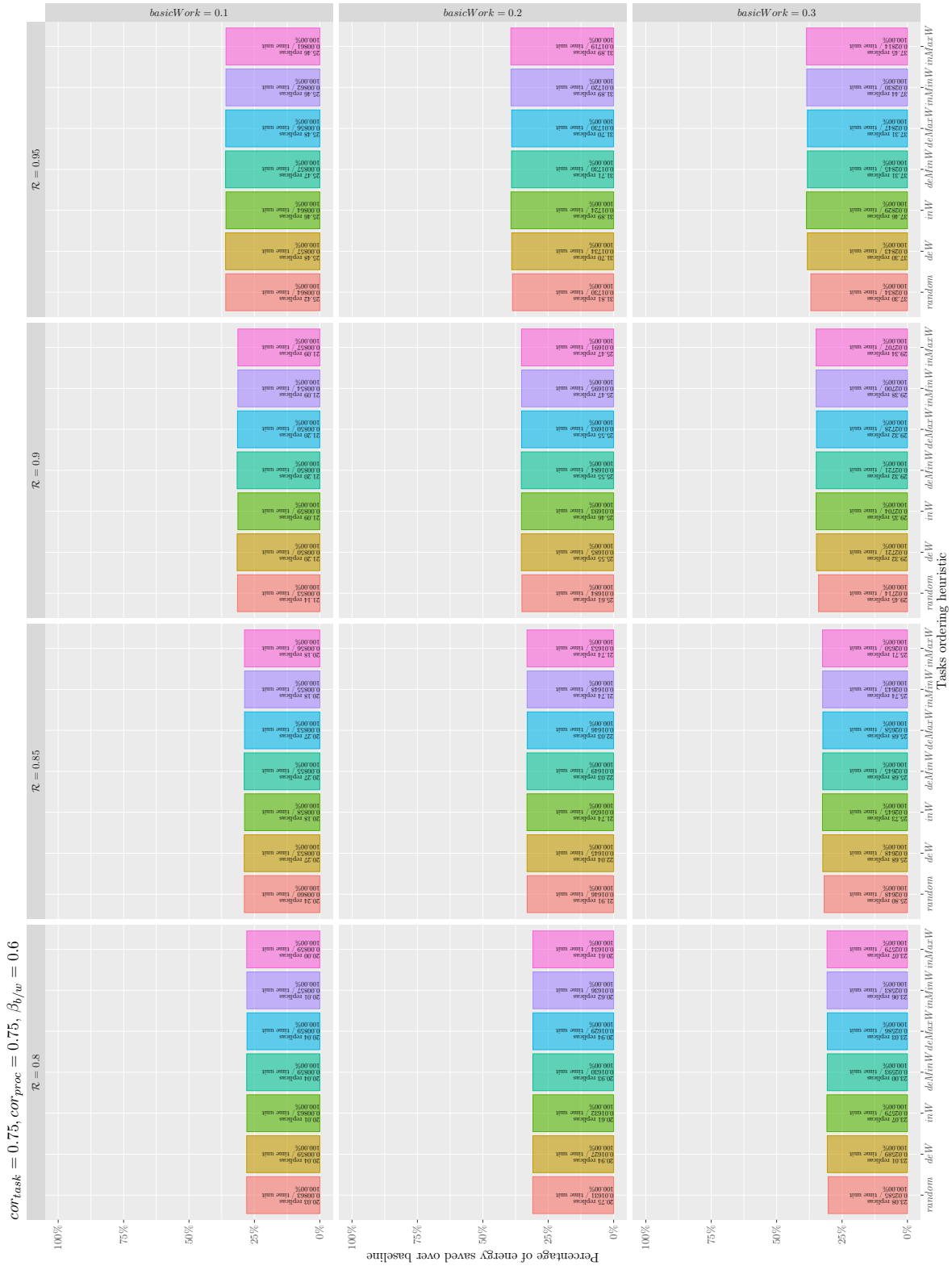


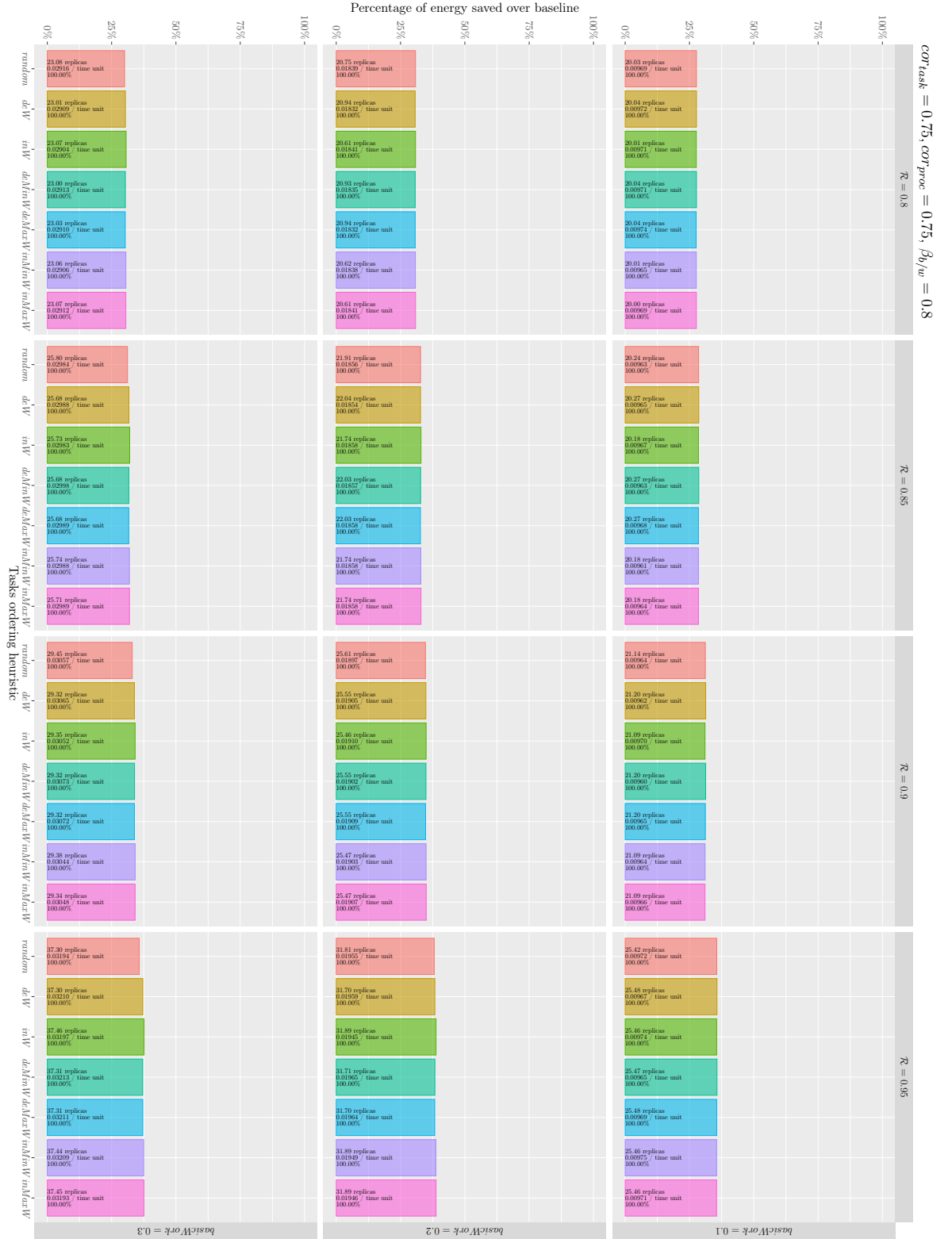




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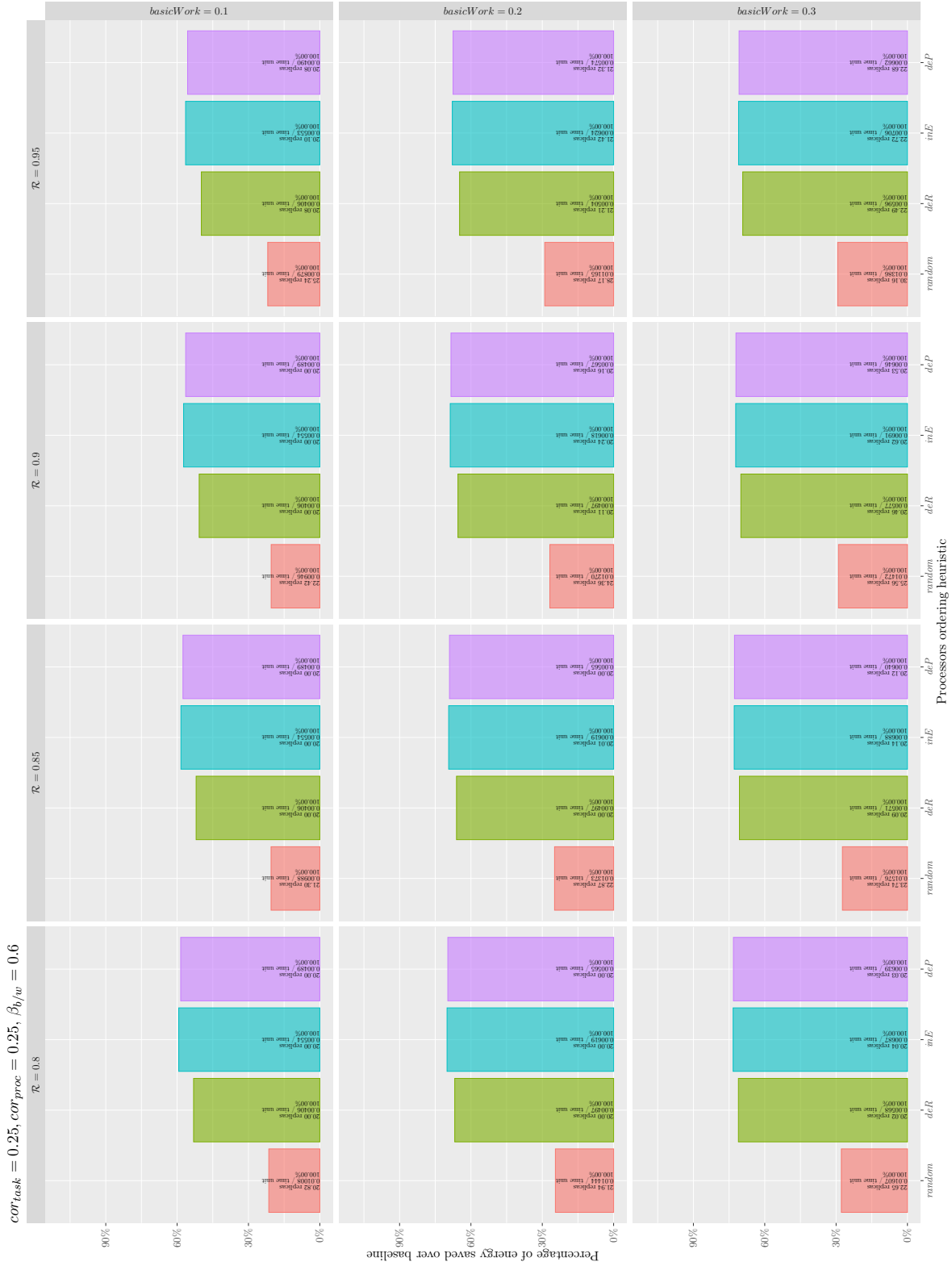
## **Processor ordering criteria for mapping phase**

The following section presents percentage of energy saved over baseline using different processor ordering criteria under big failure rate.















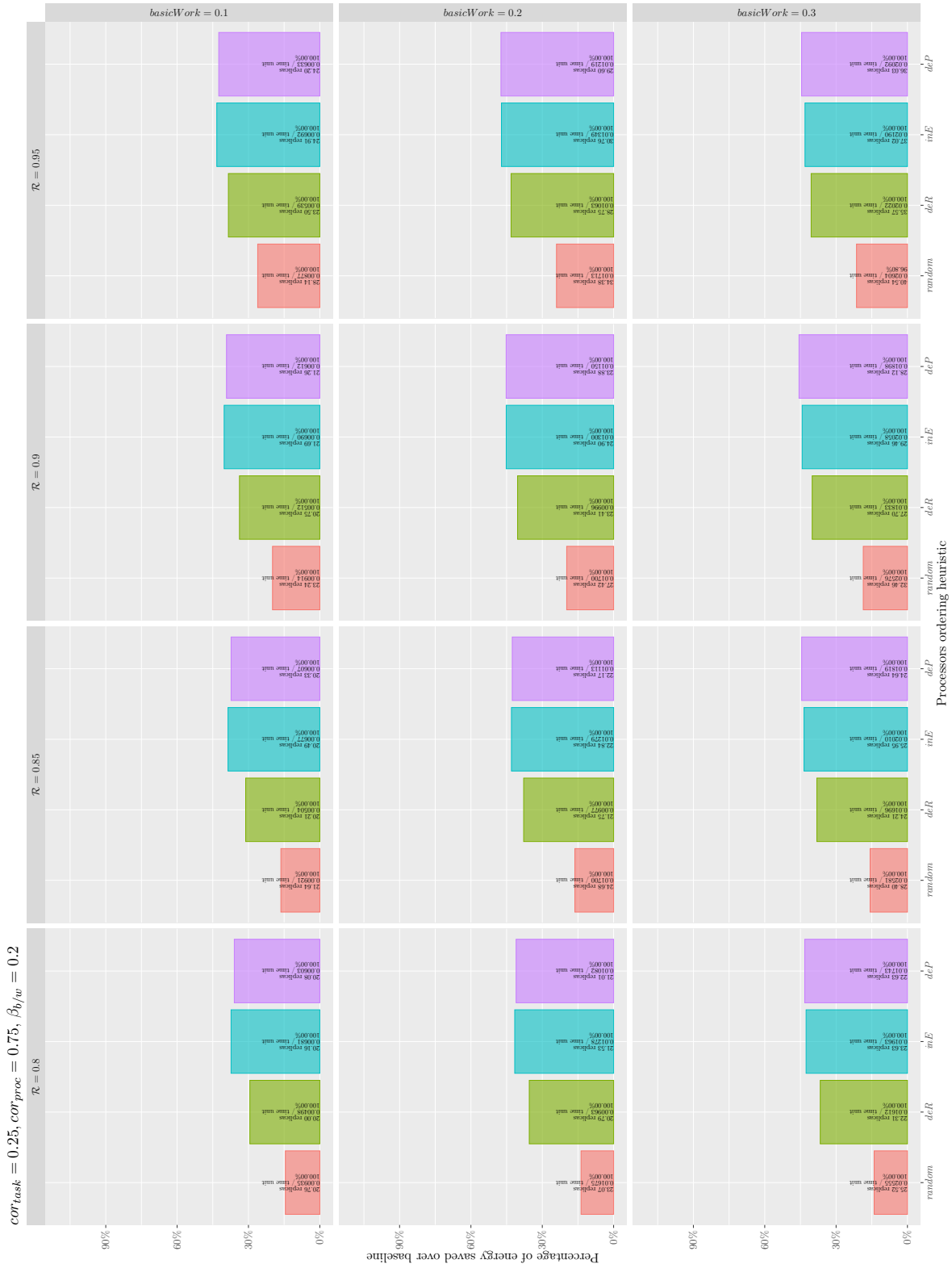




































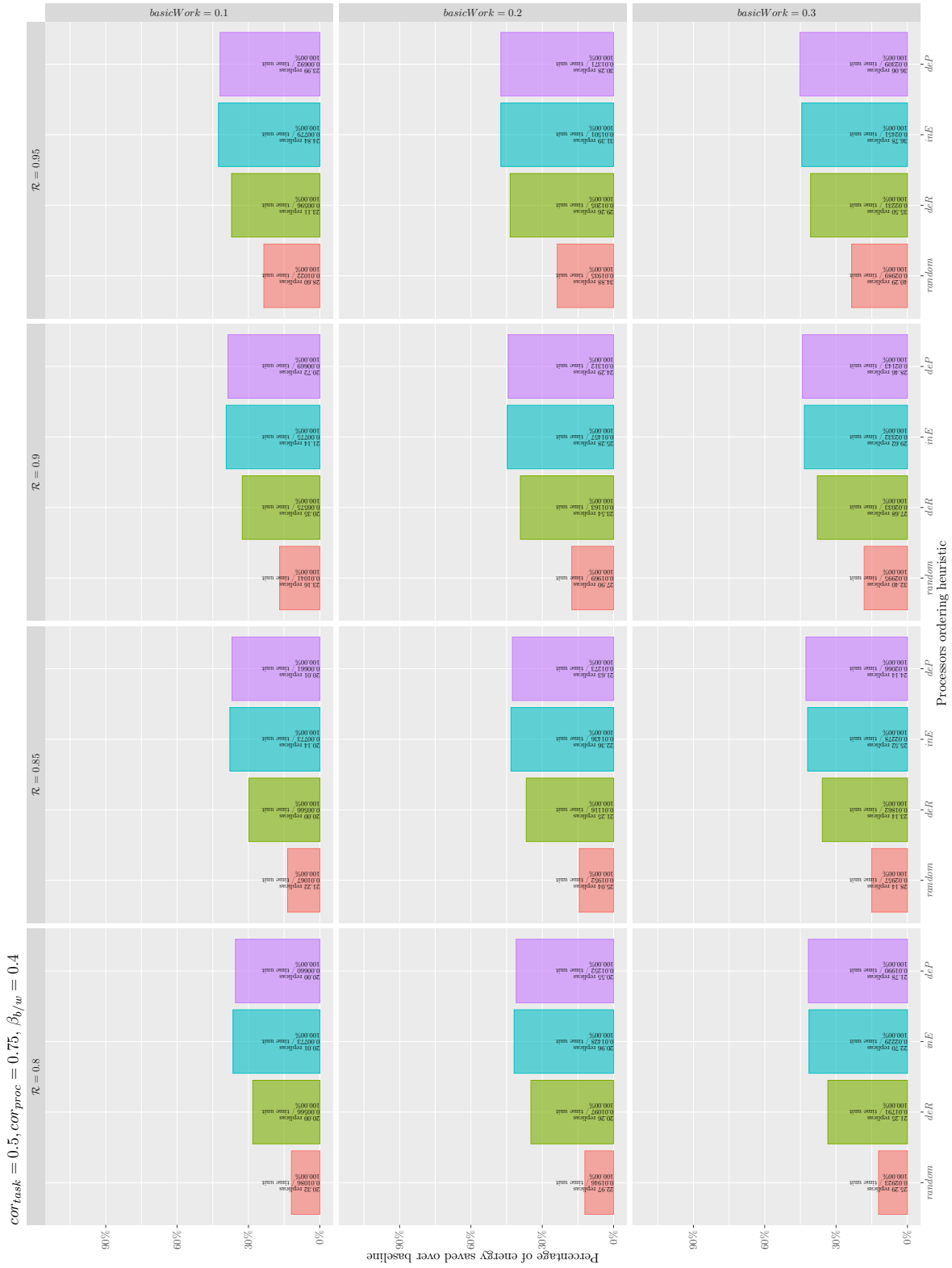




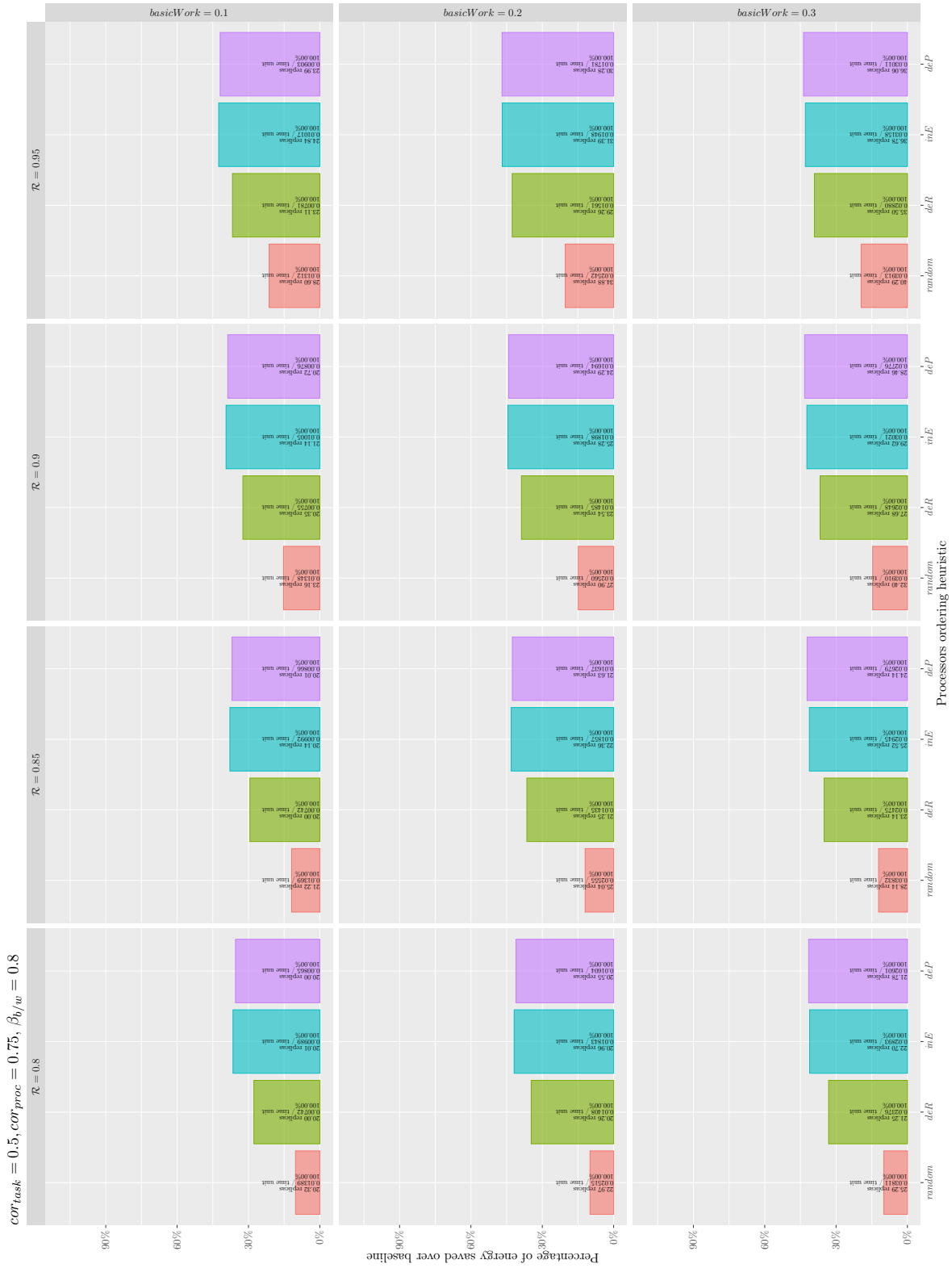


























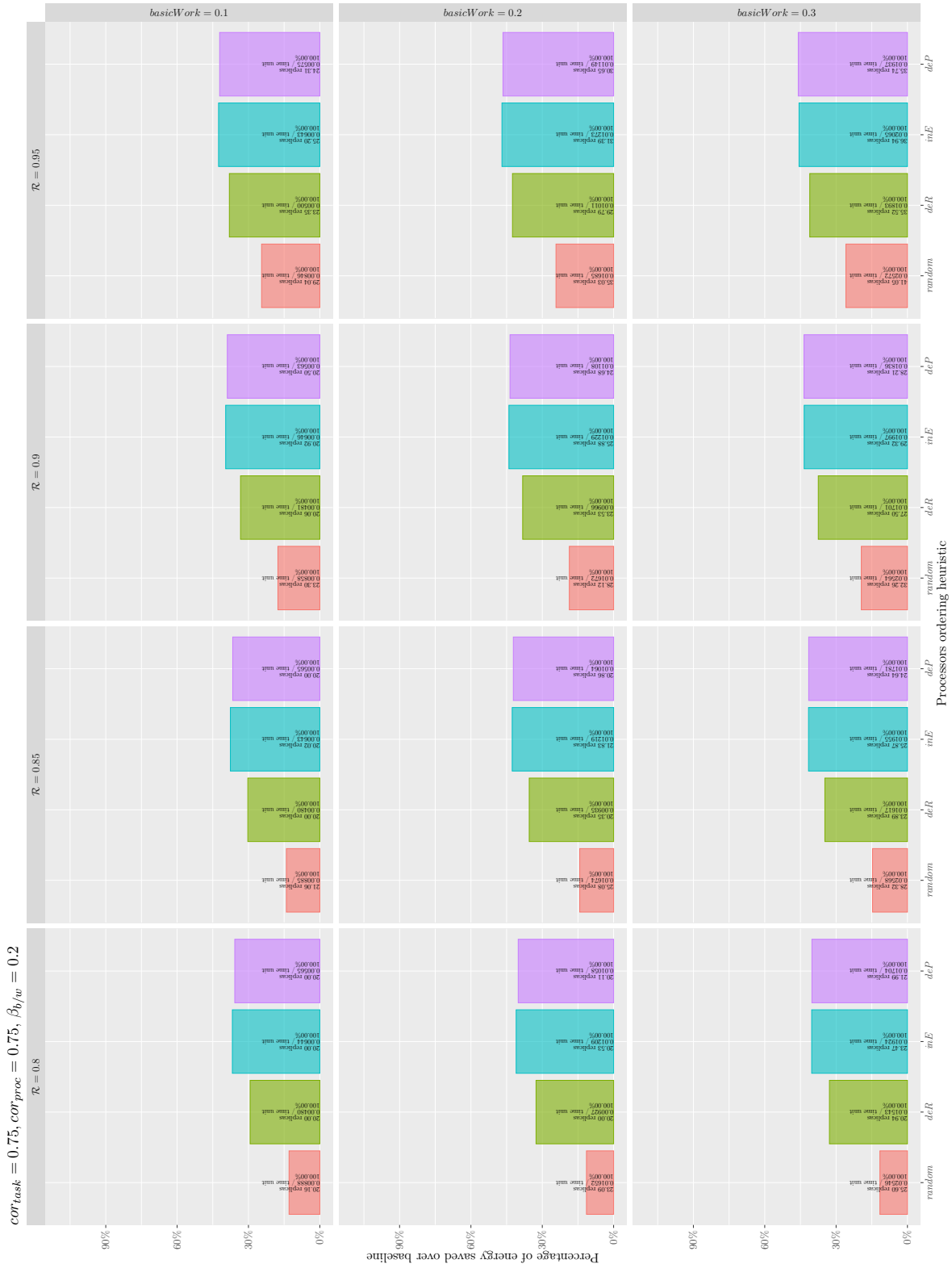






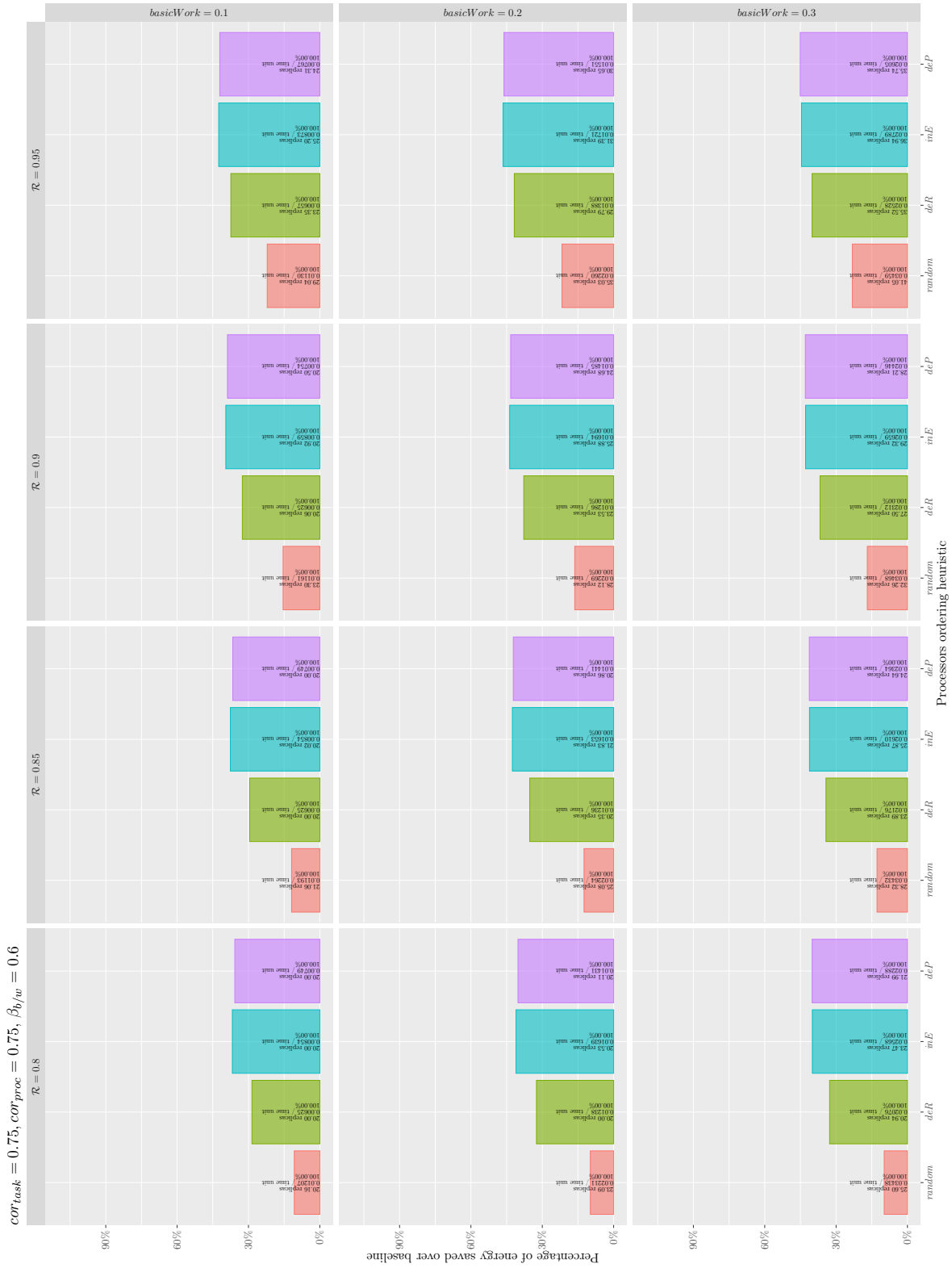












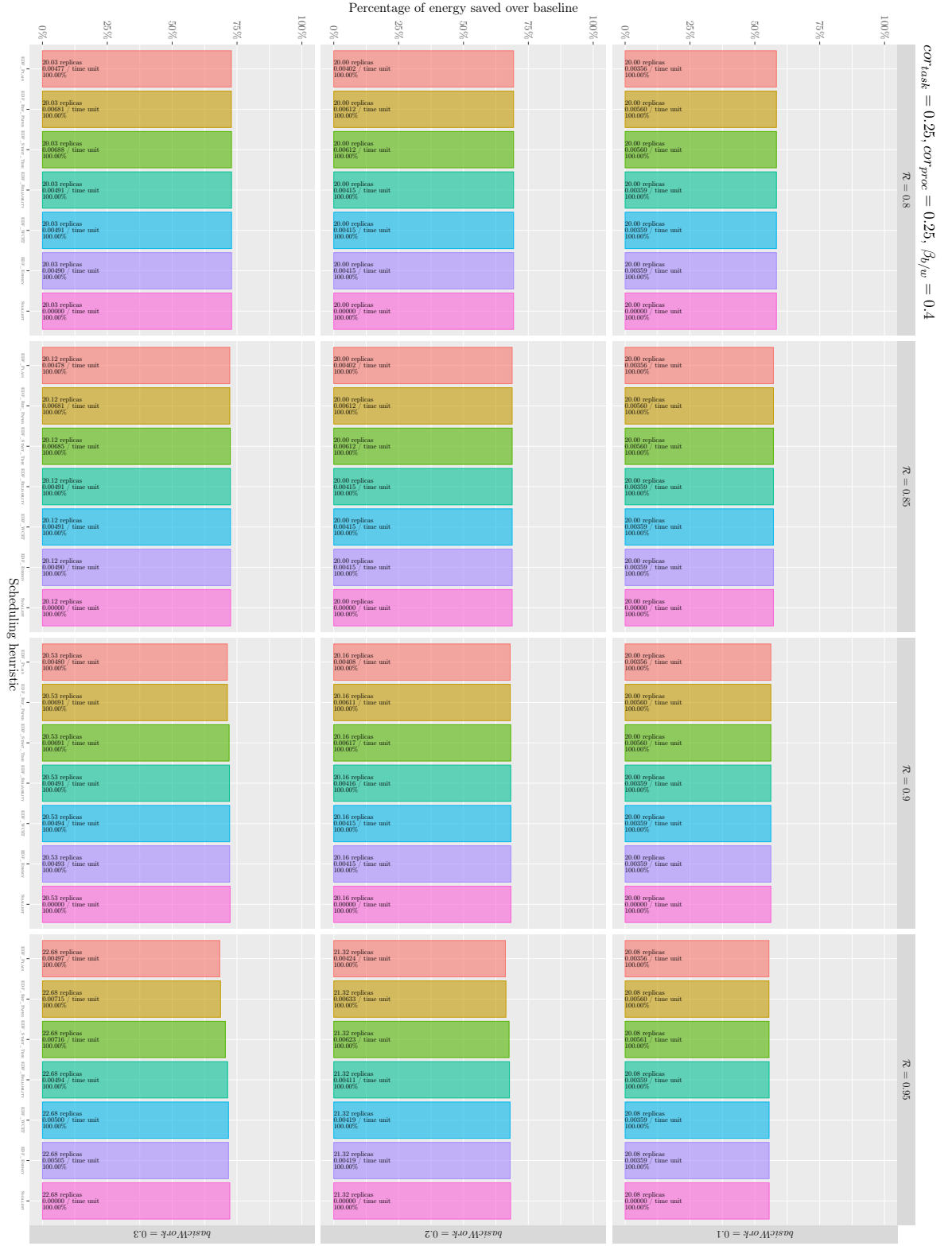


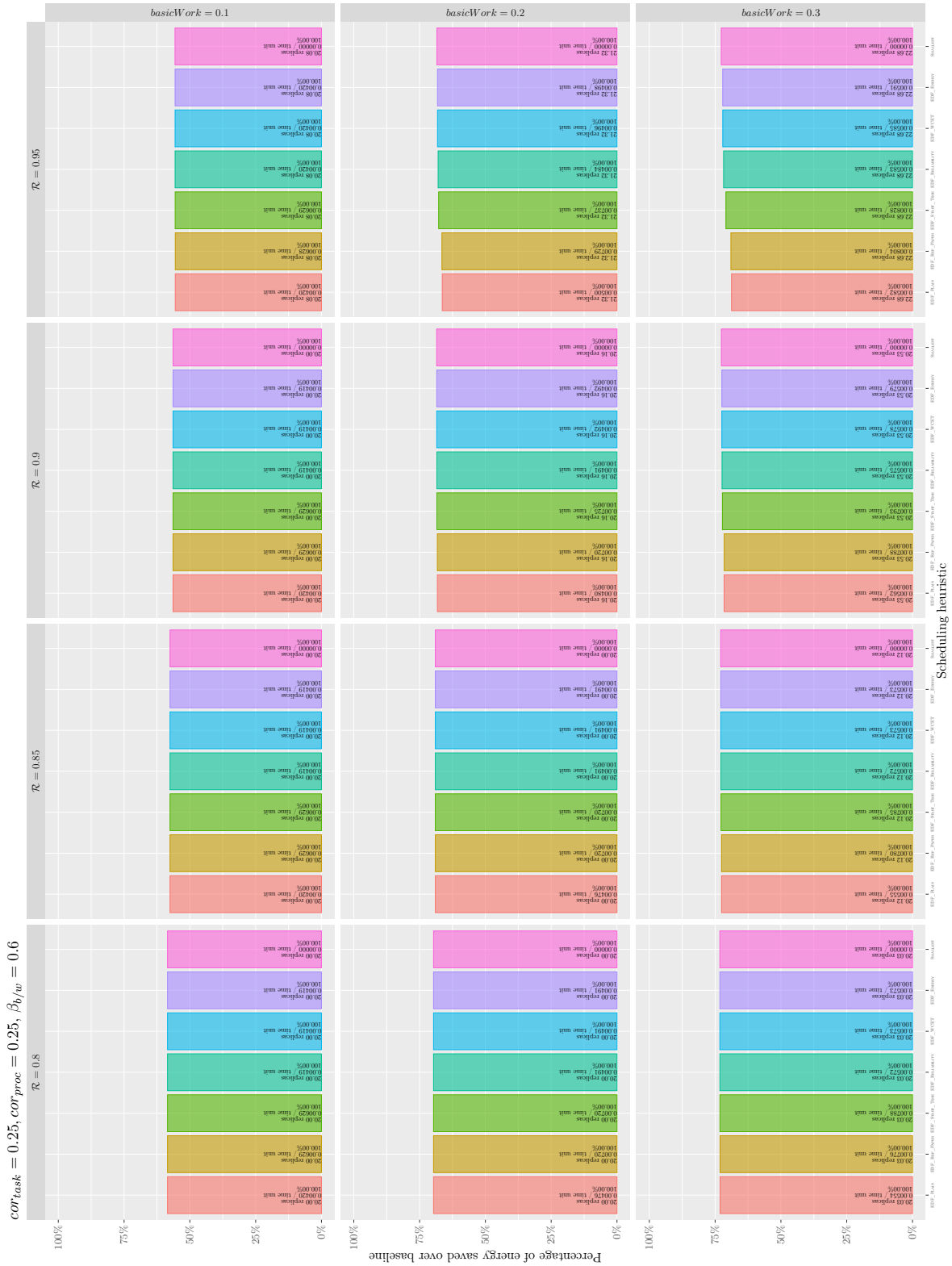


## **Scheduling heuristics**

The following section presents percentage of energy saved over baseline using different scheduling heuristics under big failure rate.

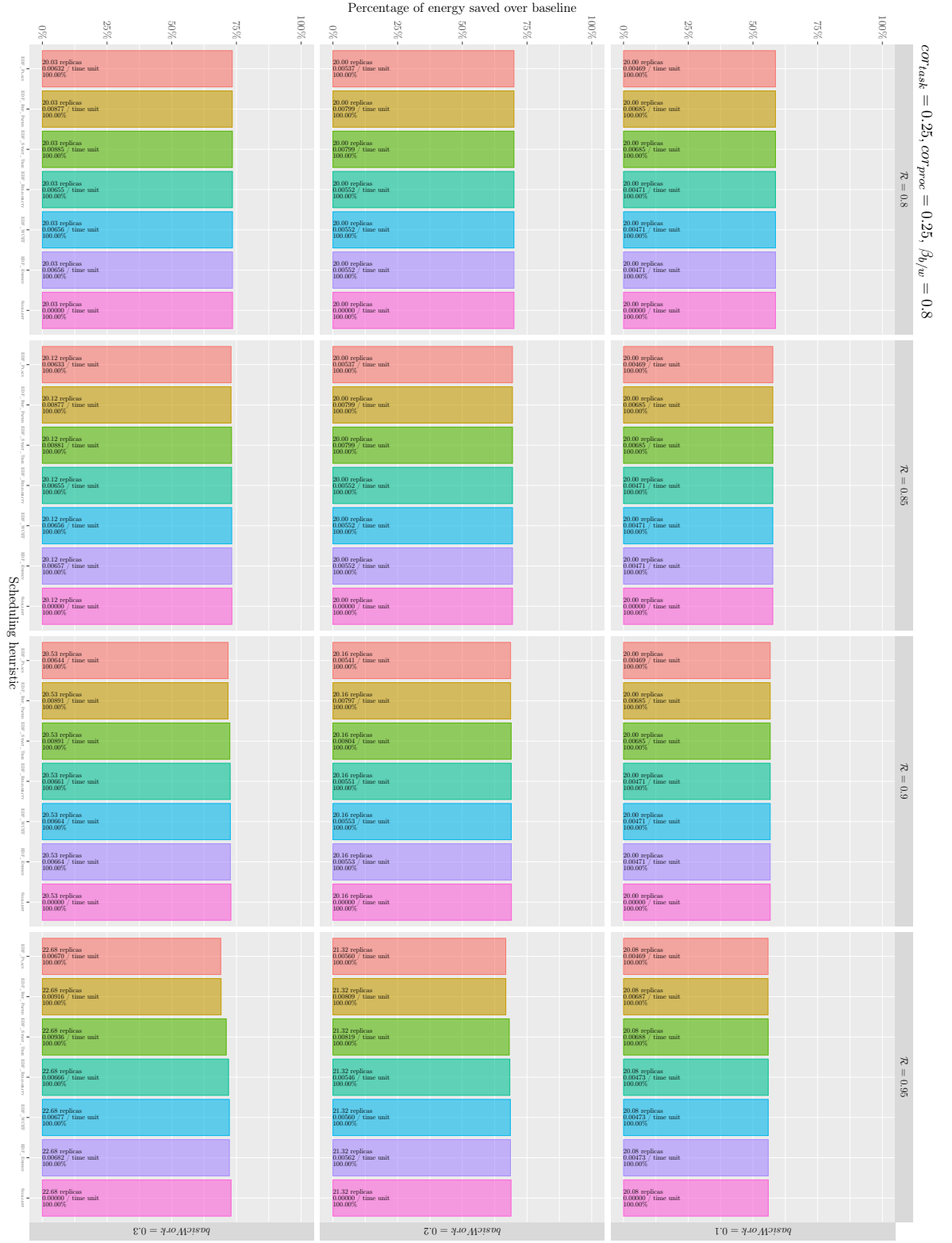


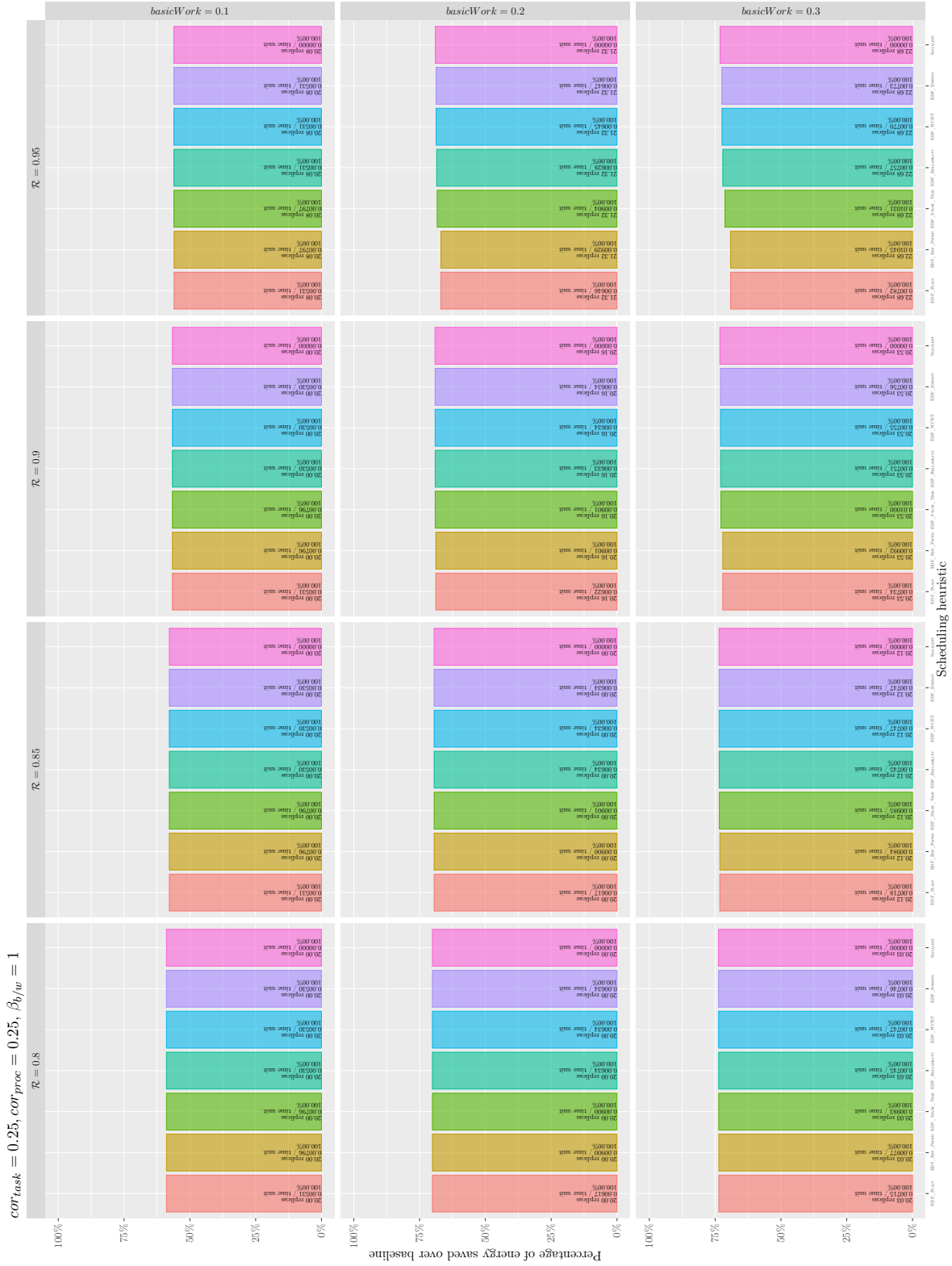


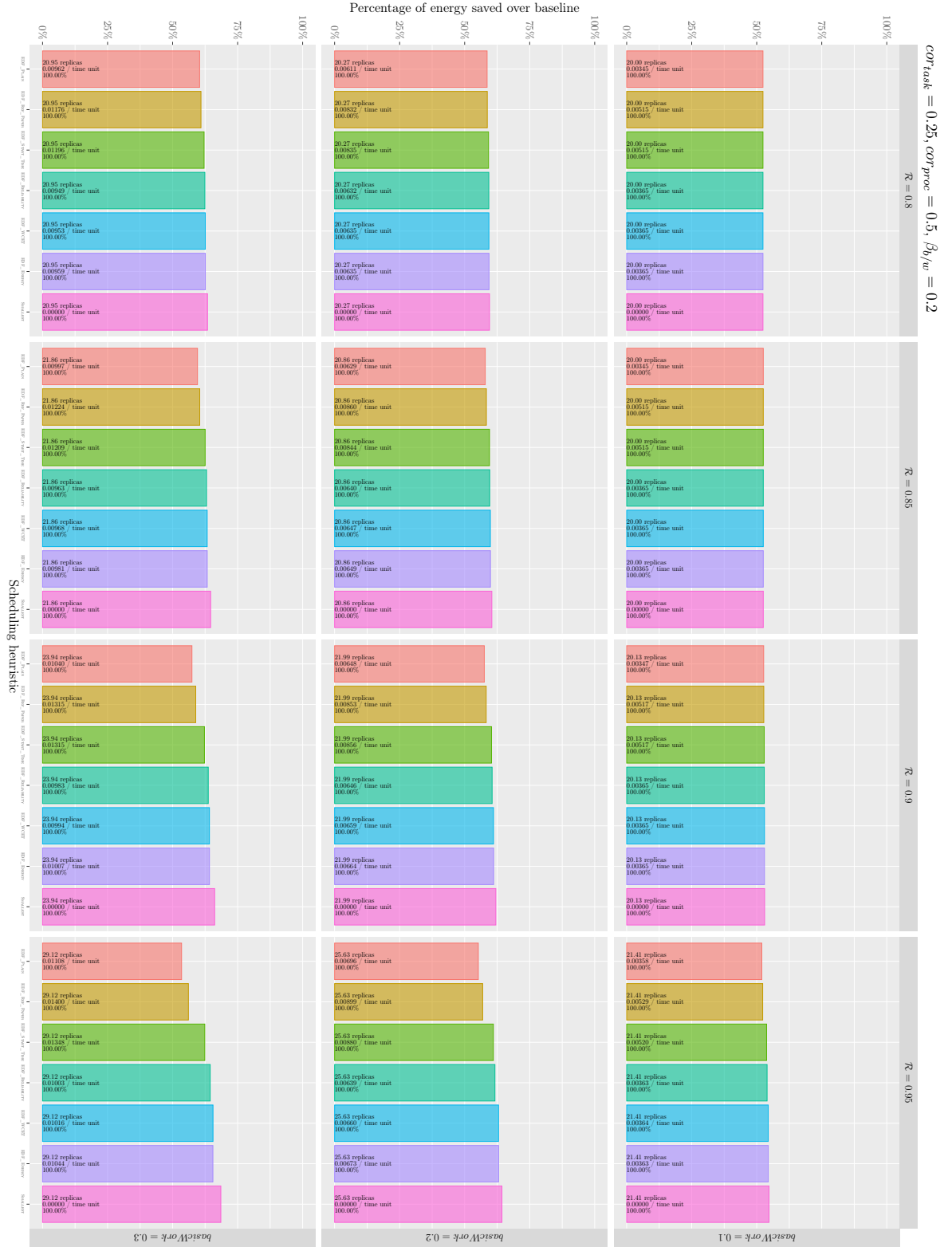


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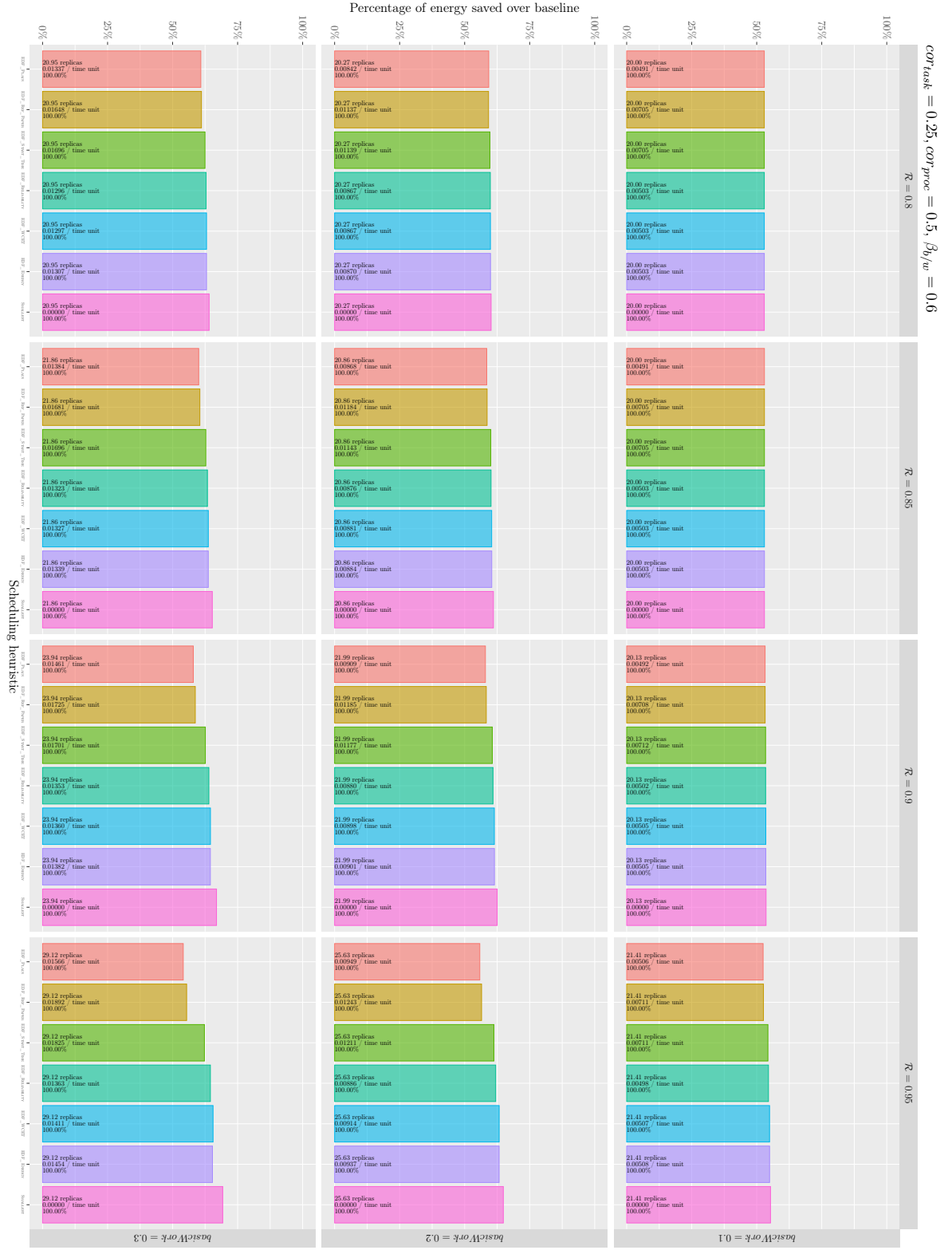


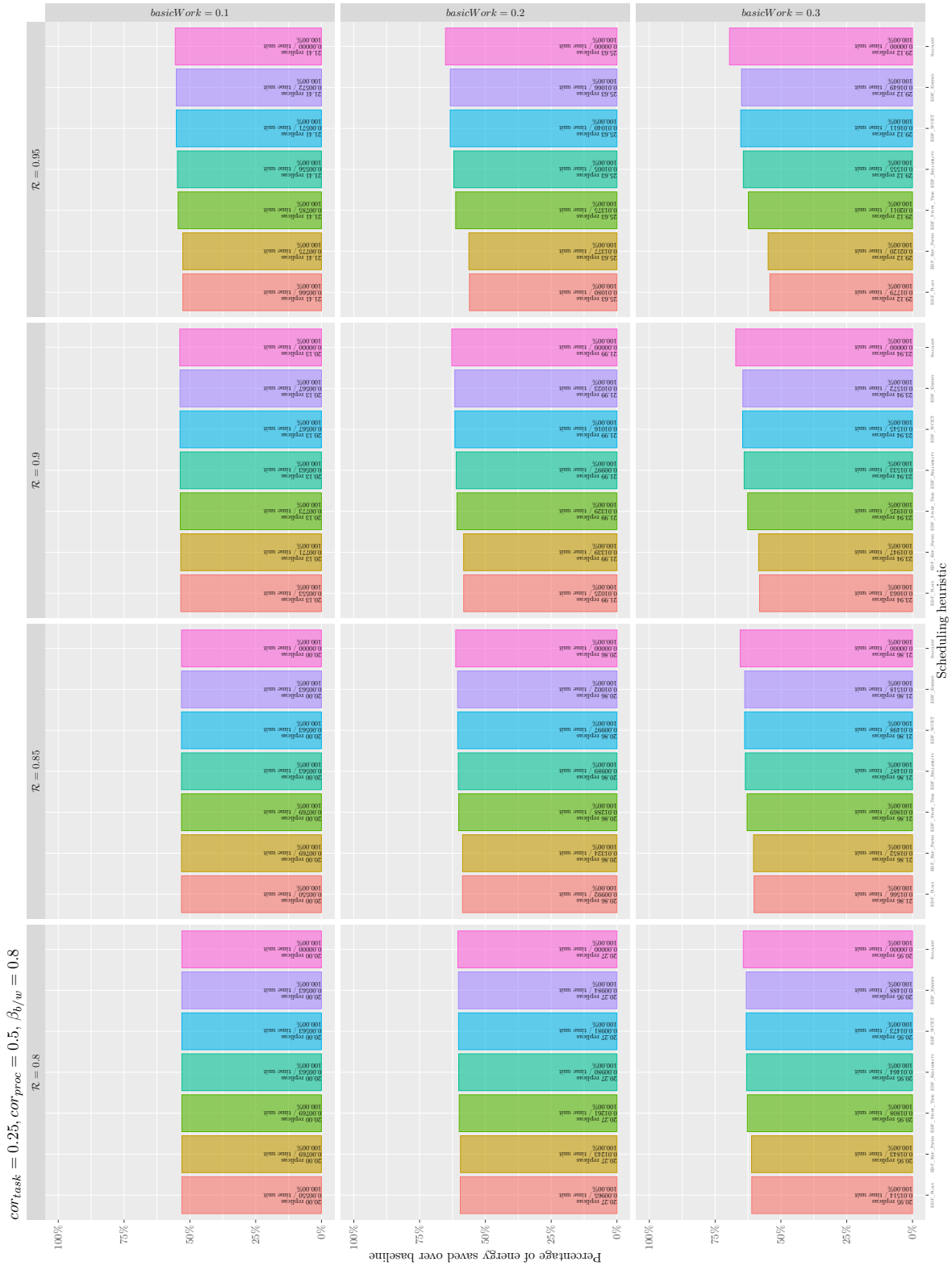


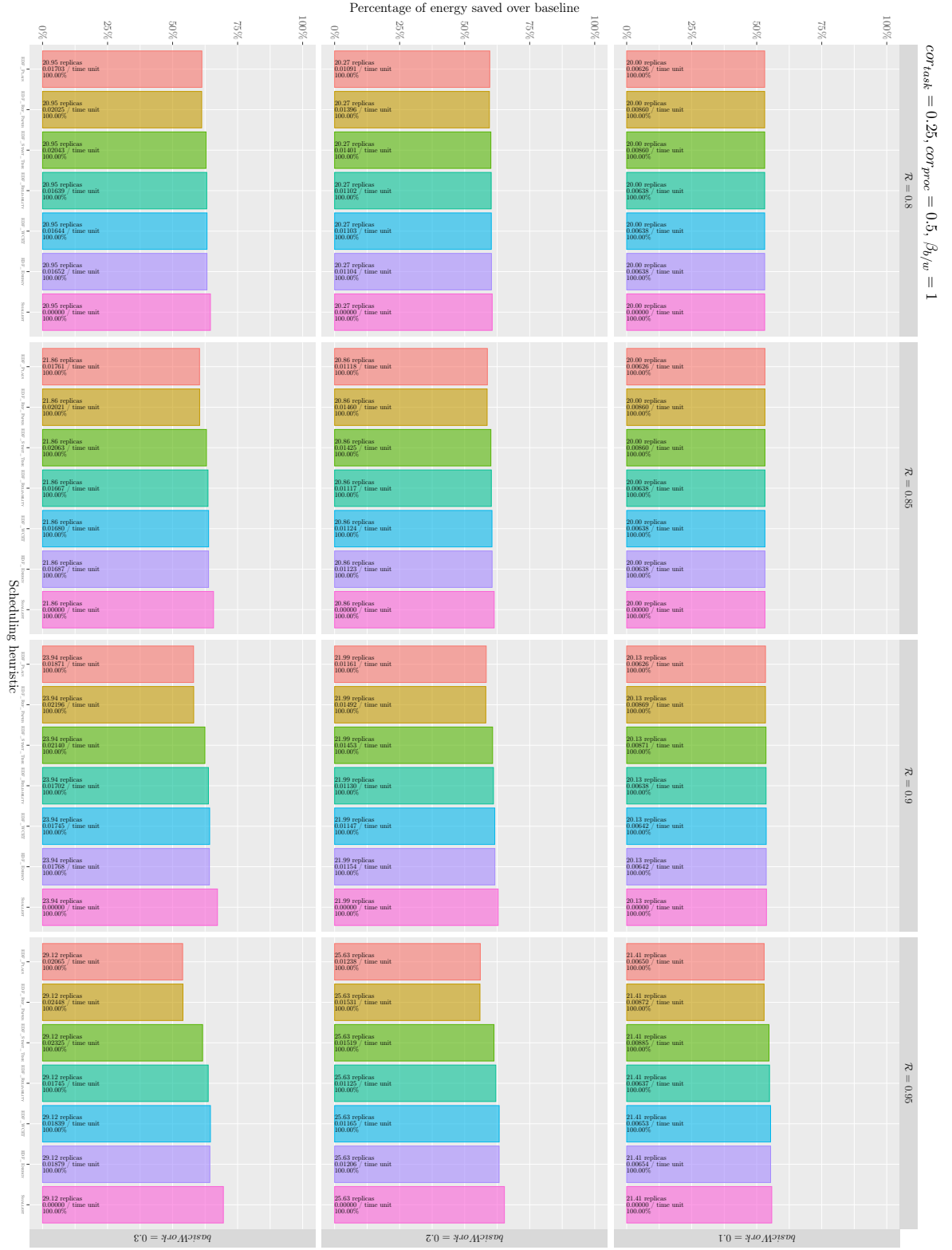


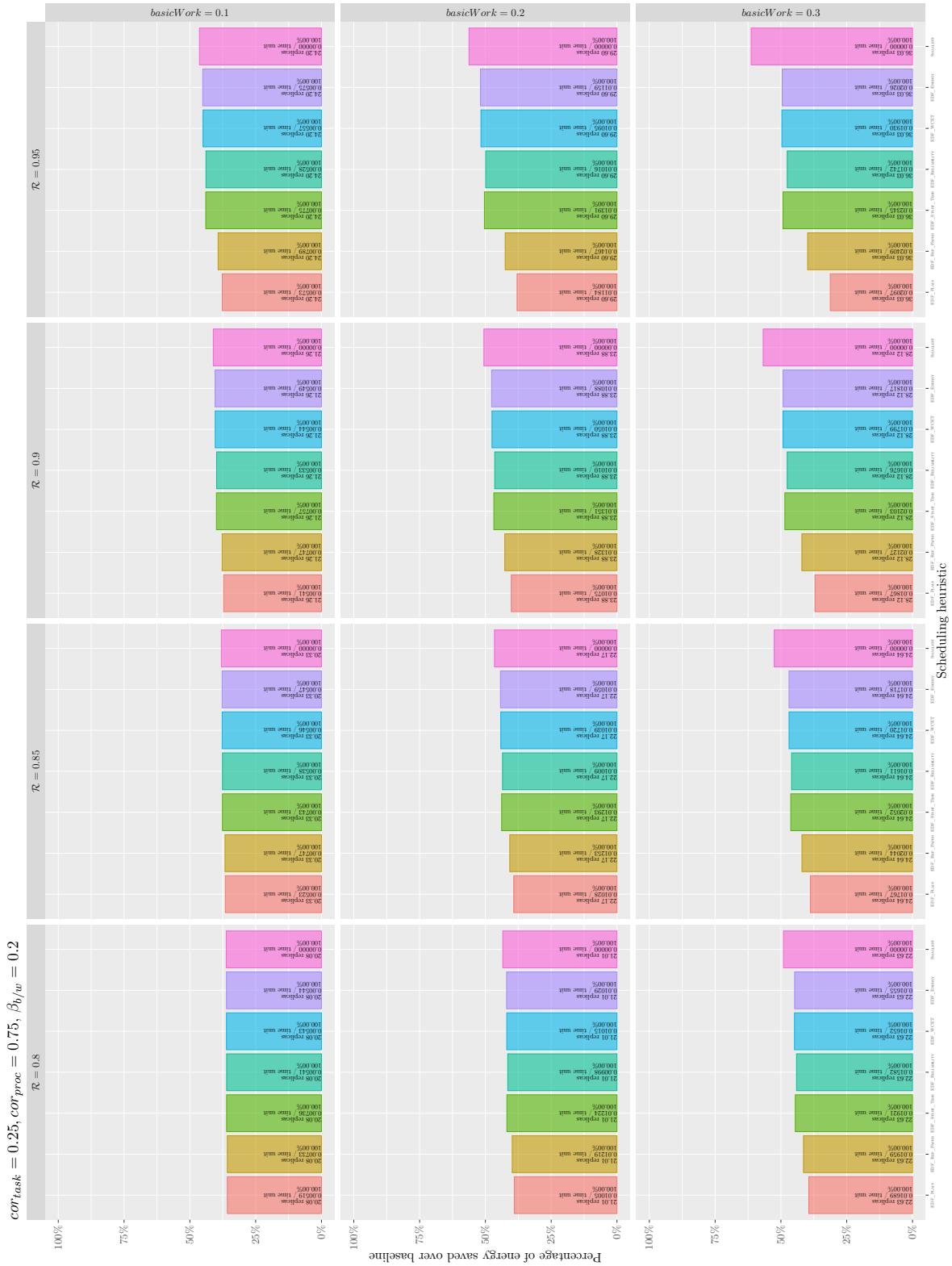






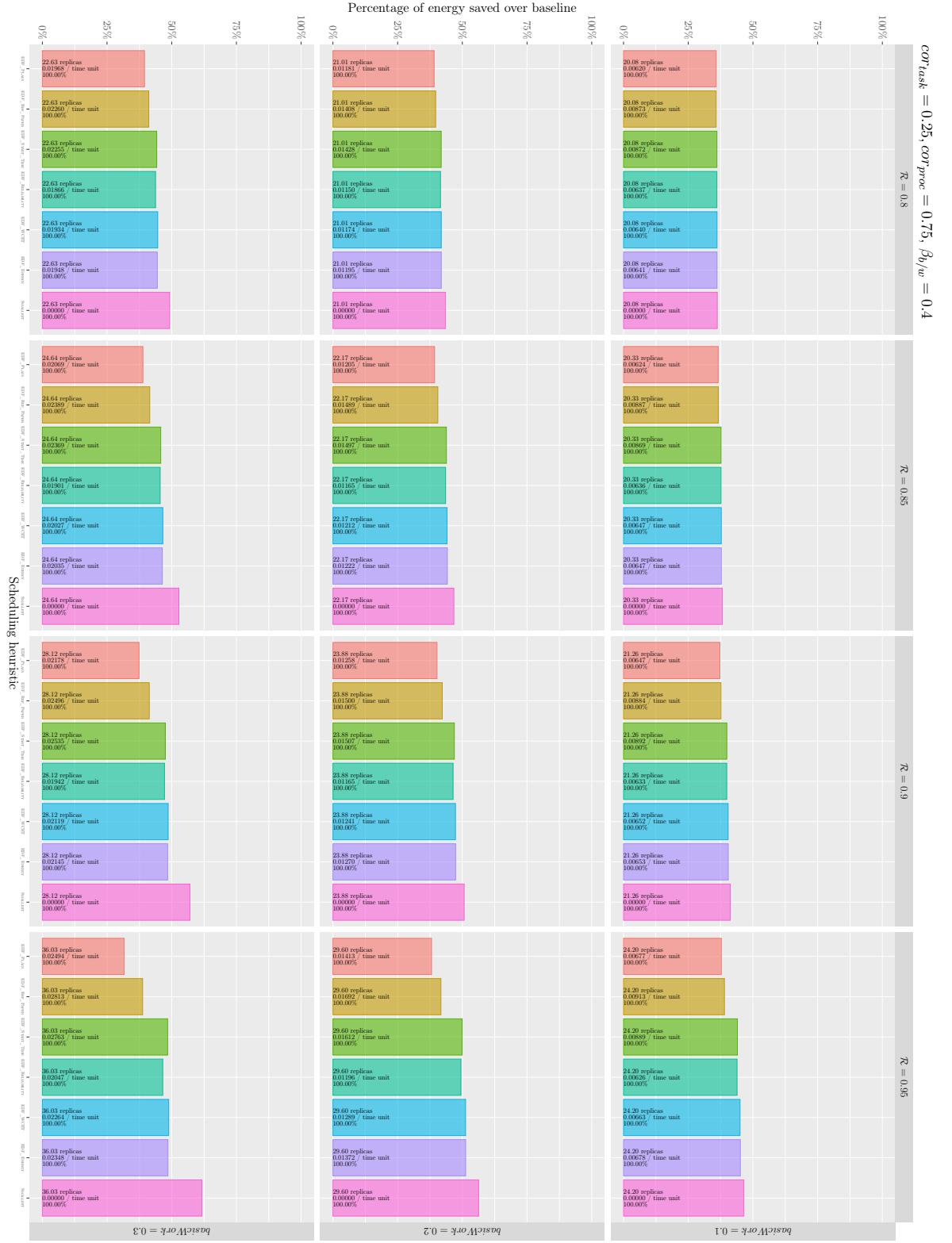


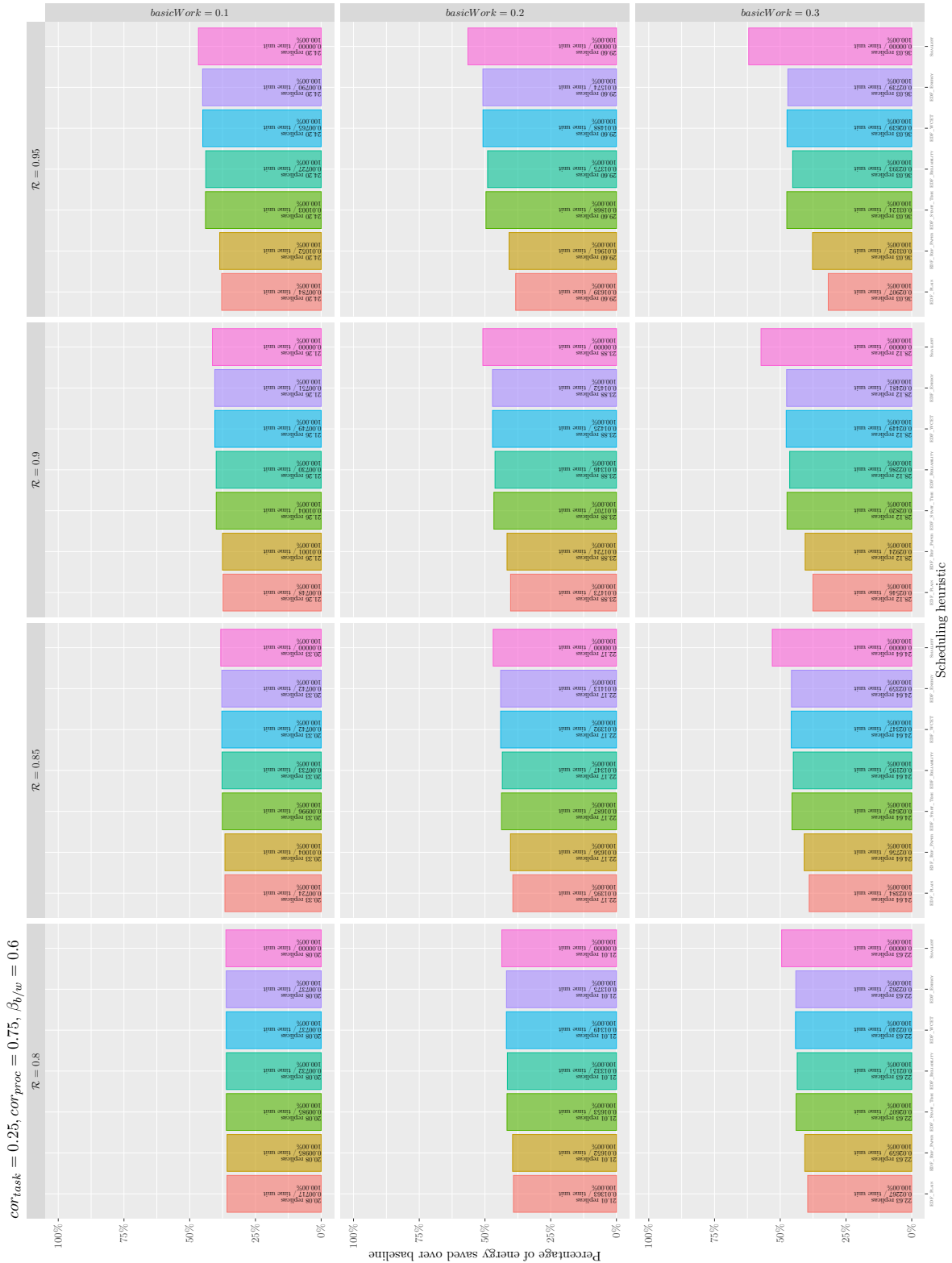




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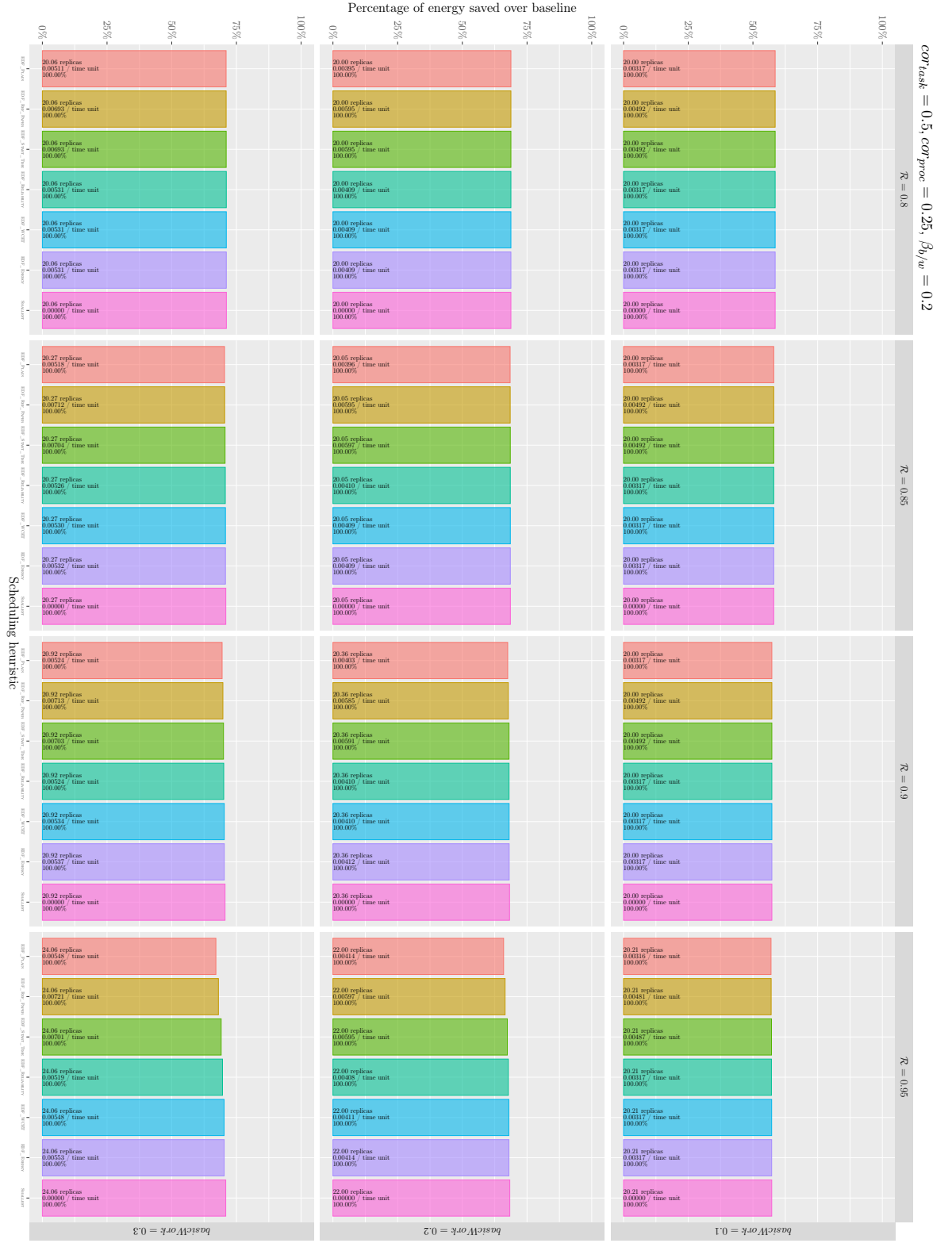




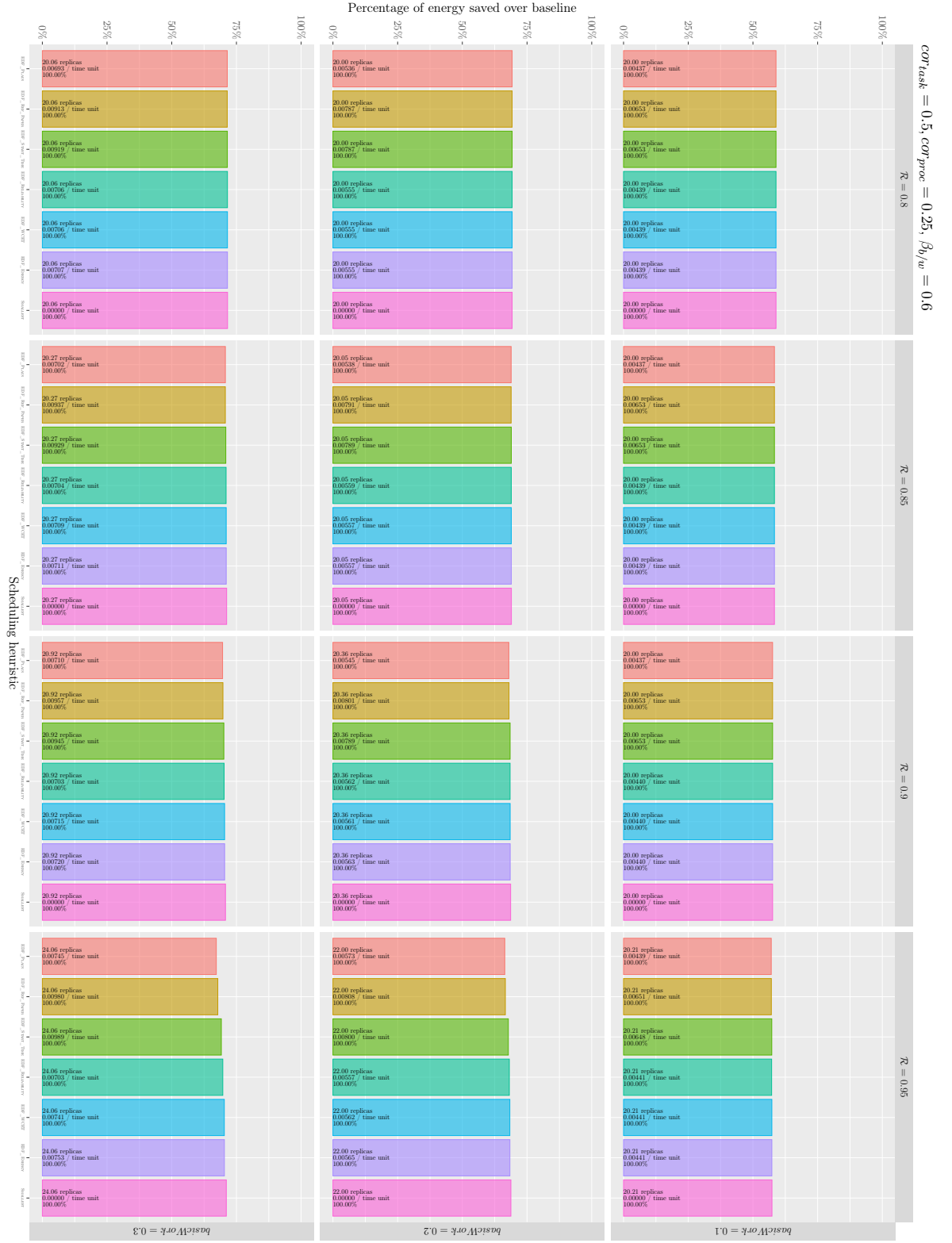


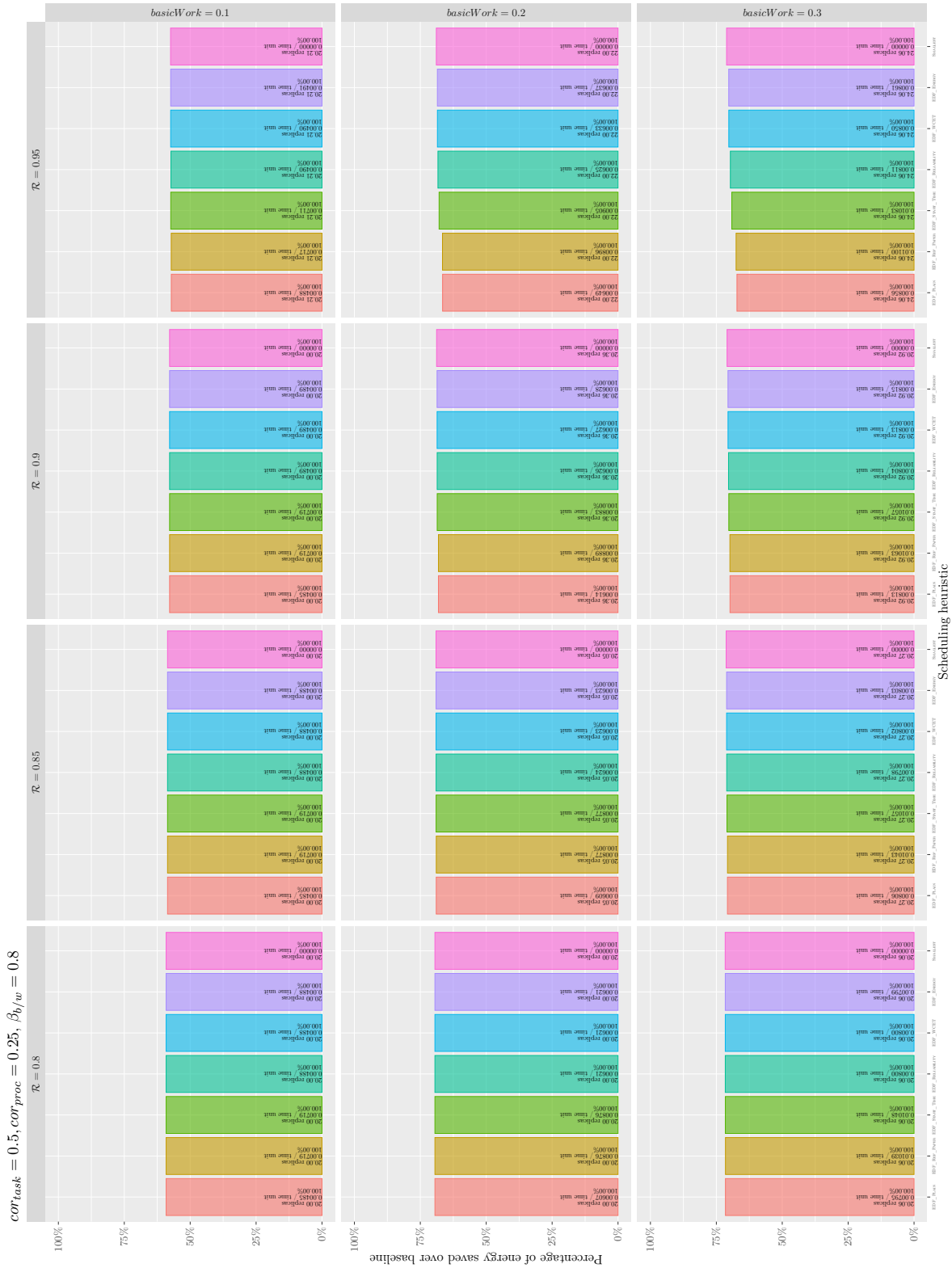












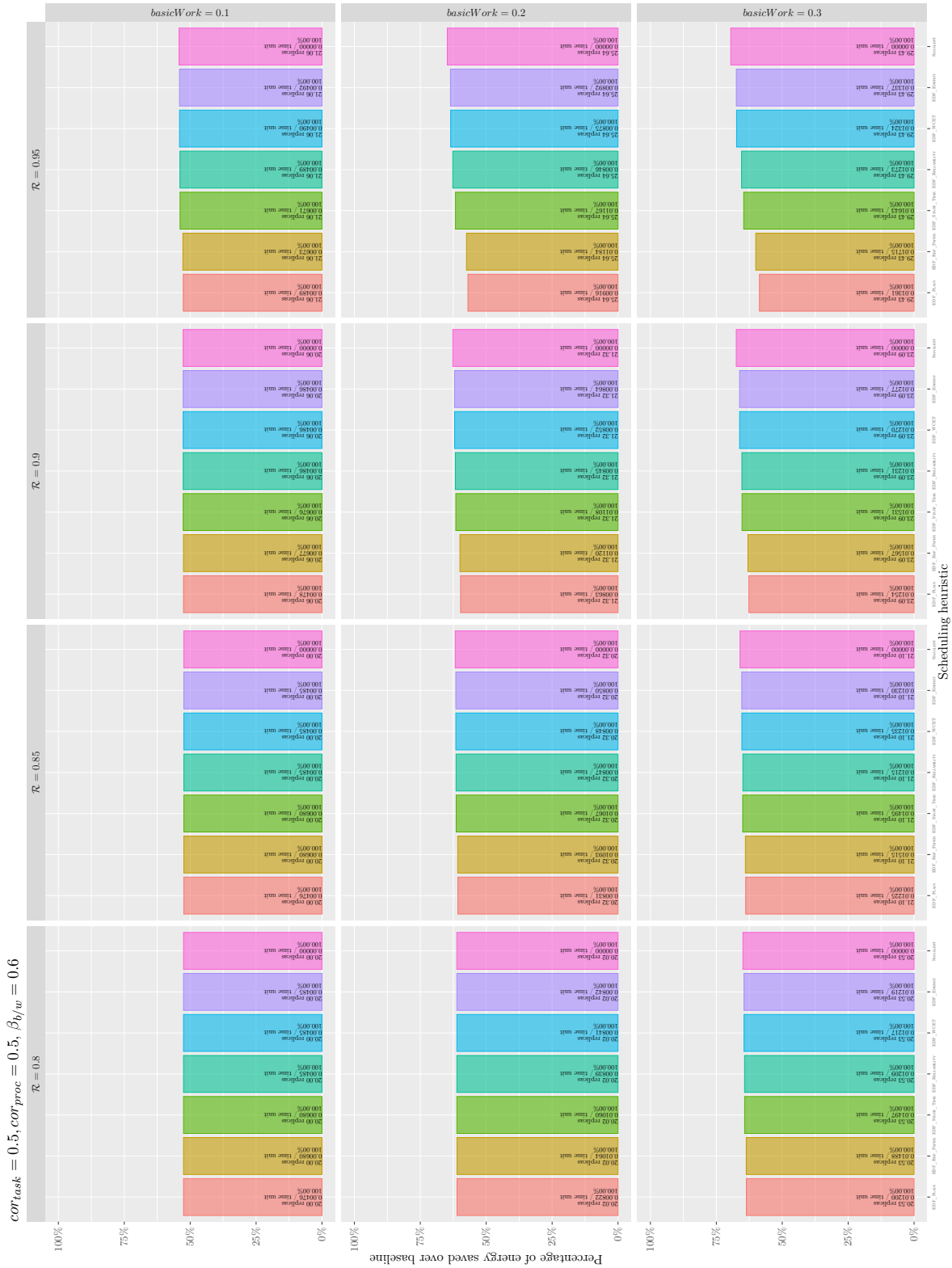
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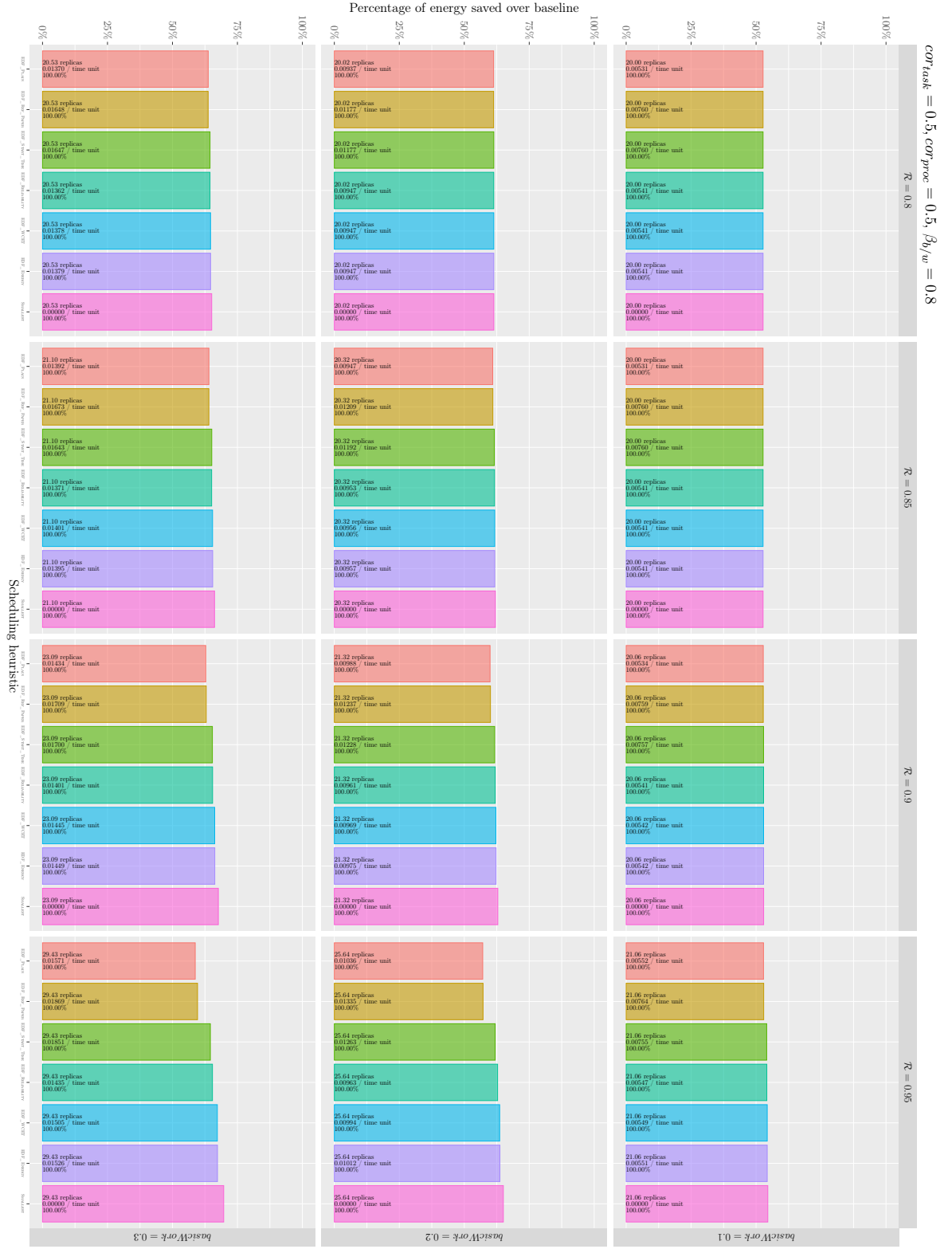


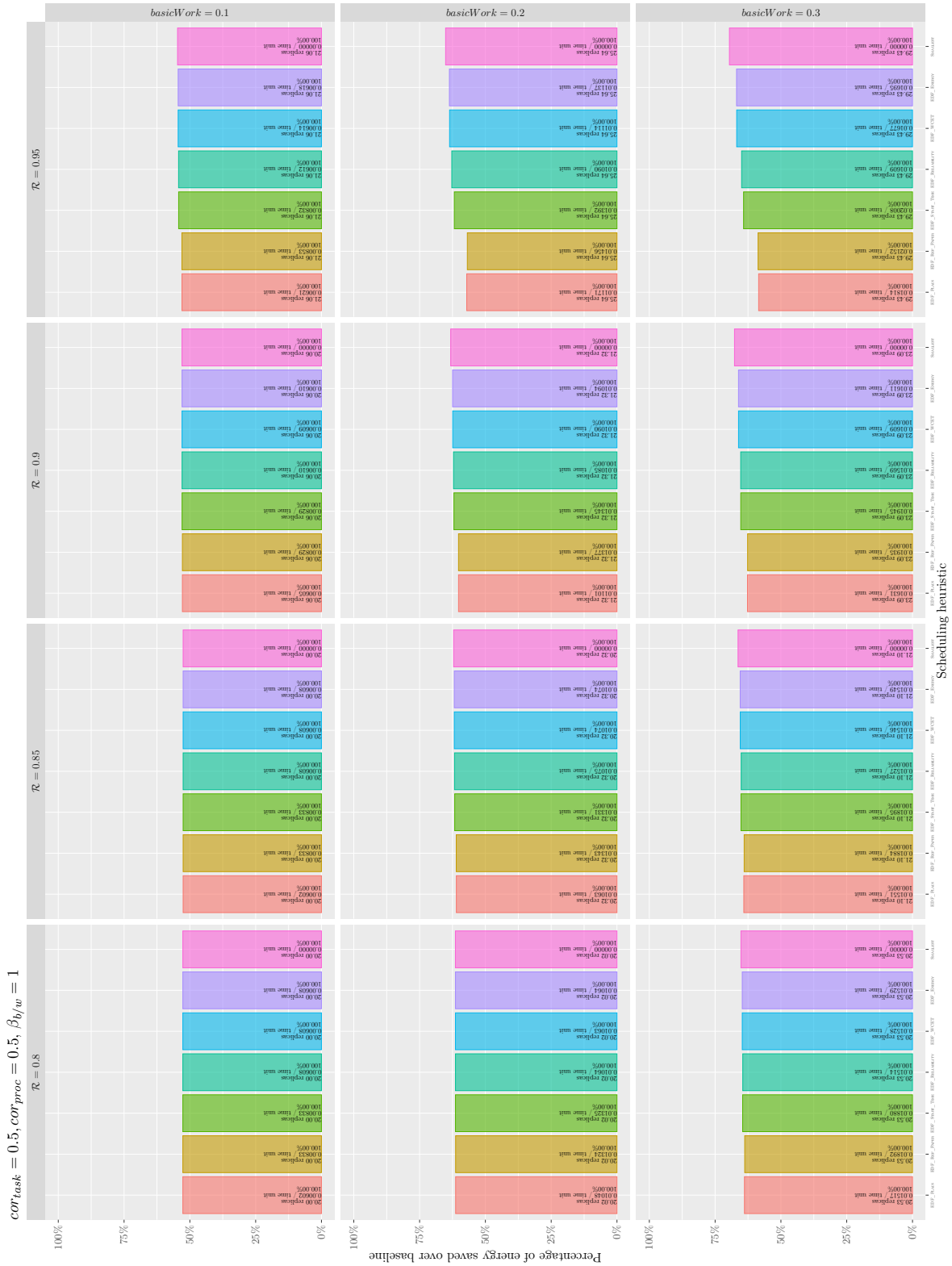


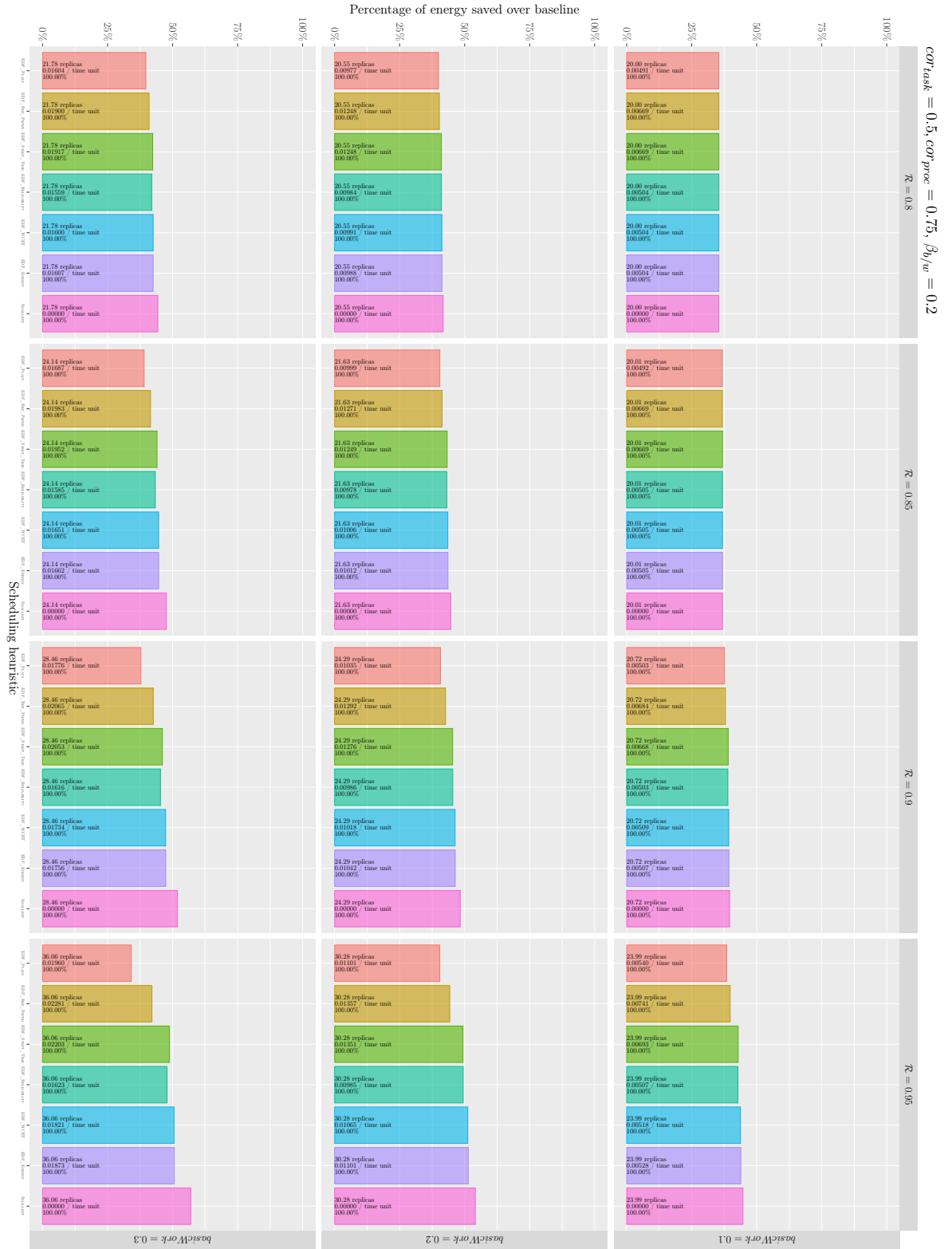


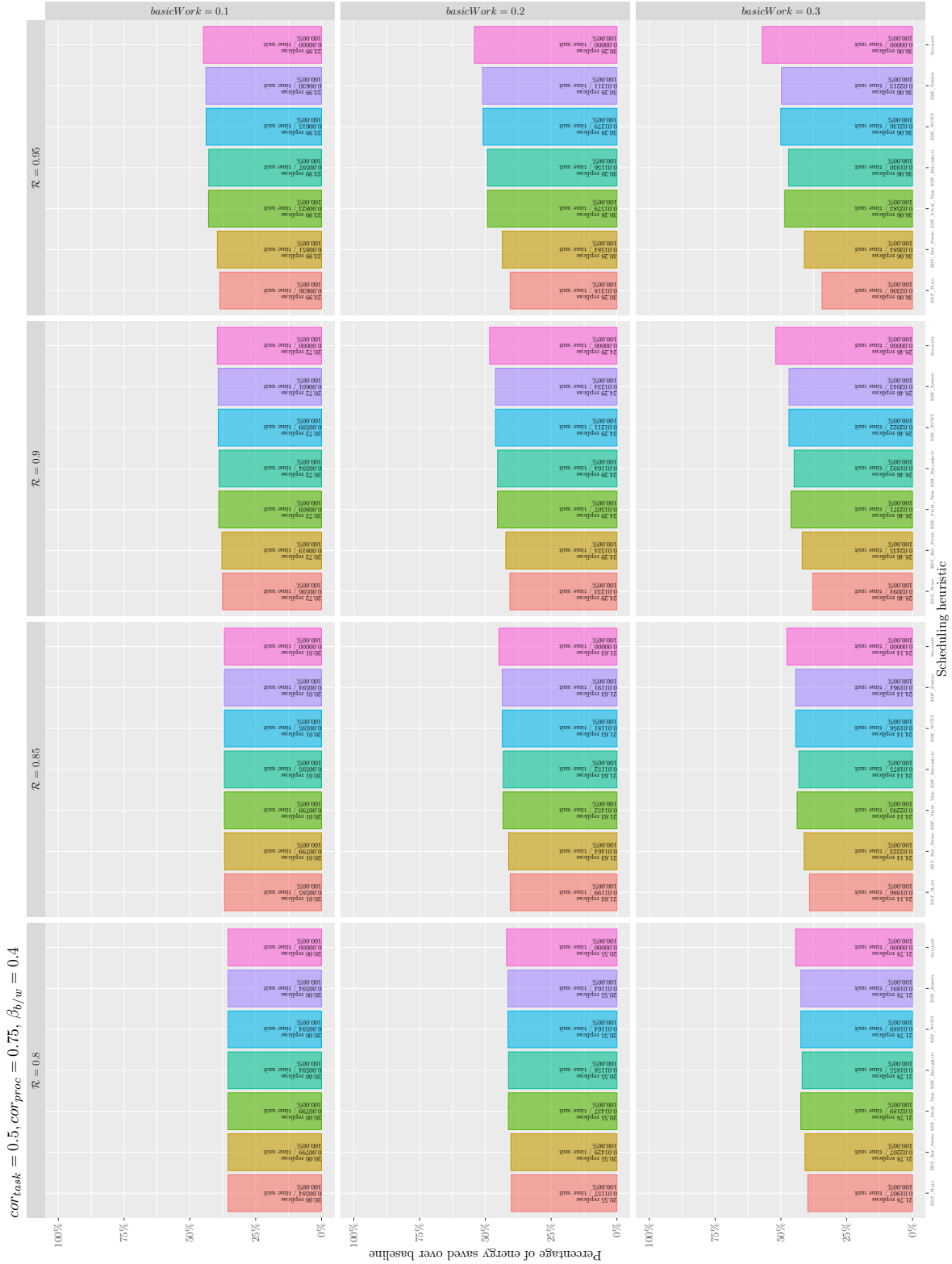






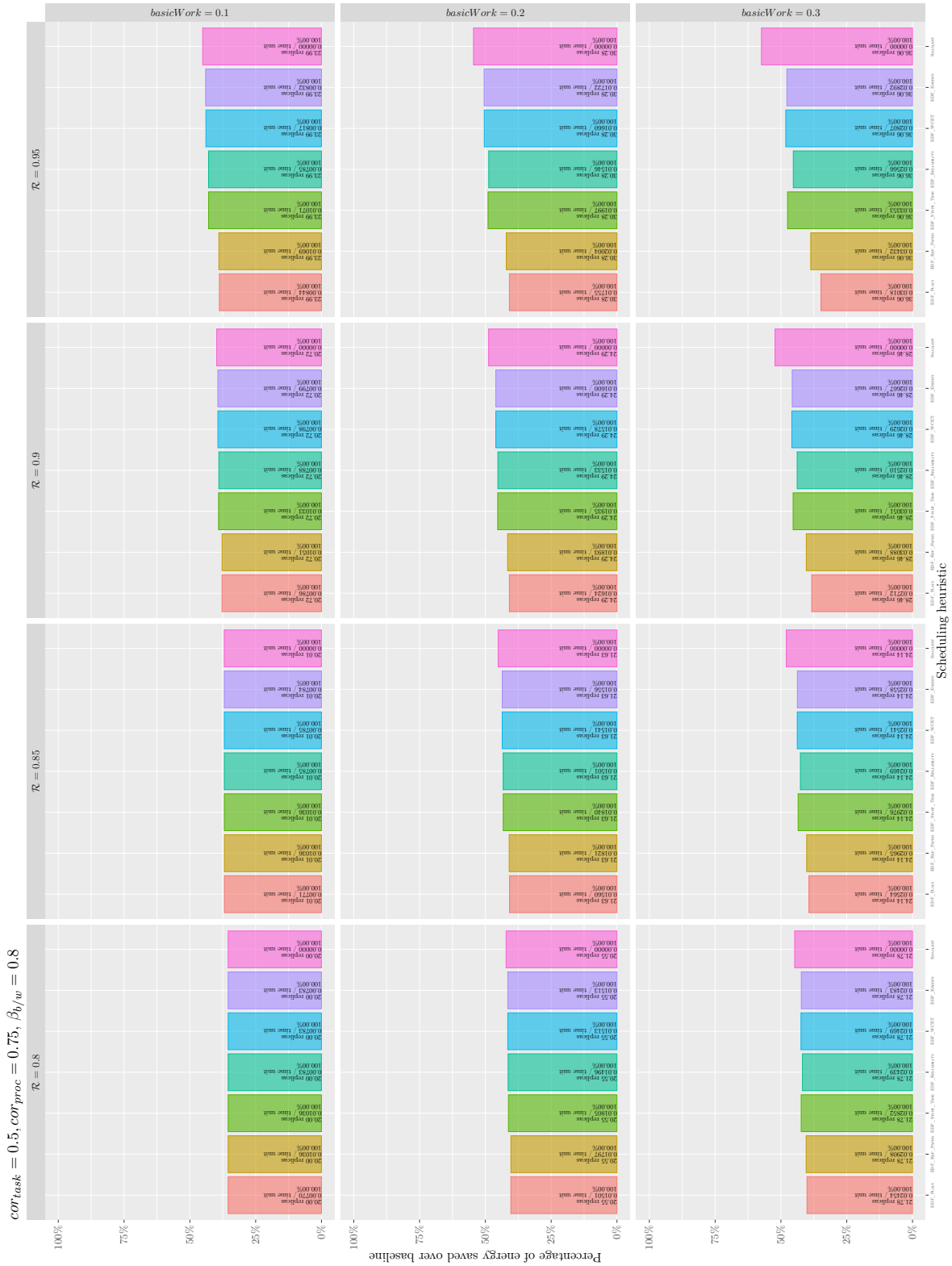


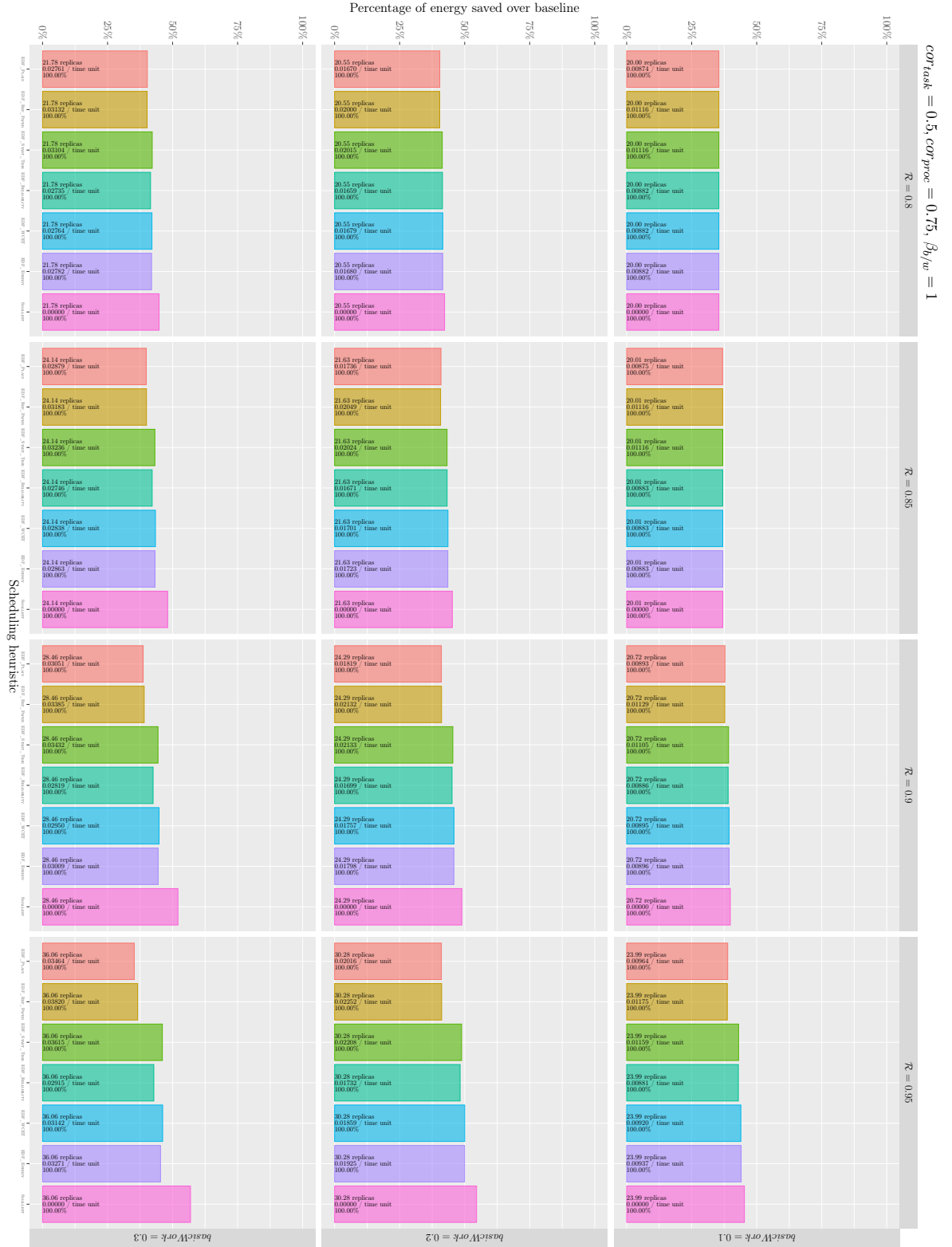


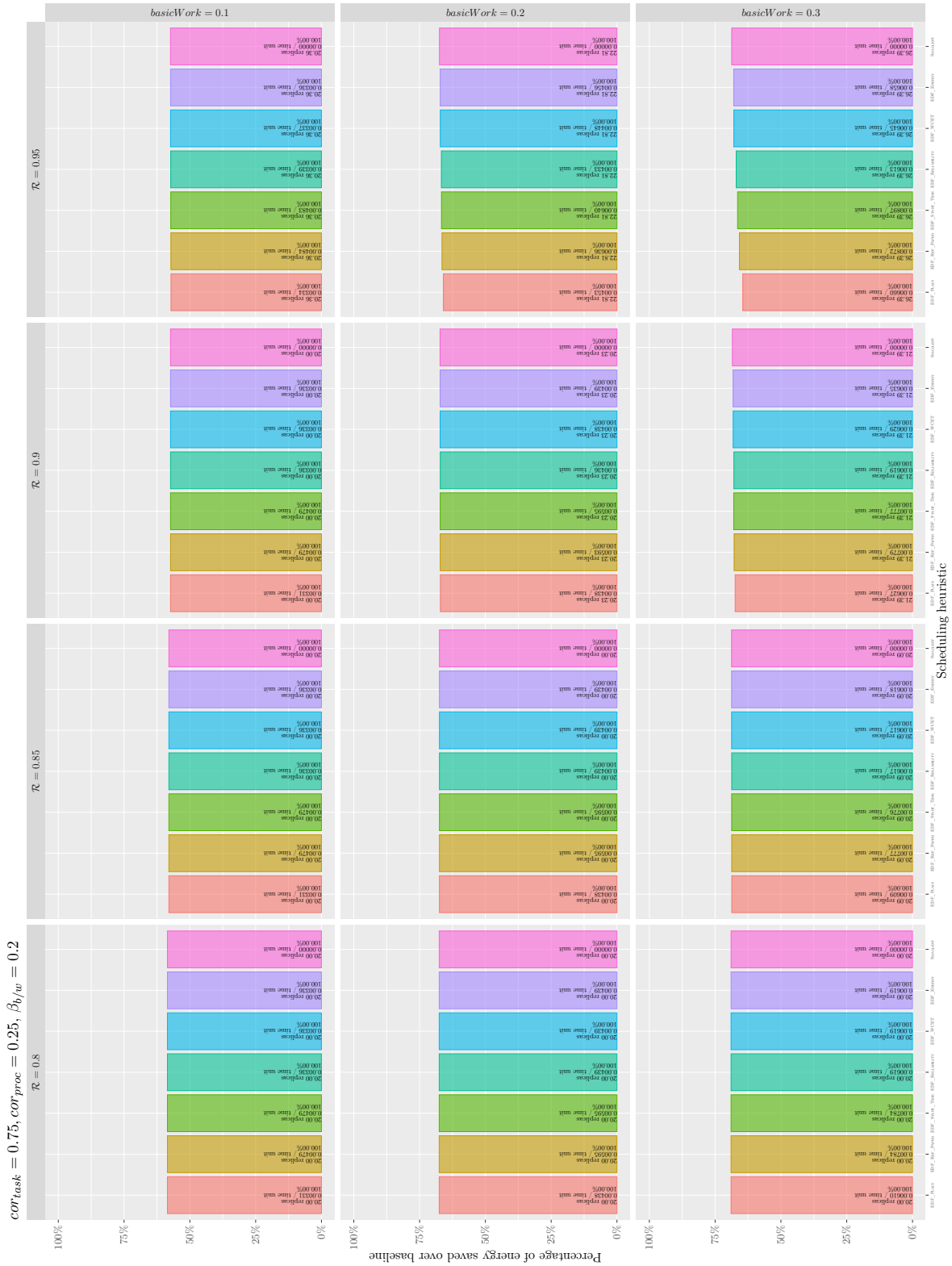


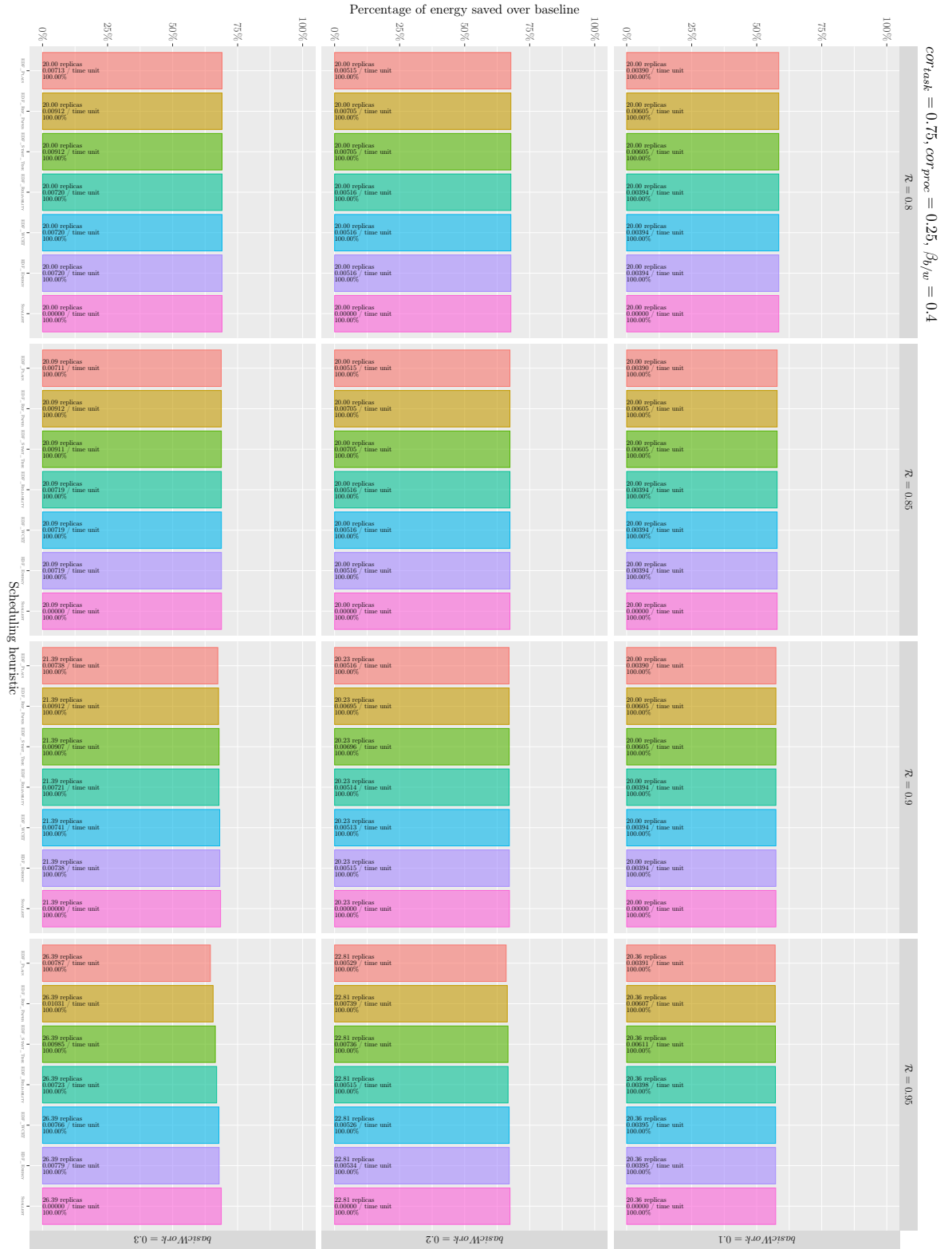


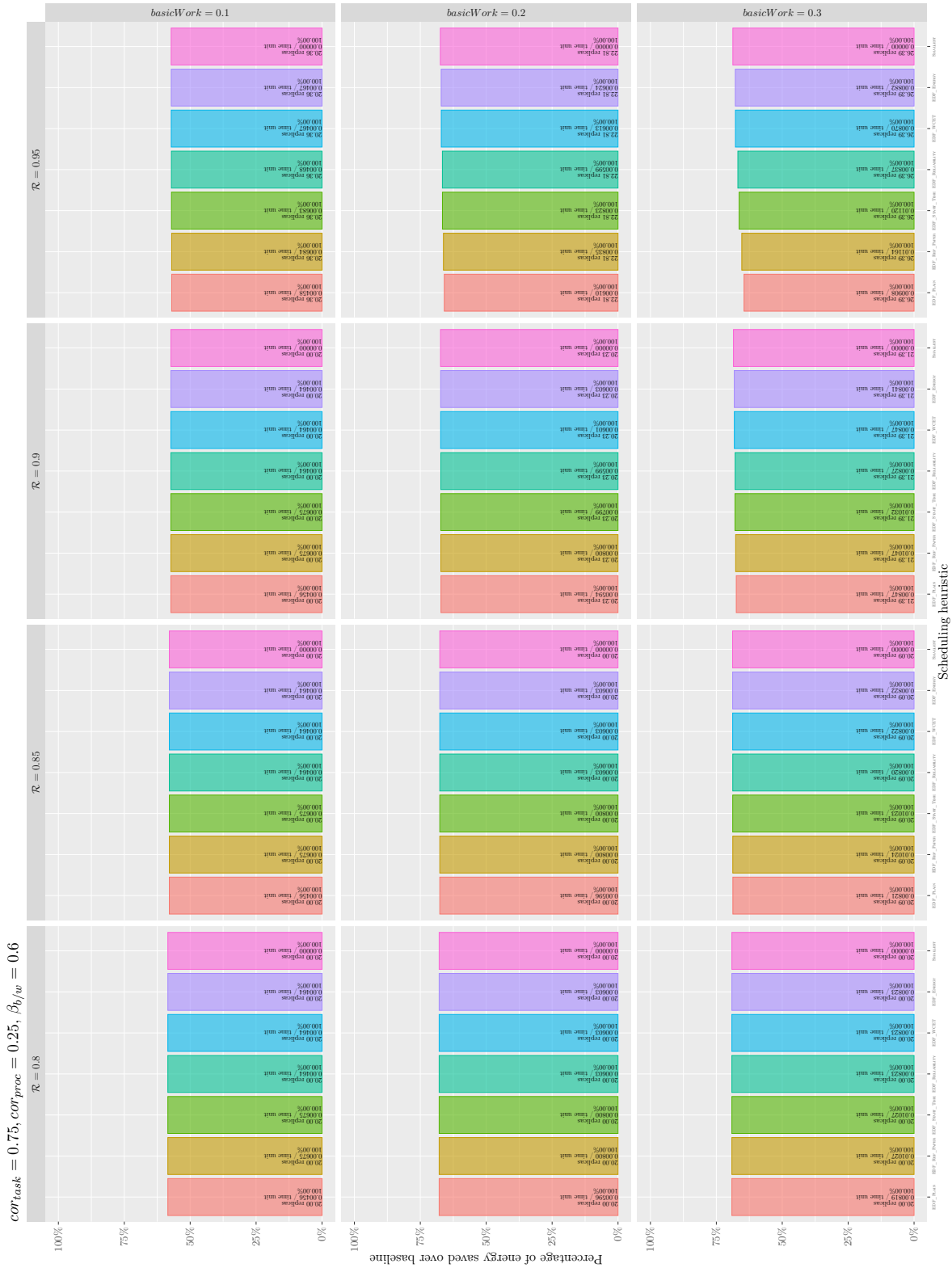




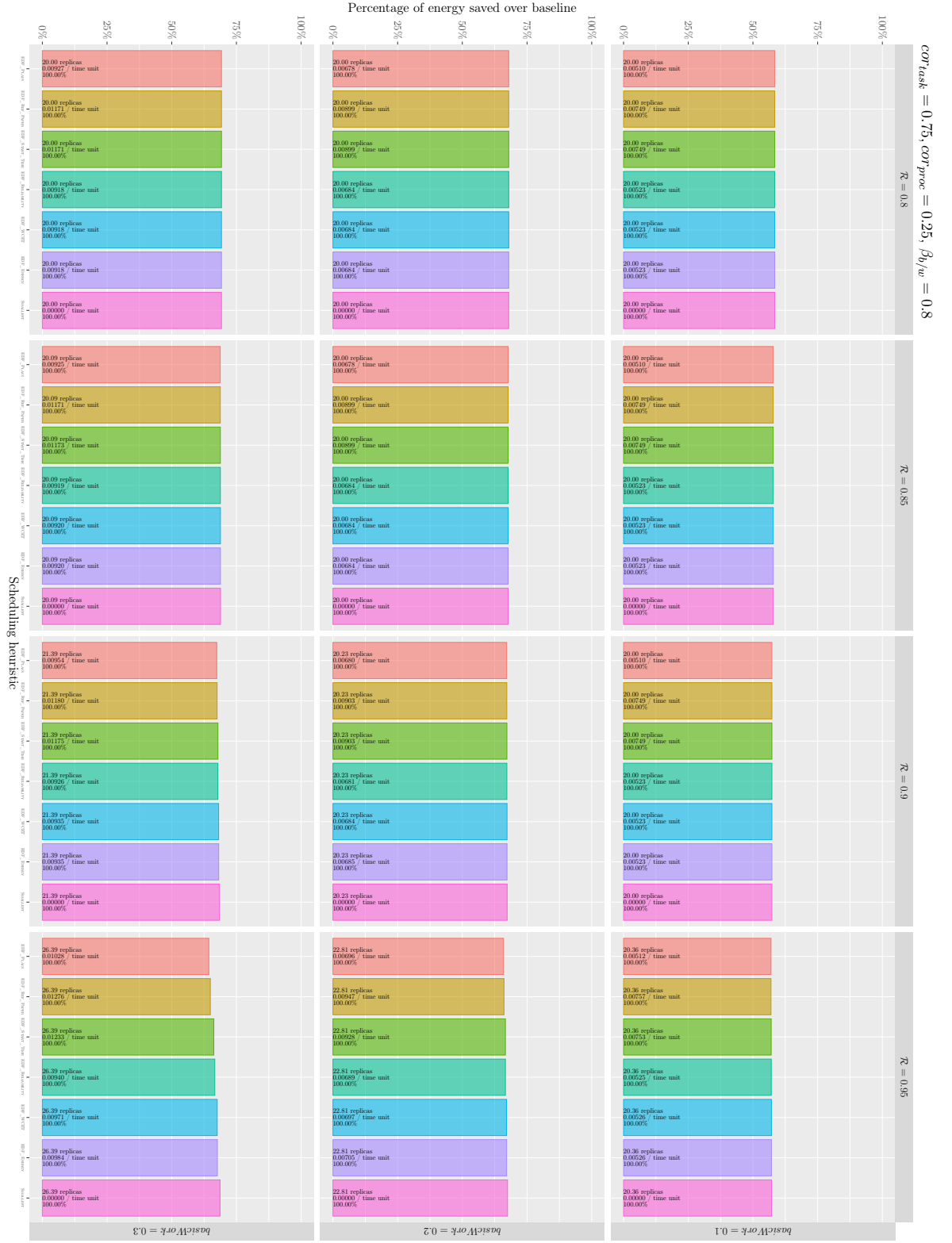


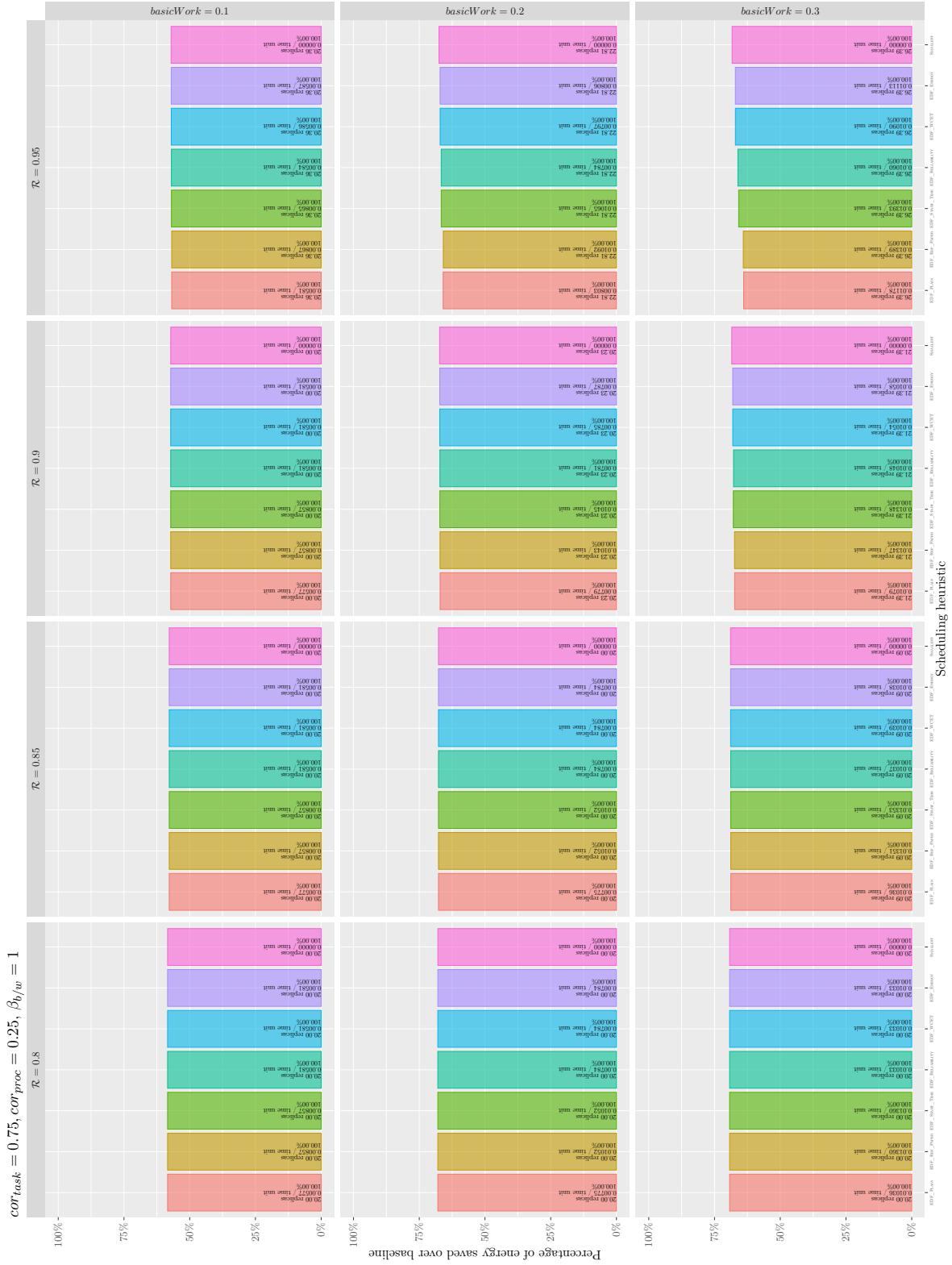




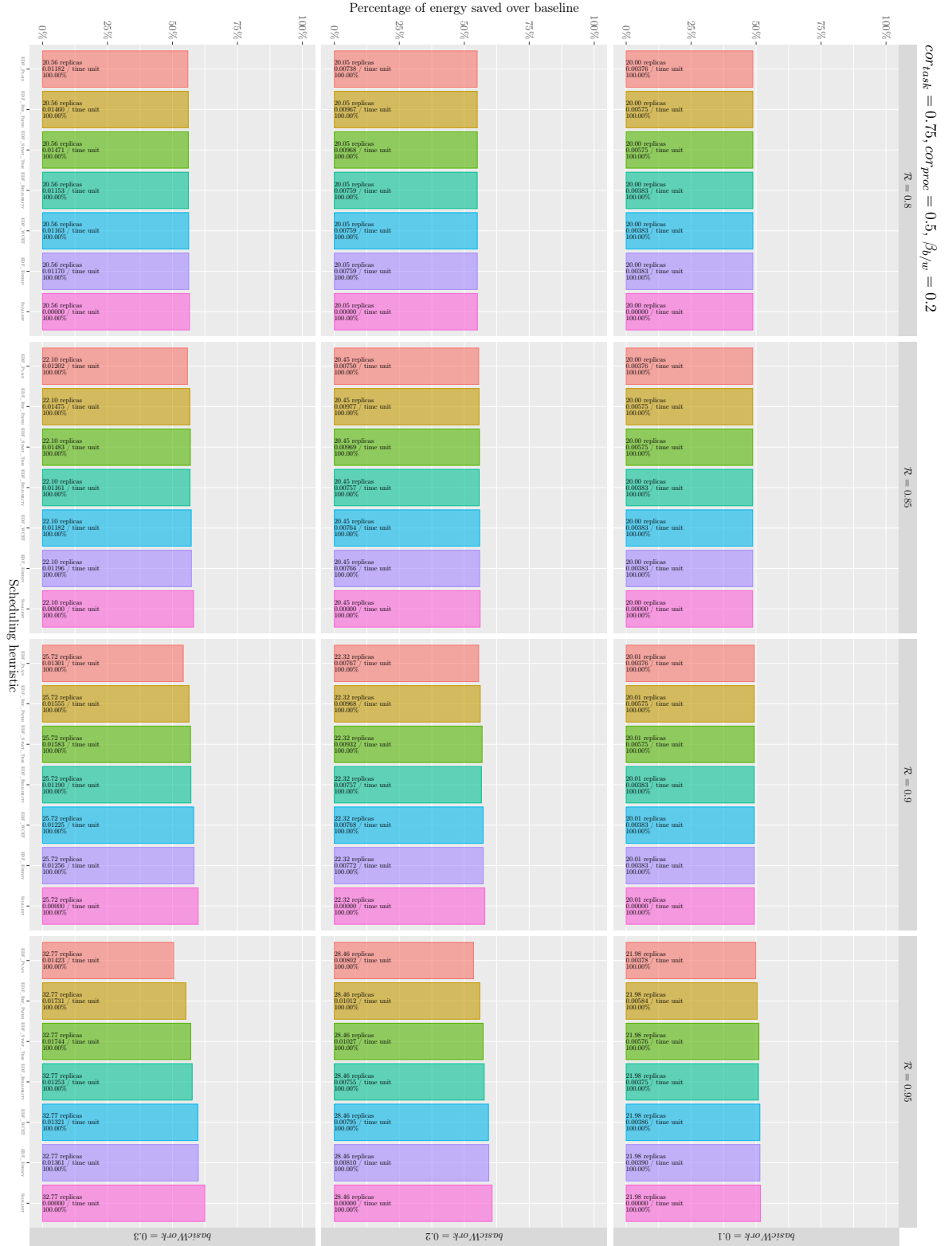


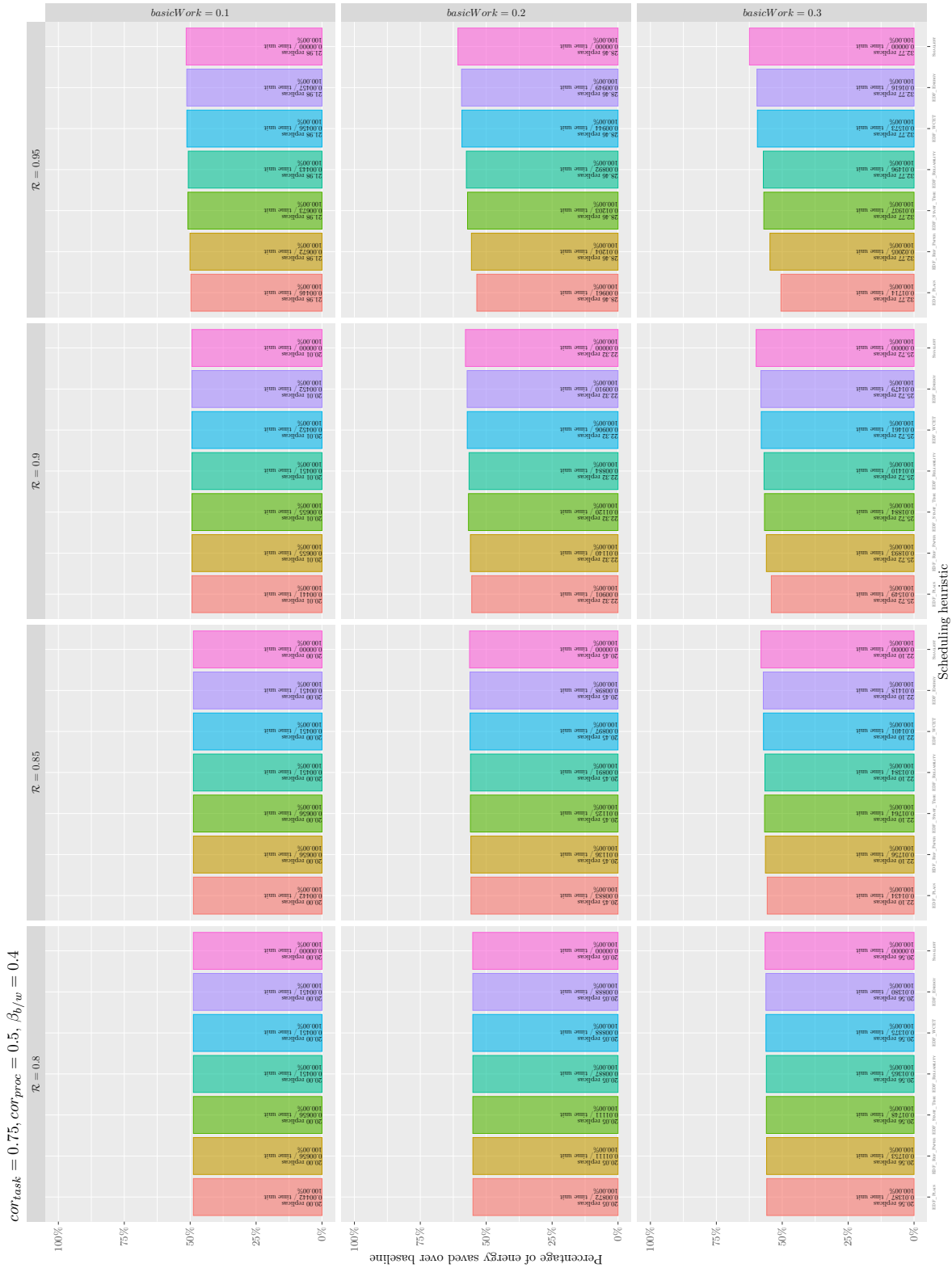
$cof_{task} = 0.75, cof_{proc} = 0.25, \beta_h/\beta_w = 0.6$



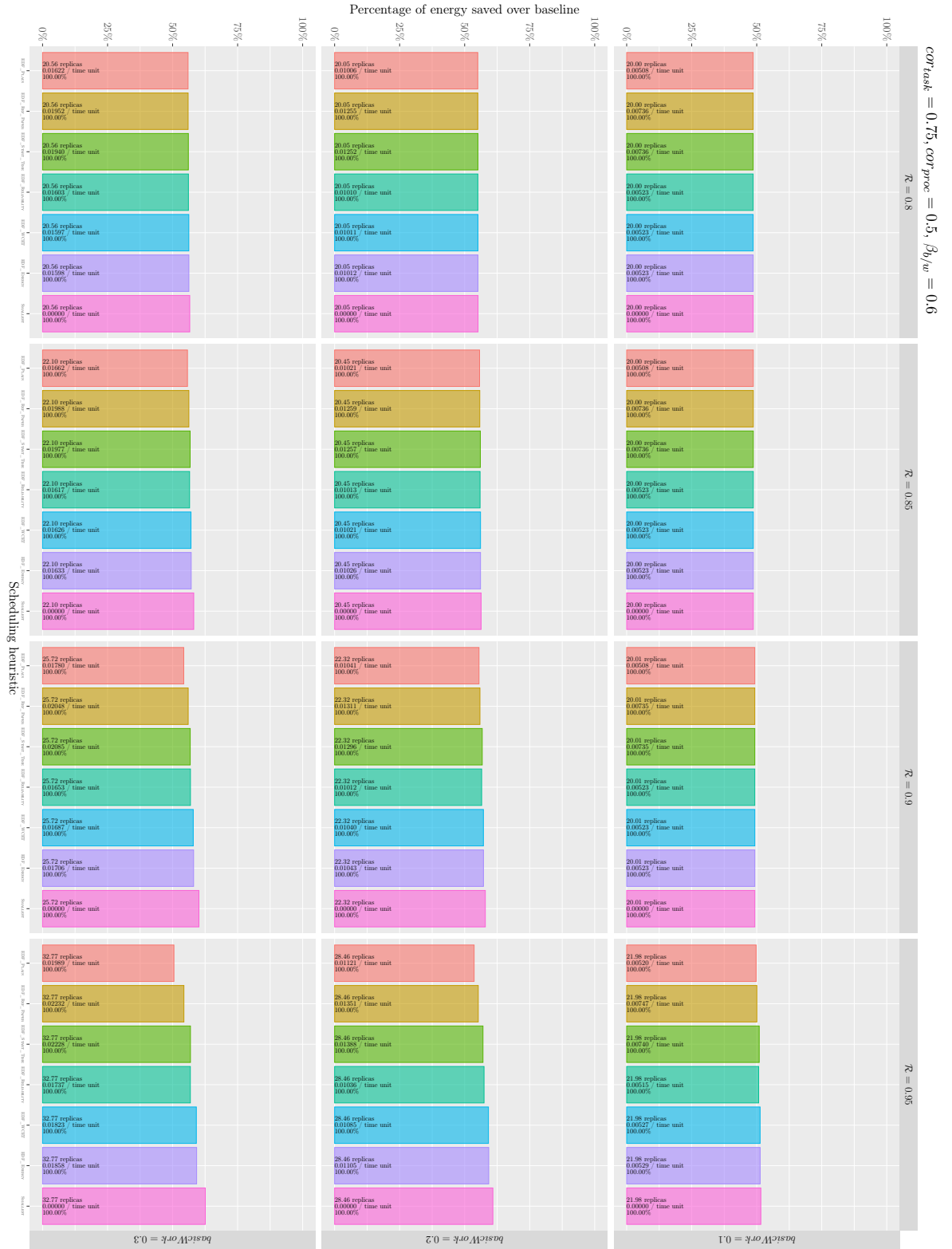


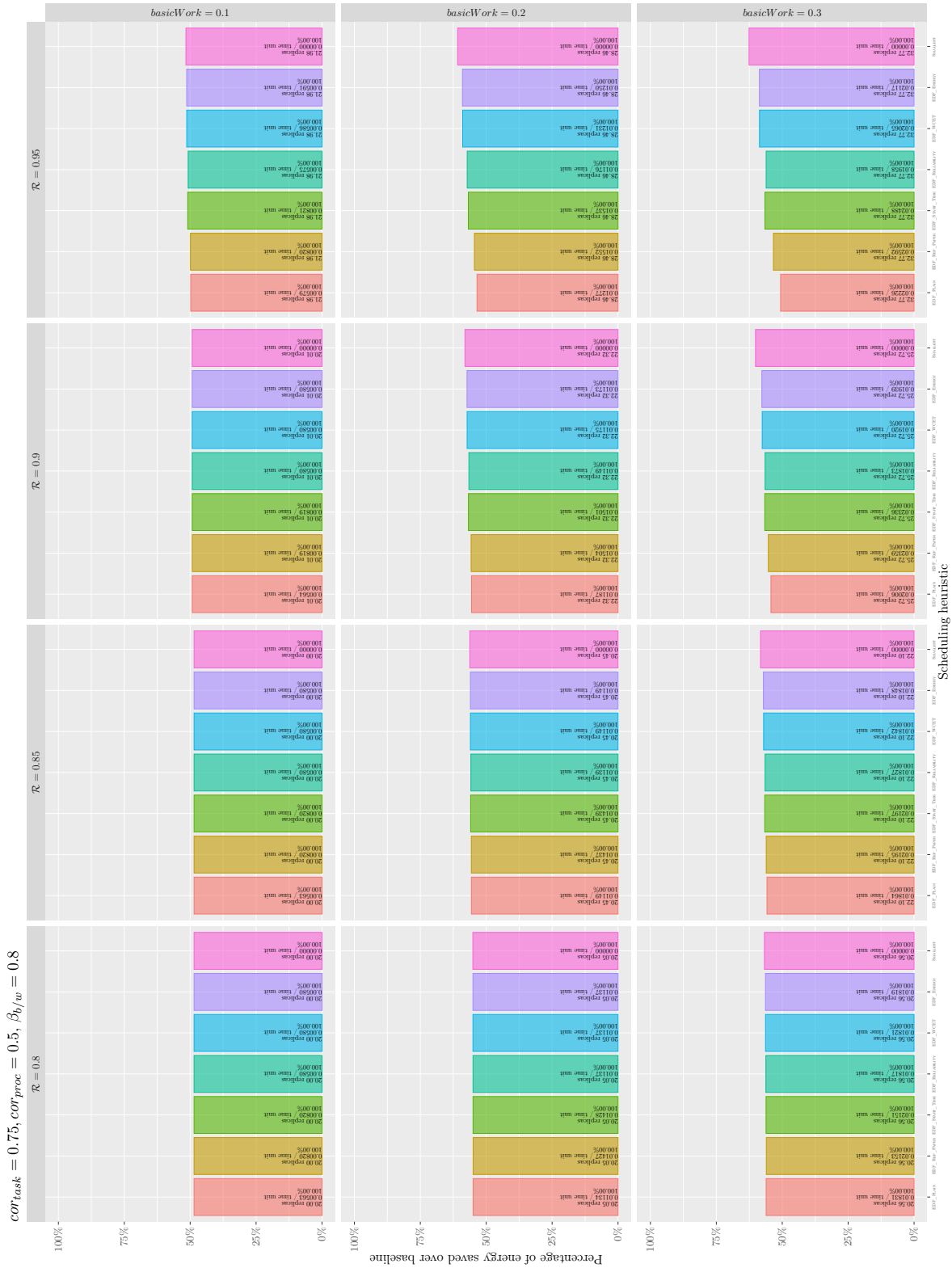




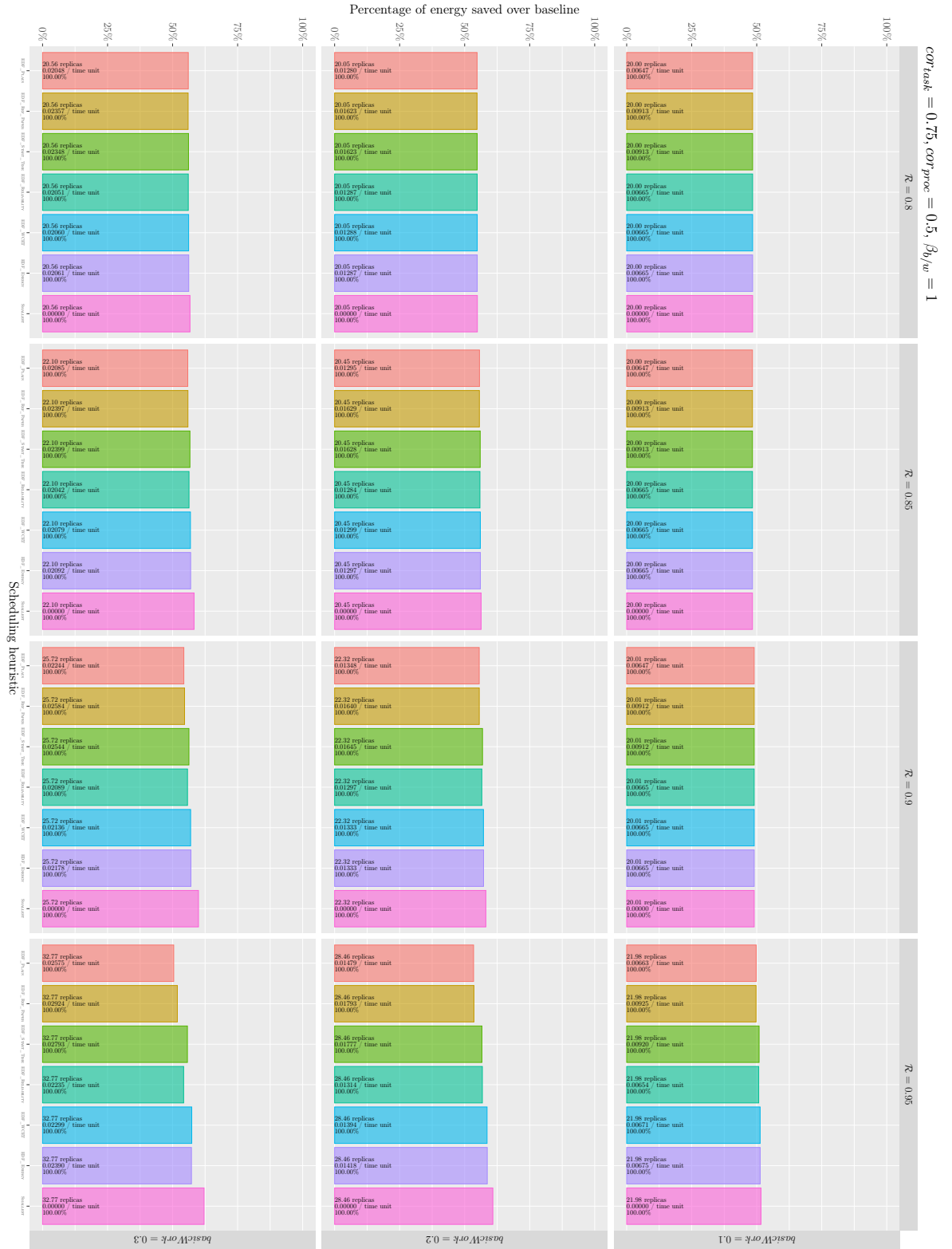


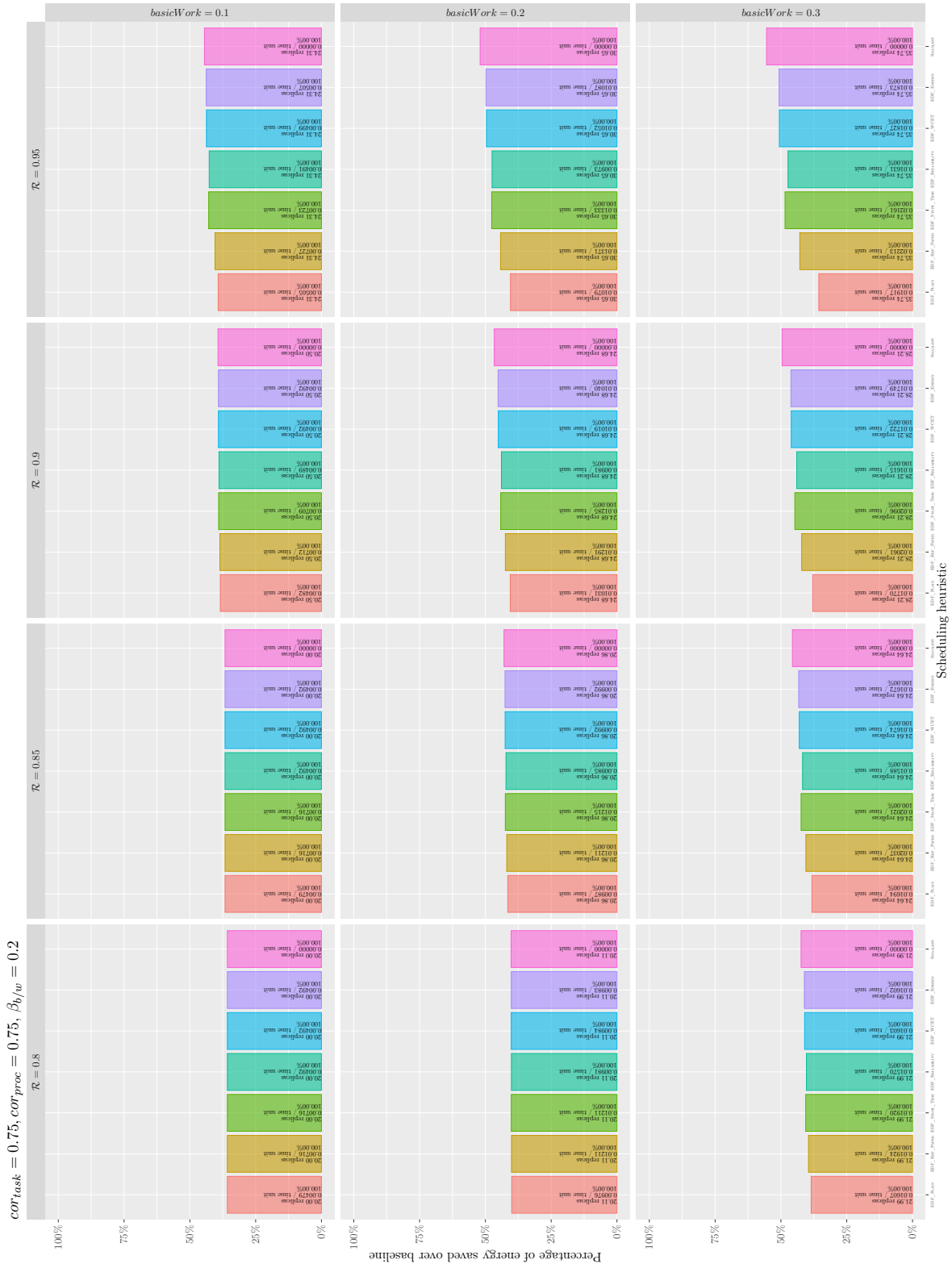
$COI_{task} = 0.75, COI_{proc} = 0.5, \beta_{h/w} = 0.4$





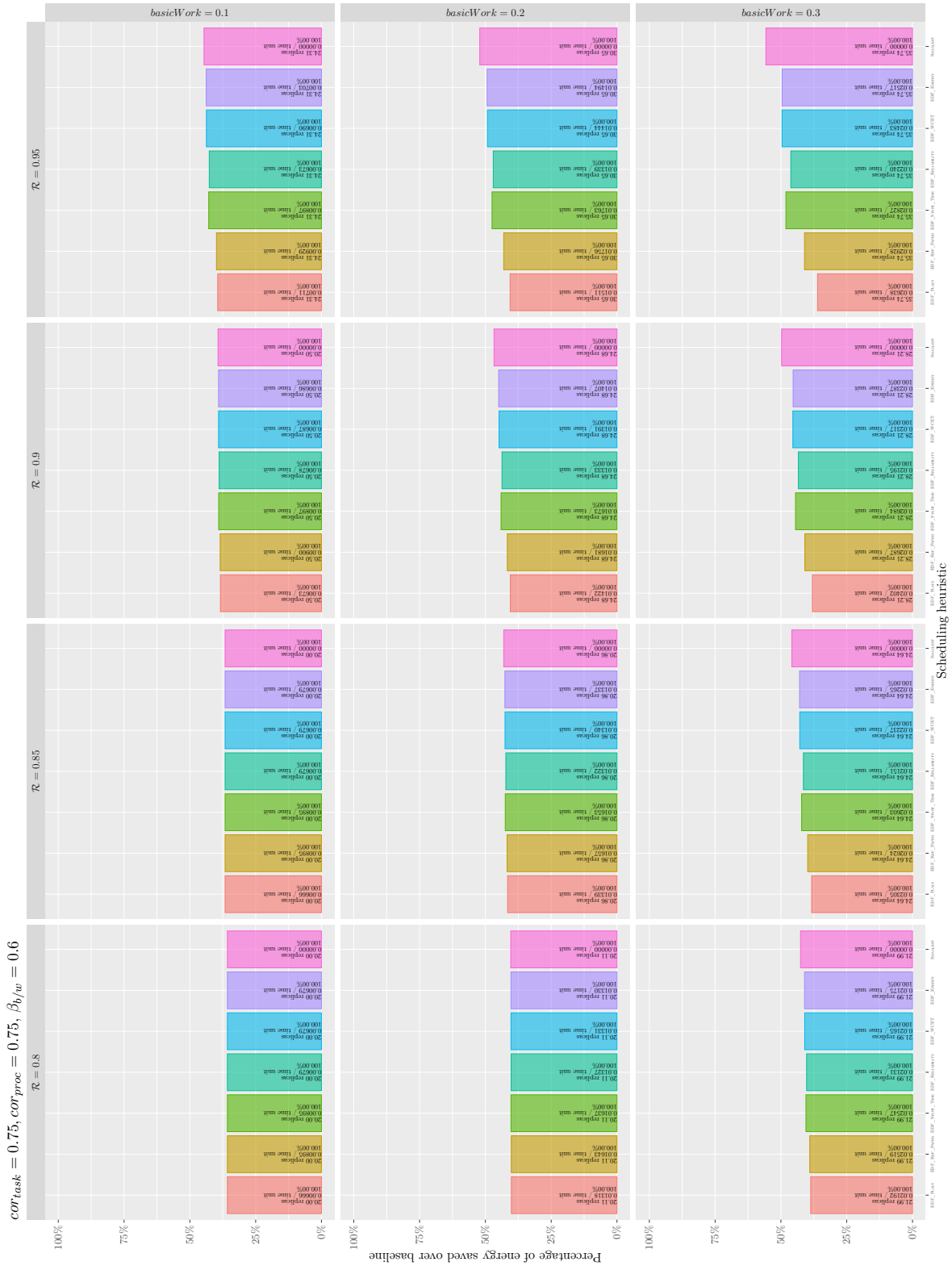
$cot_{task} = 0.75, cot_{proc} = 0.5, \beta_{h/w} = 0.8$





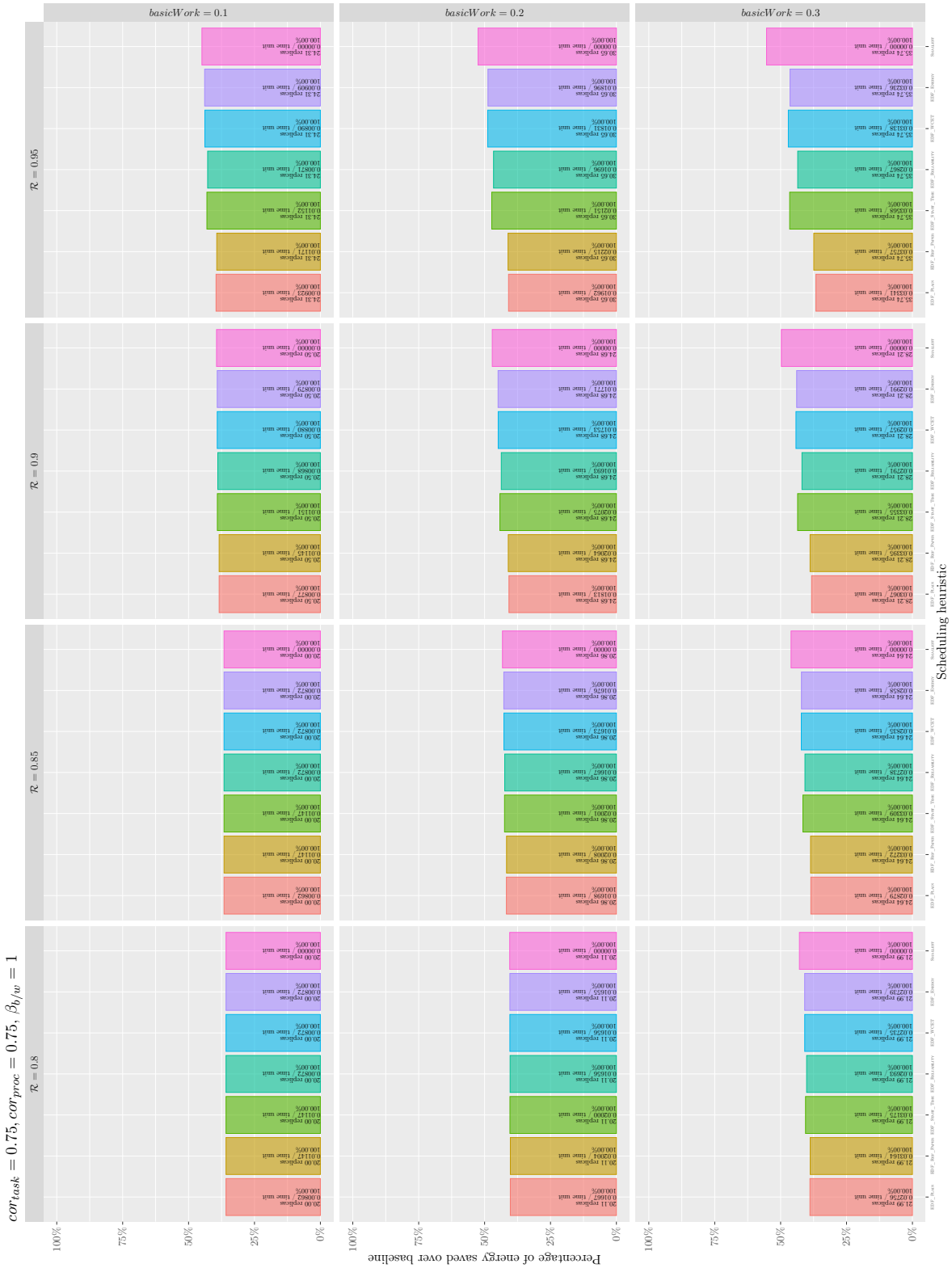
$cof_{task} = 0.75, cof_{proc} = 0.75, \beta_{p/w} = 0.2$





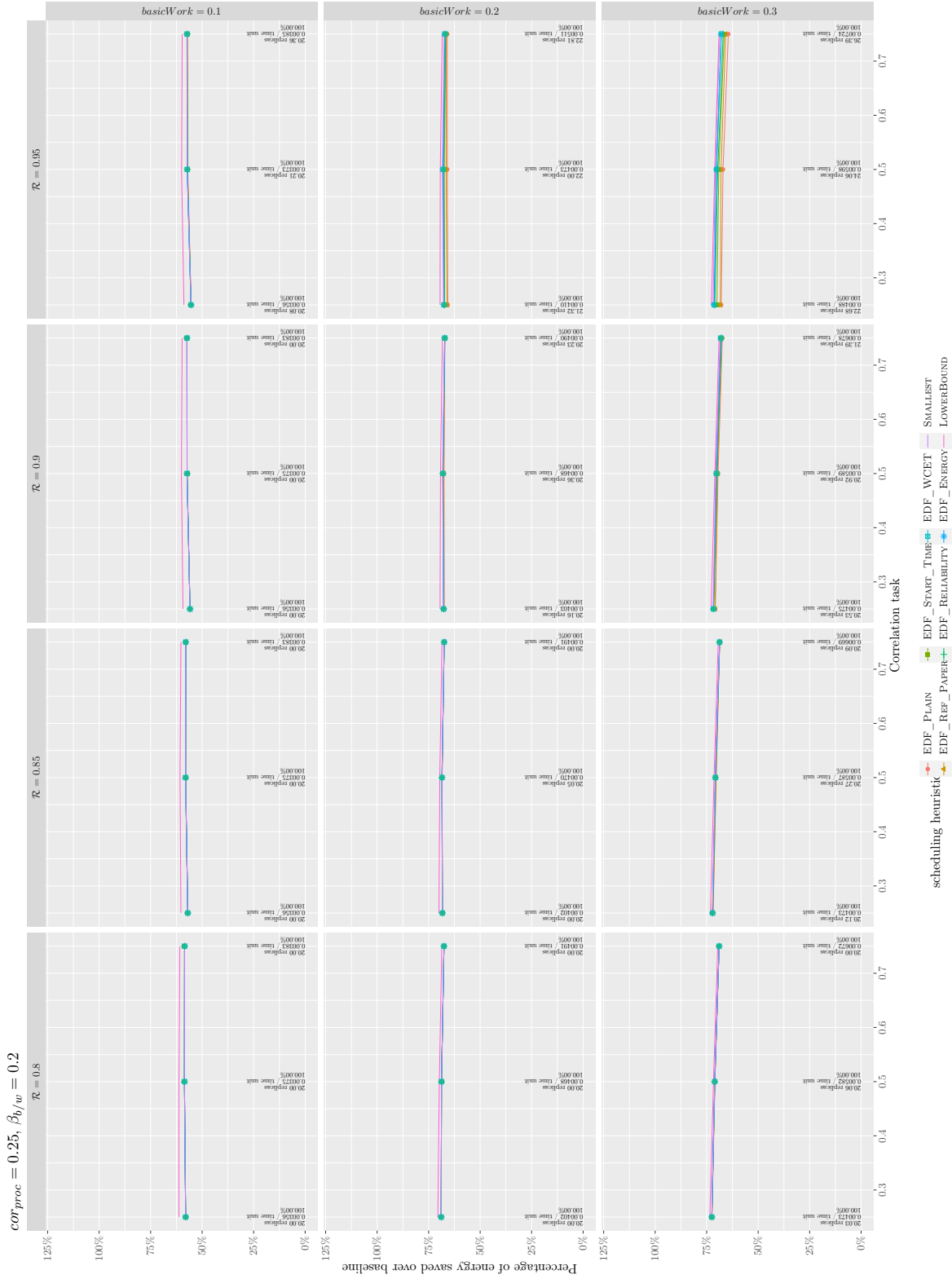




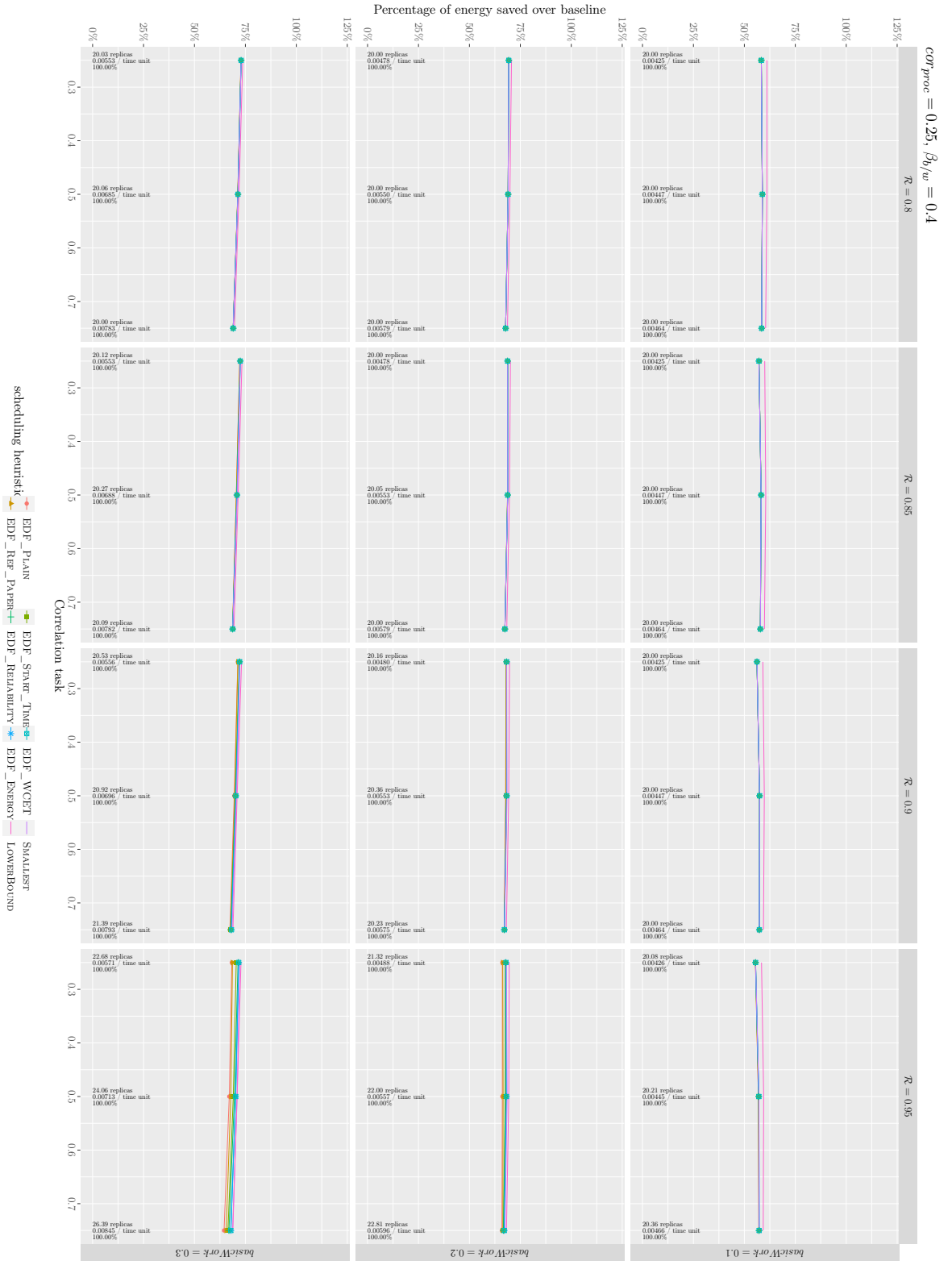


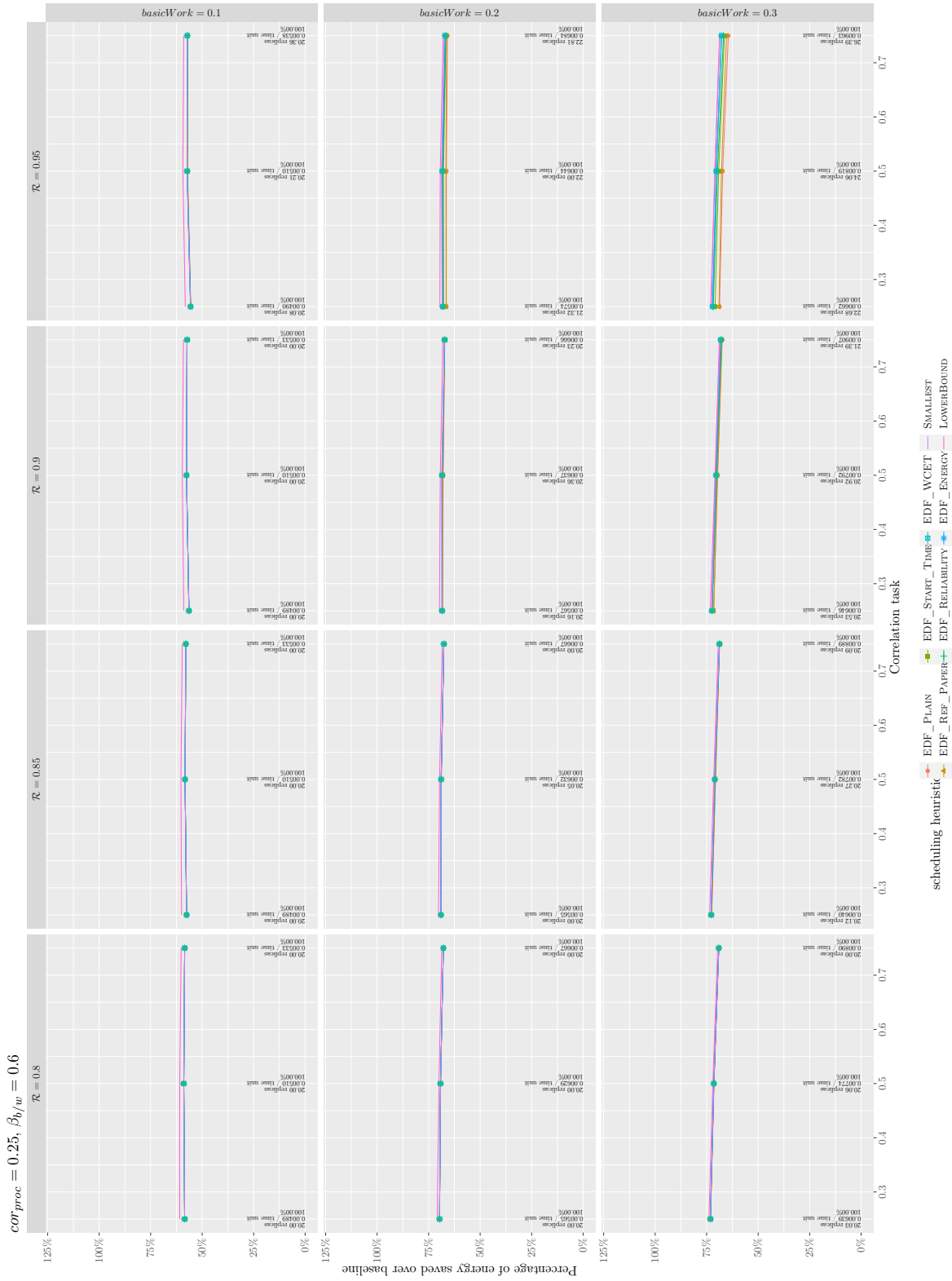
## **Task correlation**

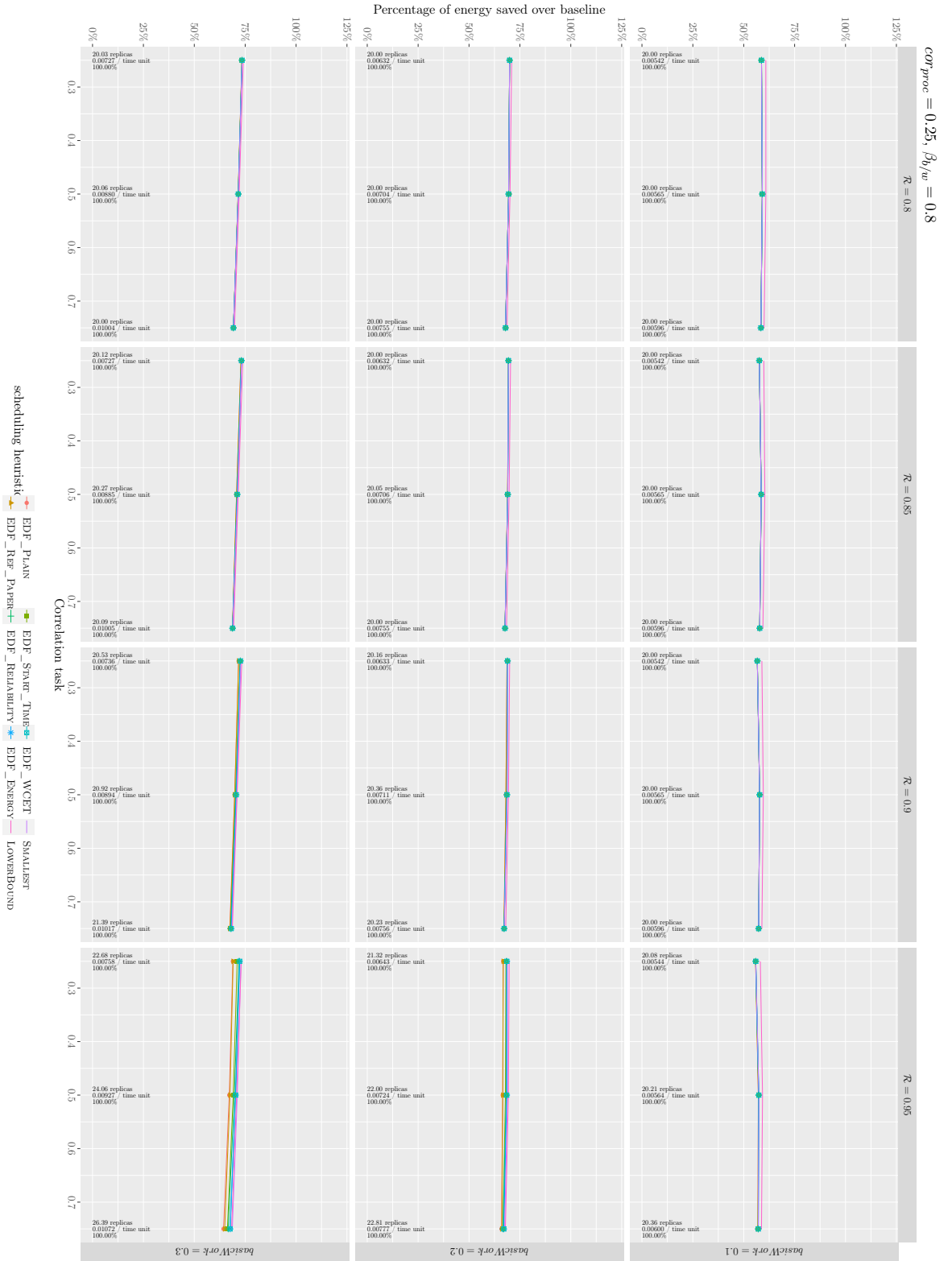
The following section presents percentage of energy saved over baseline using different scheduling strategies under big failure rate, when varying  $cor_{task}$ .

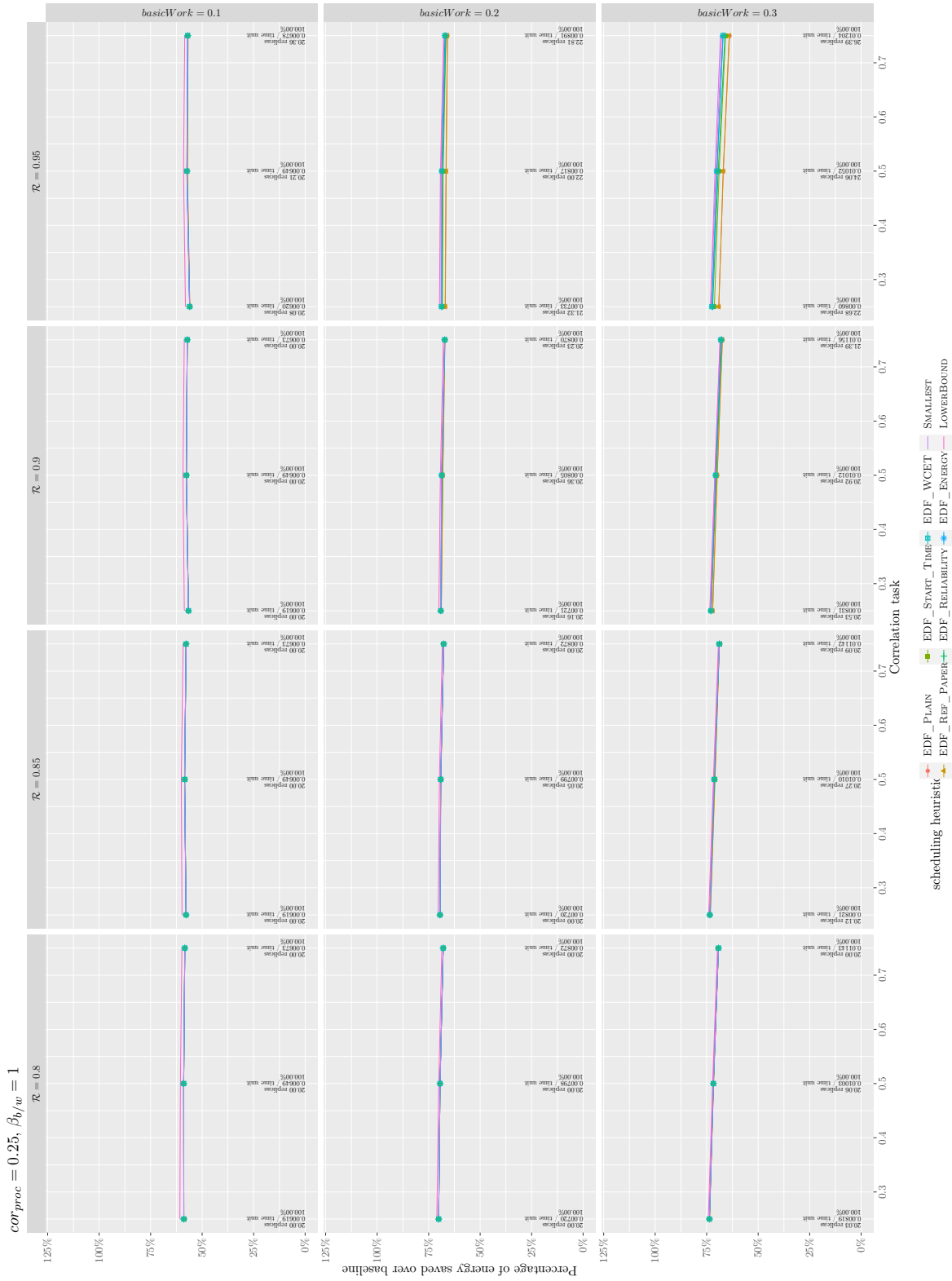


$col_{proc} = 0.25, \beta_{t/w} = 0.2$



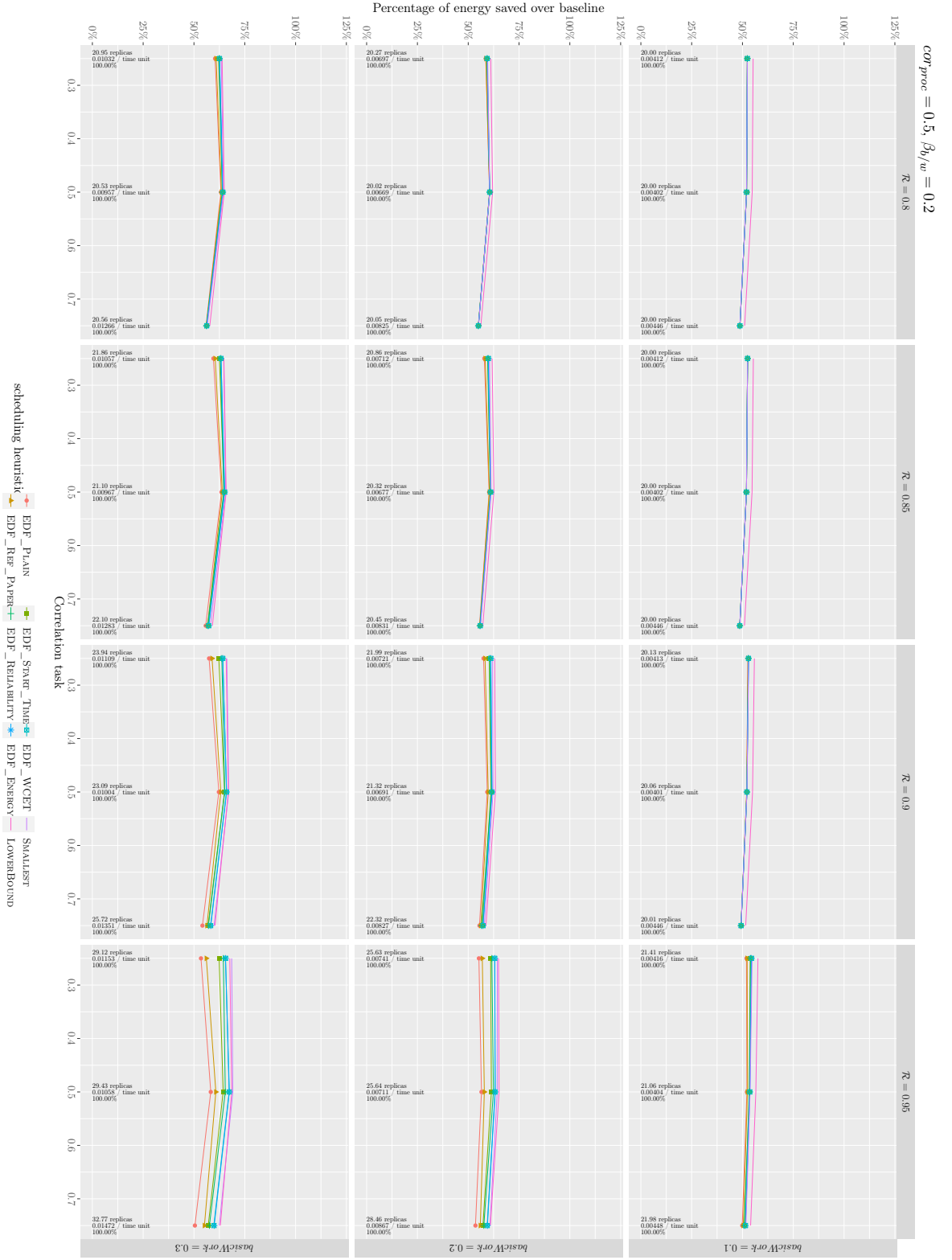


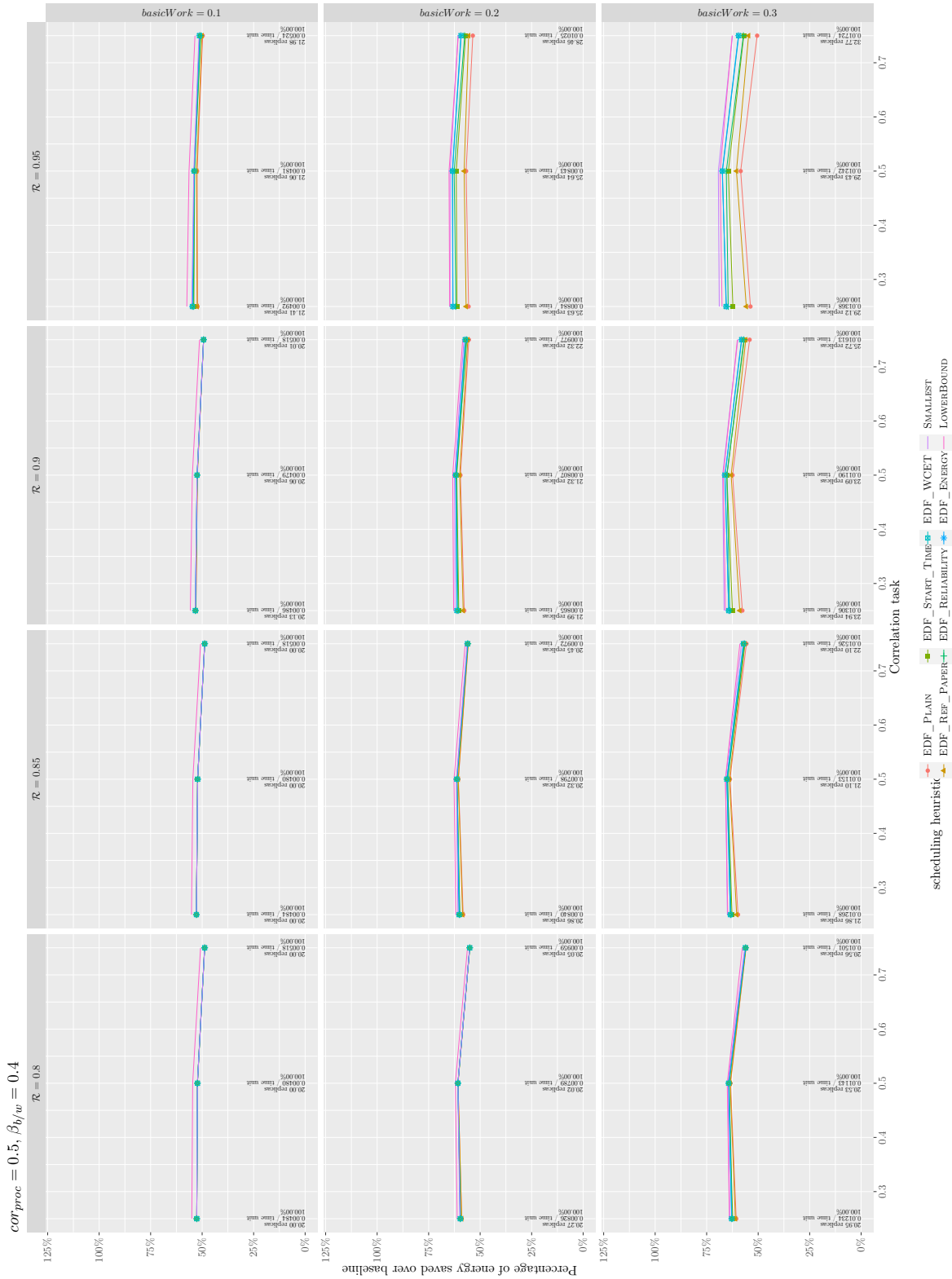




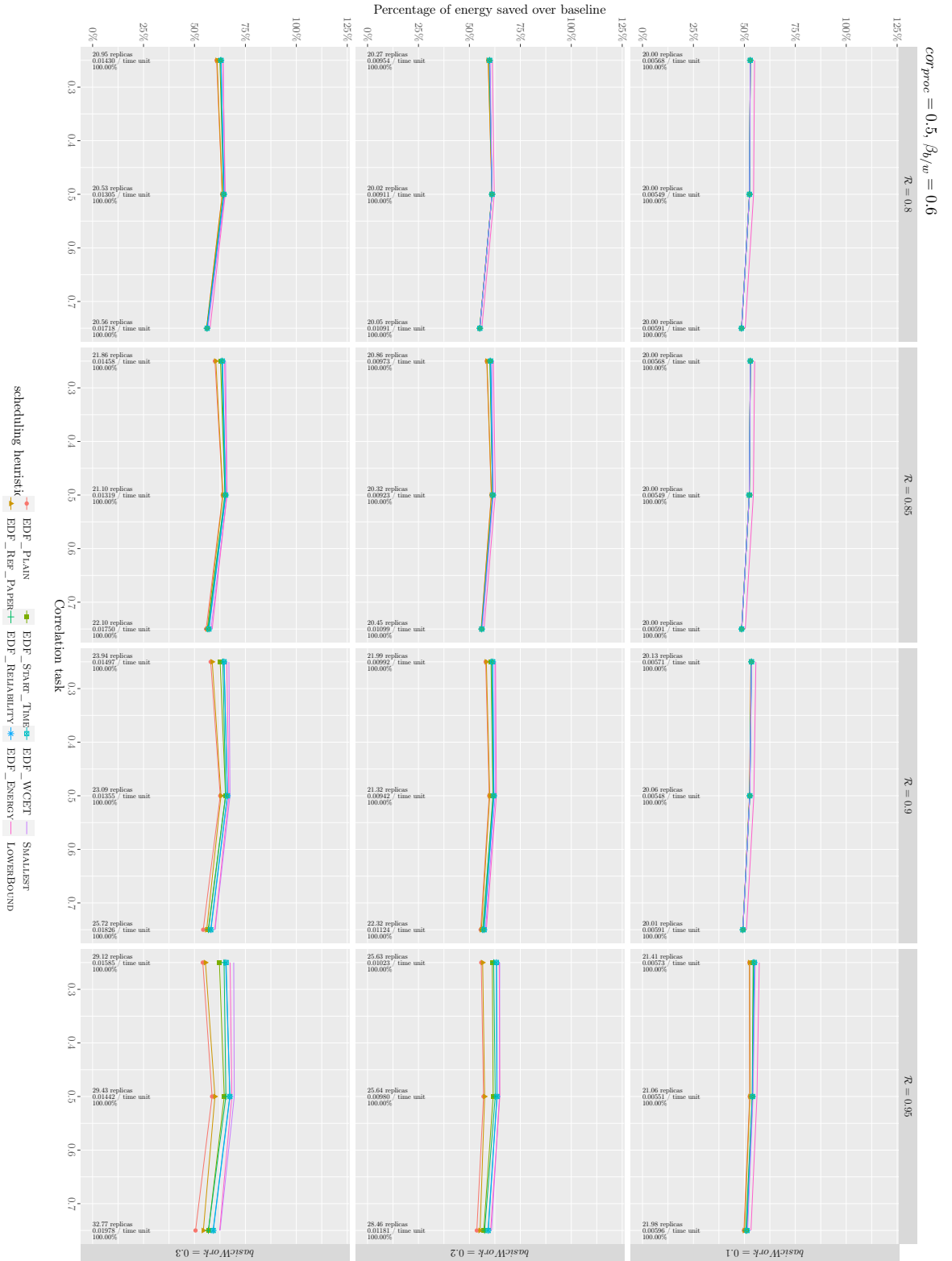
$col_{proc} = 0.25, \beta_{t/w} = 1$

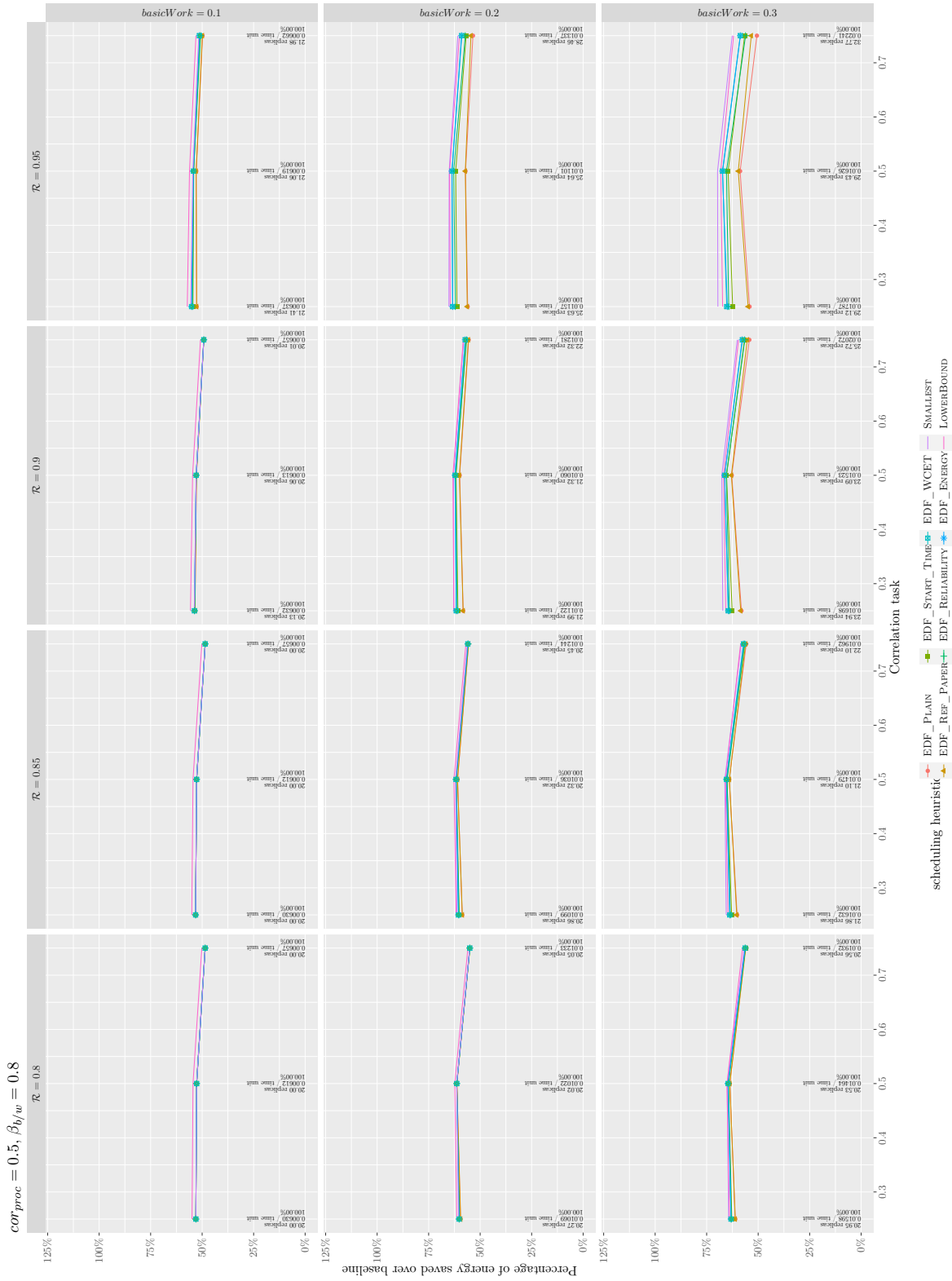


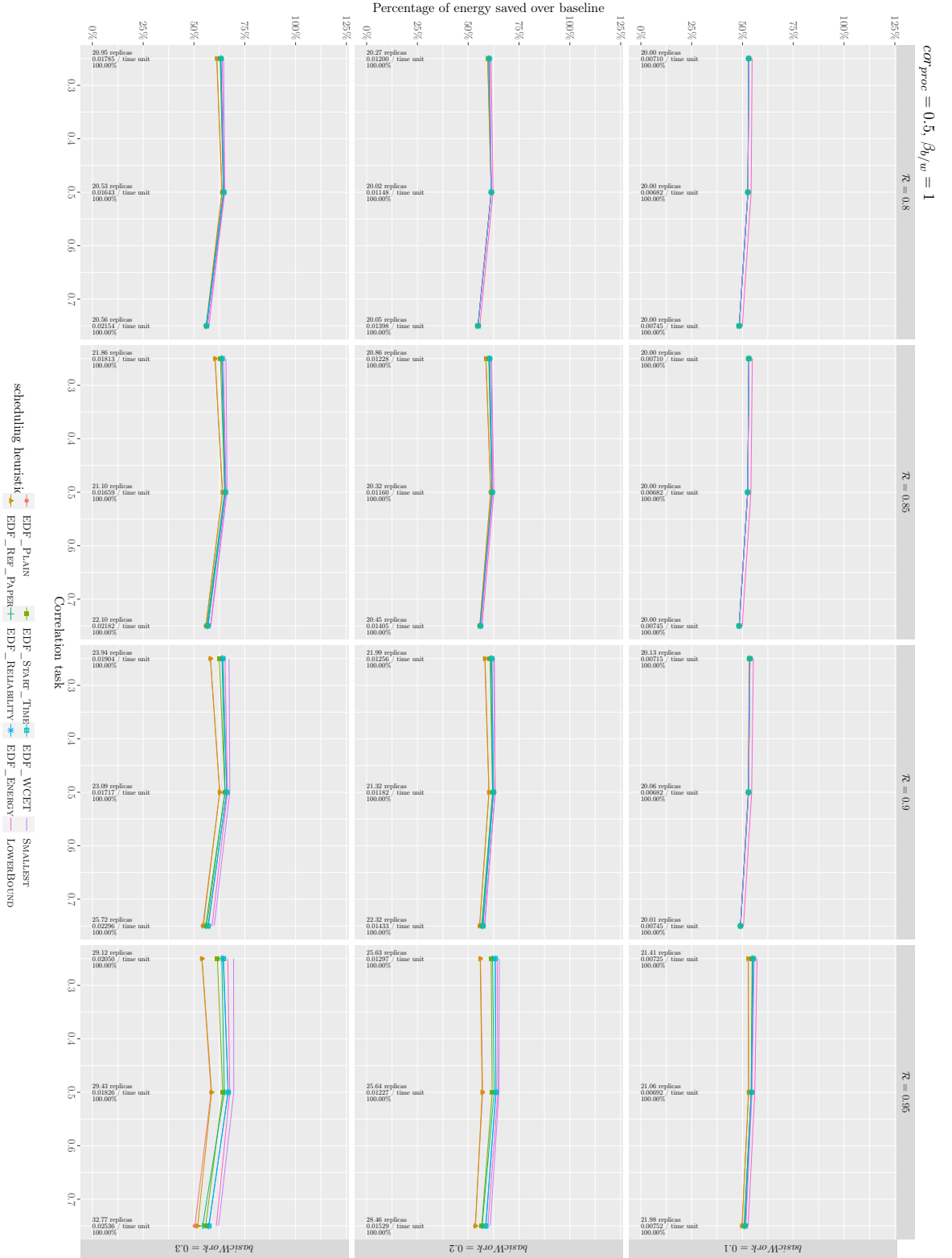


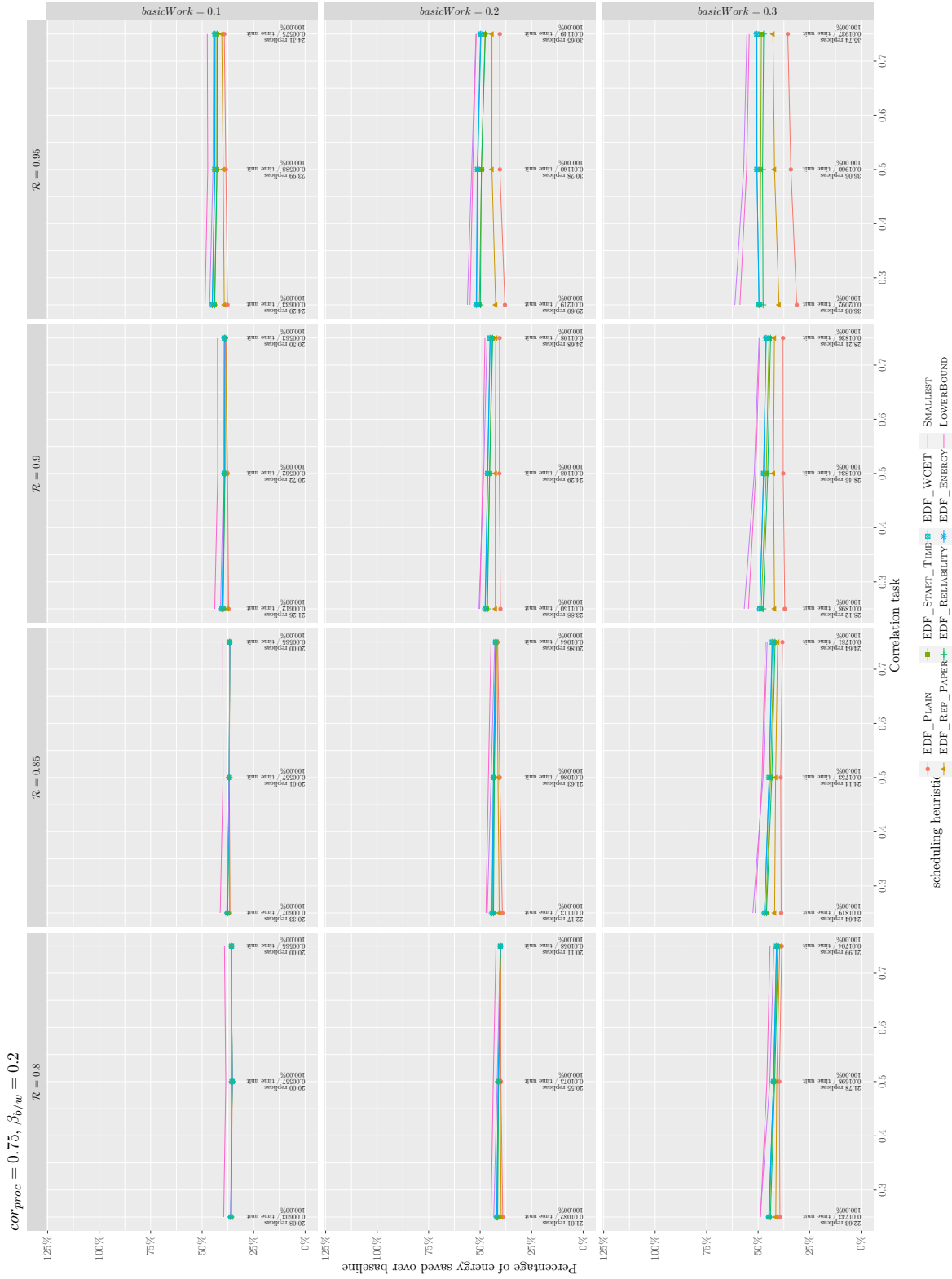


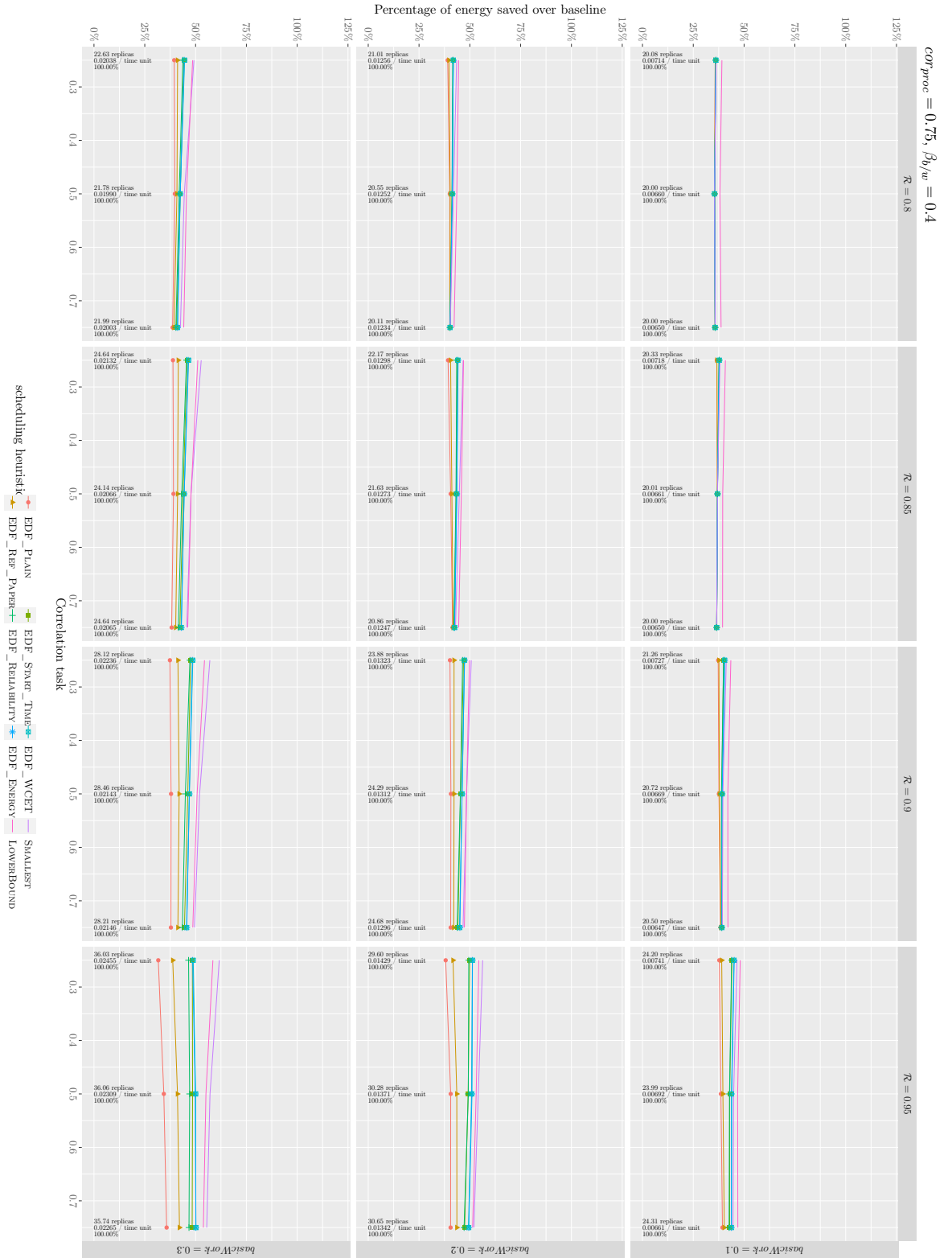
$col_{proc} = 0.5, \beta_{h/w} = 0.4$

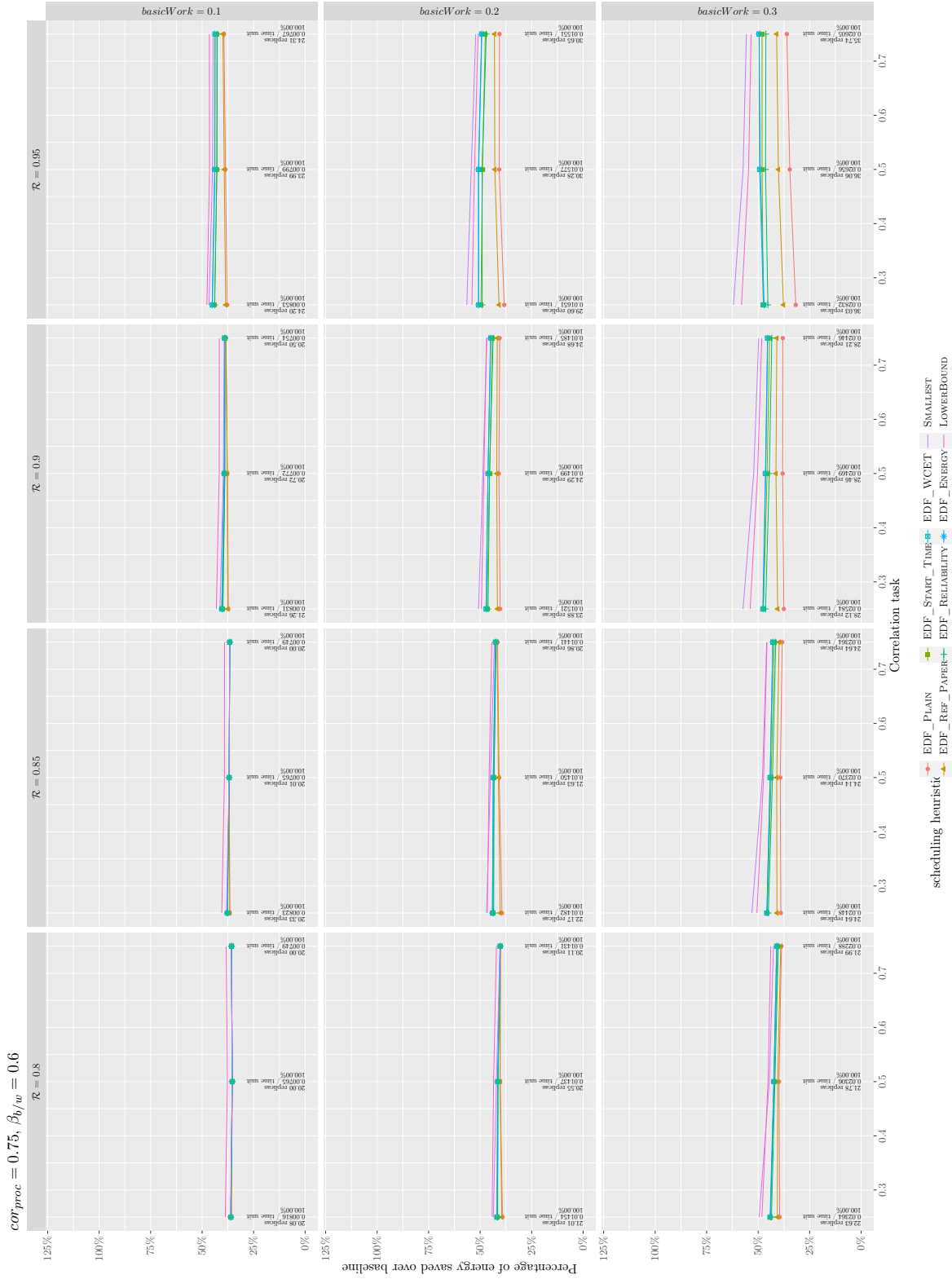






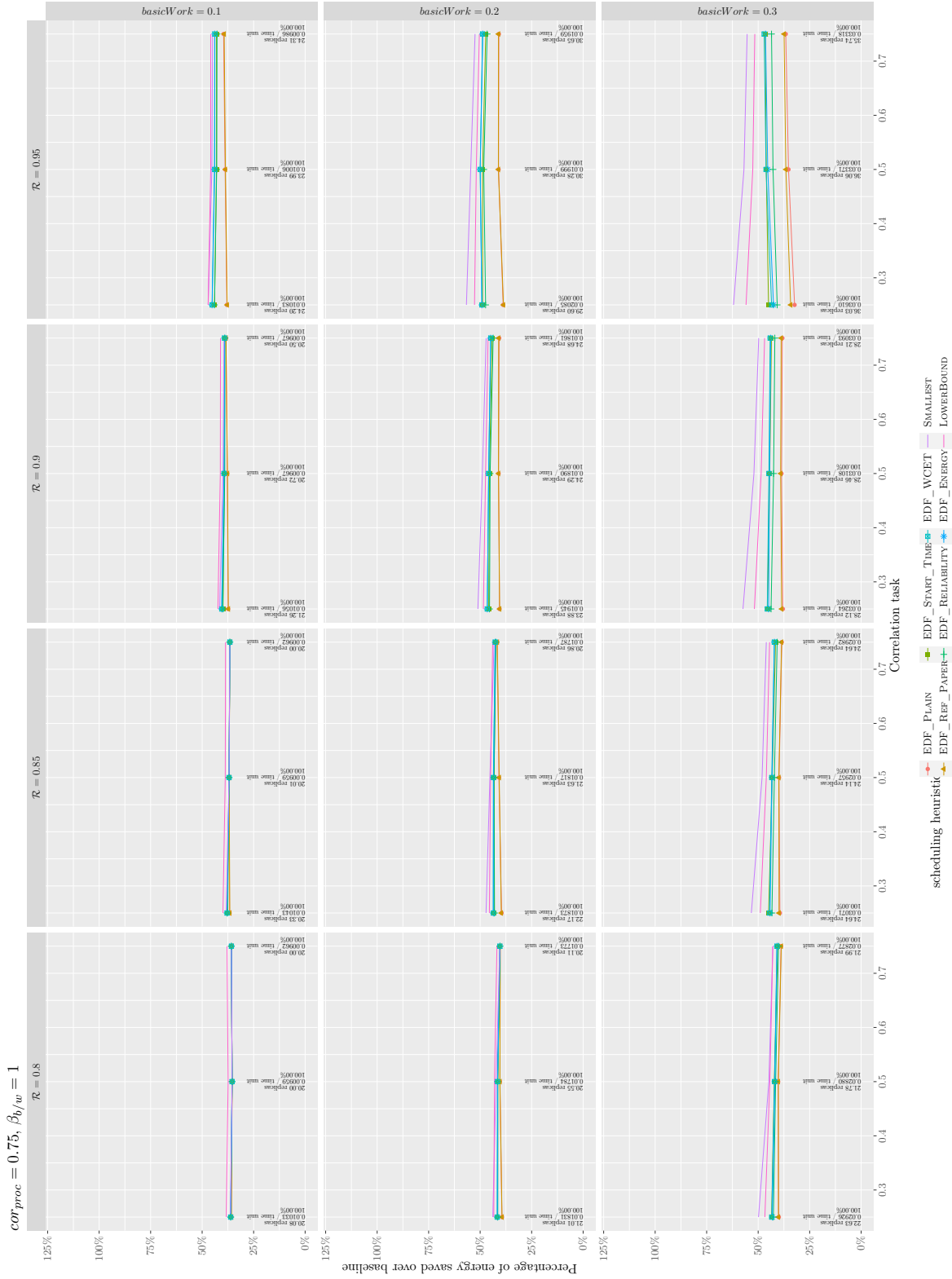








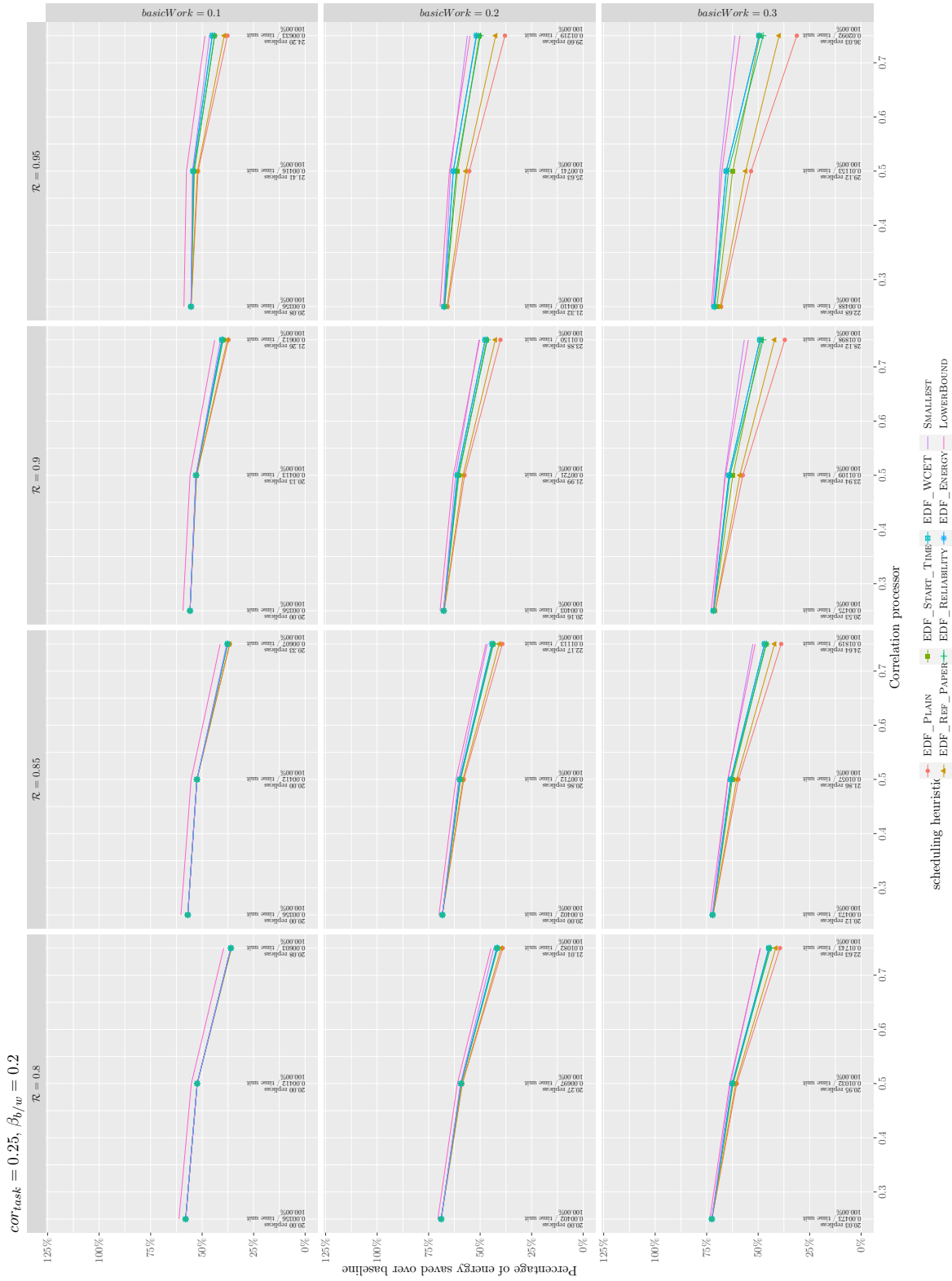




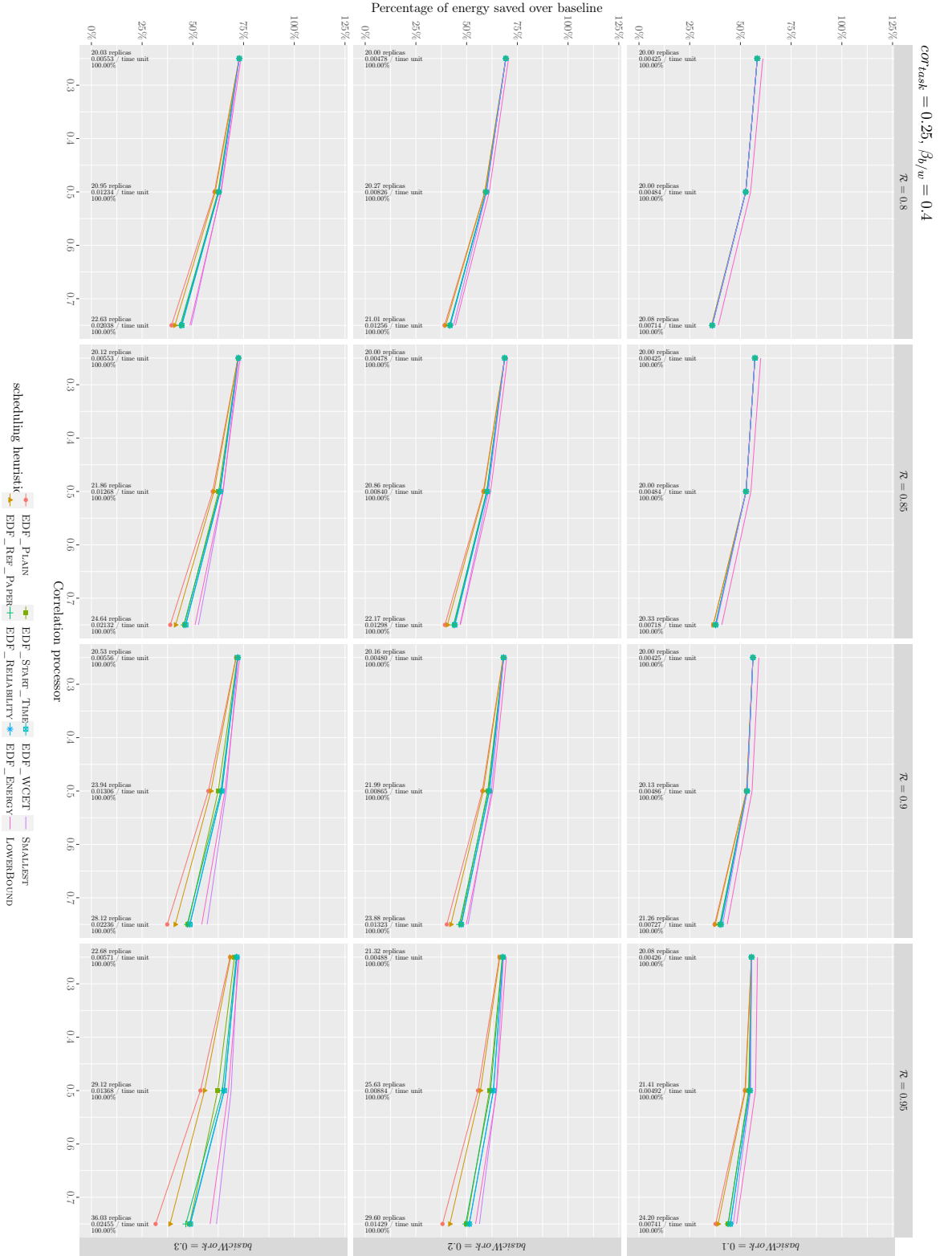
$col_{proc} = 0.75, \beta_{t/w} = 1$

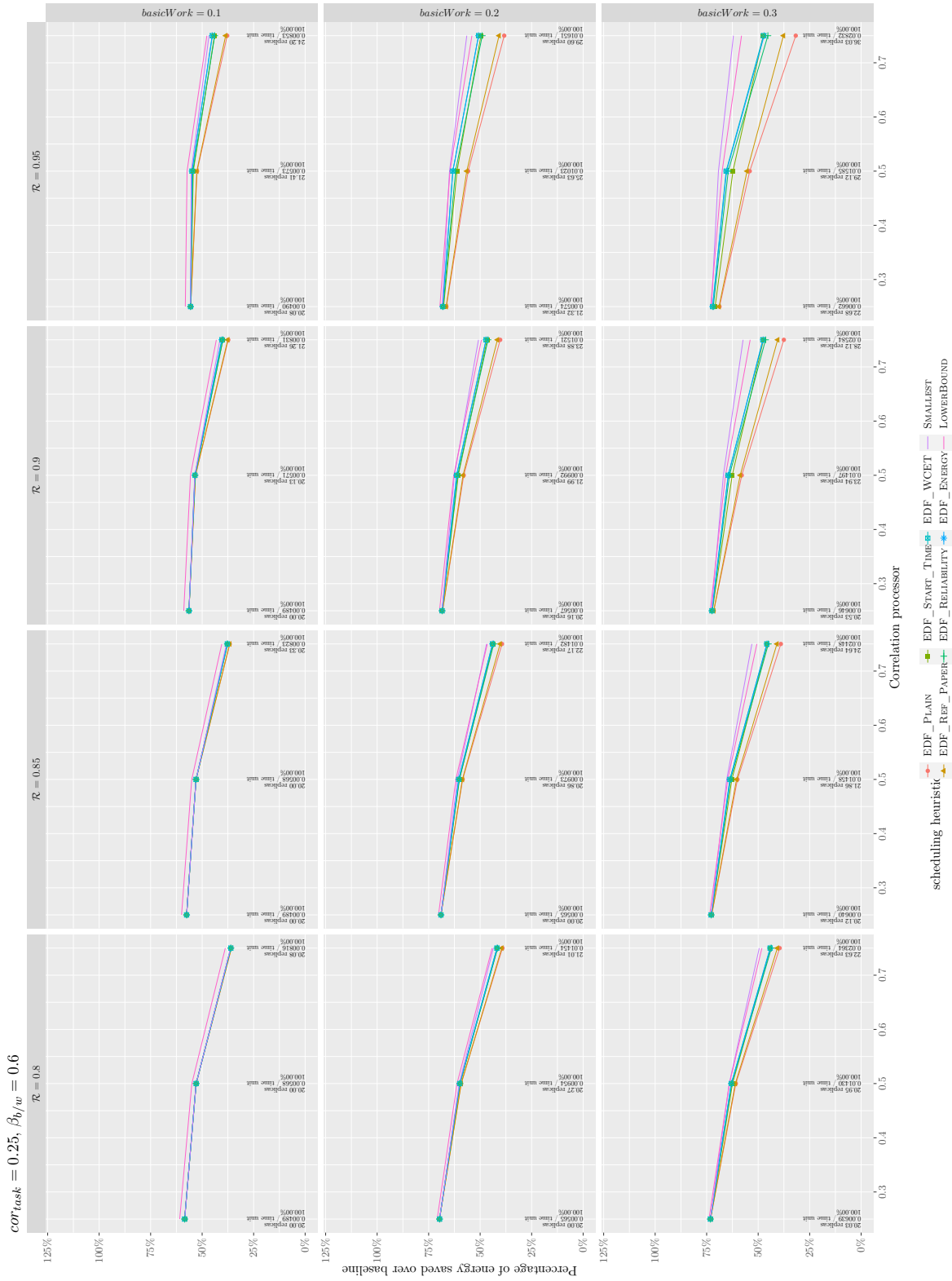
## Processor correlation

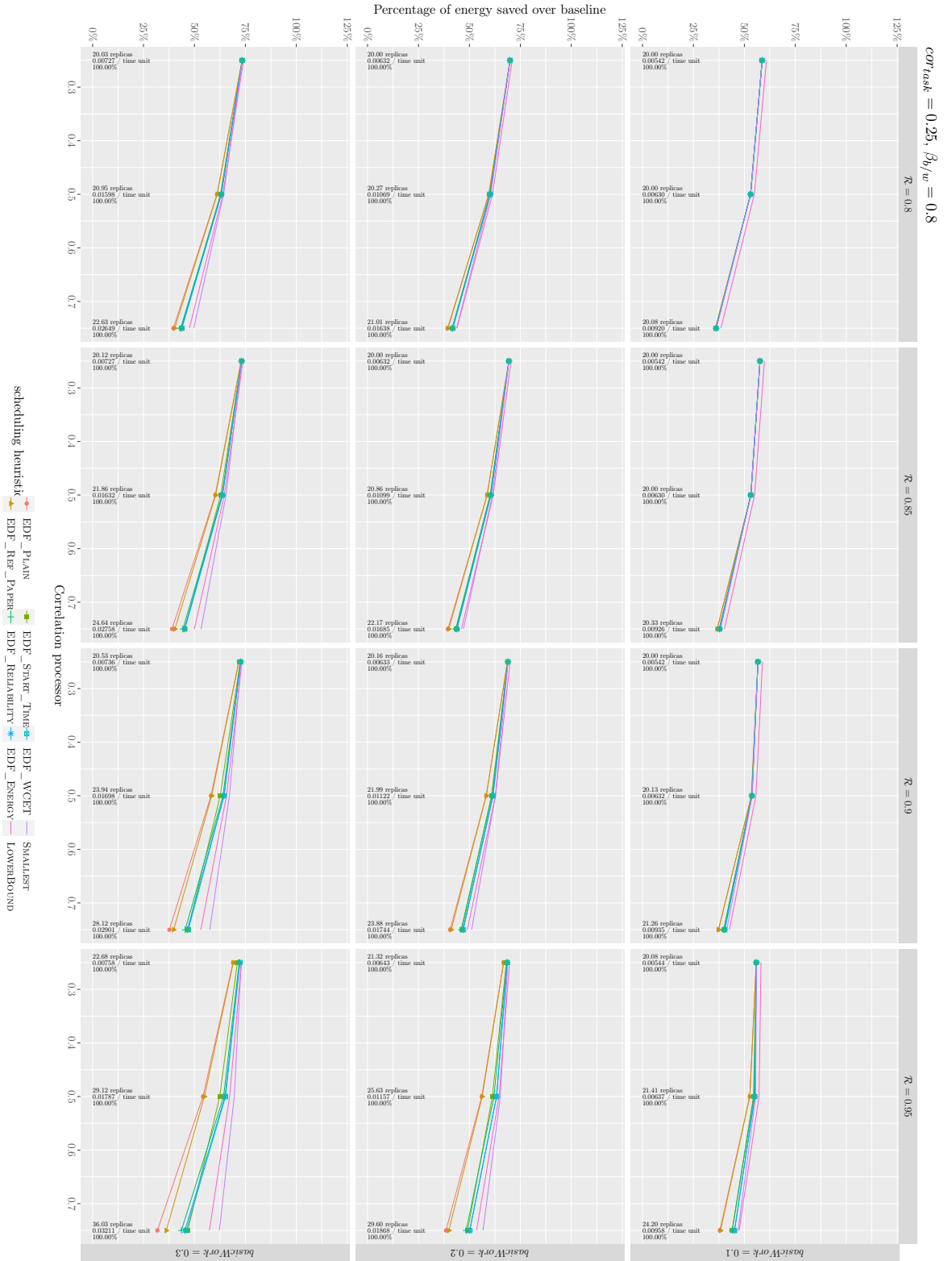
The following section presents percentage of energy saved over baseline using different scheduling strategies under big failure rate, when varying  $cor_{proc}$ .

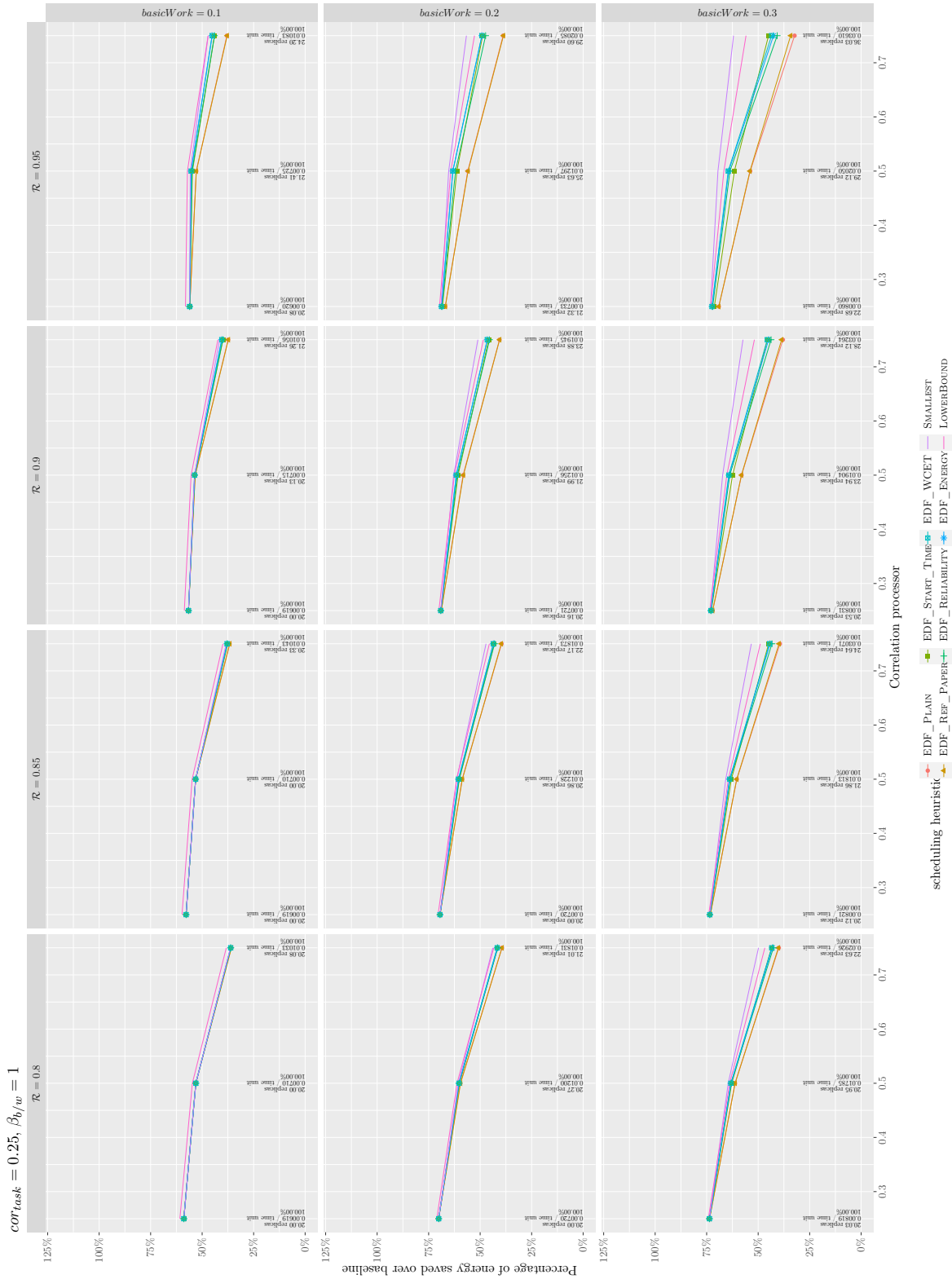


$cop_{task} = 0.25, \beta_{t/w} = 0.2$

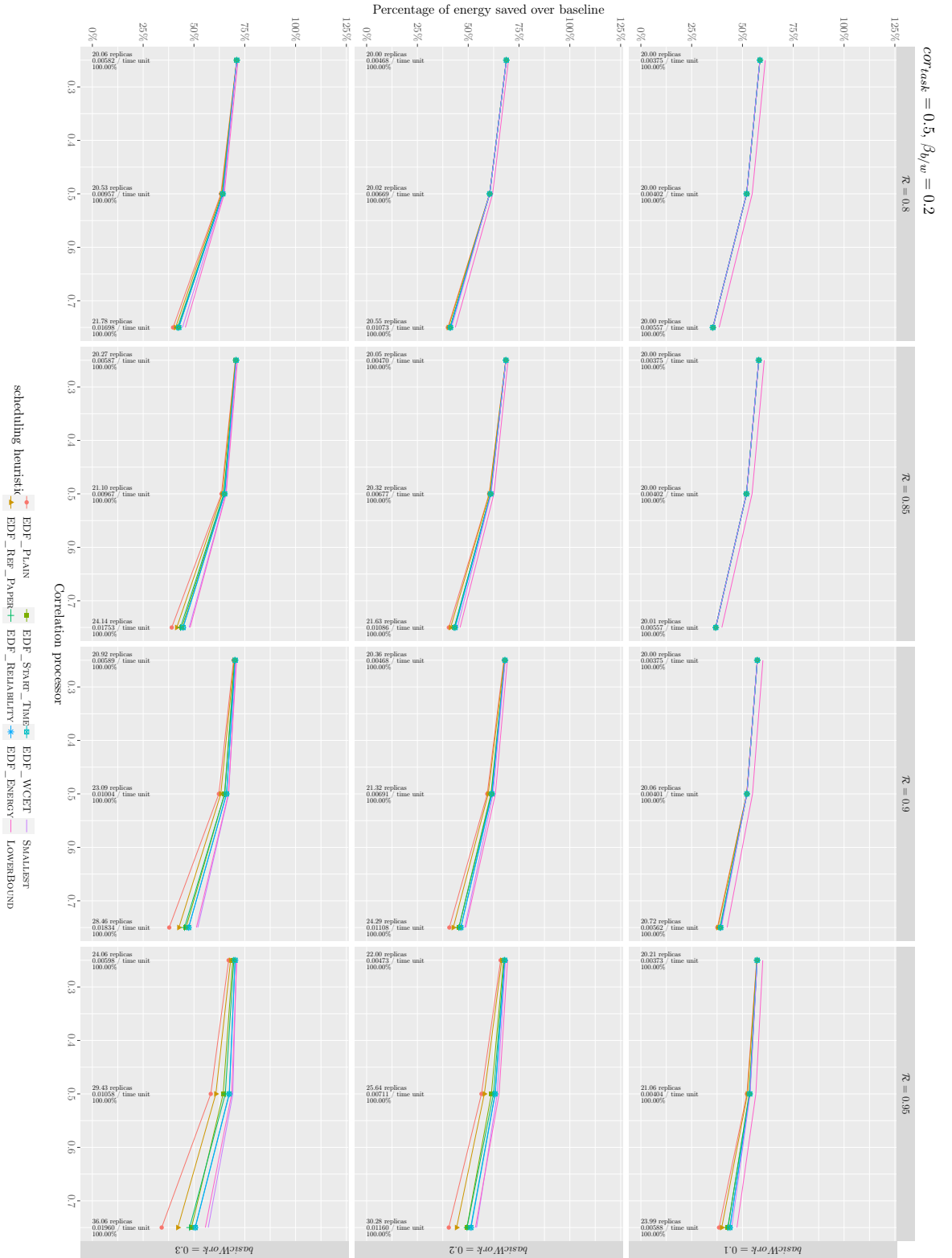


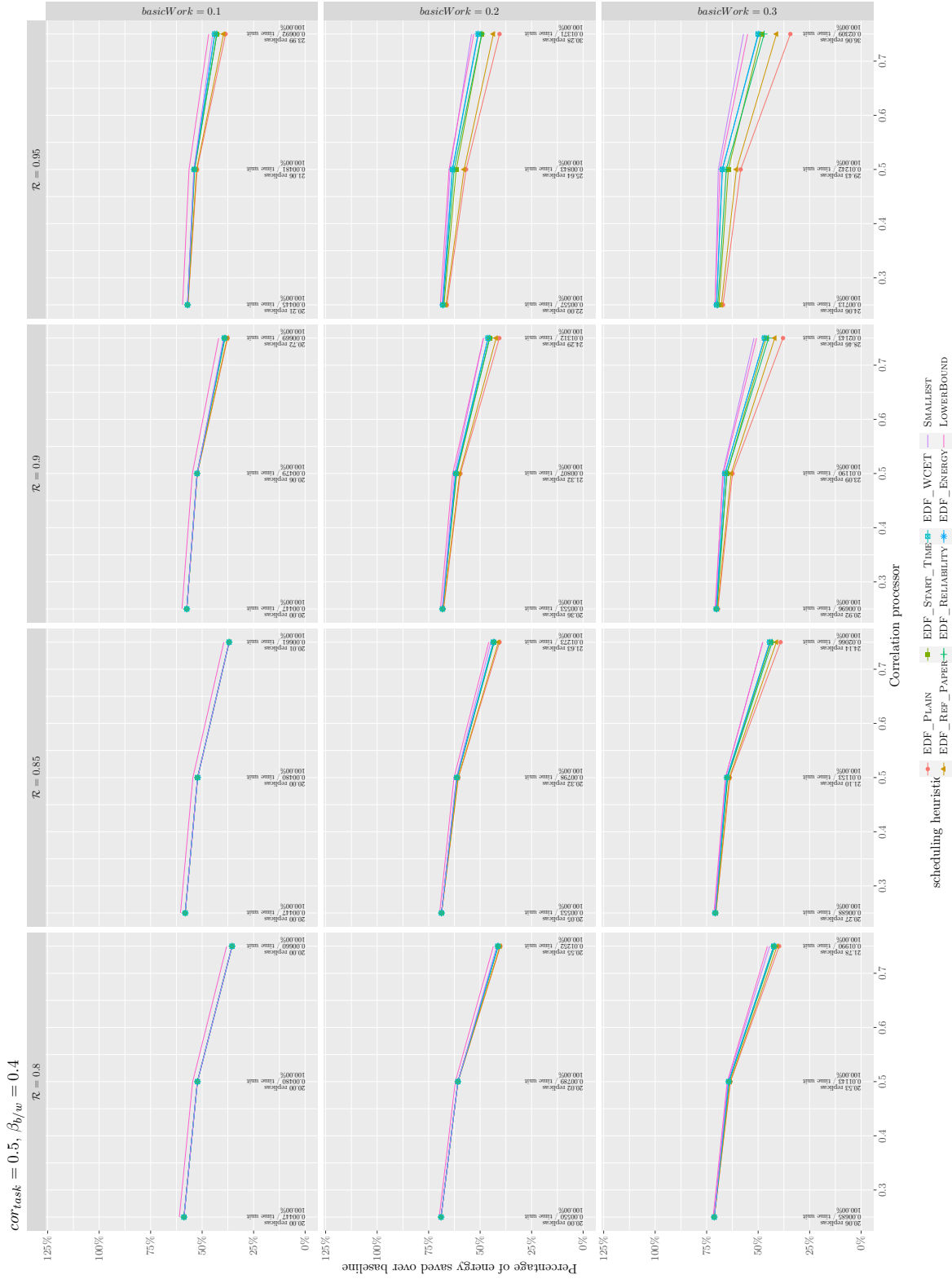




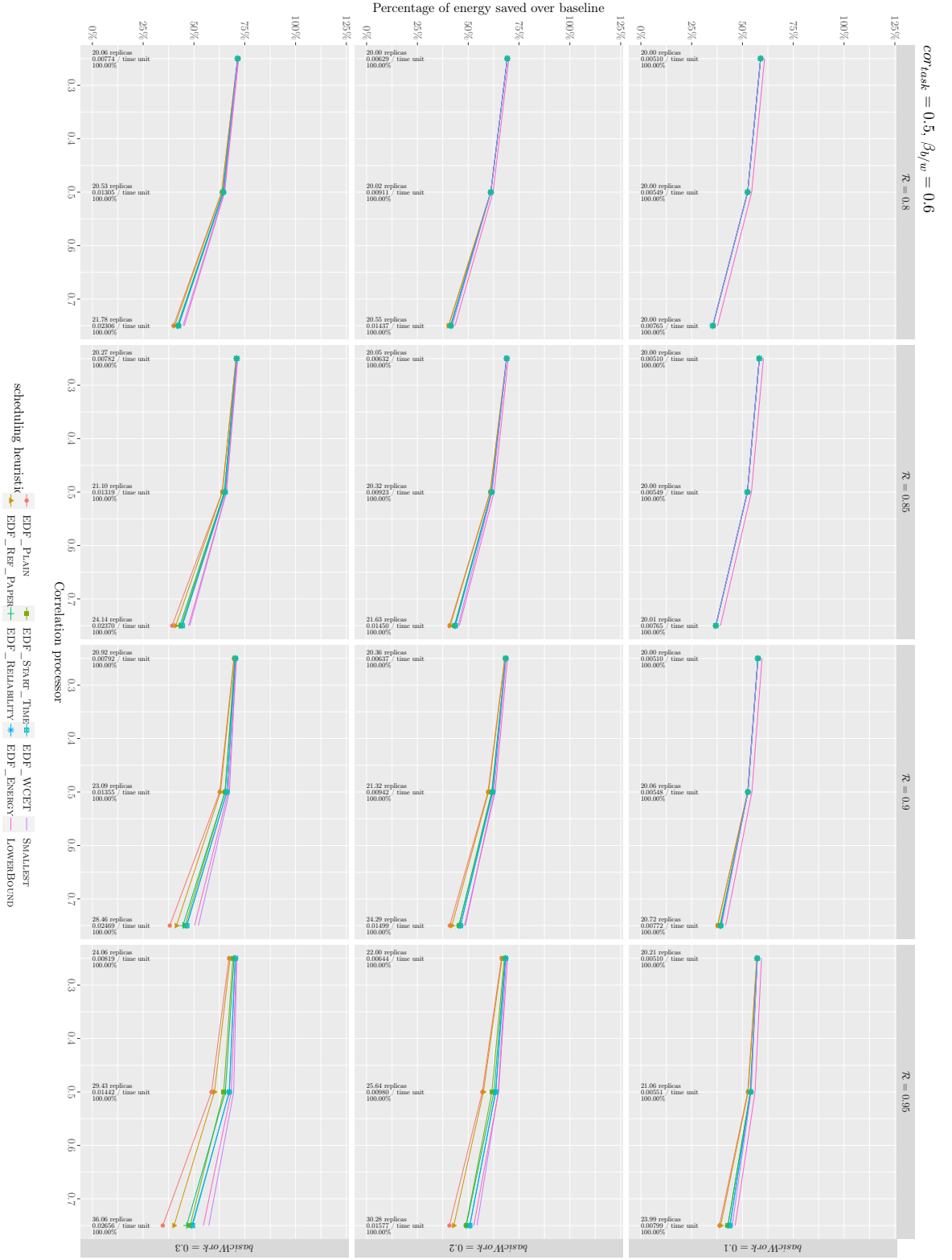


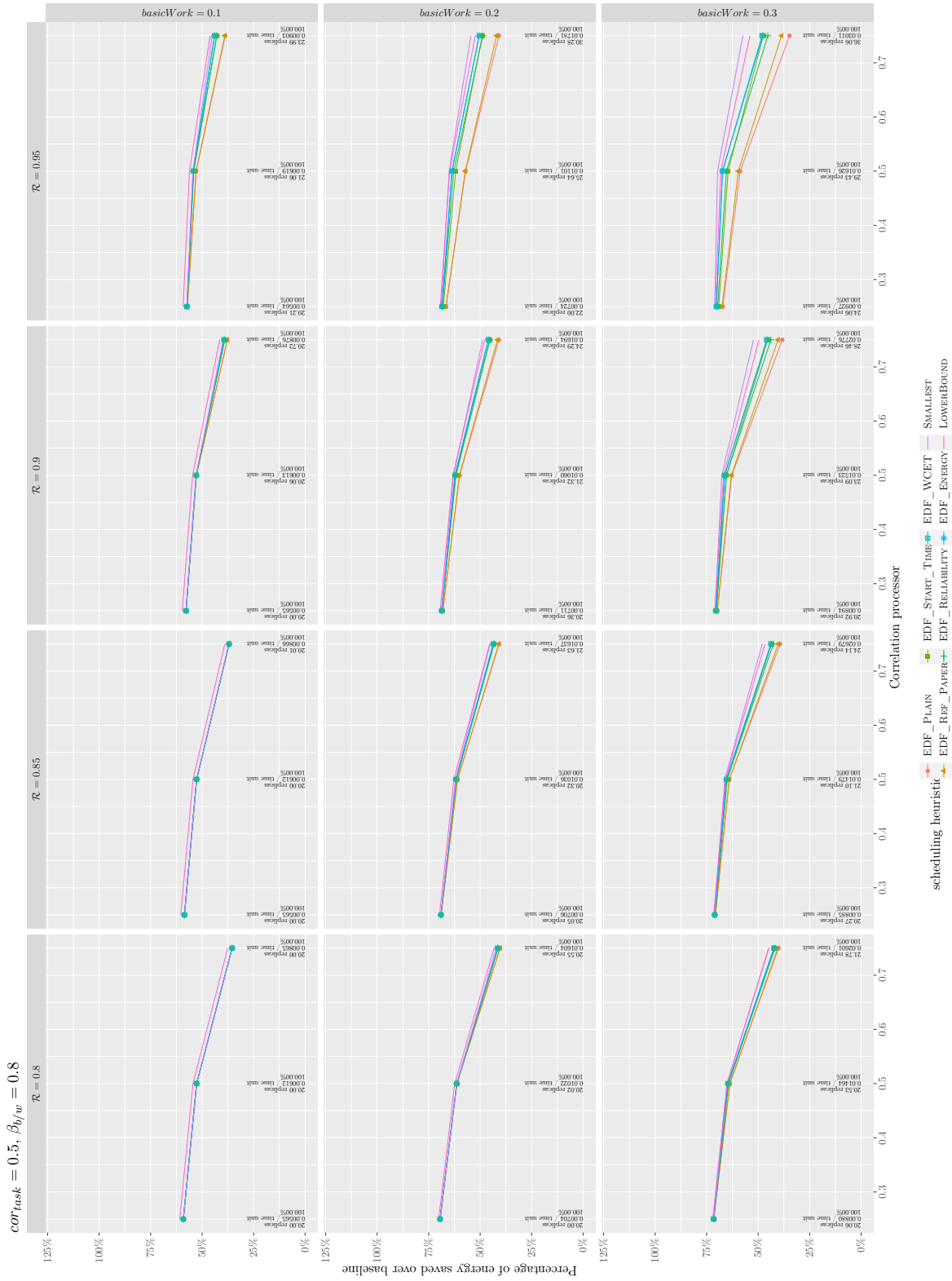


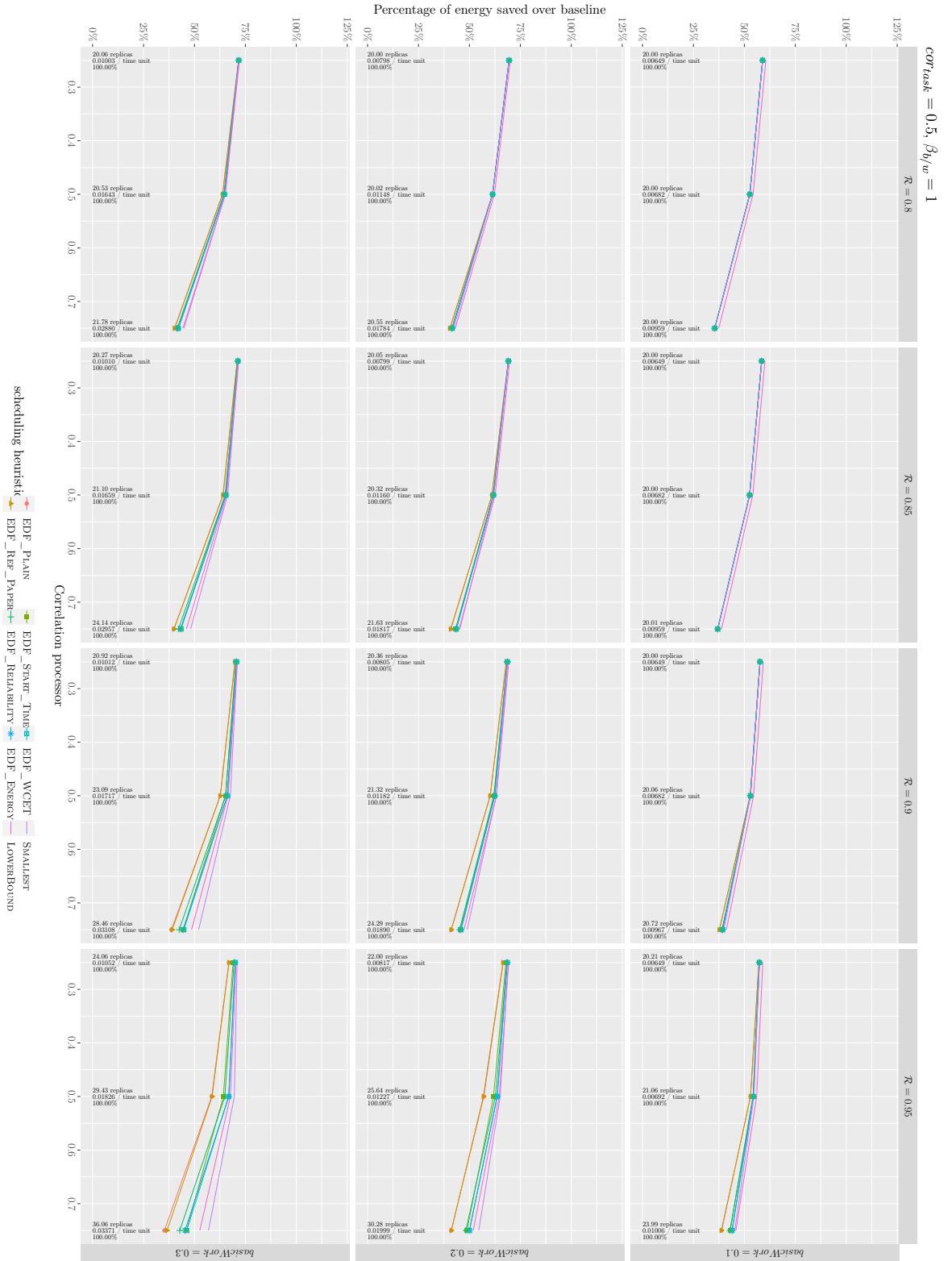


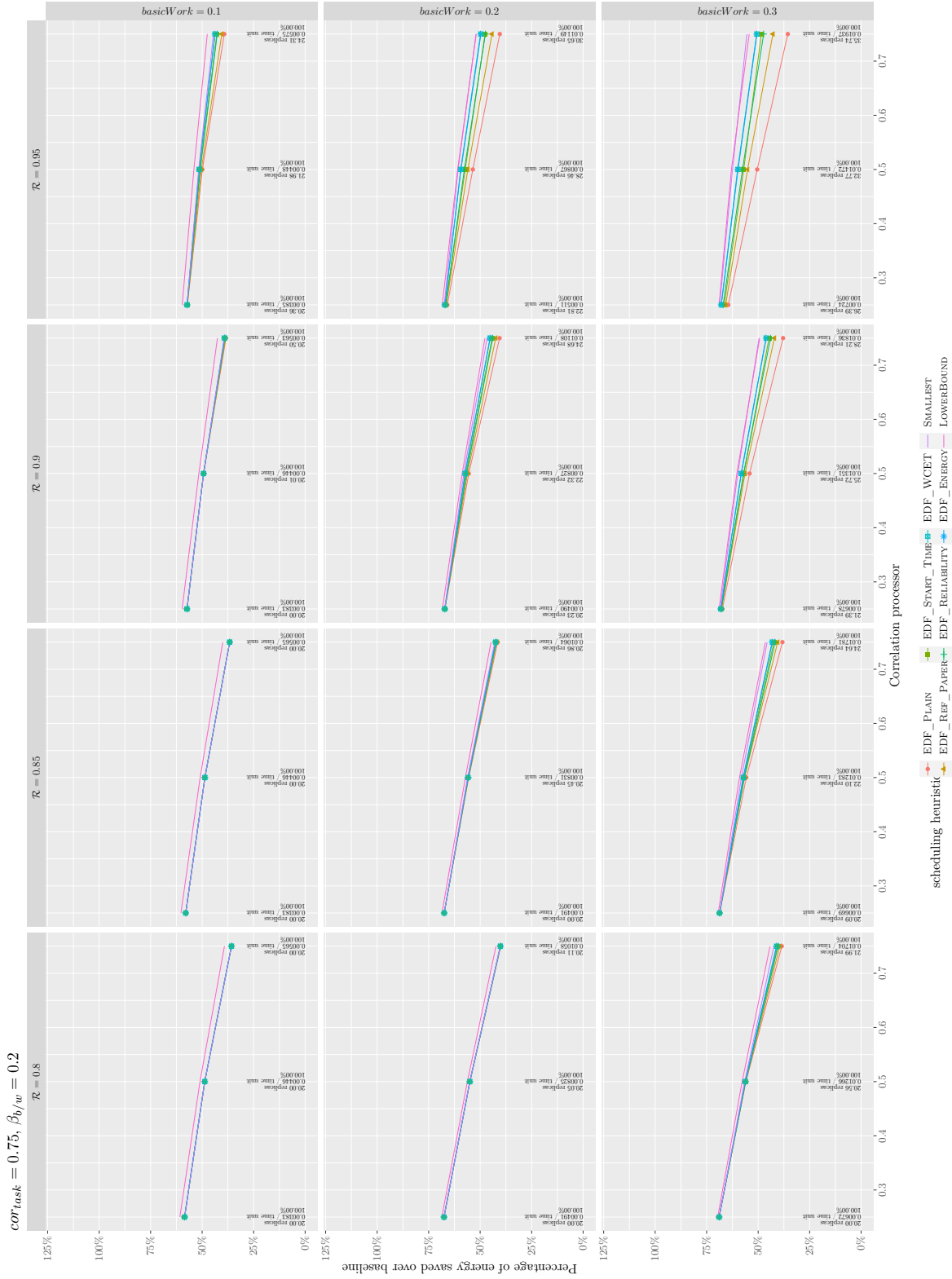


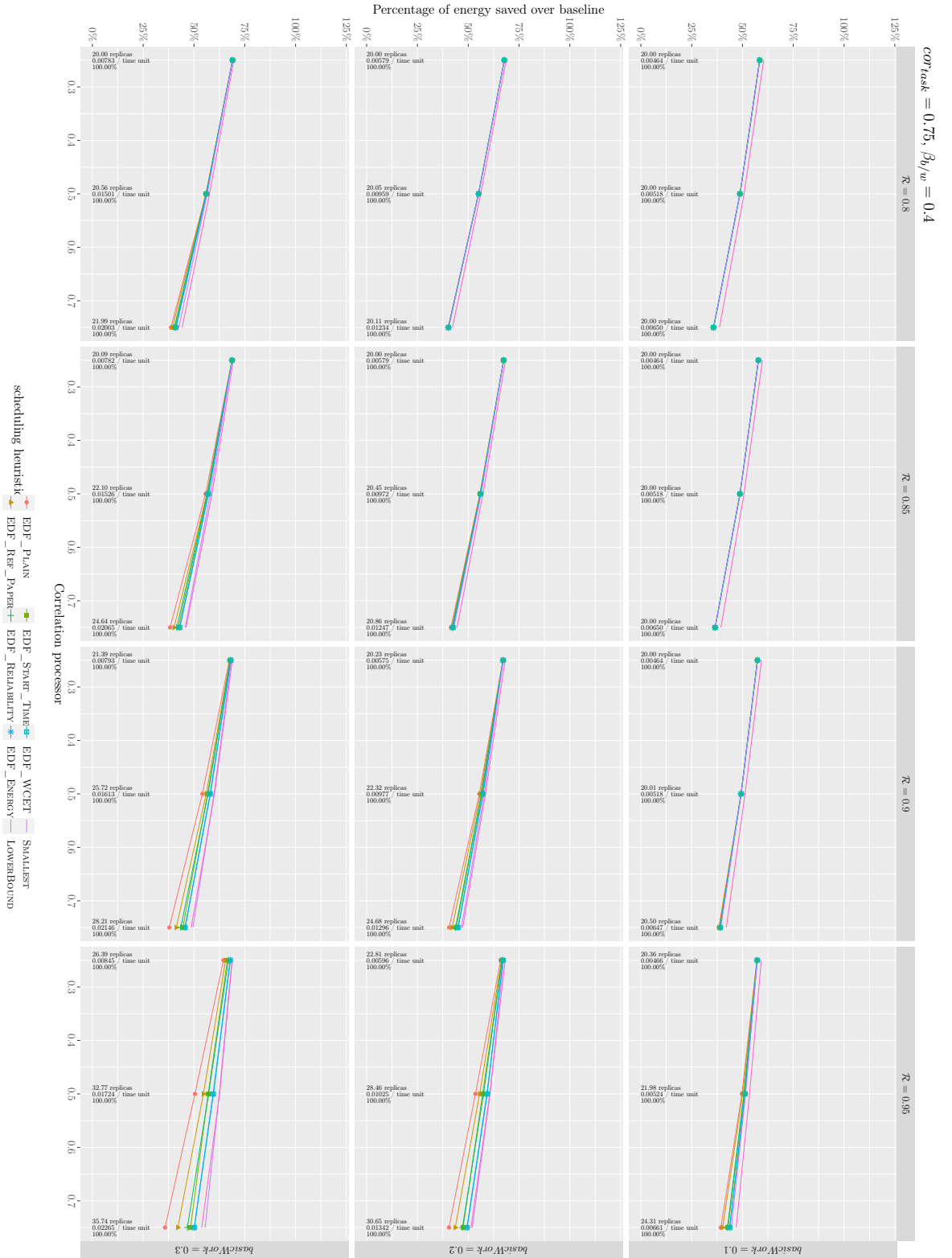
$cop_{task} = 0.5, \beta_{h/w} = 0.4$

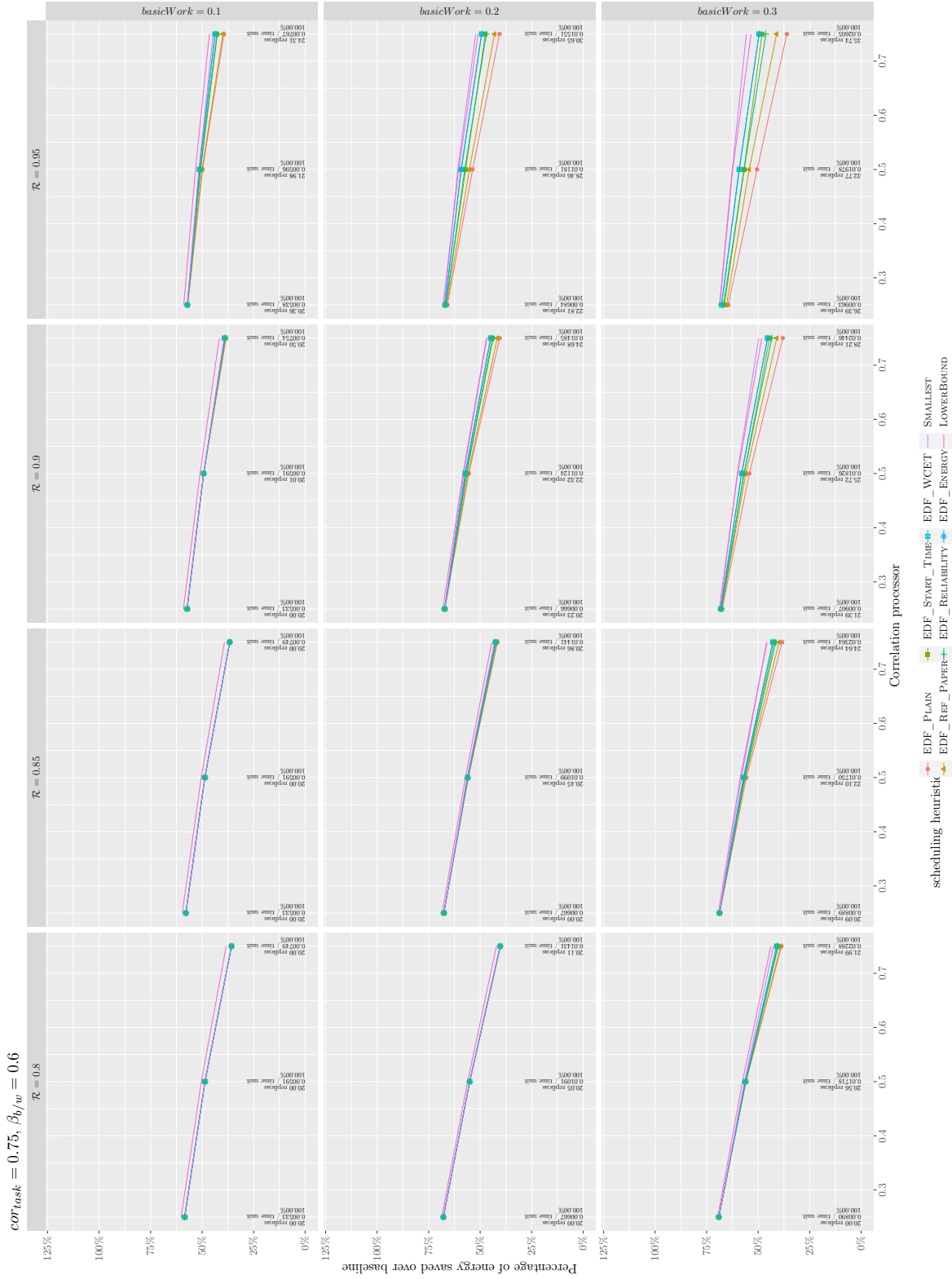




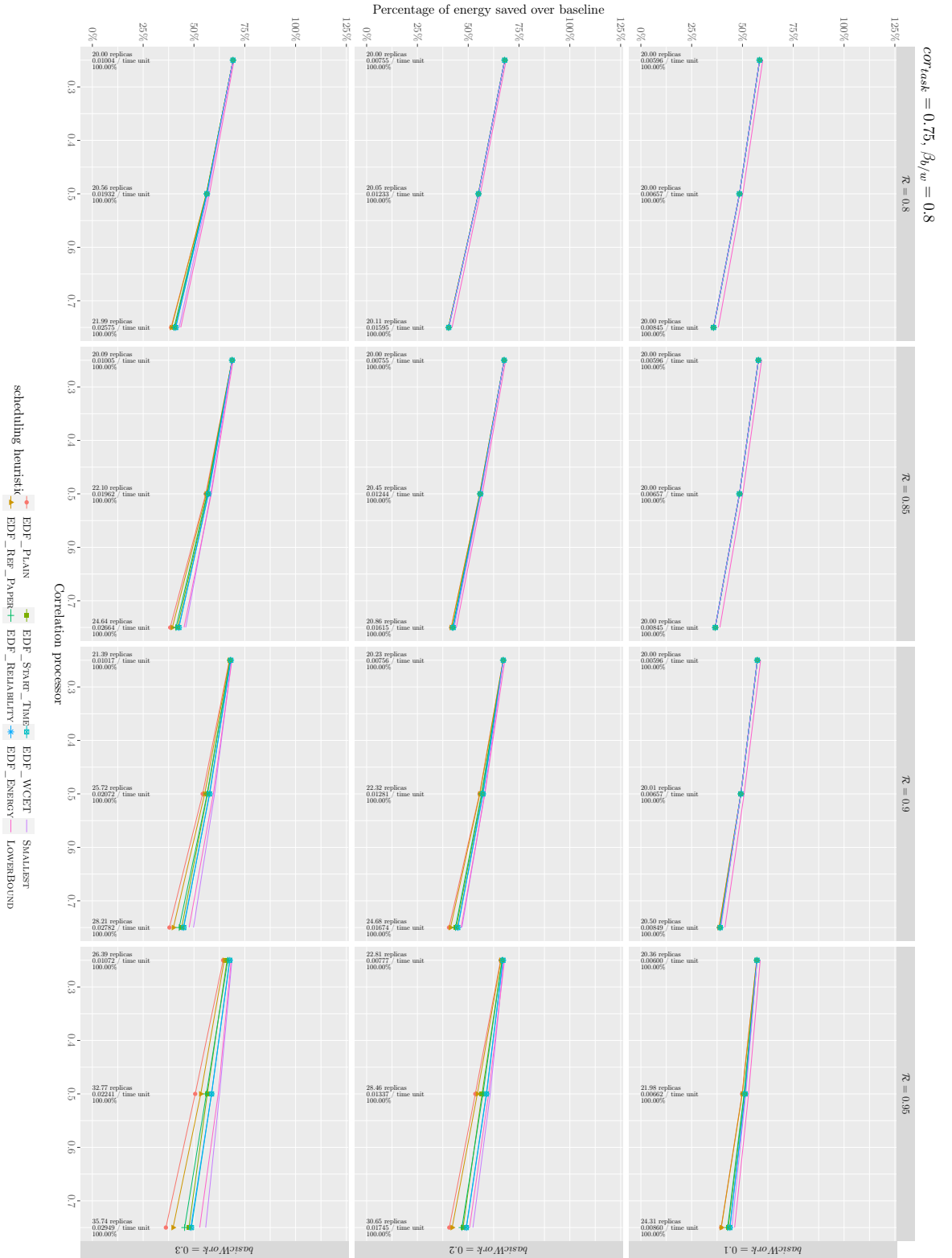


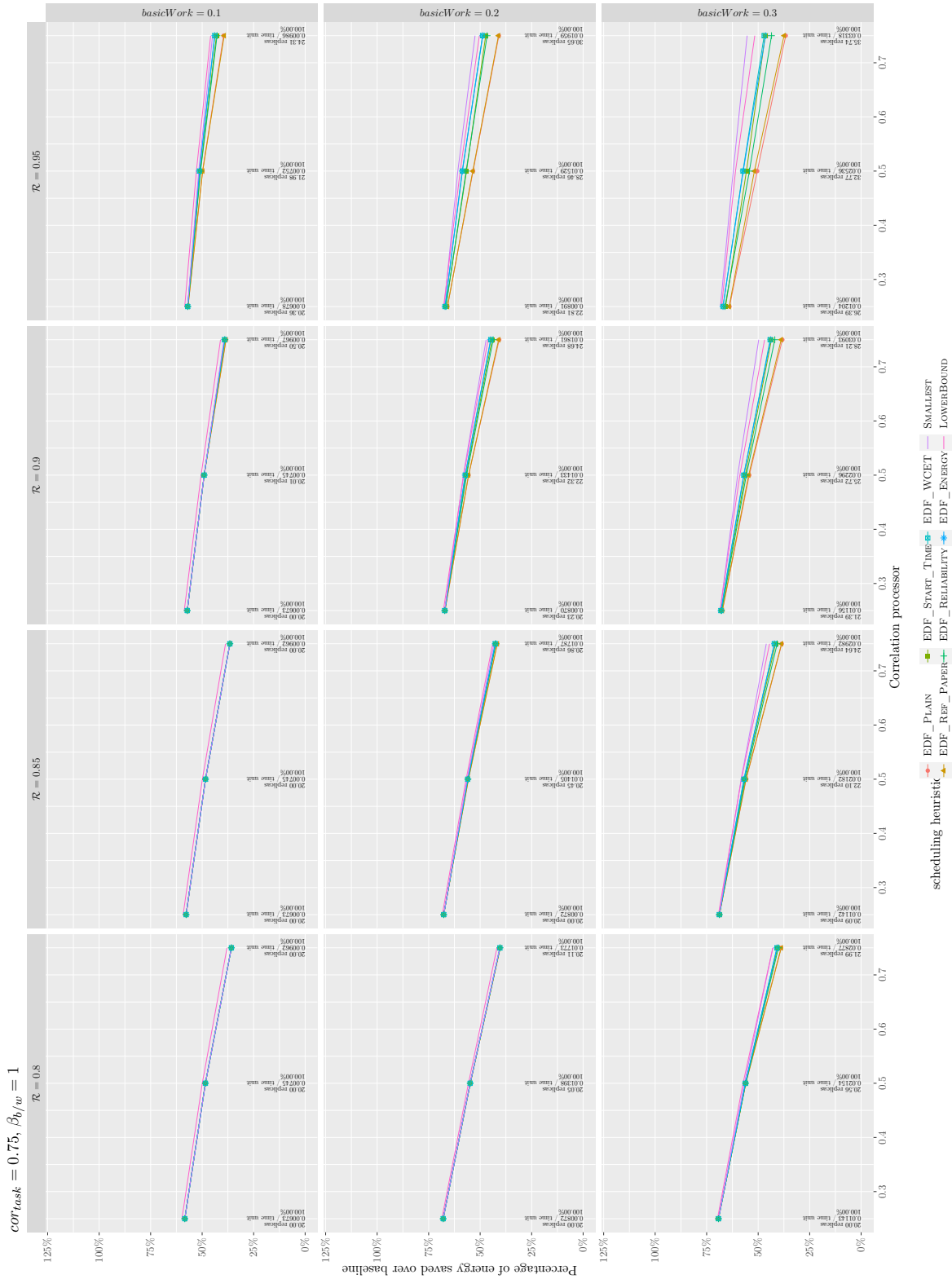












## **Task variability**

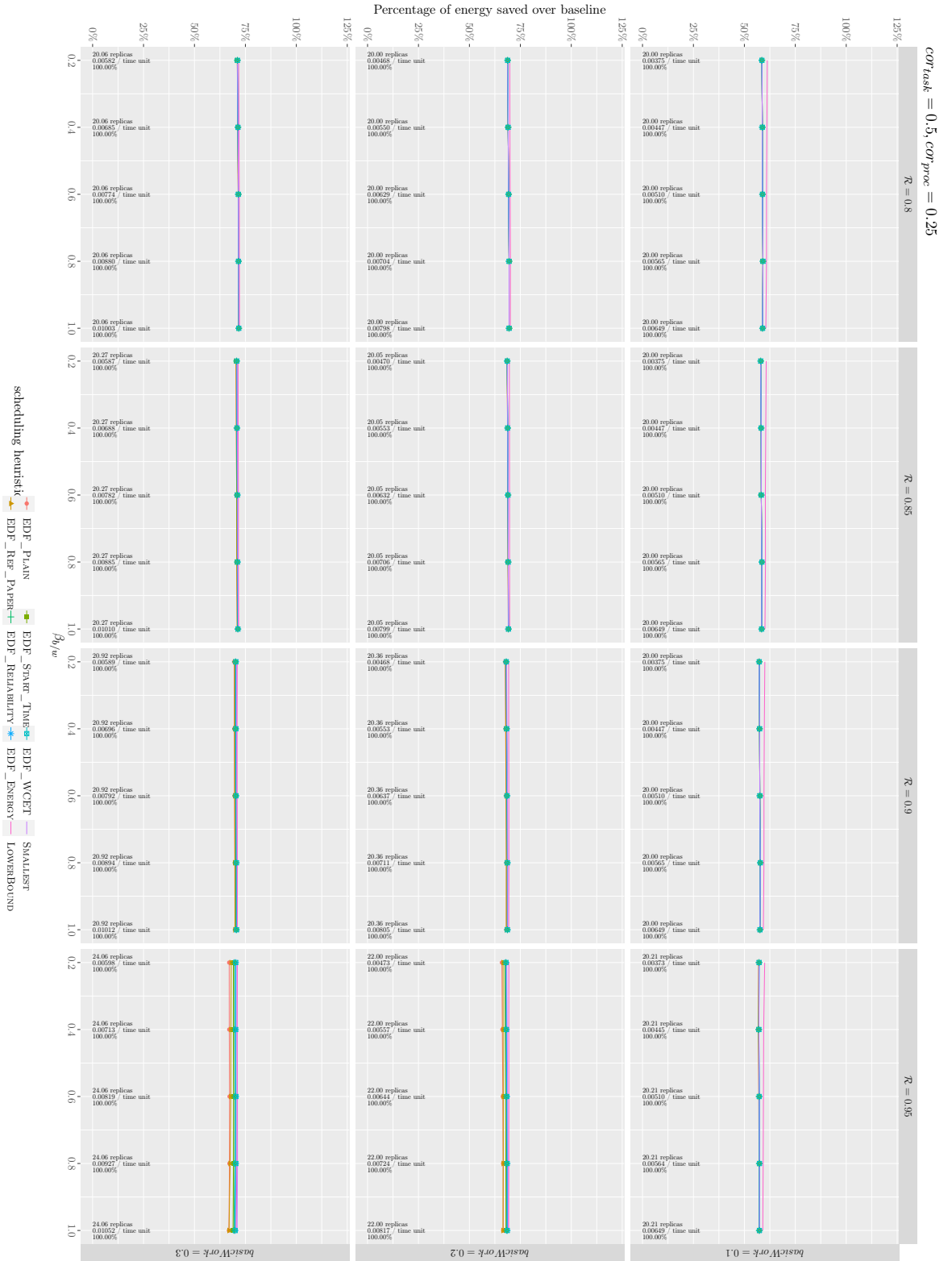
The following section presents percentage of energy saved over baseline using different scheduling strategies under big failure rate, when varying  $\beta_{b/w}$ .



$cot_{task} = 0.25, cot_{proc} = 0.25$



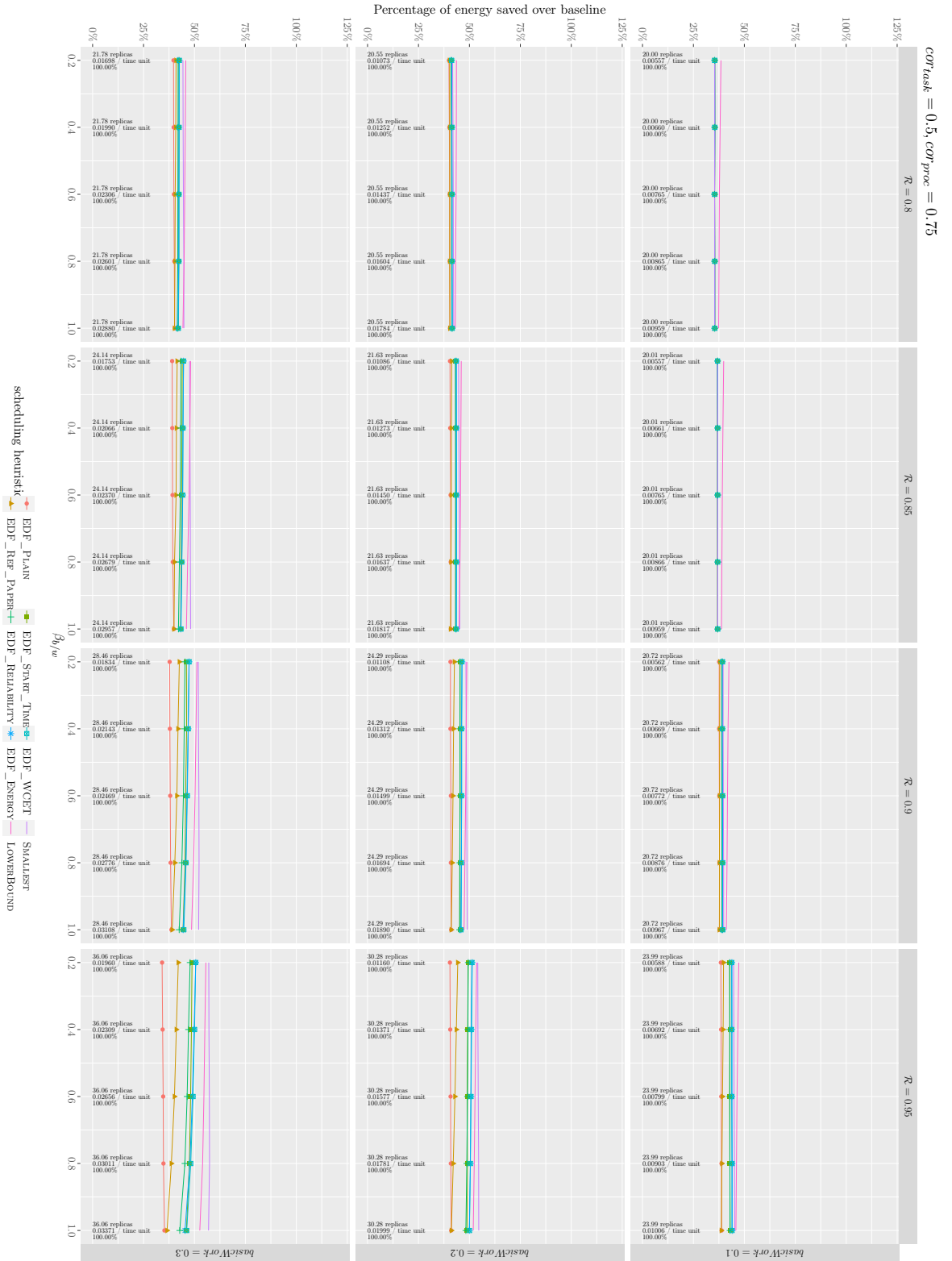






$cop_{task} = 0.5, cop_{proc} = 0.5$

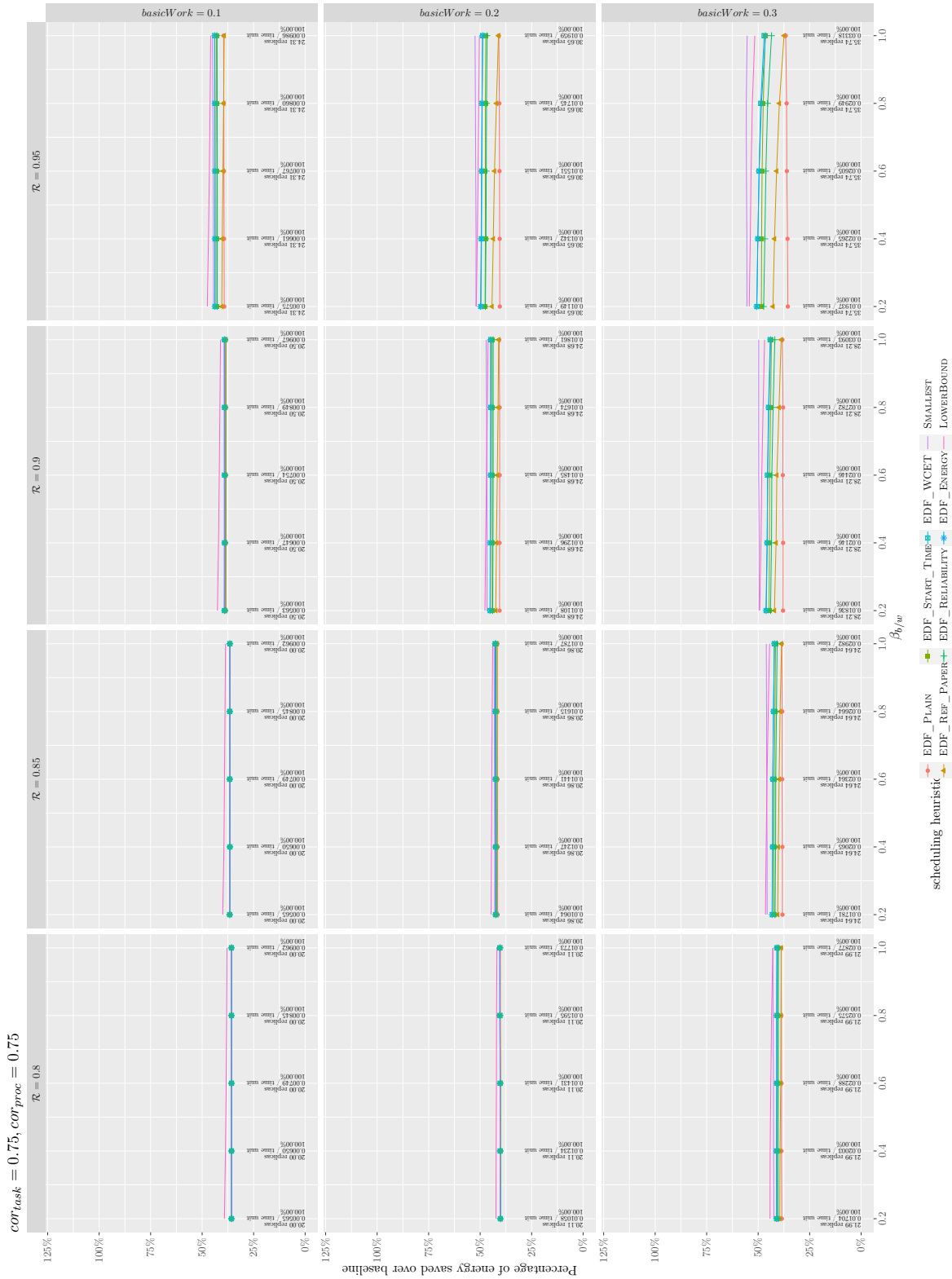






$cop_{task} = 0.75, cop_{proc} = 0.25$



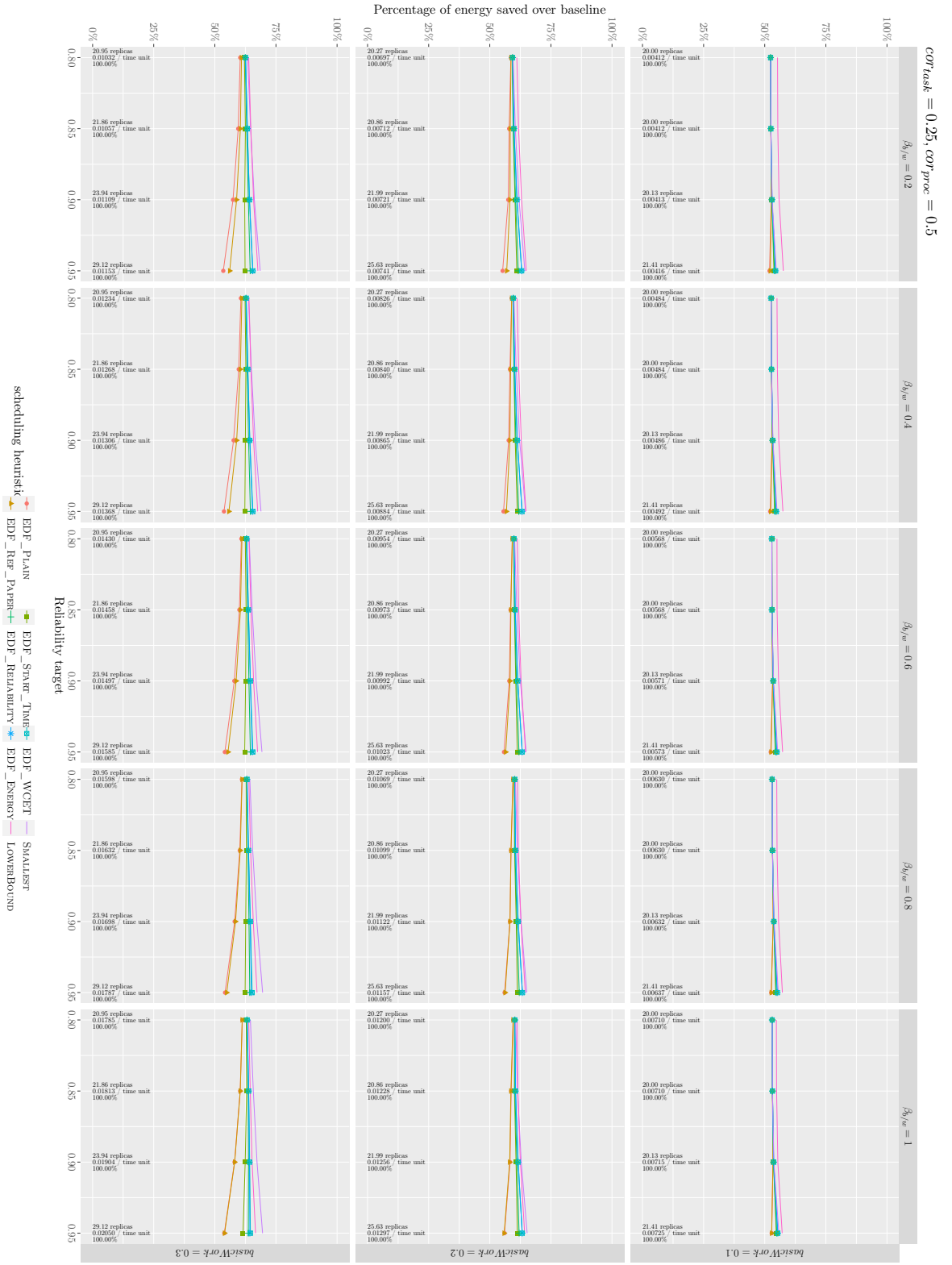


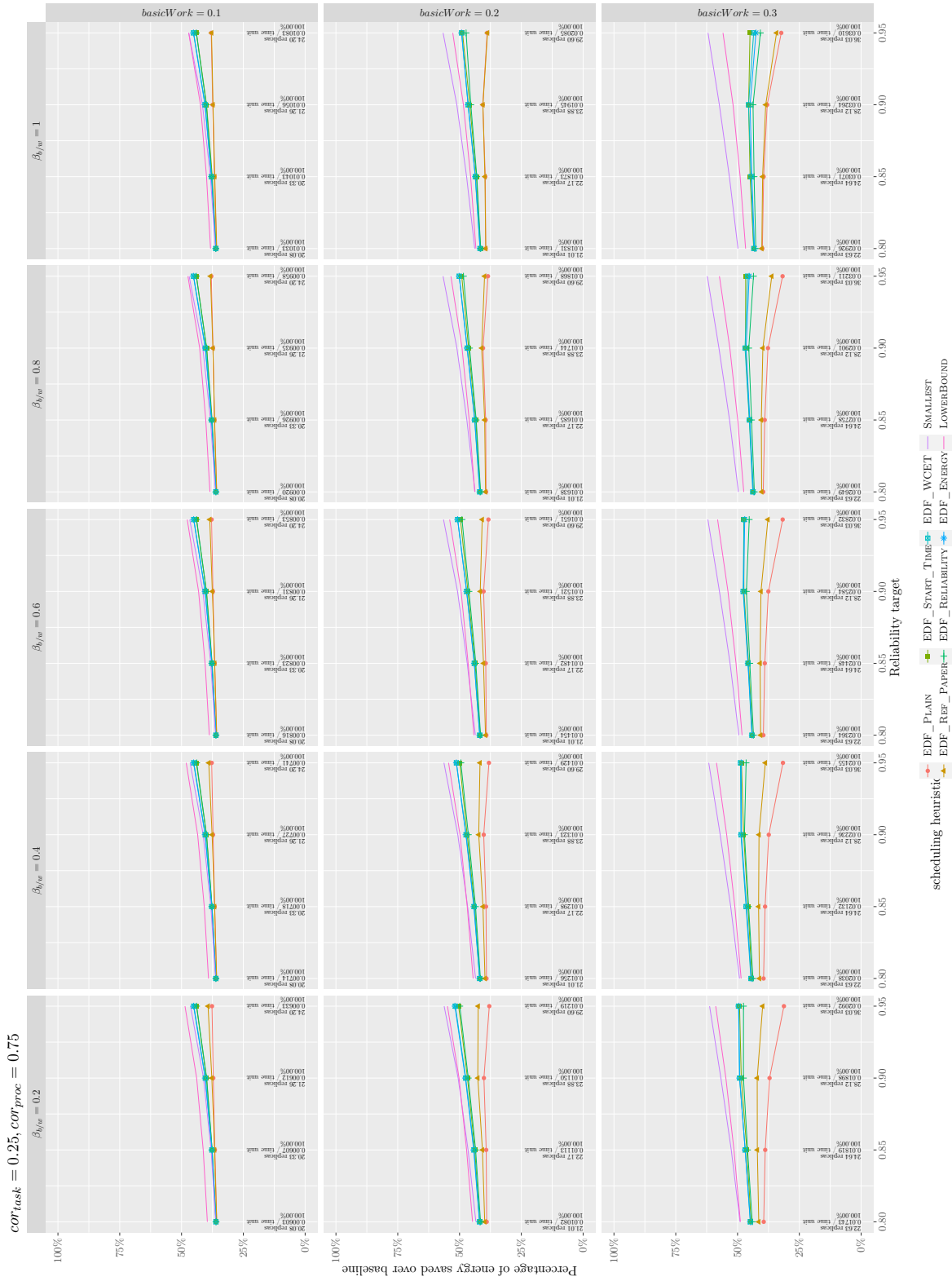
$cop_{task} = 0.75, cop_{proc} = 0.75$

## Utilization and reliability threshold

The following section presents percentage of energy saved over baseline using different scheduling strategies under big failure rate, when varying *basicWork* and  $\mathcal{R}$ .

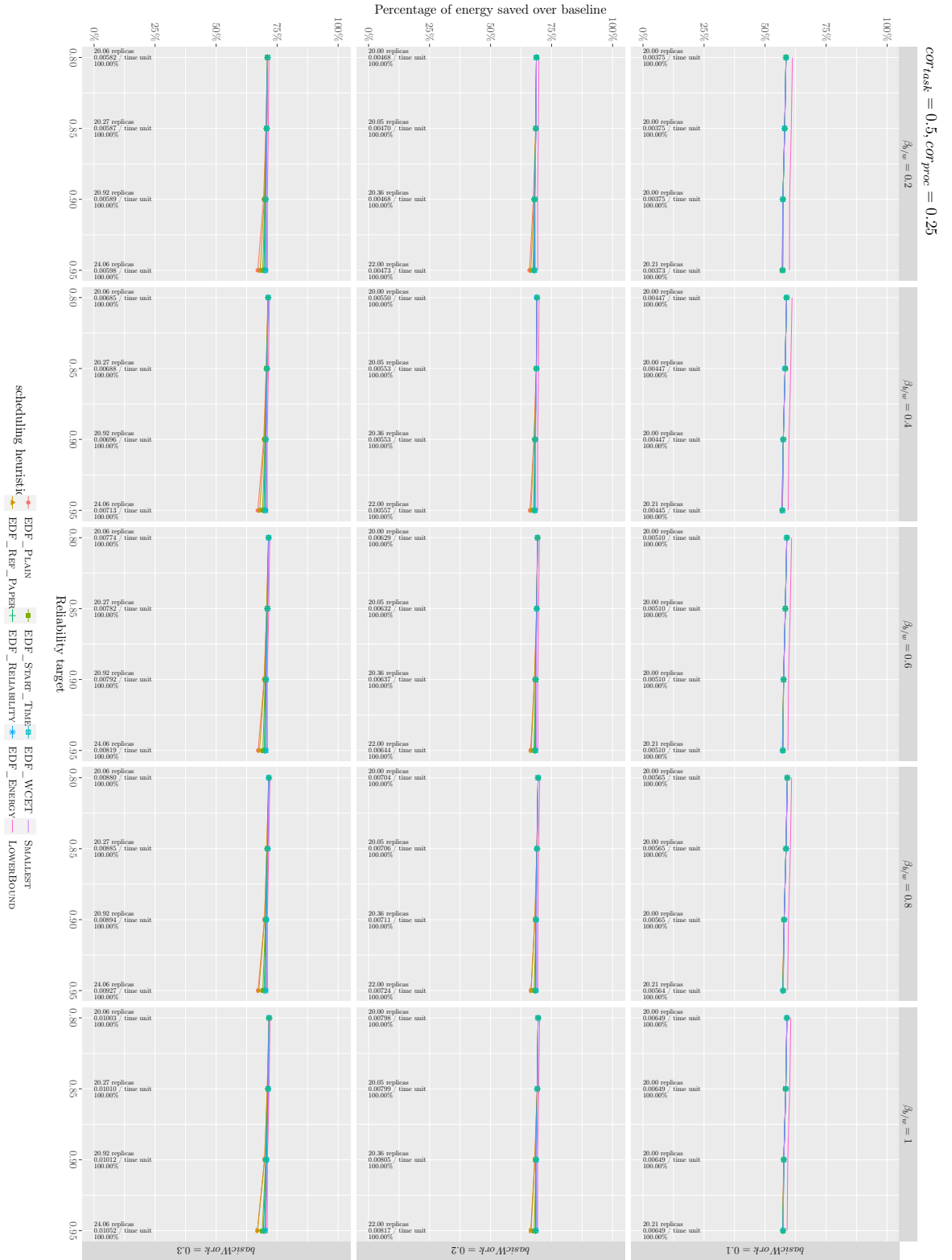




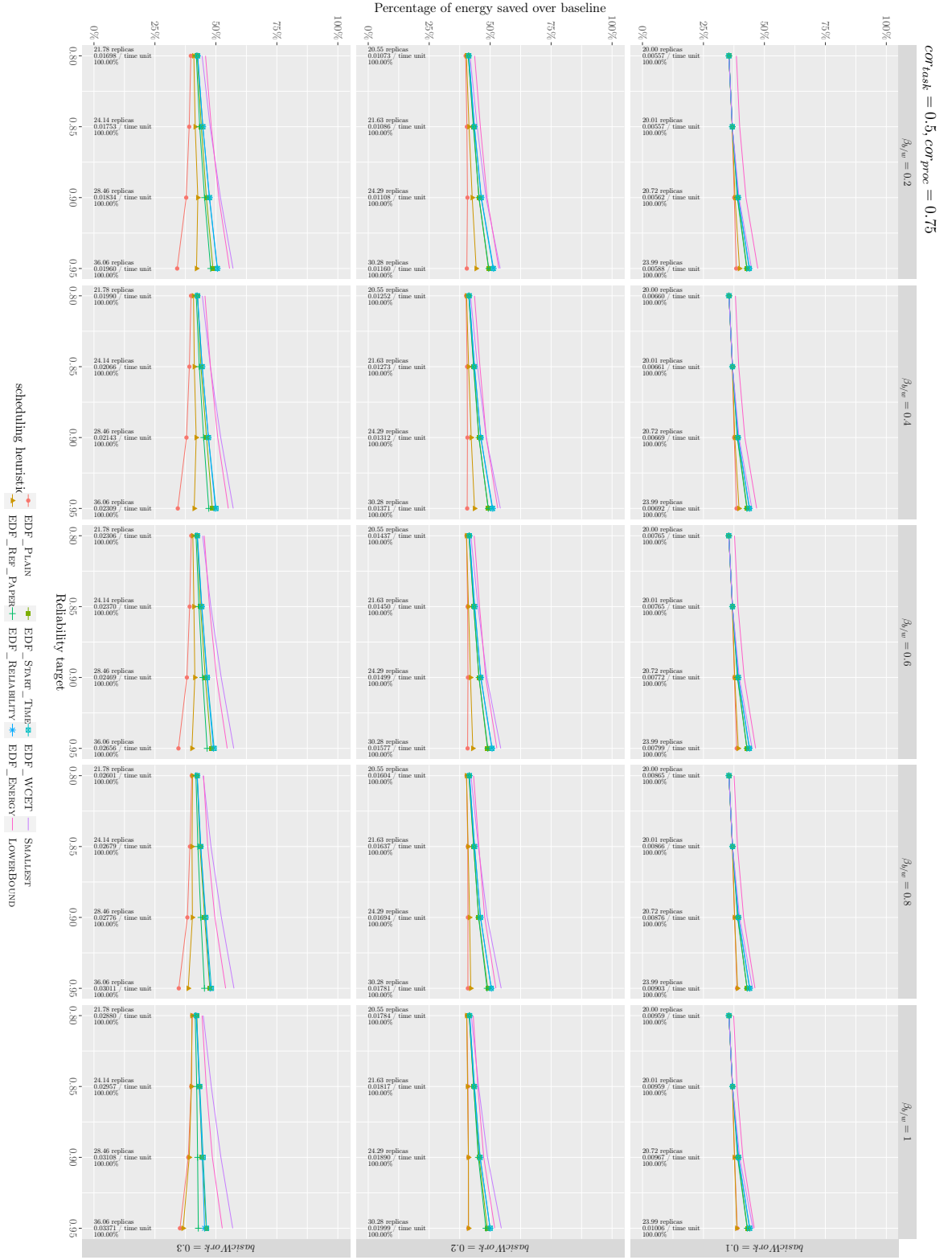


$cot_{task} = 0.25, cot_{proc} = 0.75$



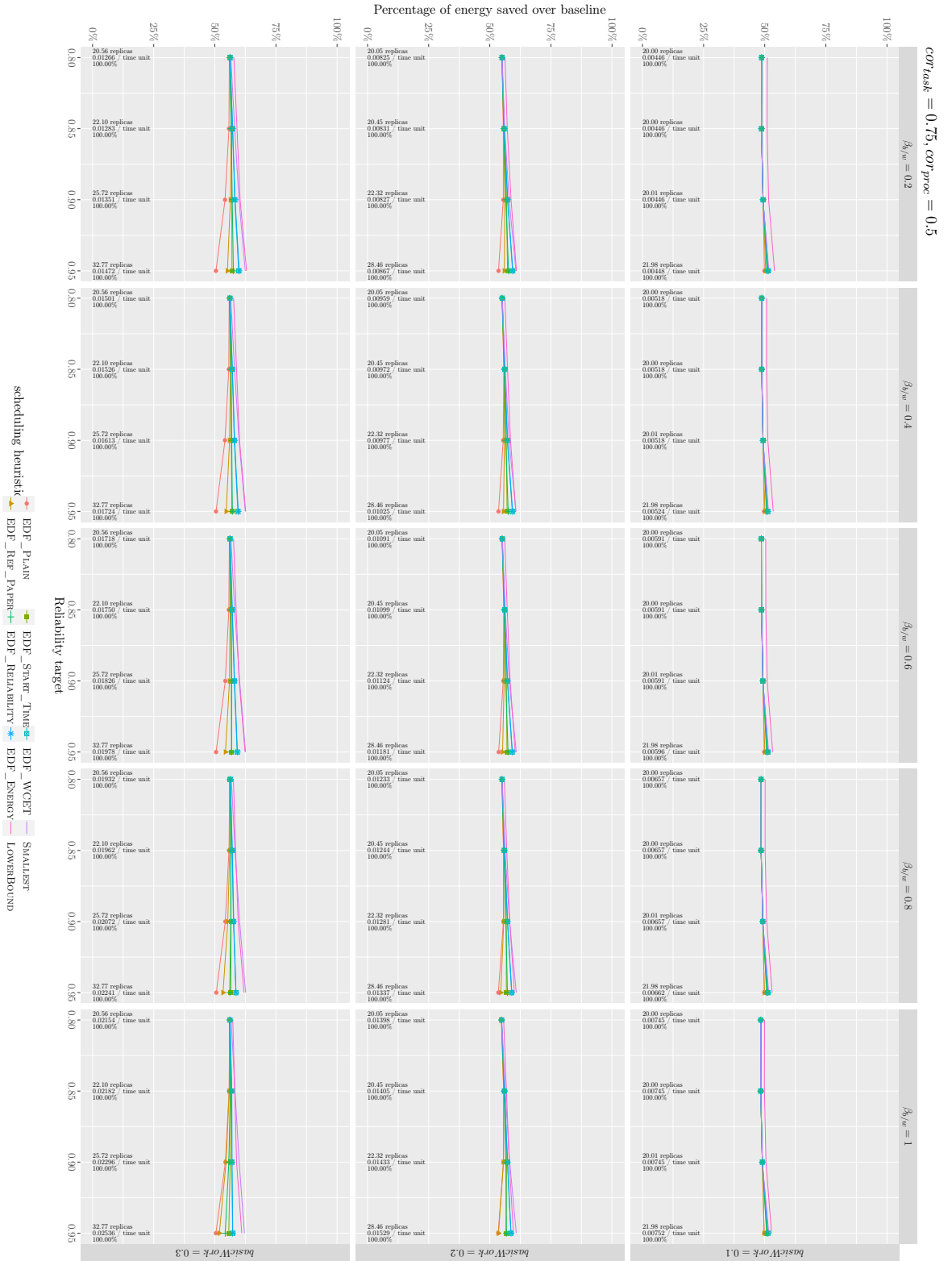


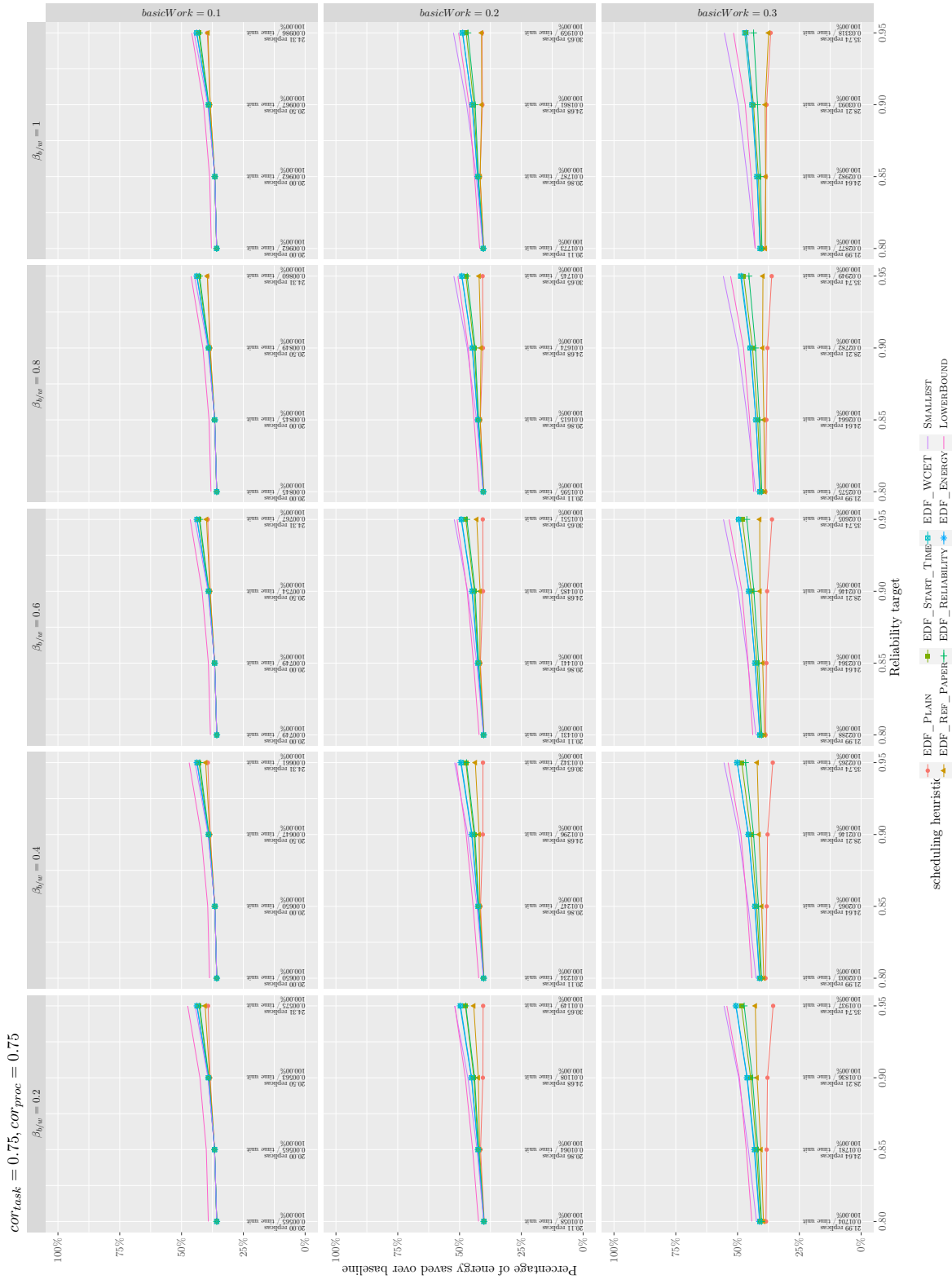






$cot_{task} = 0.75, cot_{proc} = 0.25$





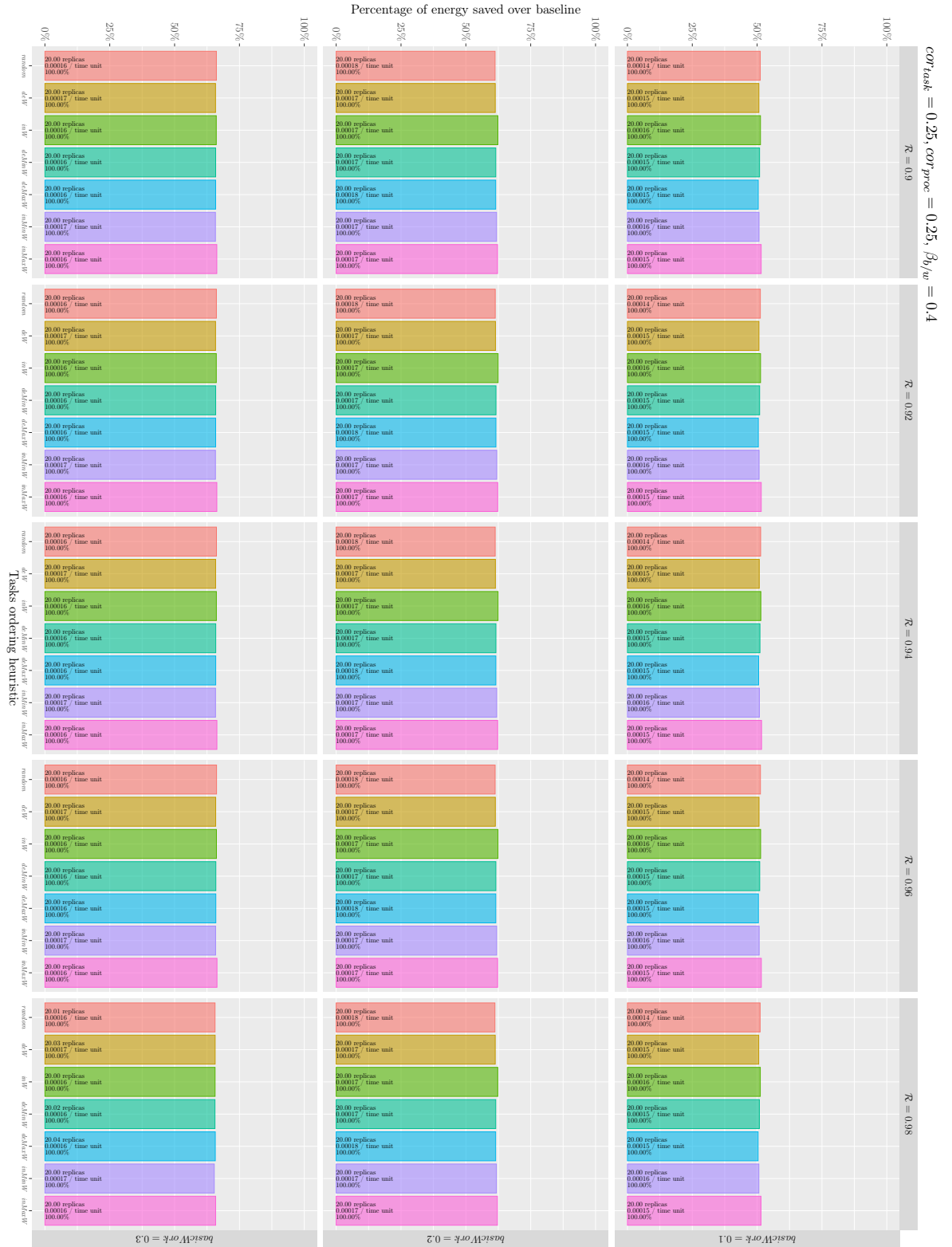
$cop_{task} = 0.75, cop_{proc} = 0.75$

## **Task ordering criteria for mapping phase**

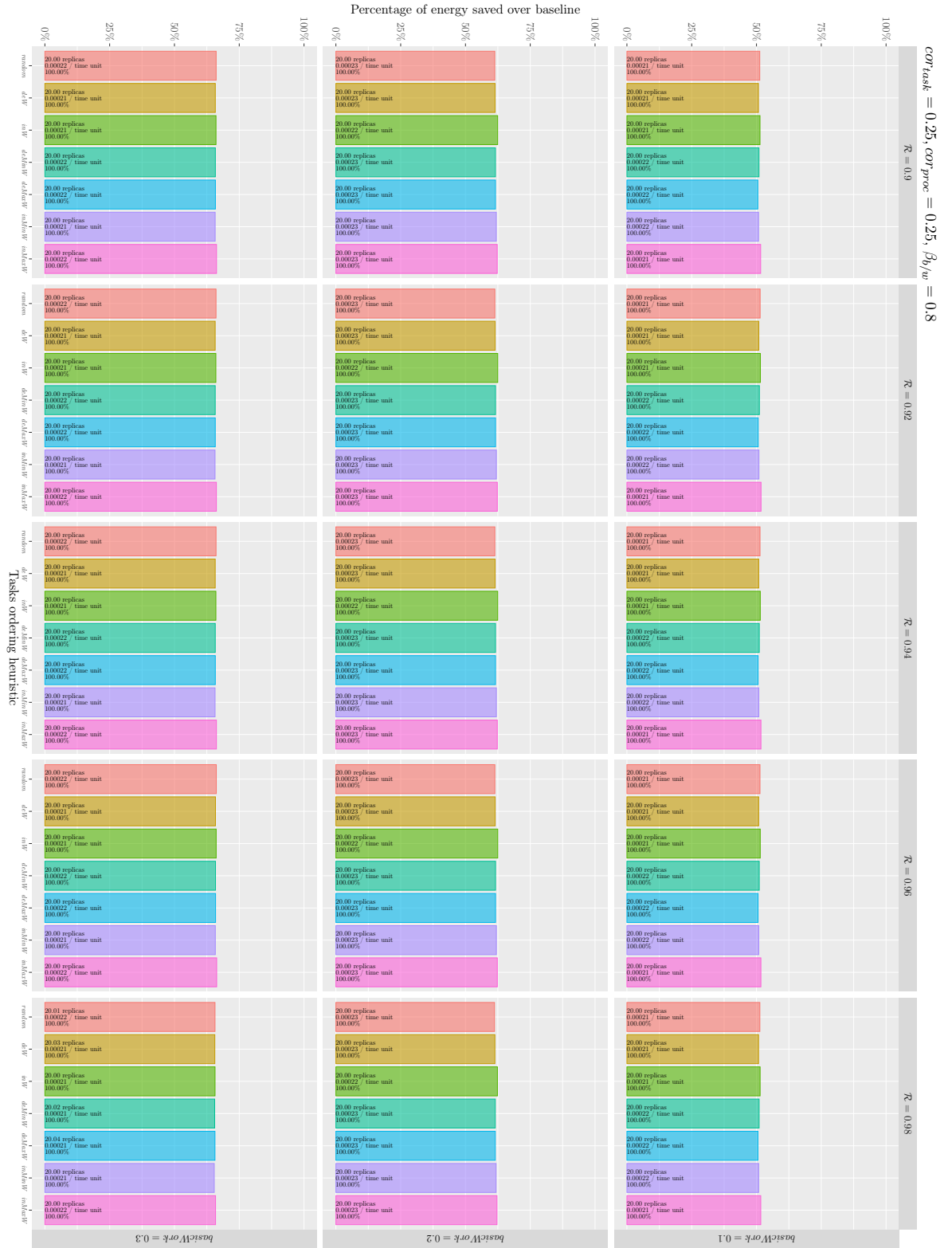
The following section presents percentage of energy saved over baseline using different task ordering criteria under small failure rate.





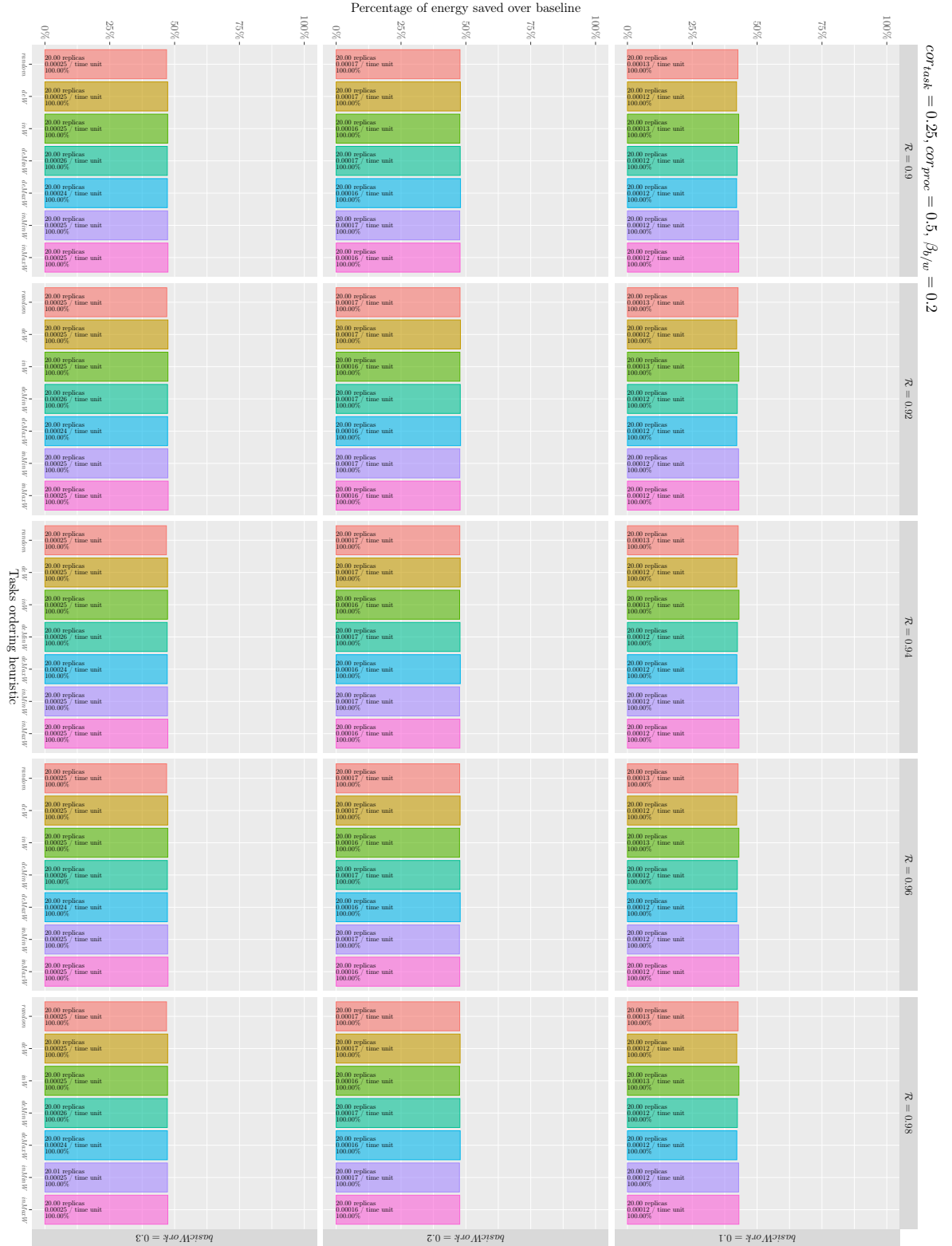




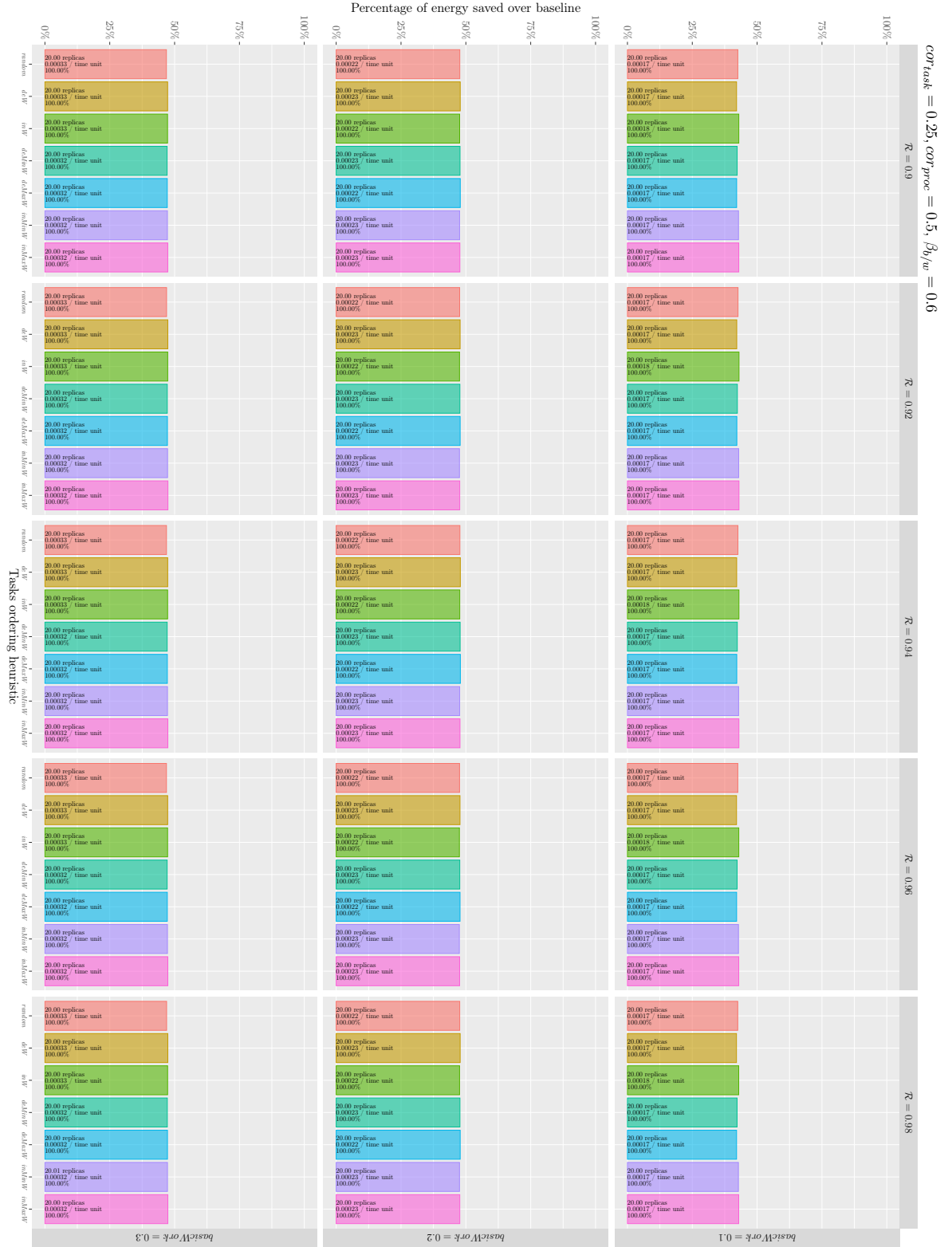


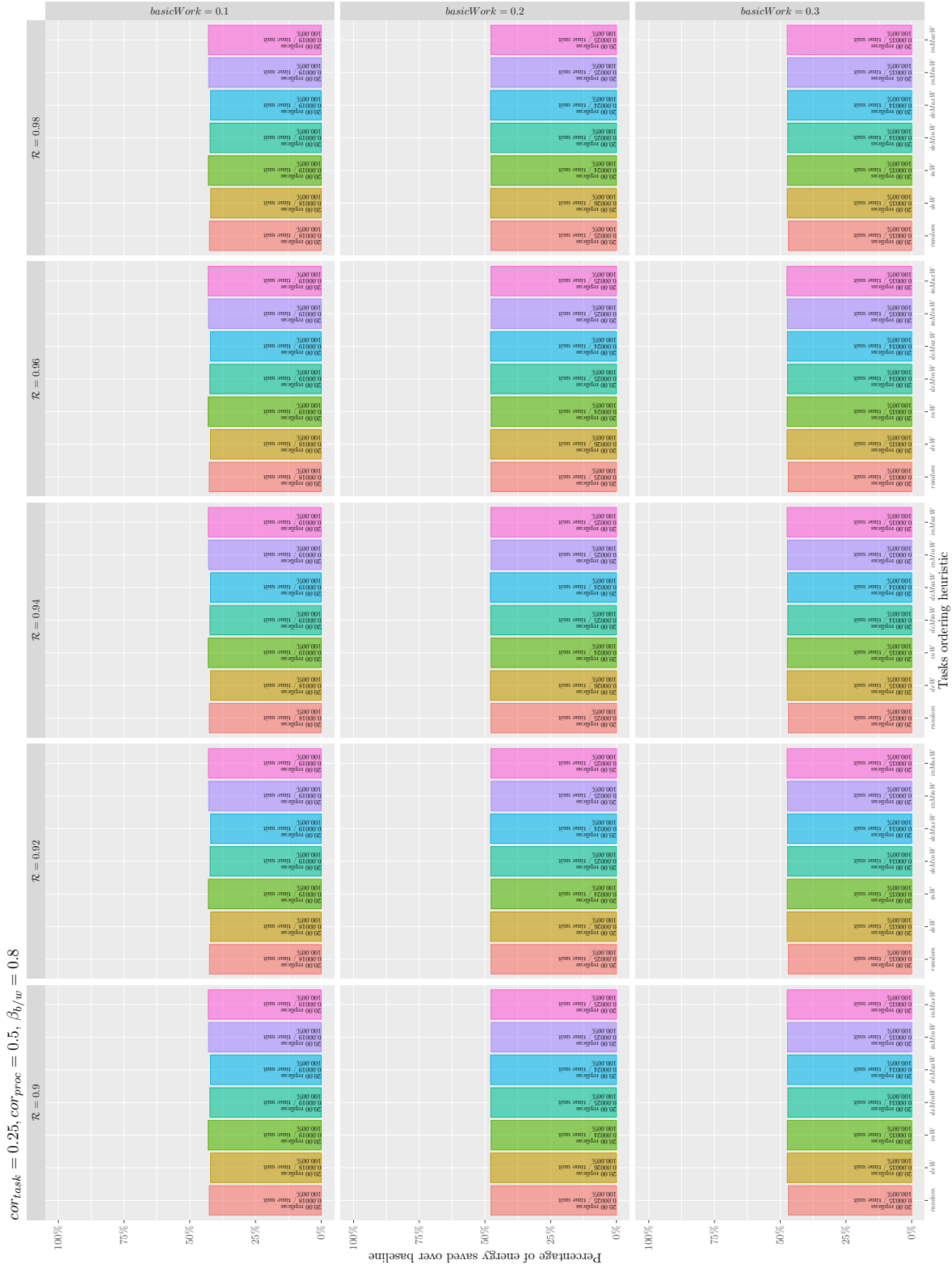


$cop_{task} = 0.25, cop_{proc} = 0.25, \beta_{rw} = 1$



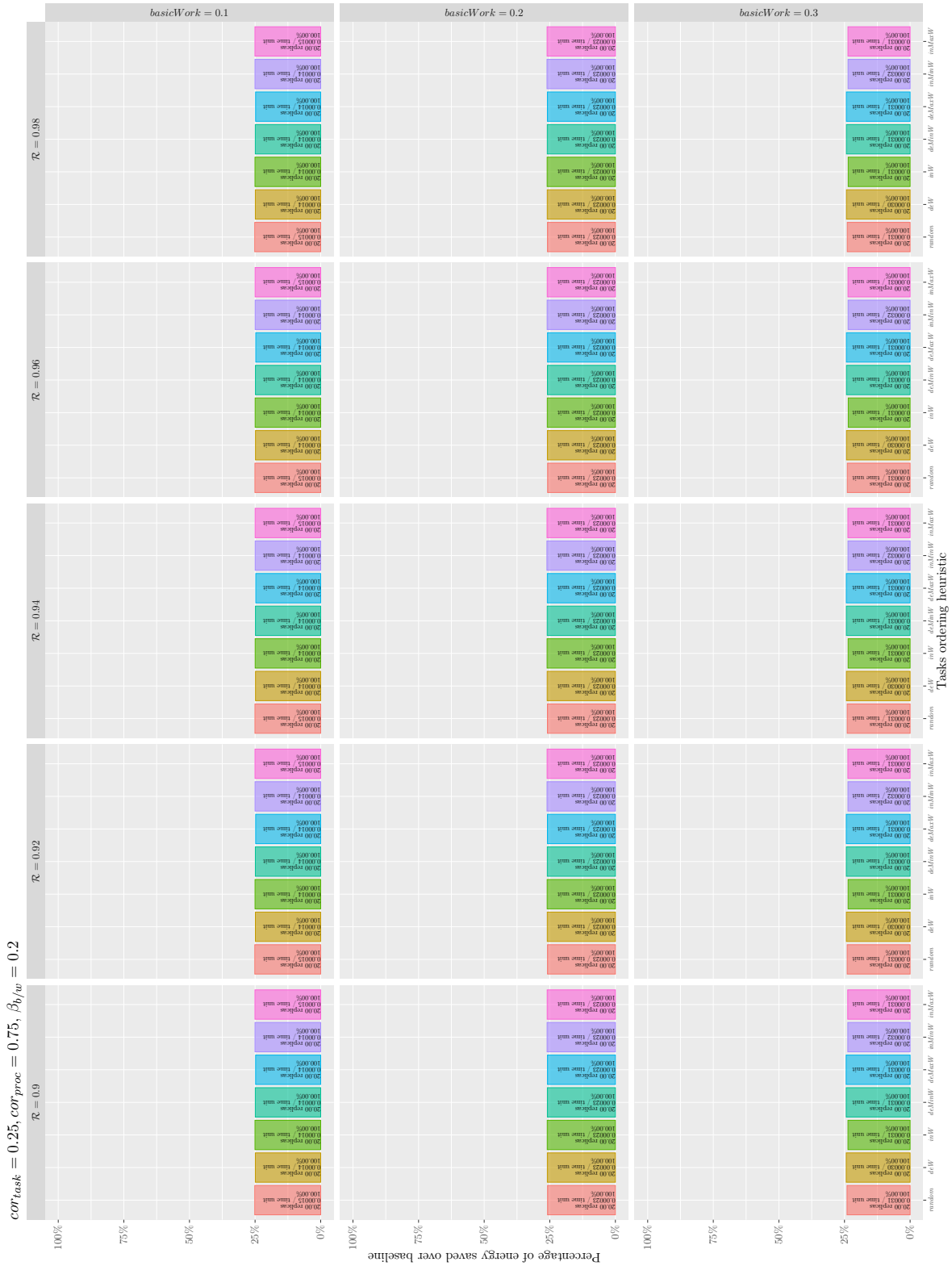






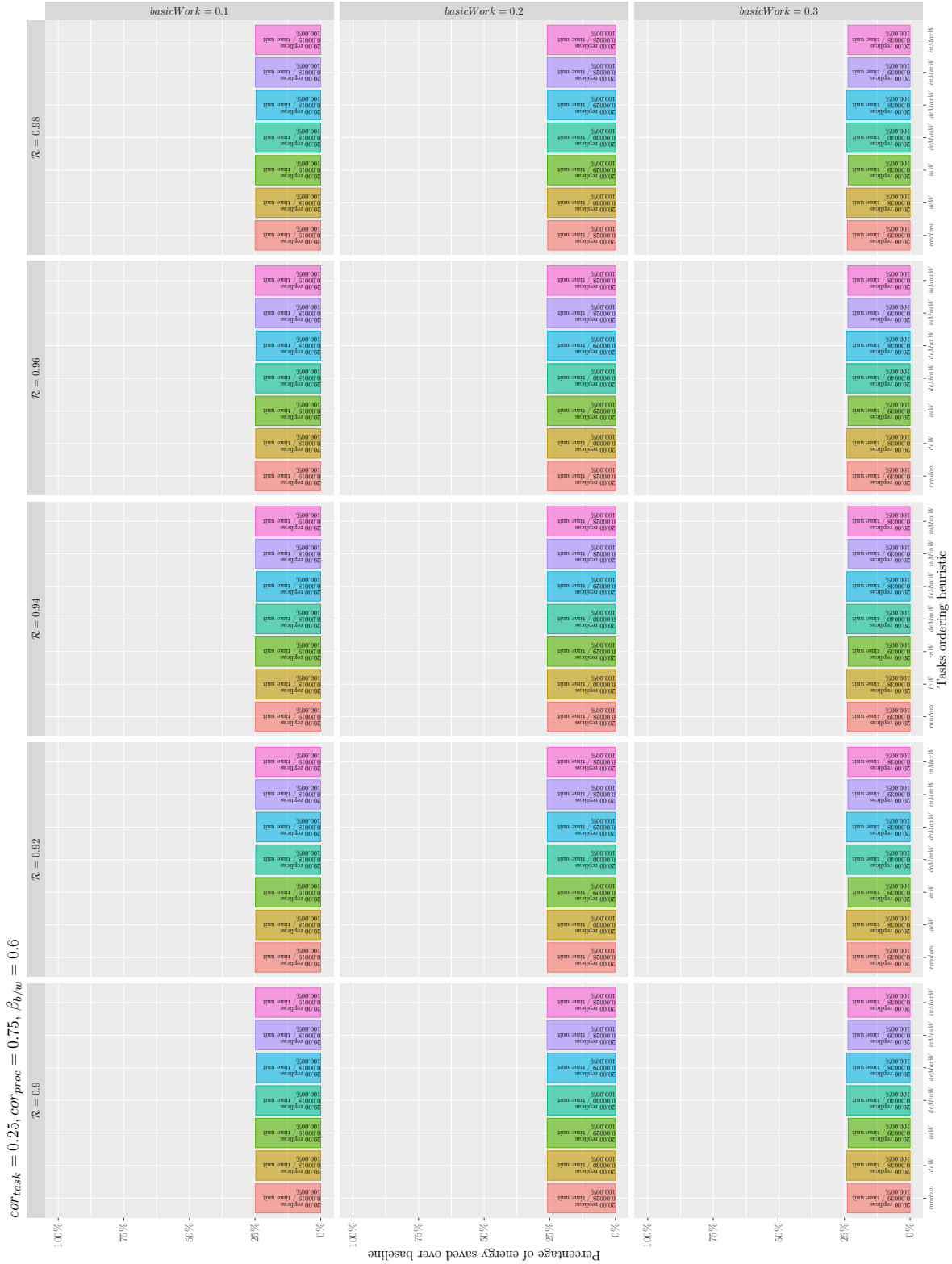




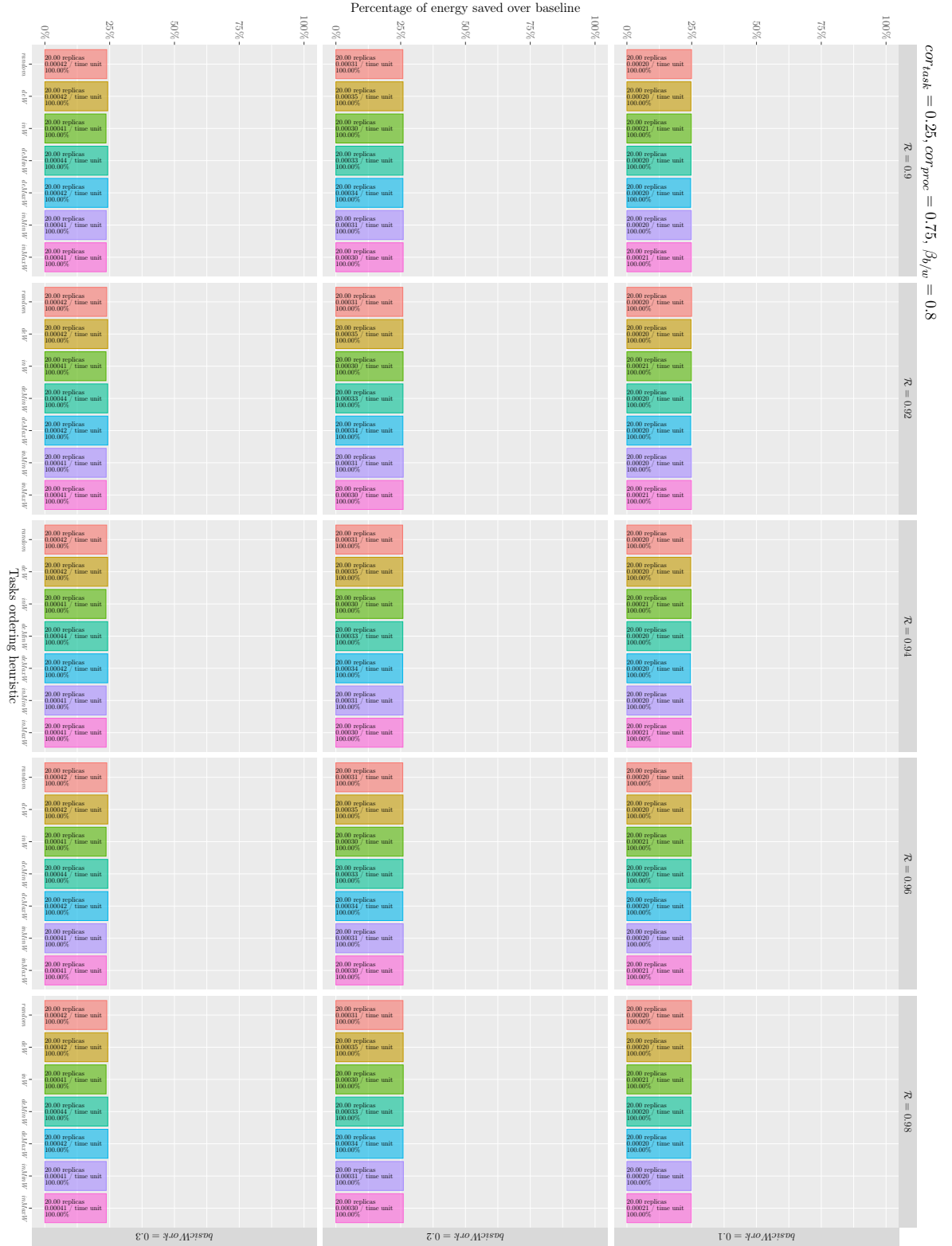


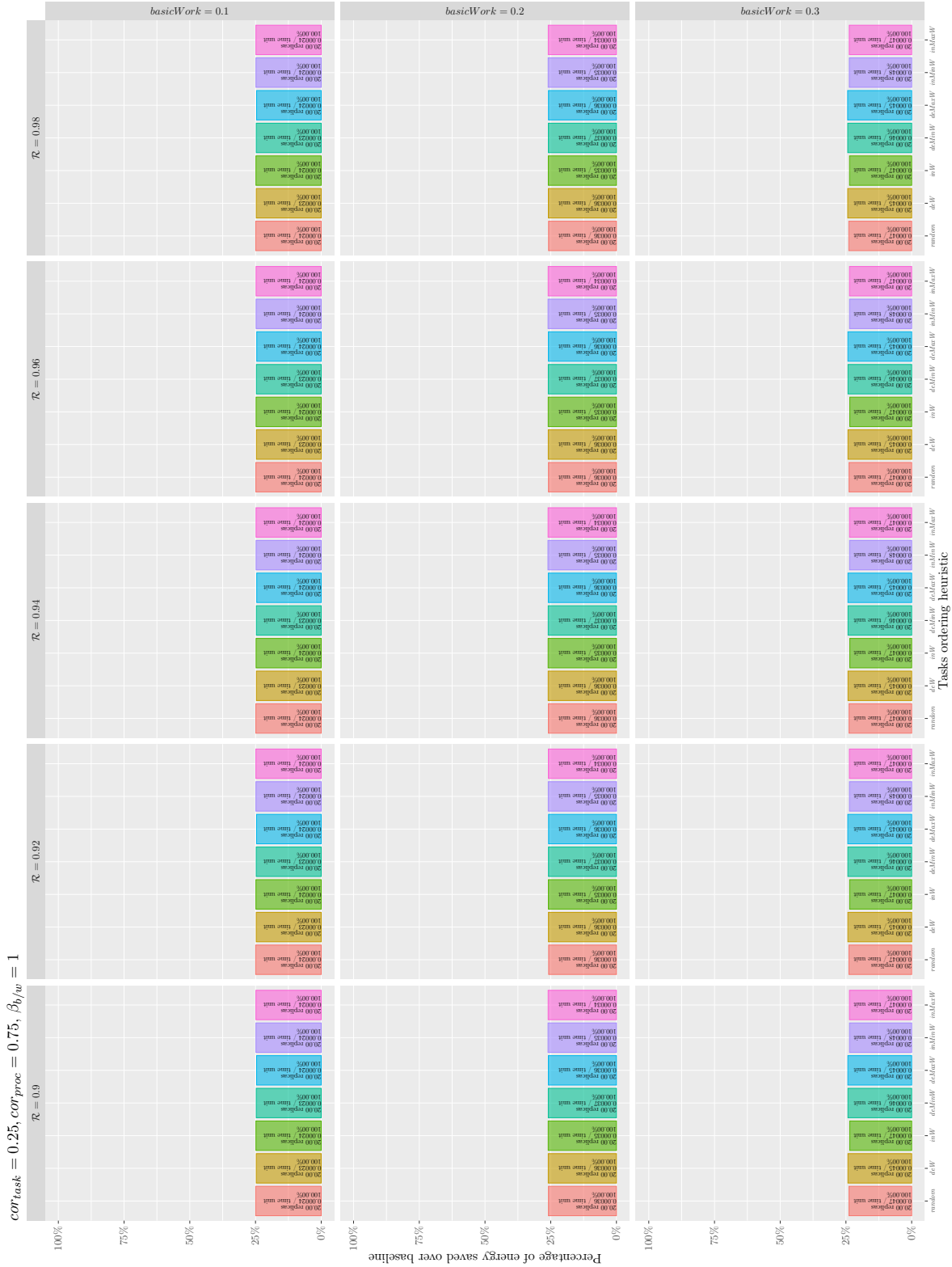
$COI_{task} = 0.25, COI_{proc} = 0.75, \beta_{fu} = 0.2$



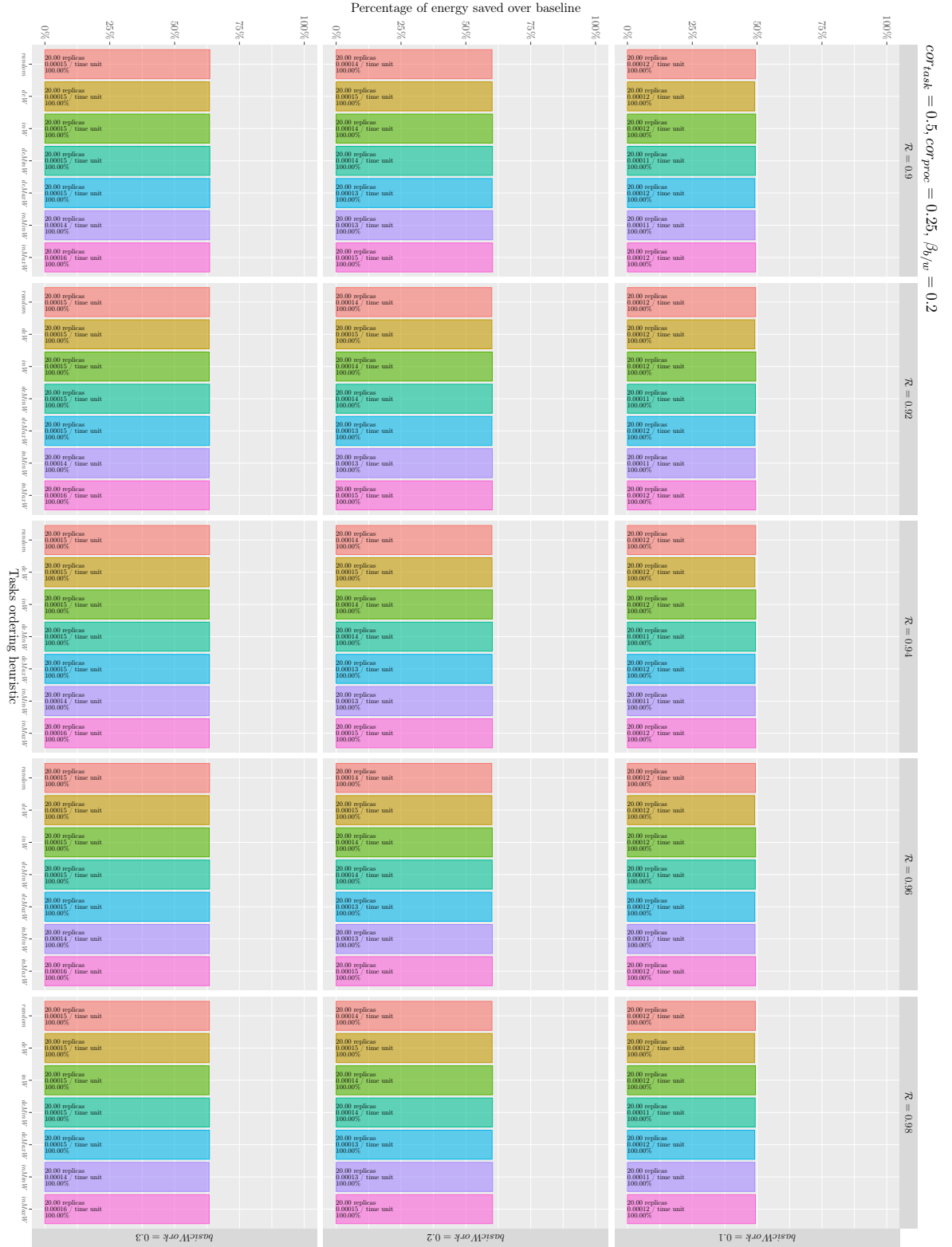


$cop_{task} = 0.25, cop_{proc} = 0.75, \beta_{rw} = 0.6$



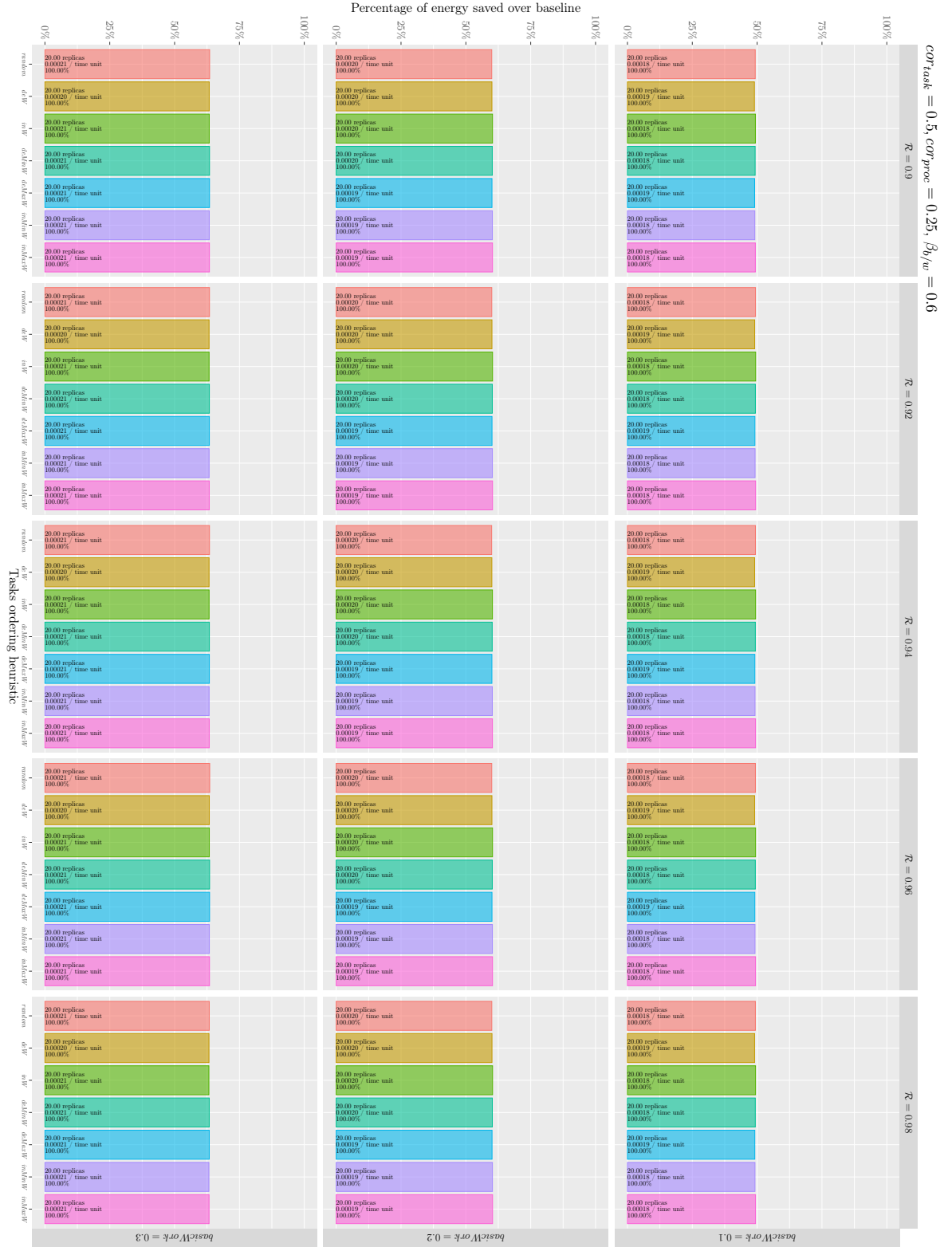


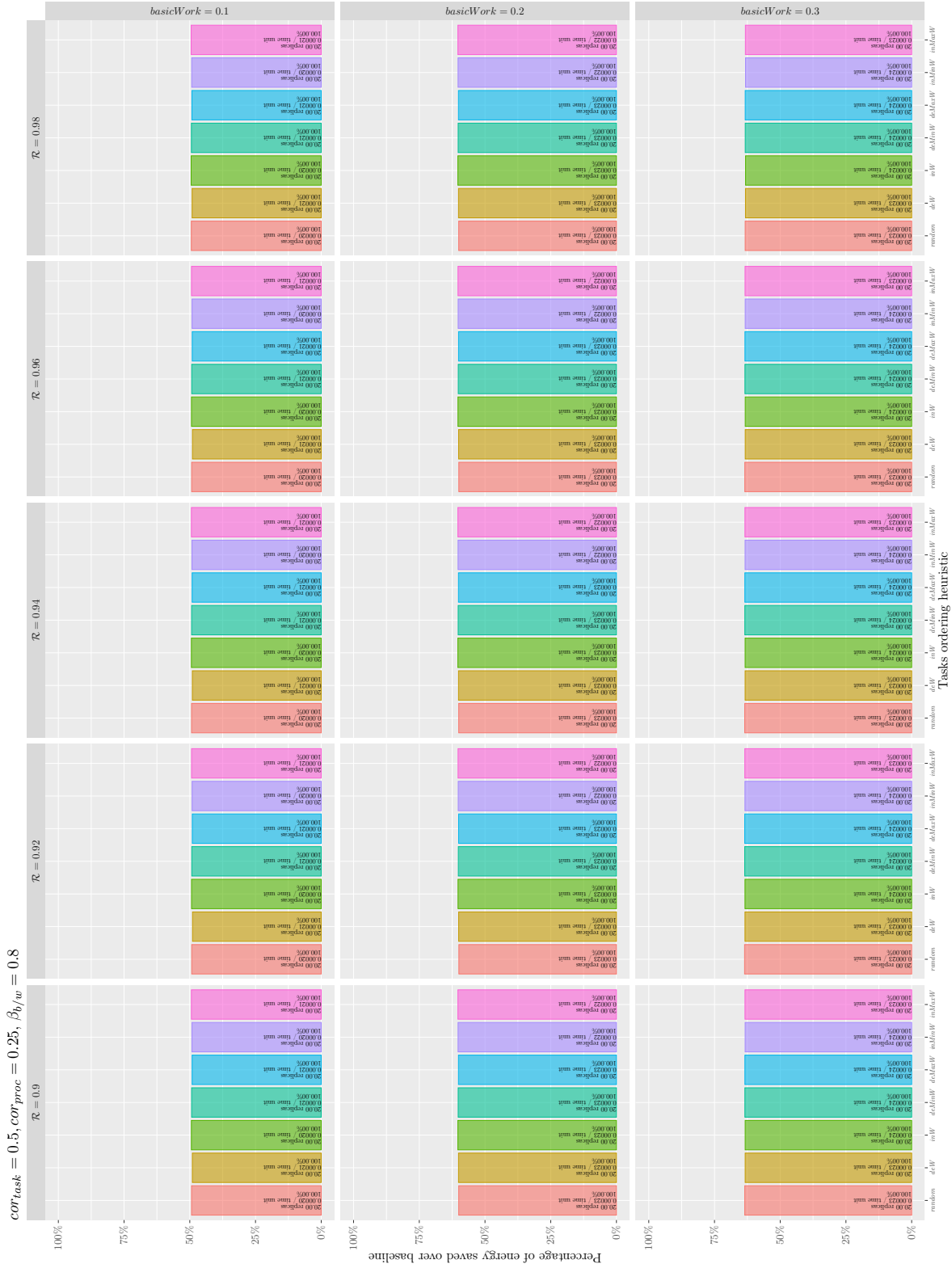
$cop_{task} = 0.25, cop_{proc} = 0.75, \beta_{i/w} = 1$



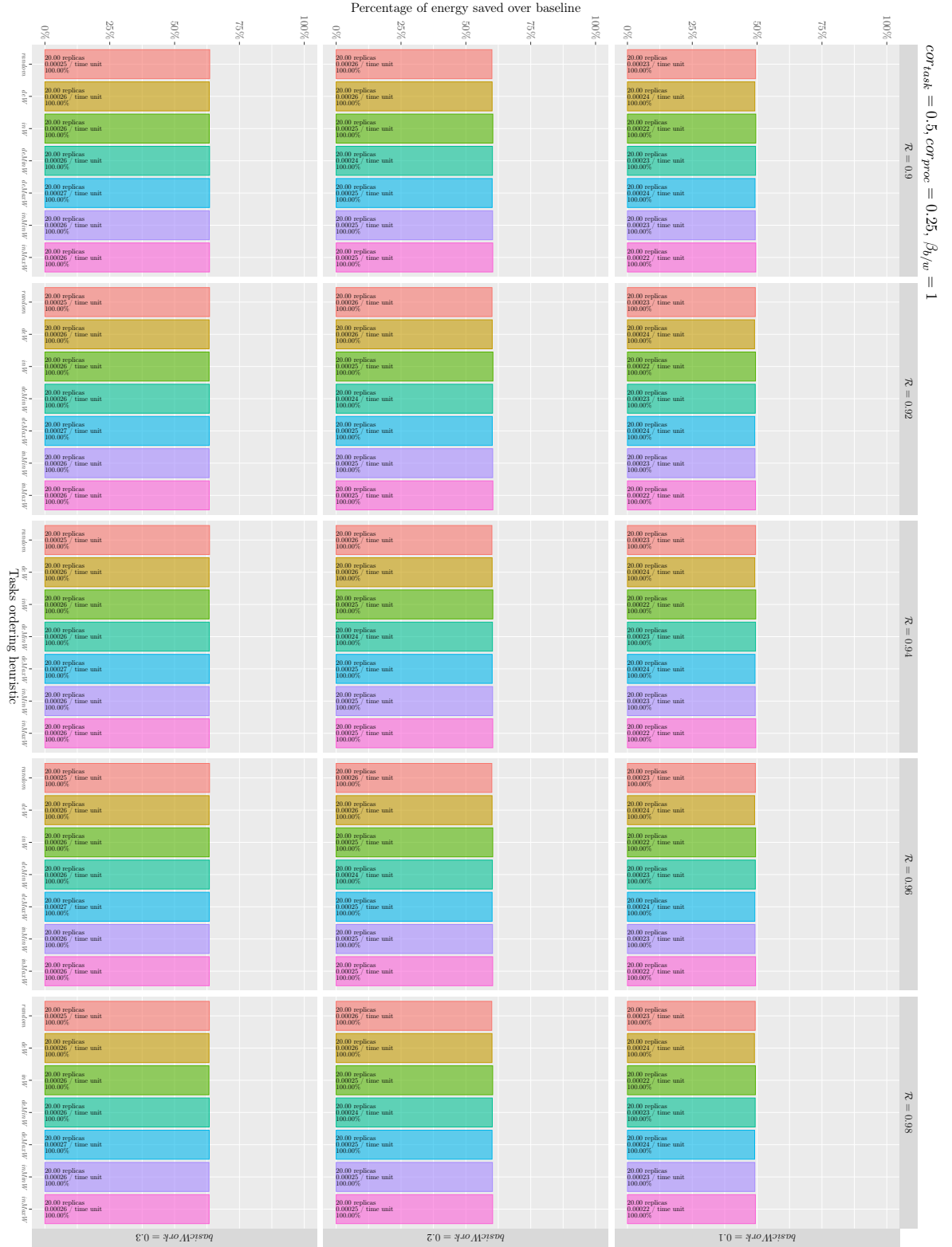




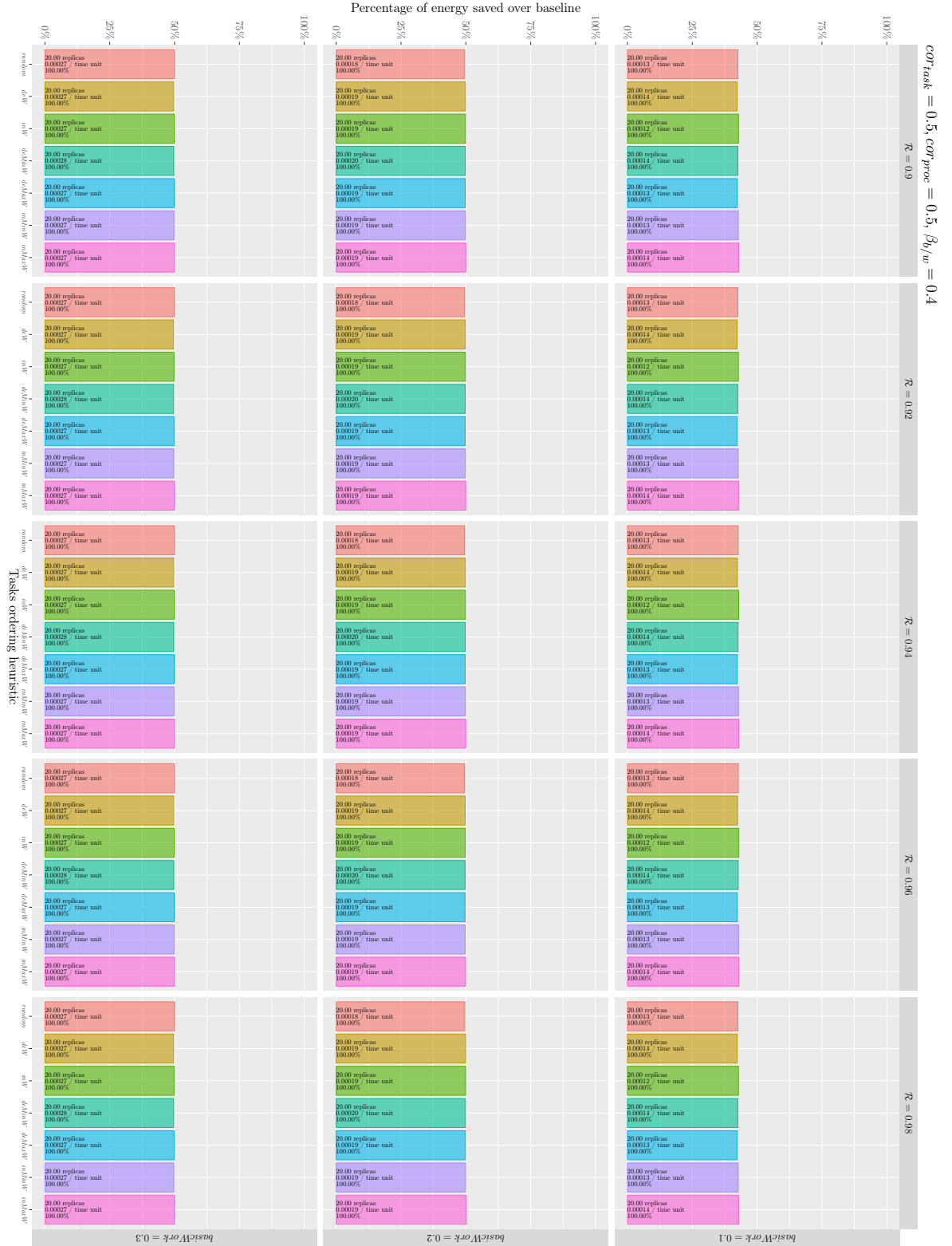




$cot_{task} = 0.5, cot_{proc} = 0.25, \beta_{hw} = 0.8$





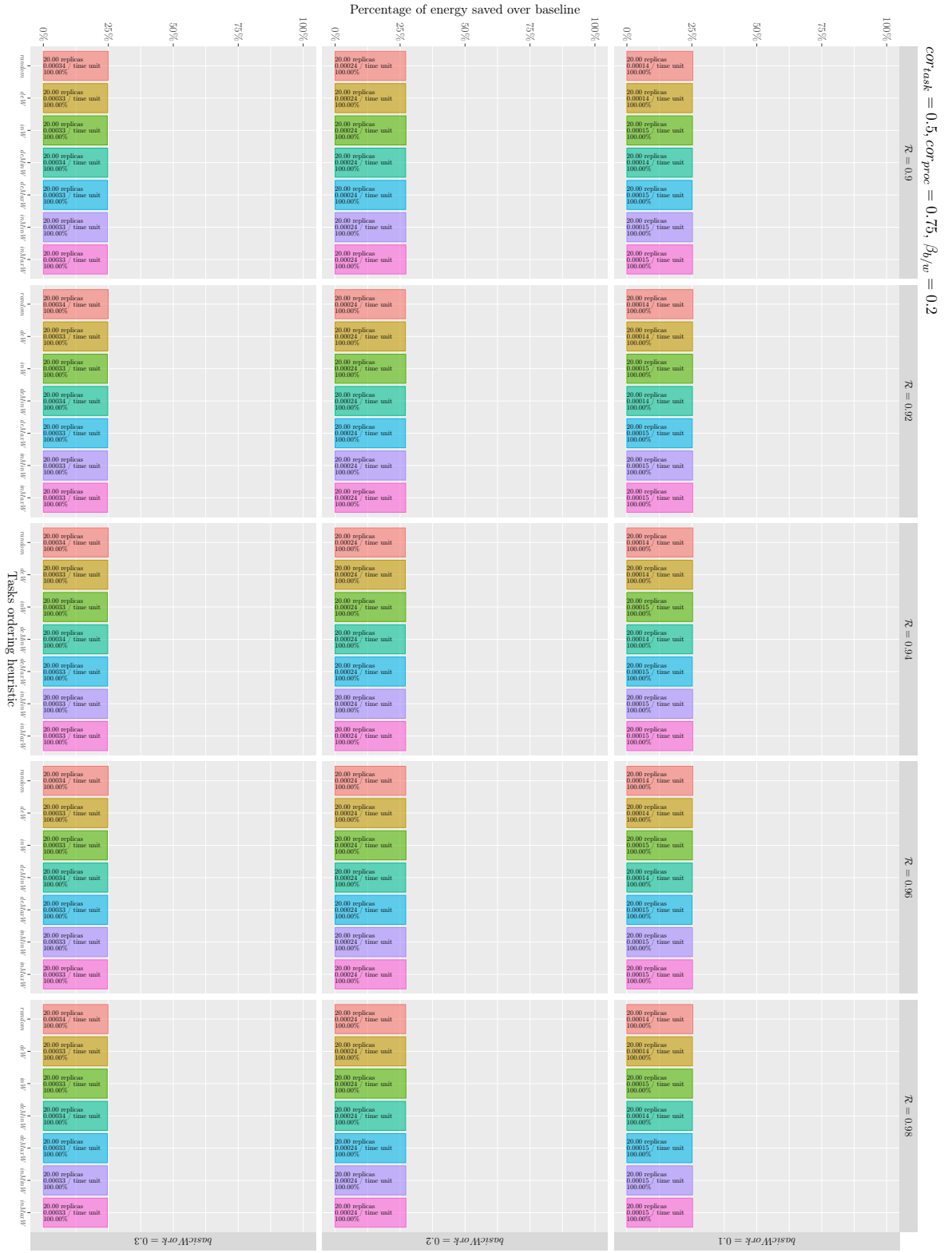


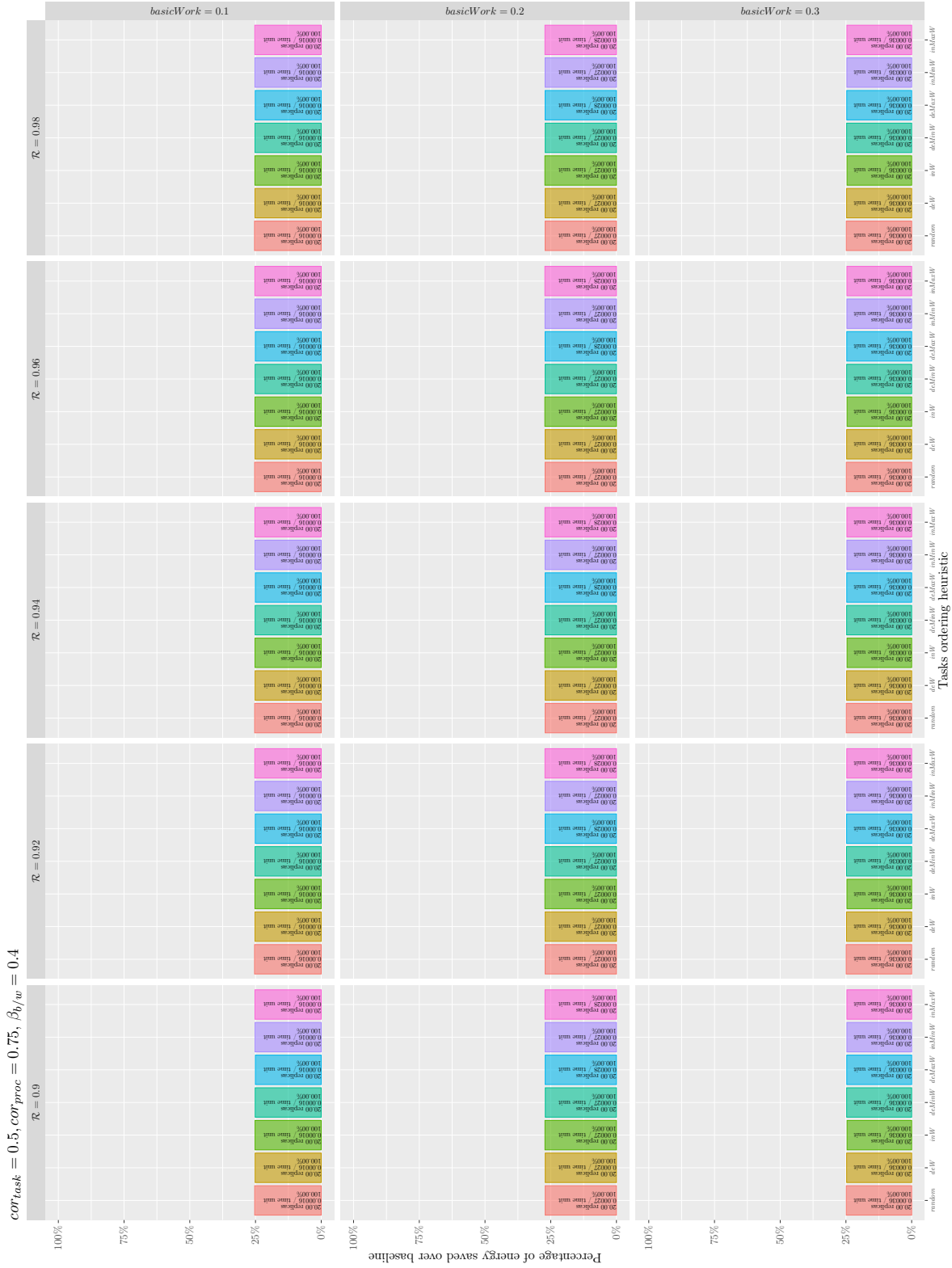


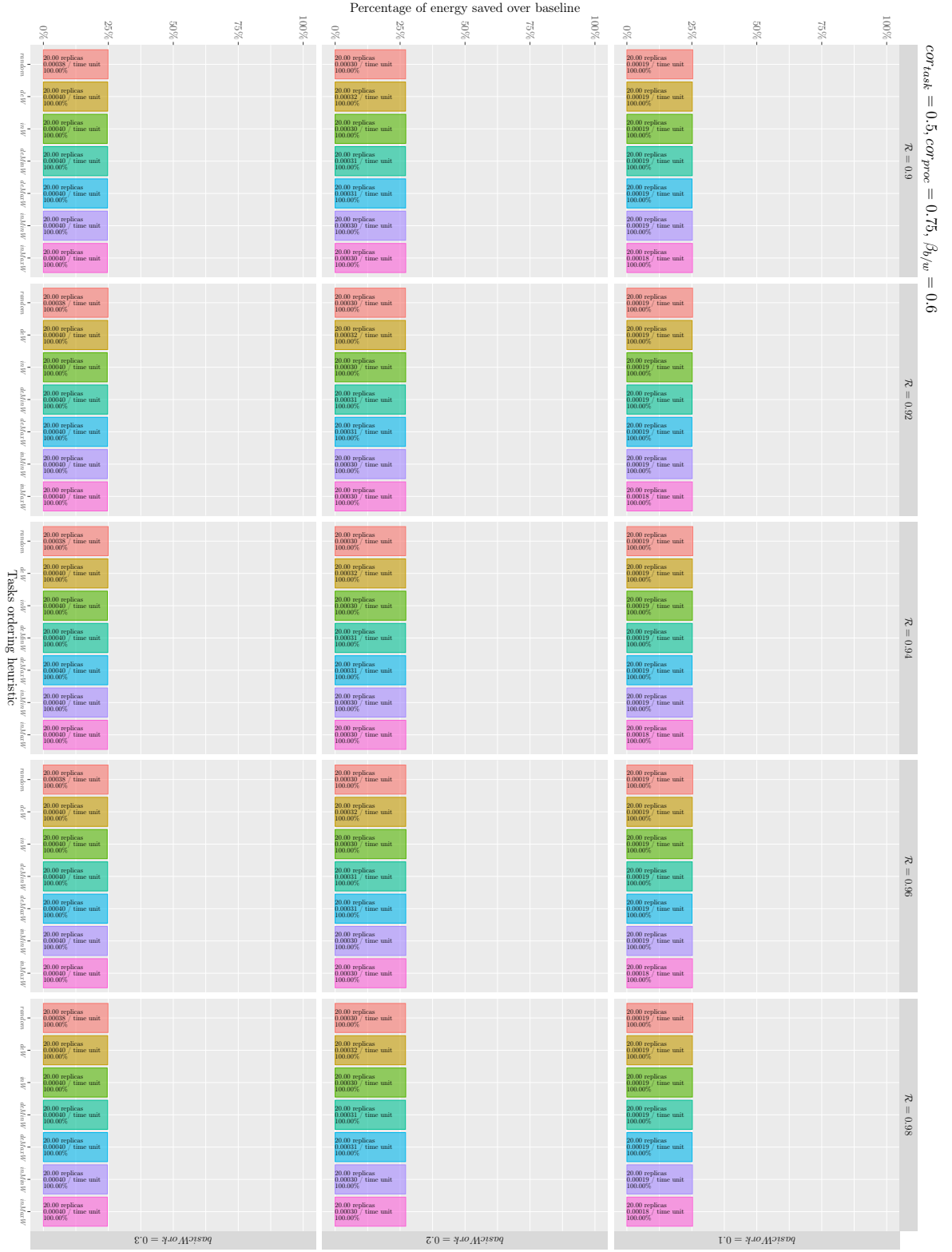


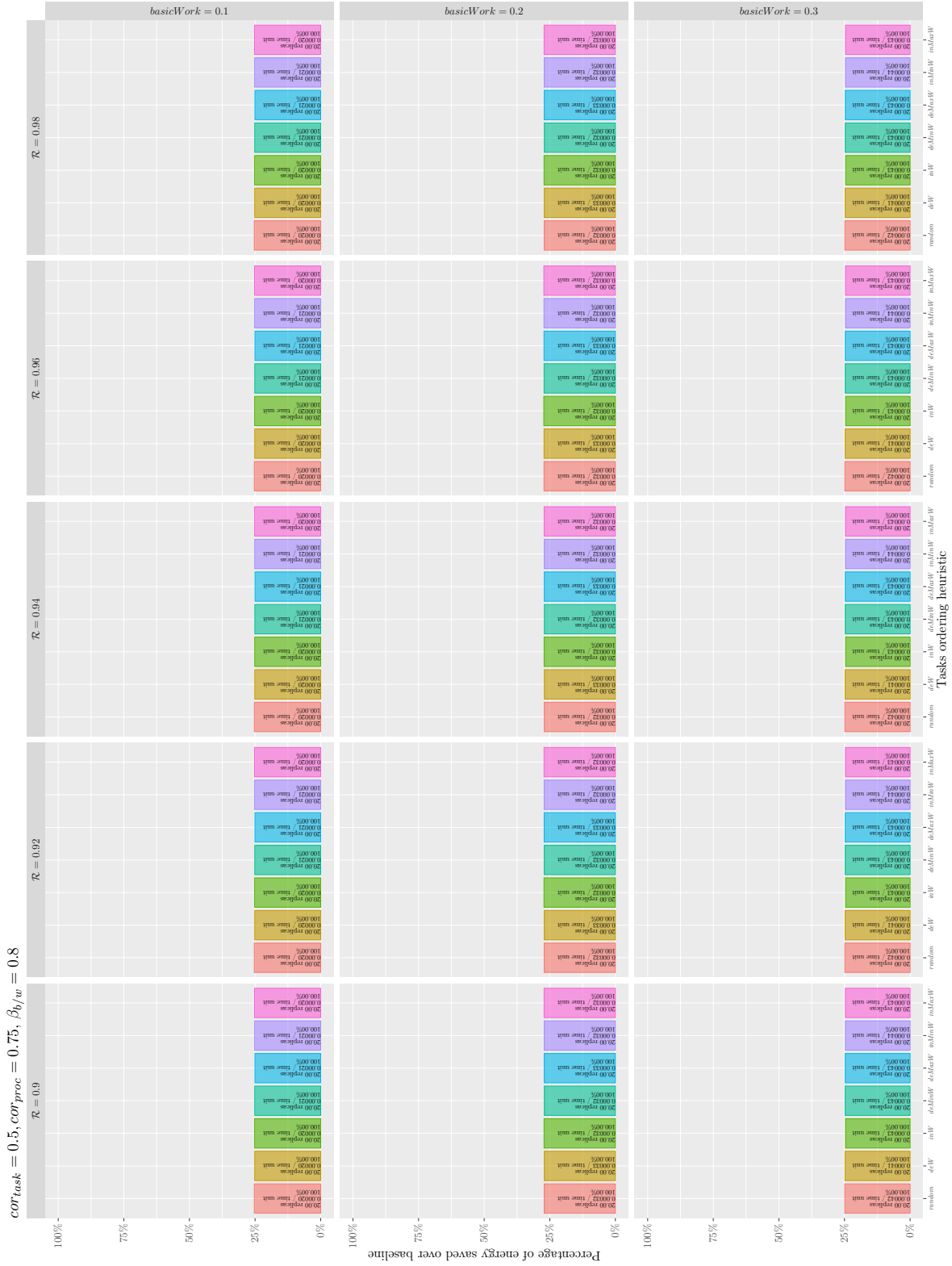








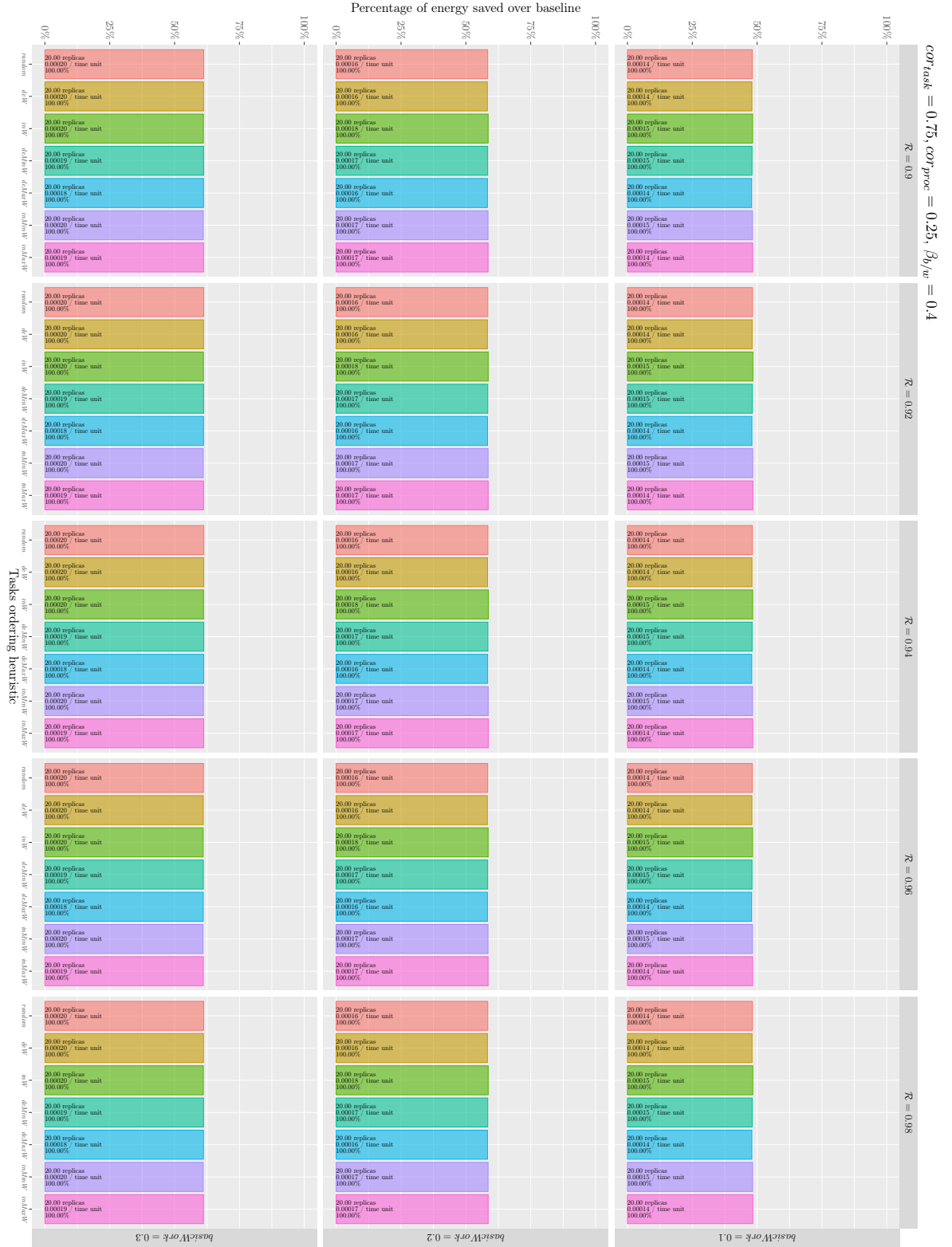


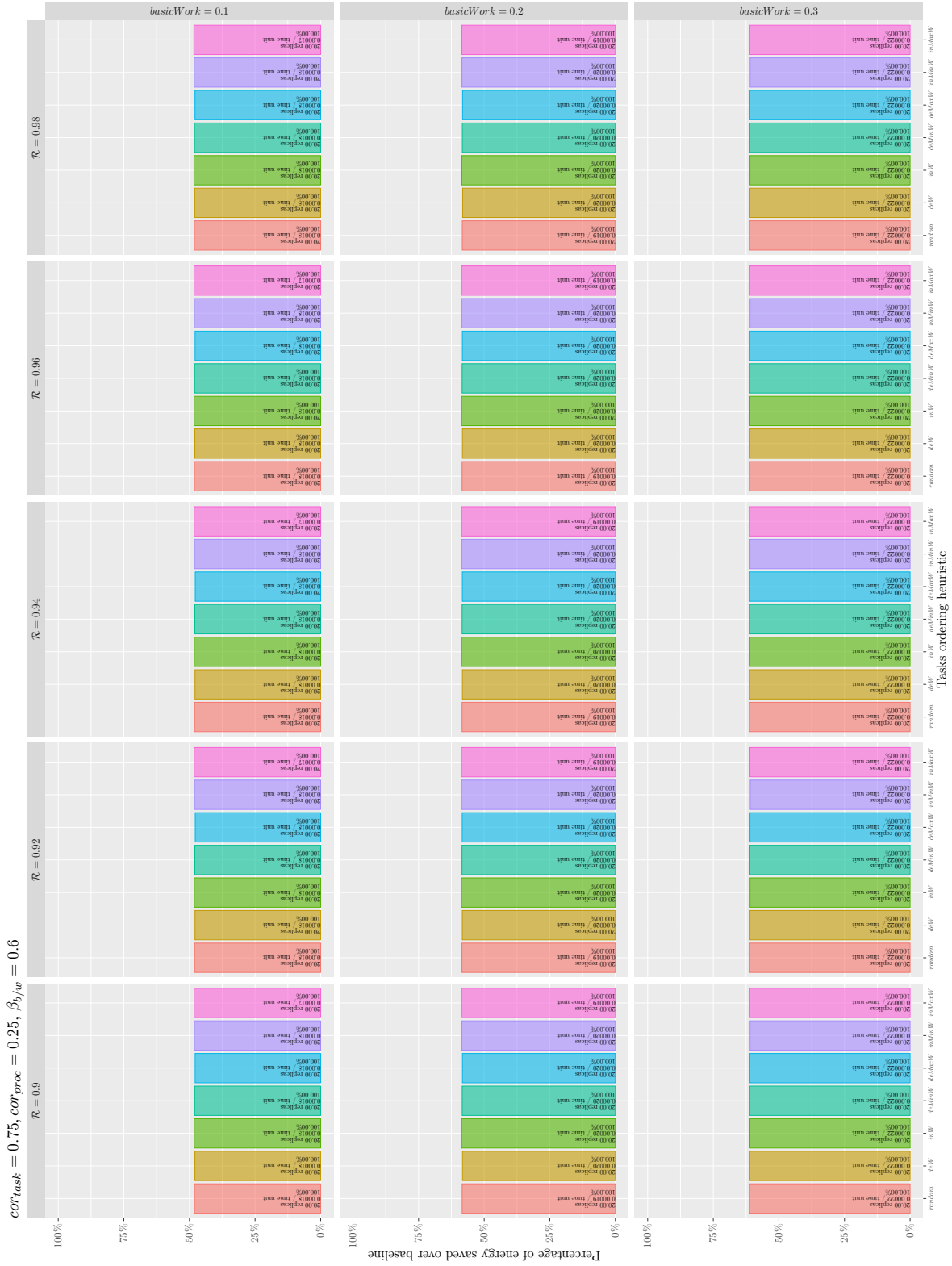


$cop_{task} = 0.5, cop_{proc} = 0.75, \beta_{h/w} = 0.8$

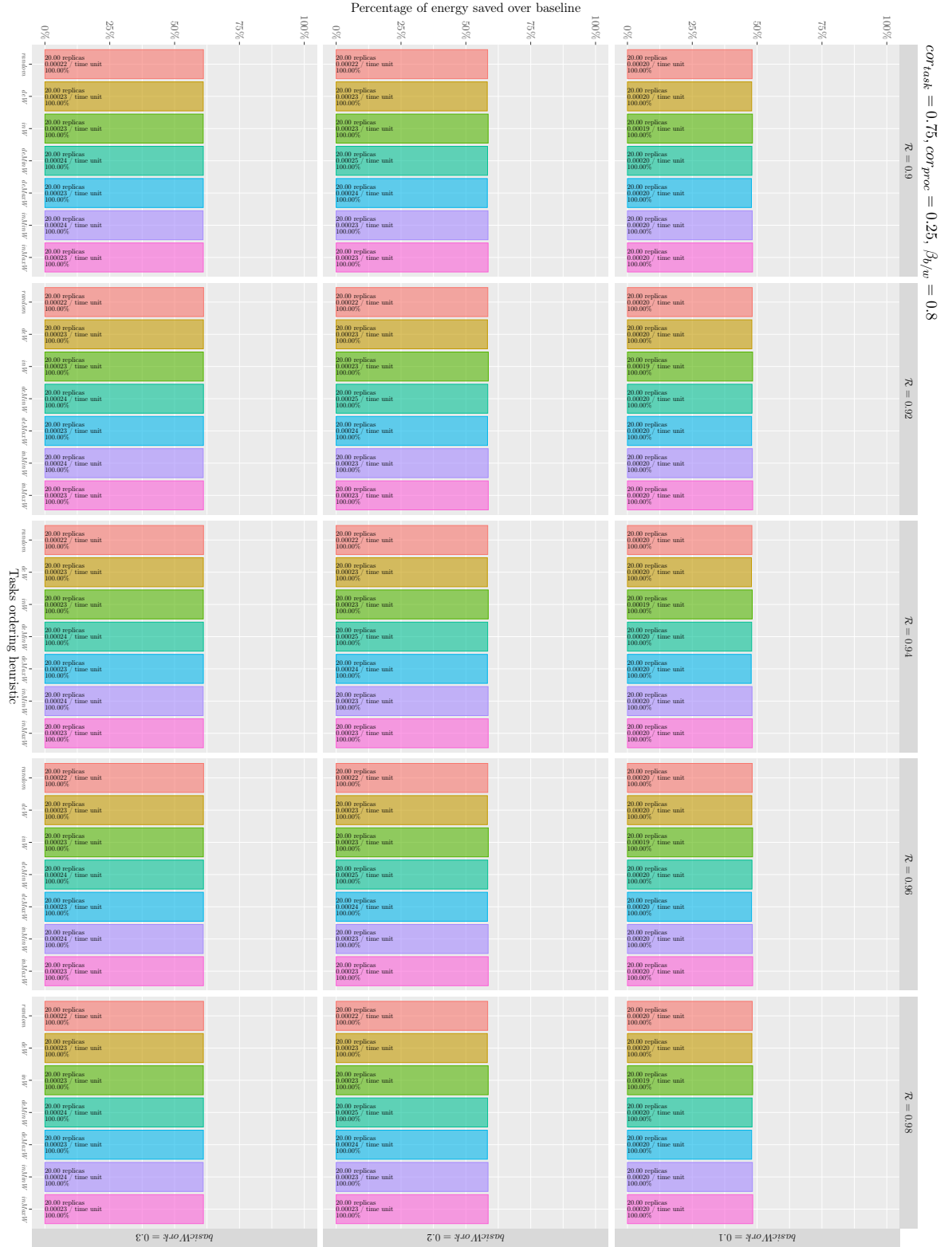














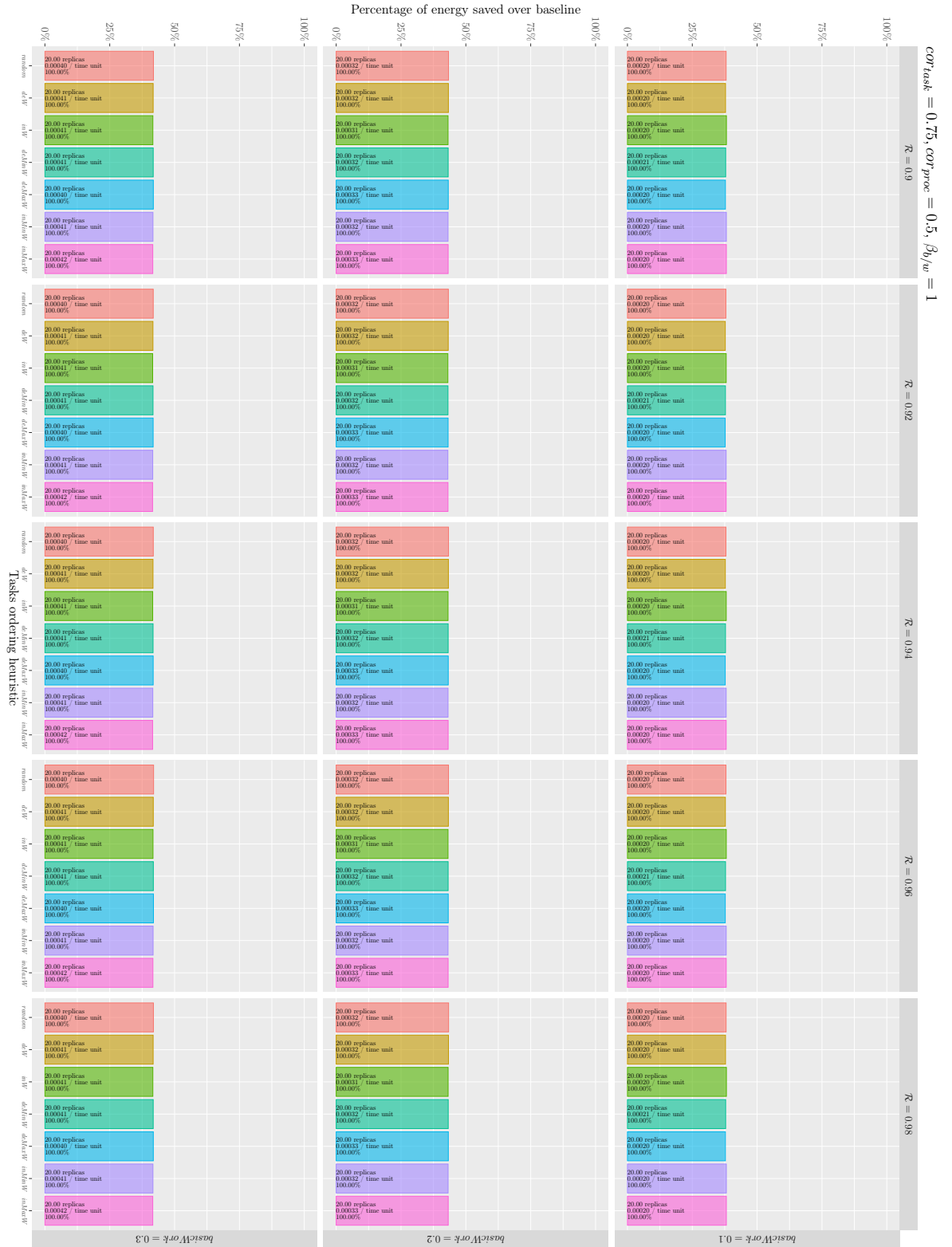






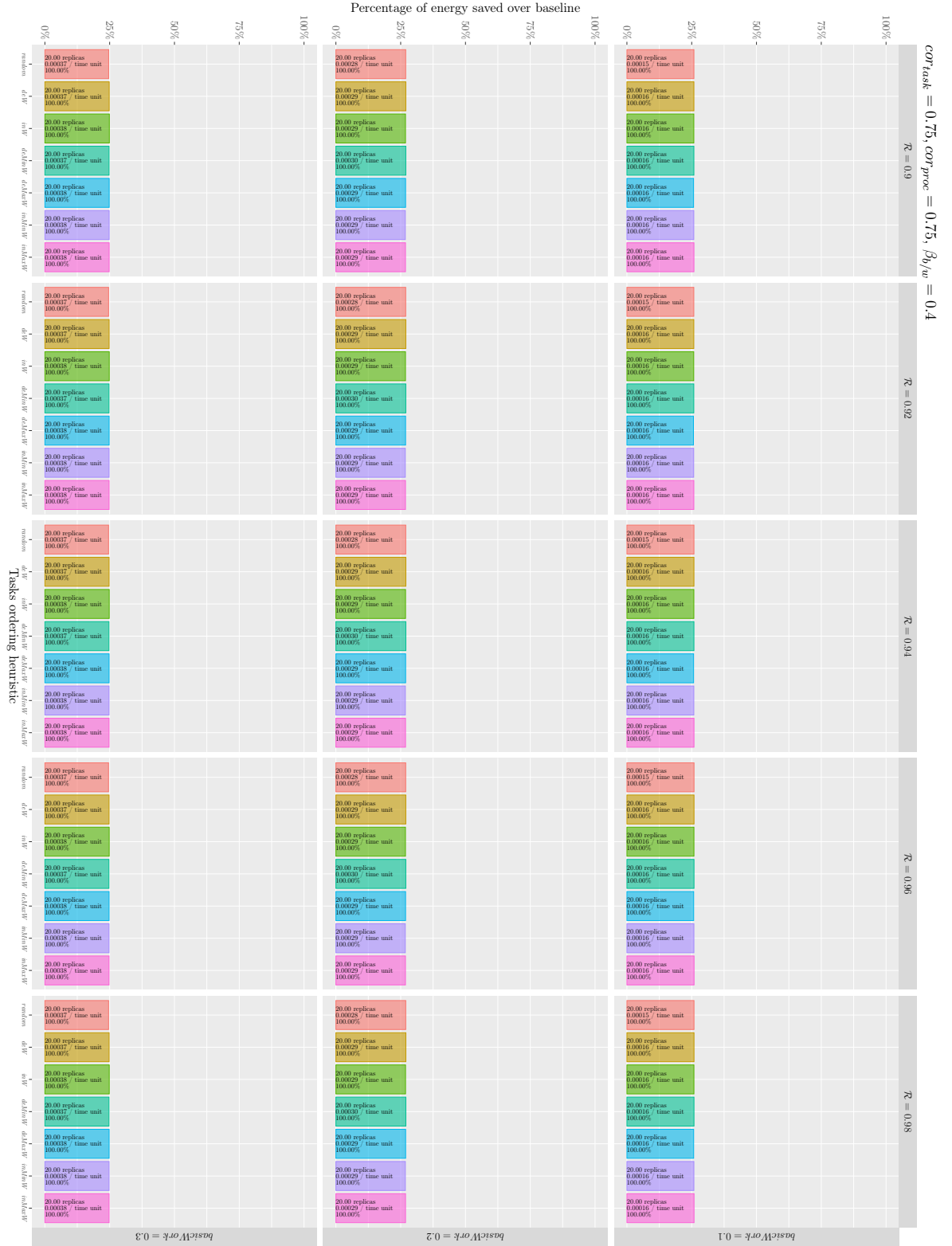


$cot_{task} = 0.75, cot_{proc} = 0.5, \beta_{hw} = 0.8$



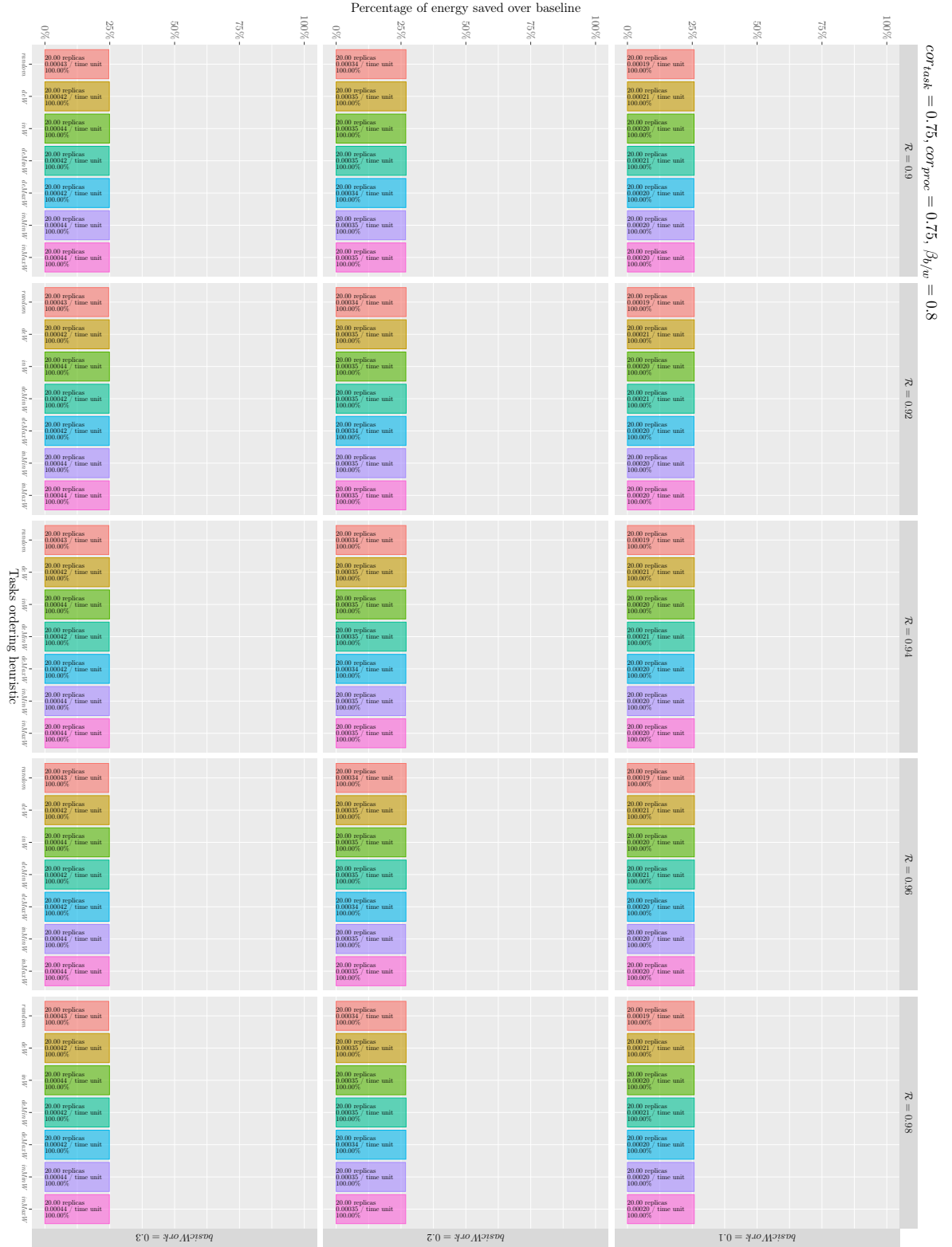








$COI_{task} = 0.75, COI_{proc} = 0.75, \beta_{i,w} = 0.6$





## **Processor ordering criteria for mapping phase**

The following section presents percentage of energy saved over baseline using different processor ordering criteria under small failure rate.



$con_{task} = 0.25, con_{proc} = 0.25, \beta_{b/w} = 0.2$















$cor_{task} = 0.25, cor_{proc} = 0.5, \beta_h/w = 0.4$









$cor_{task} = 0.25, cor_{proc} = 0.75, \beta_{b/w} = 0.2$











$con_{task} = 0.25, con_{proc} = 0.75, \beta_{b/w} = 1$





$COI_{task} = 0.5, COI_{proc} = 0.25, \beta_h/w = 0.4$











$COI_{task} = 0.5, COI_{proc} = 0.5, \beta_{h/w} = 0.2$





$COI_{task} = 0.5, COI_{proc} = 0.5, \beta_{h/w} = 0.6$





$COI_{task} = 0.5, COI_{proc} = 0.5, \beta_{h/w} = 1$





$COI_{task} = 0.5, COI_{proc} = 0.75, \beta_h/w = 0.4$











$con_{task} = 0.75, con_{proc} = 0.25, \beta_{b/w} = 0.2$





$con_{task} = 0.75, con_{proc} = 0.25, \beta_{b/w} = 0.6$





$con_{task} = 0.75, con_{proc} = 0.25, \beta_{b/w} = 1$







$con_{task} = 0.75, con_{proc} = 0.5, \beta_h/w = 0.4$





$\text{cor}_{task} = 0.75, \text{cor}_{proc} = 0.5, \beta_{i/w} = 0.8$



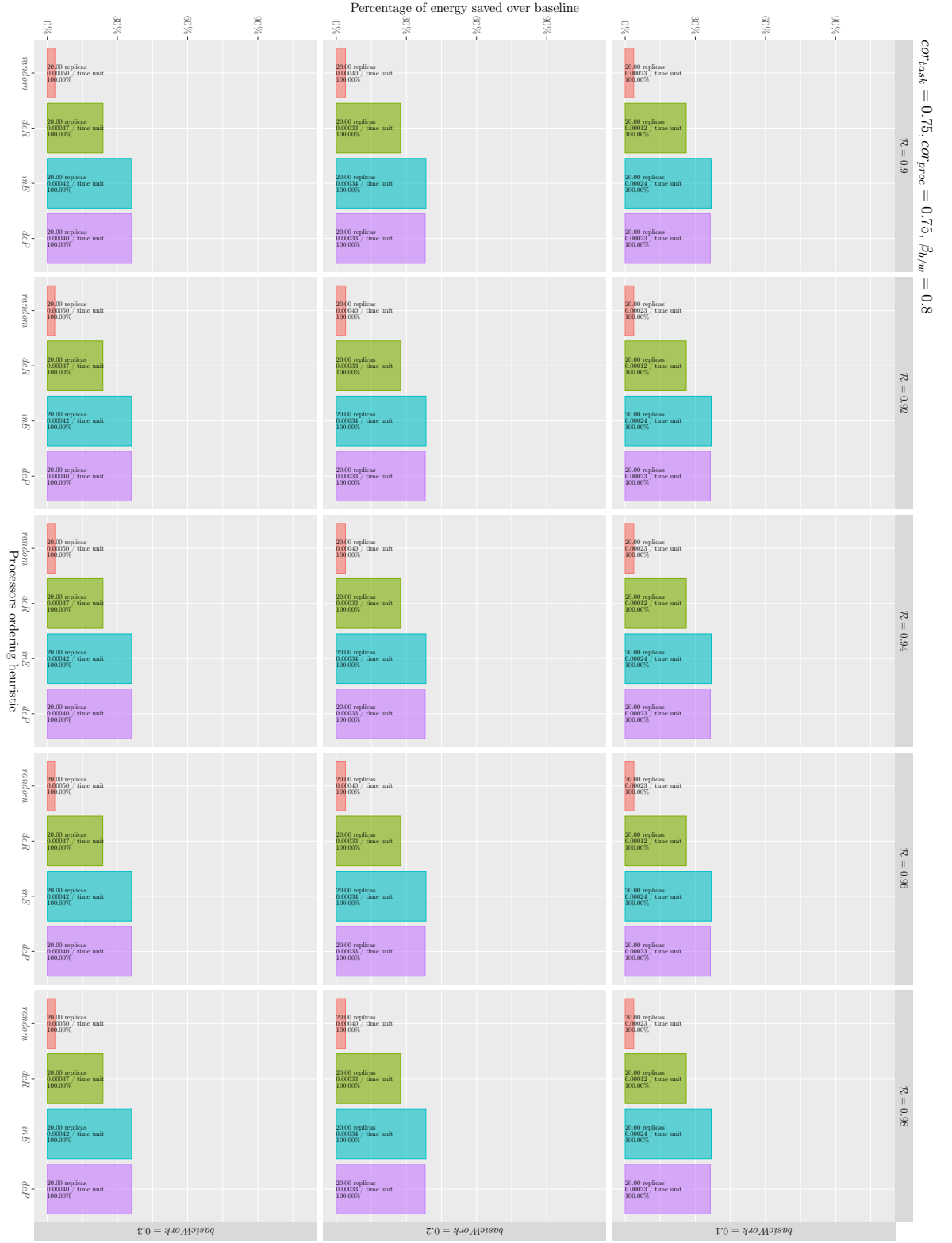


$con_{task} = 0.75, con_{proc} = 0.75, \beta_{b/w} = 0.2$







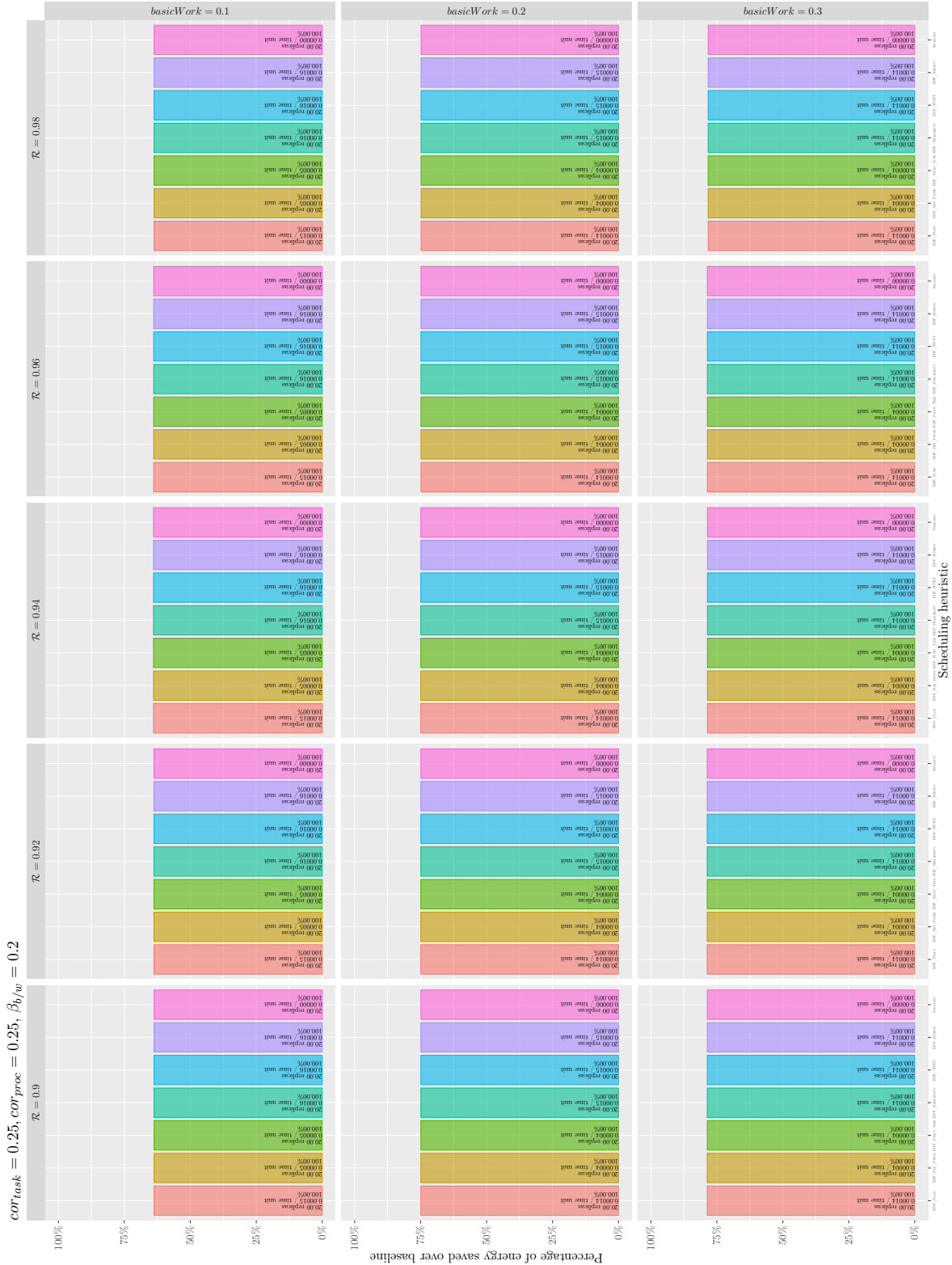


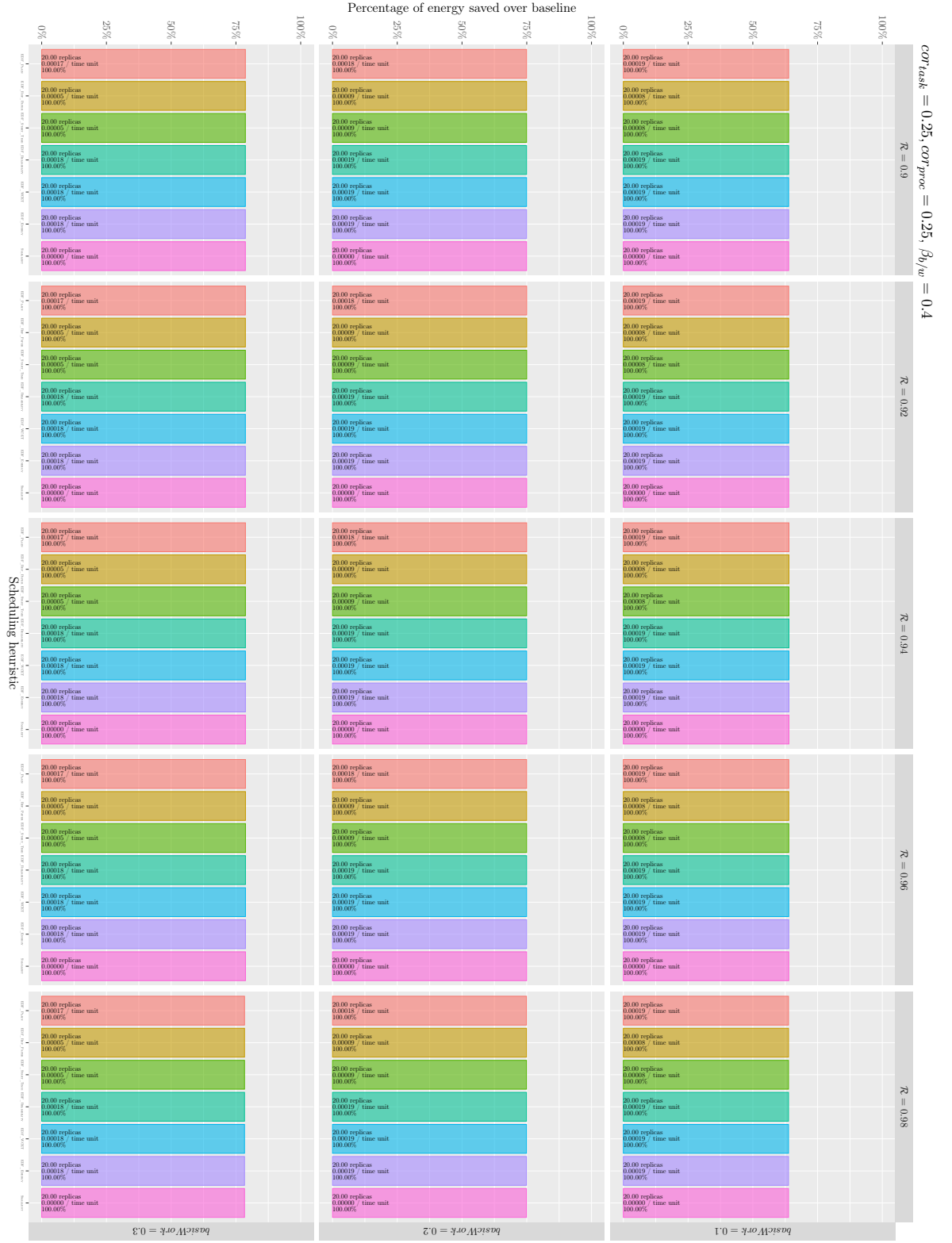


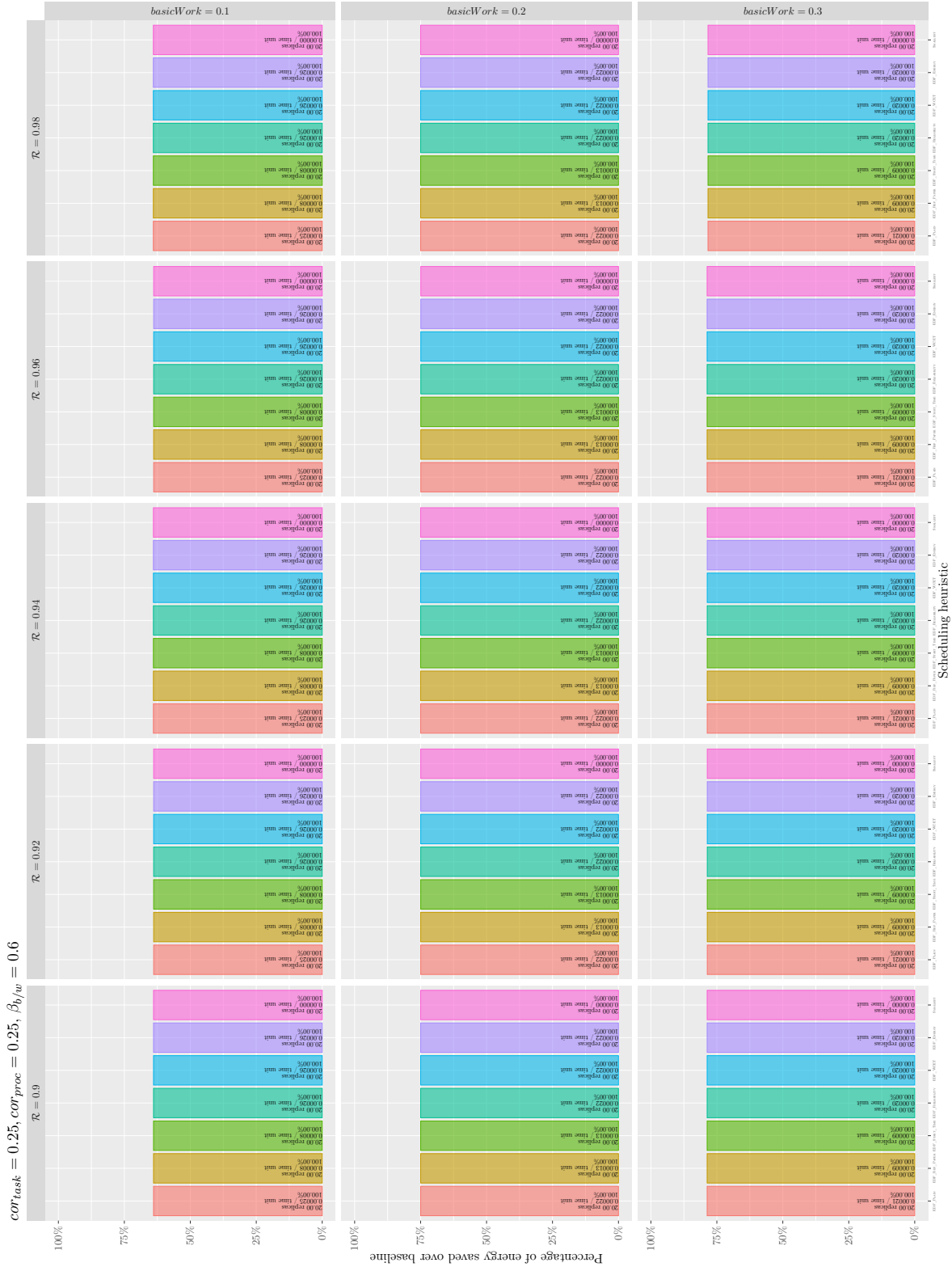
$con_{task} = 0.75, con_{proc} = 0.75, \beta_{b/w} = 1$

## **Scheduling heuristics**

The following section presents percentage of energy saved over baseline using different scheduling heuristics under small failure rate.

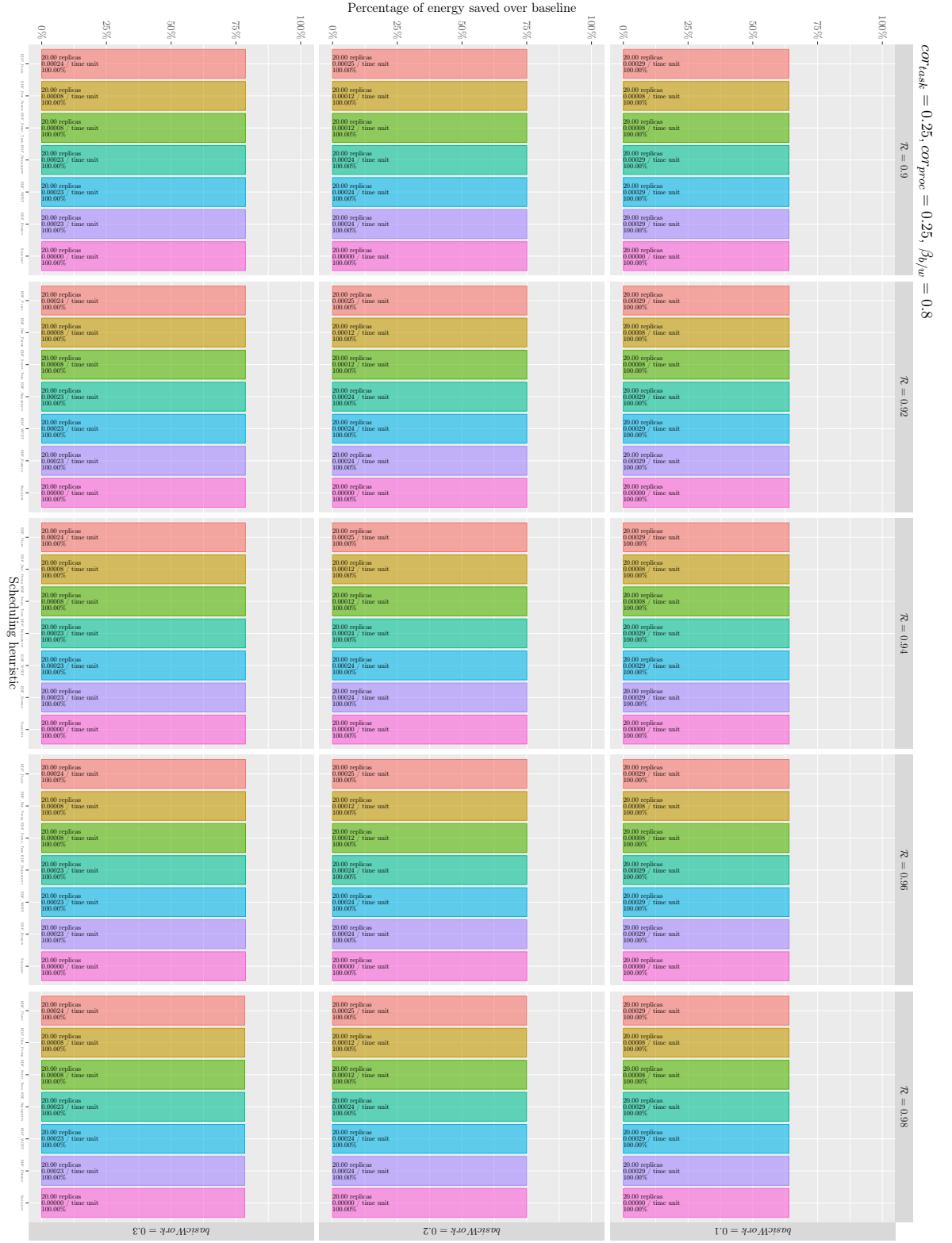


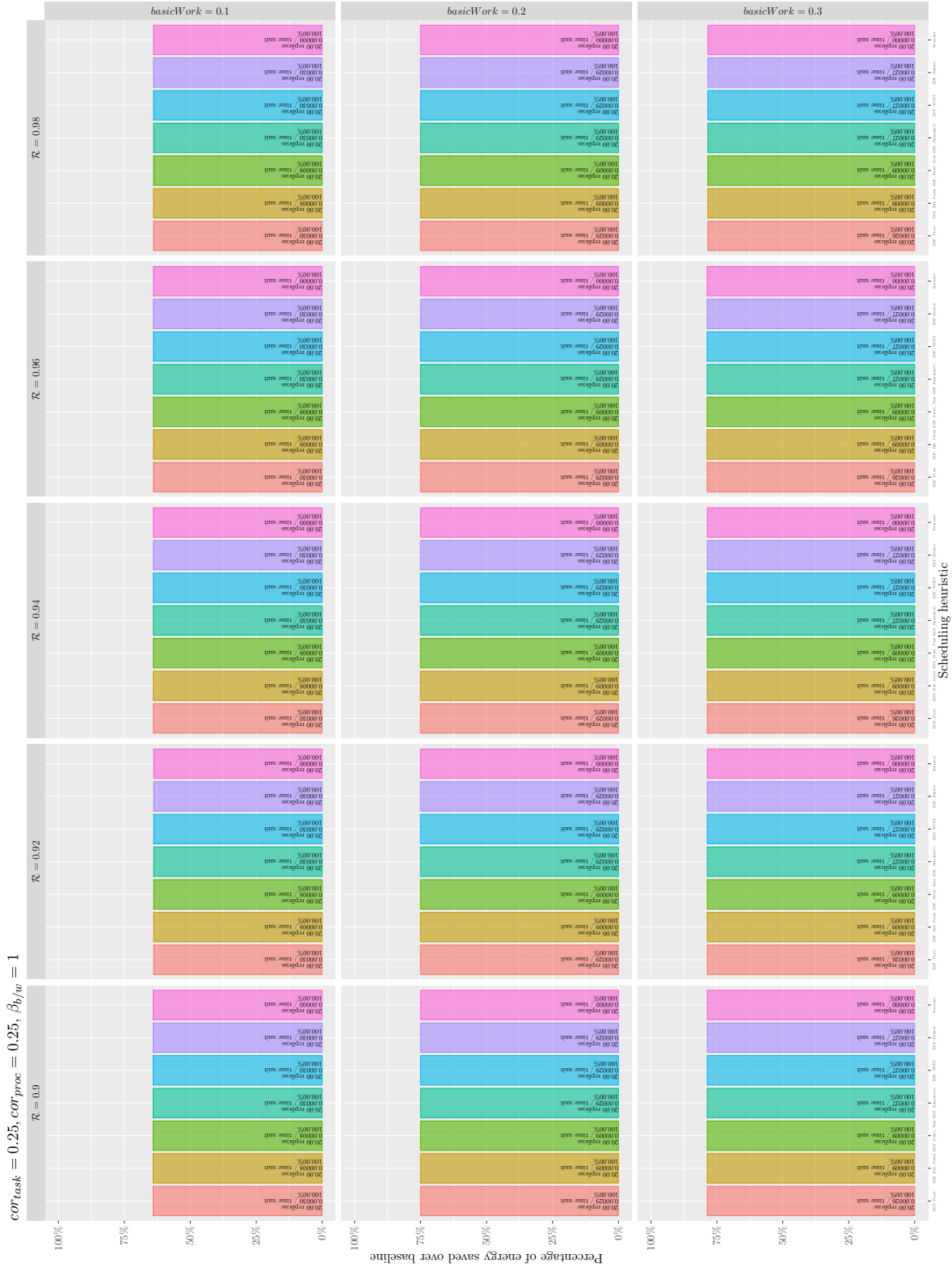




$\text{cot}_{task} = 0.25, \text{cot}_{proc} = 0.25, \beta_{h,w} = 0.6$

Scheduling heuristic

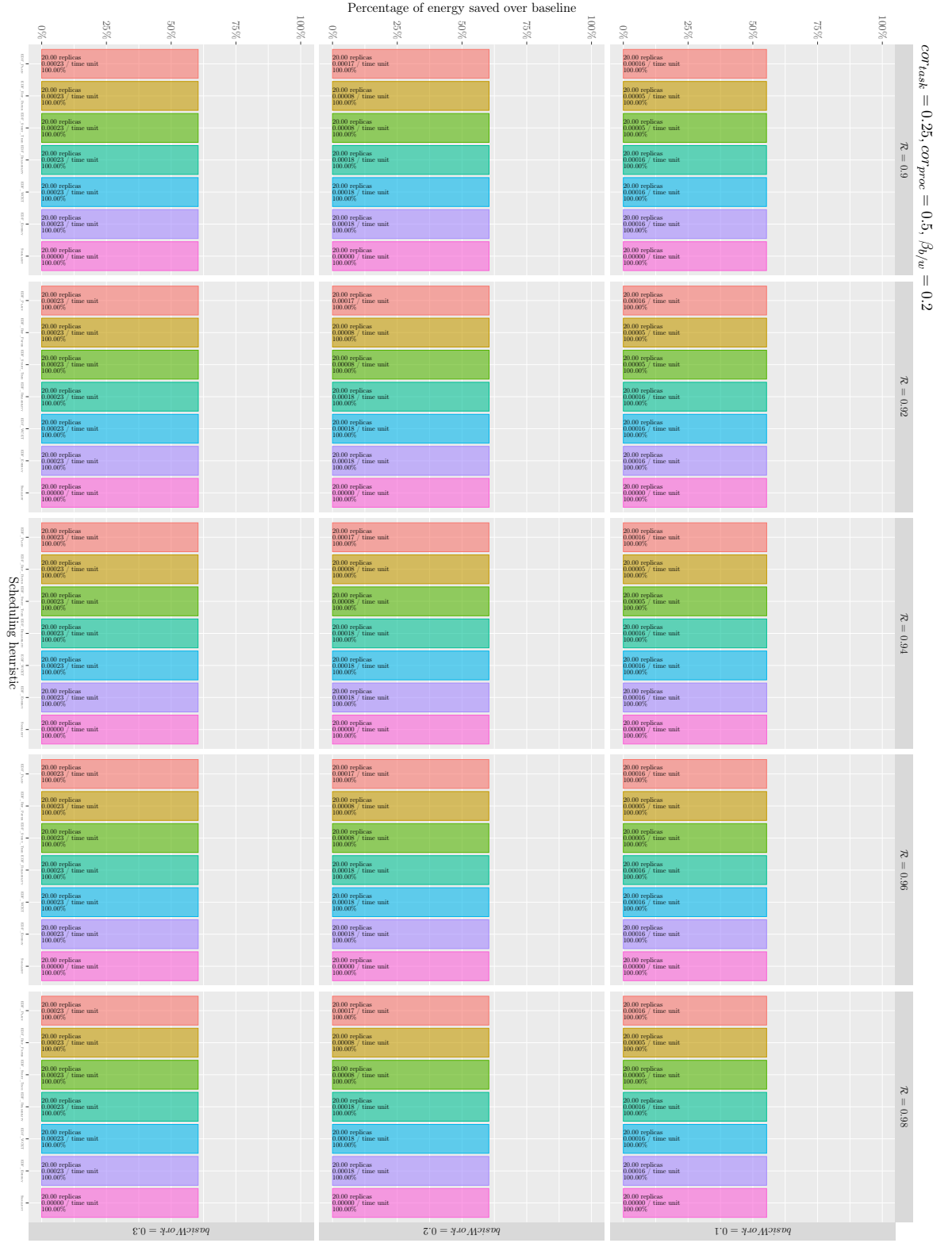


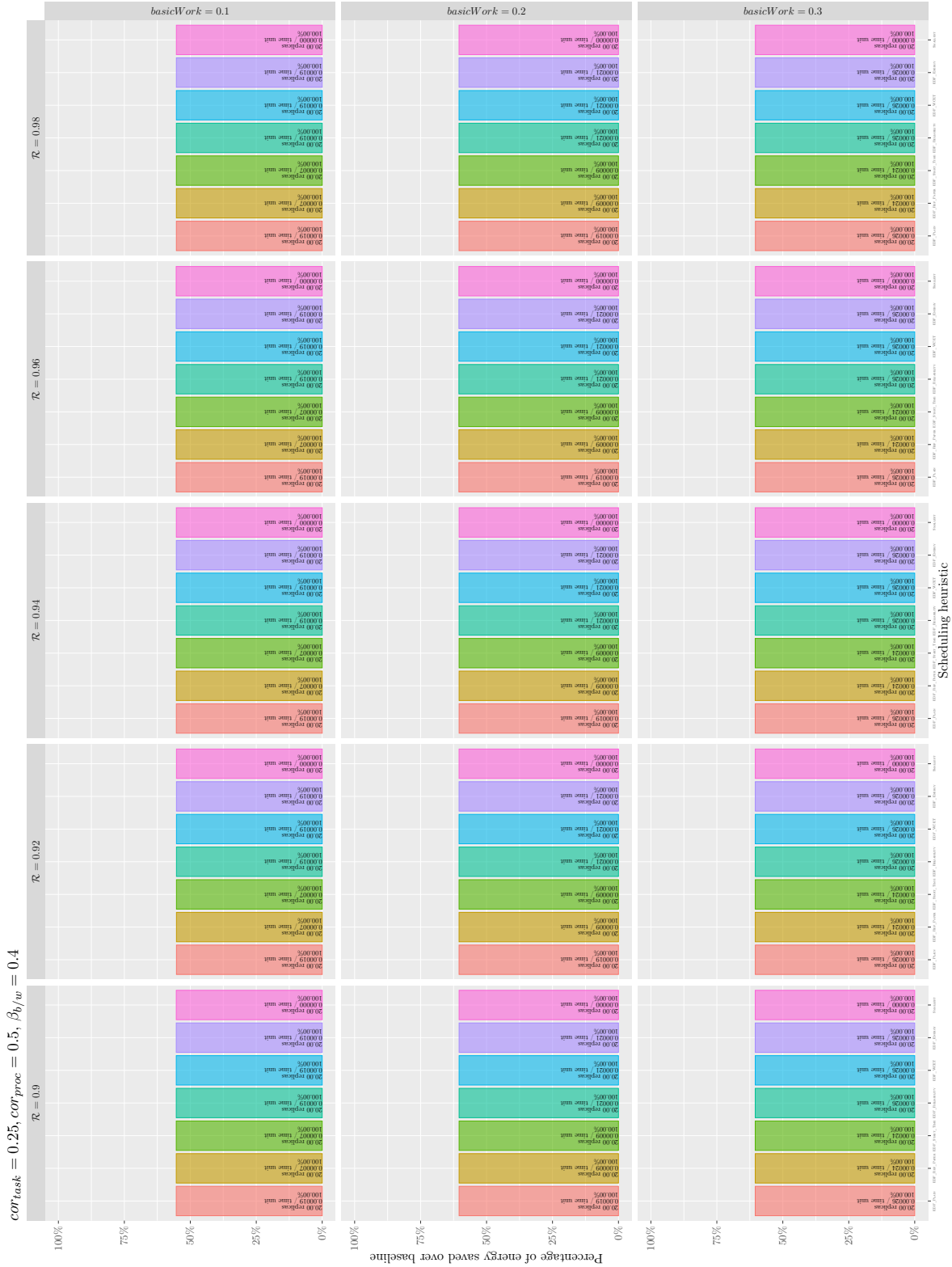


Scheduling heuristic

$$cot_{task} = 0.25, cot_{proc} = 0.25, \beta_{y/w} = 1$$

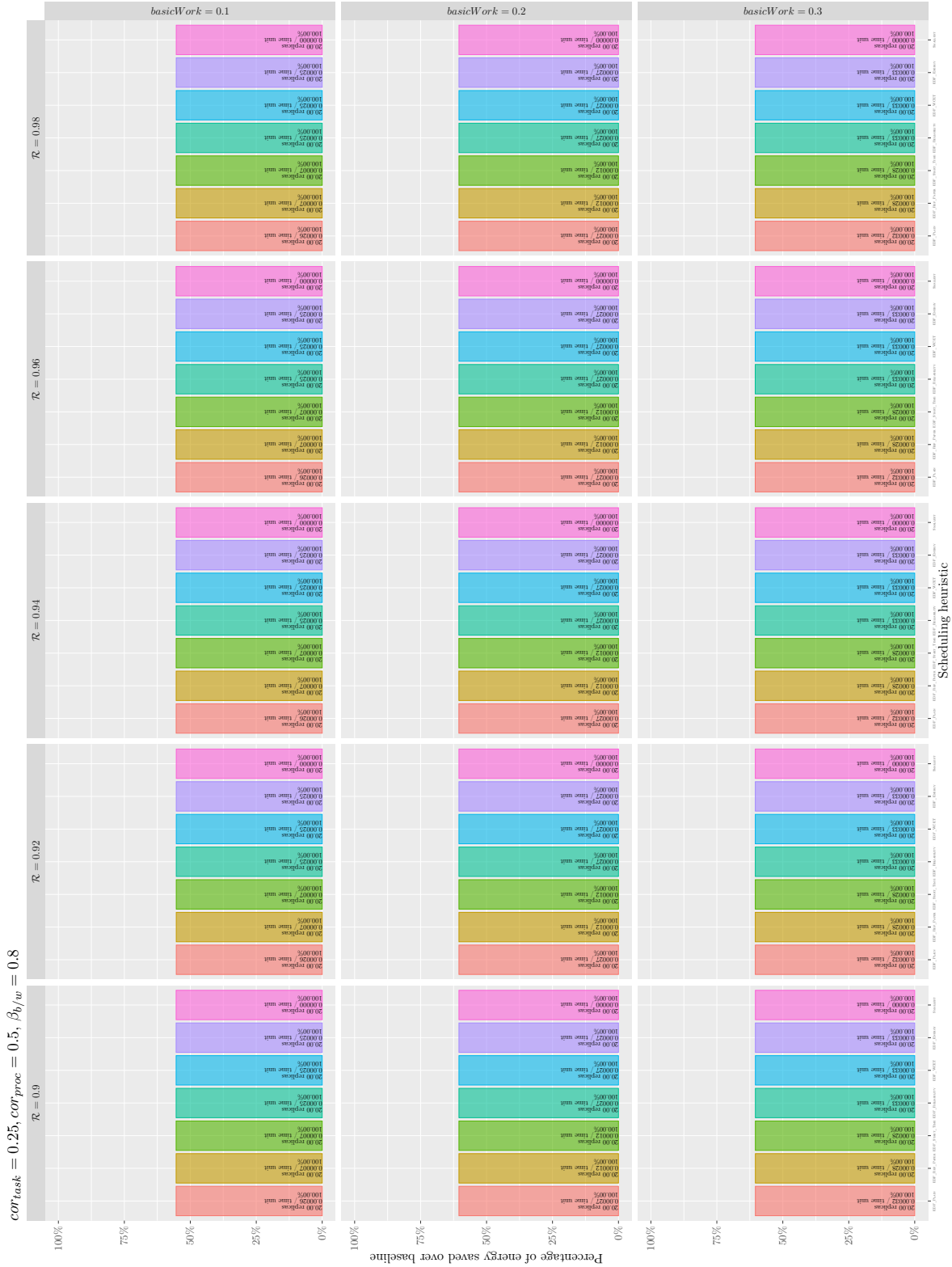






$COI_{task} = 0.25, COI_{proc} = 0.5, \beta_{h/w} = 0.4$





$COF_{task} = 0.25, COF_{proc} = 0.5, \beta_{h/w} = 0.8$

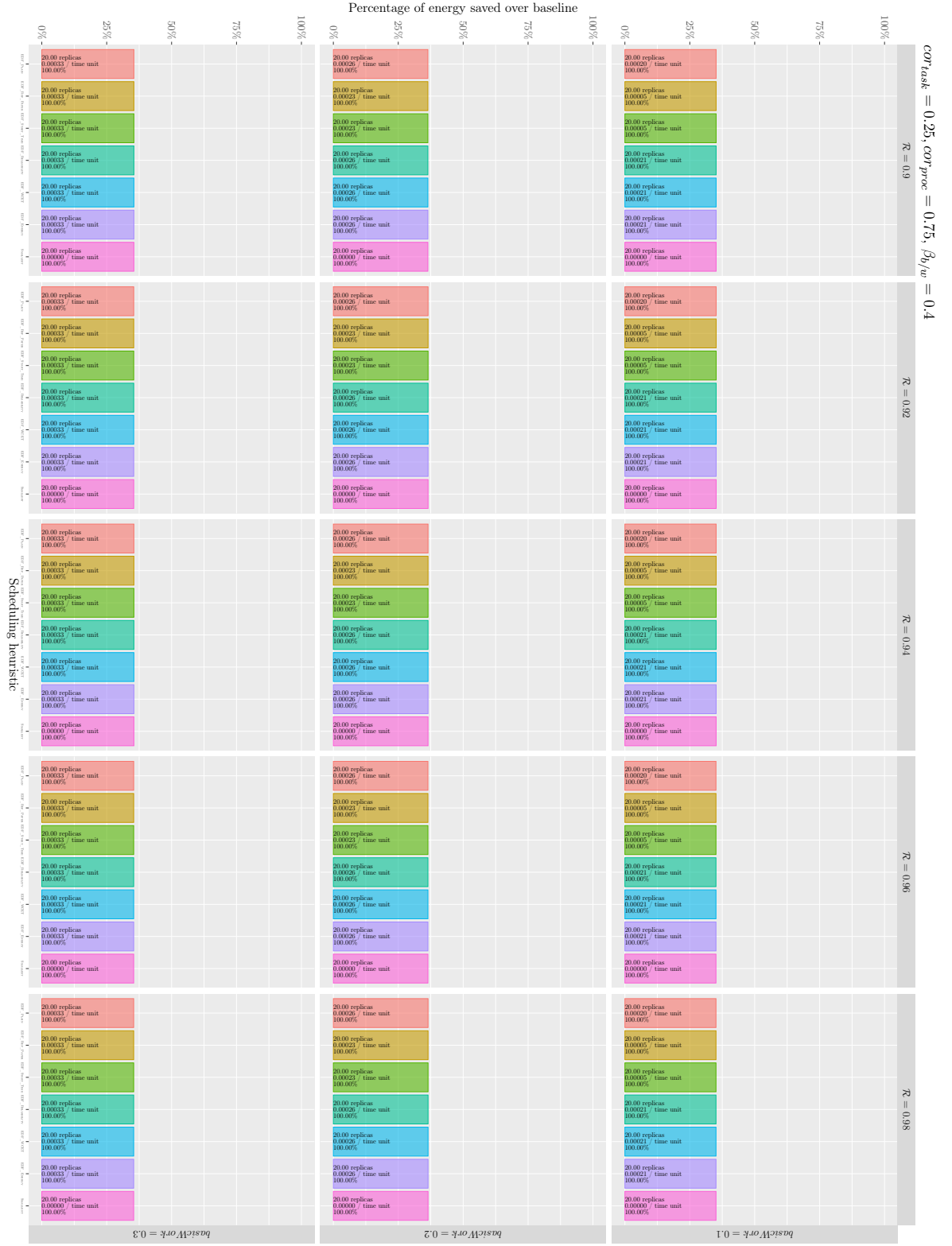
Scheduling heuristic





Scheduling heuristic

$\text{cot}_{task} = 0.25, \text{cot}_{proc} = 0.75, \beta_{hw} = 0.2$

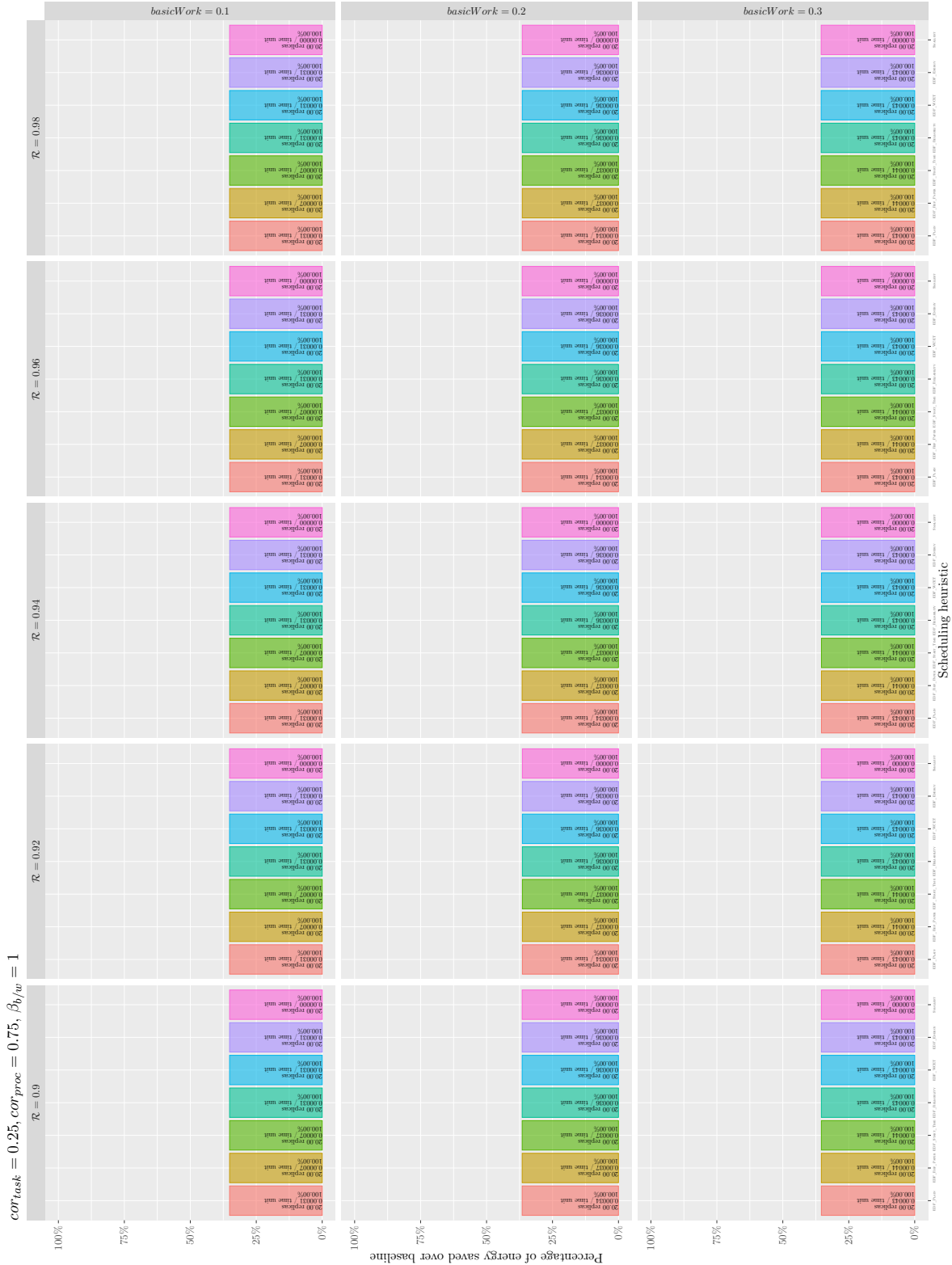




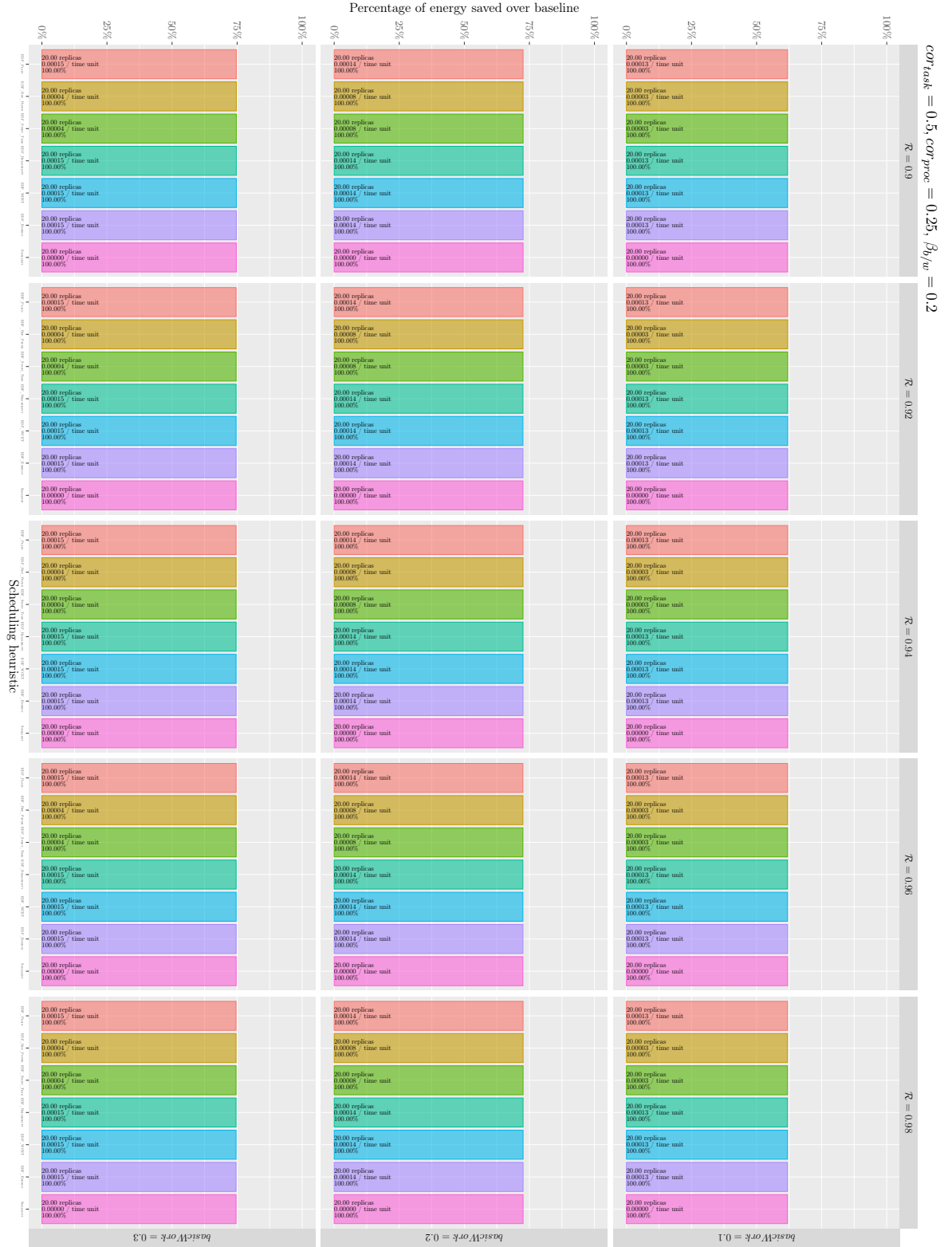
$COI_{task} = 0.25, COI_{proc} = 0.75, \beta_{h_i} = 0.6$

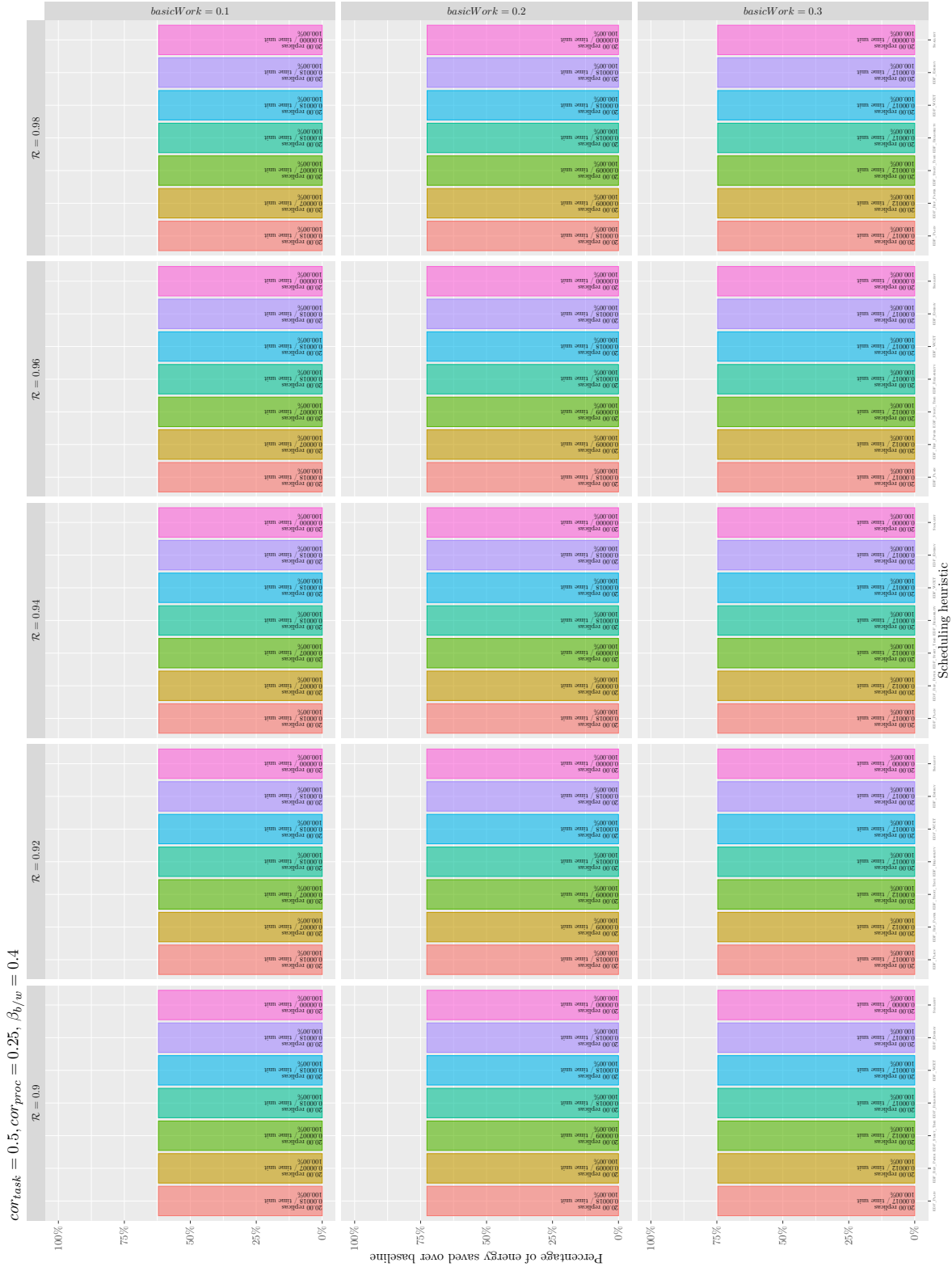




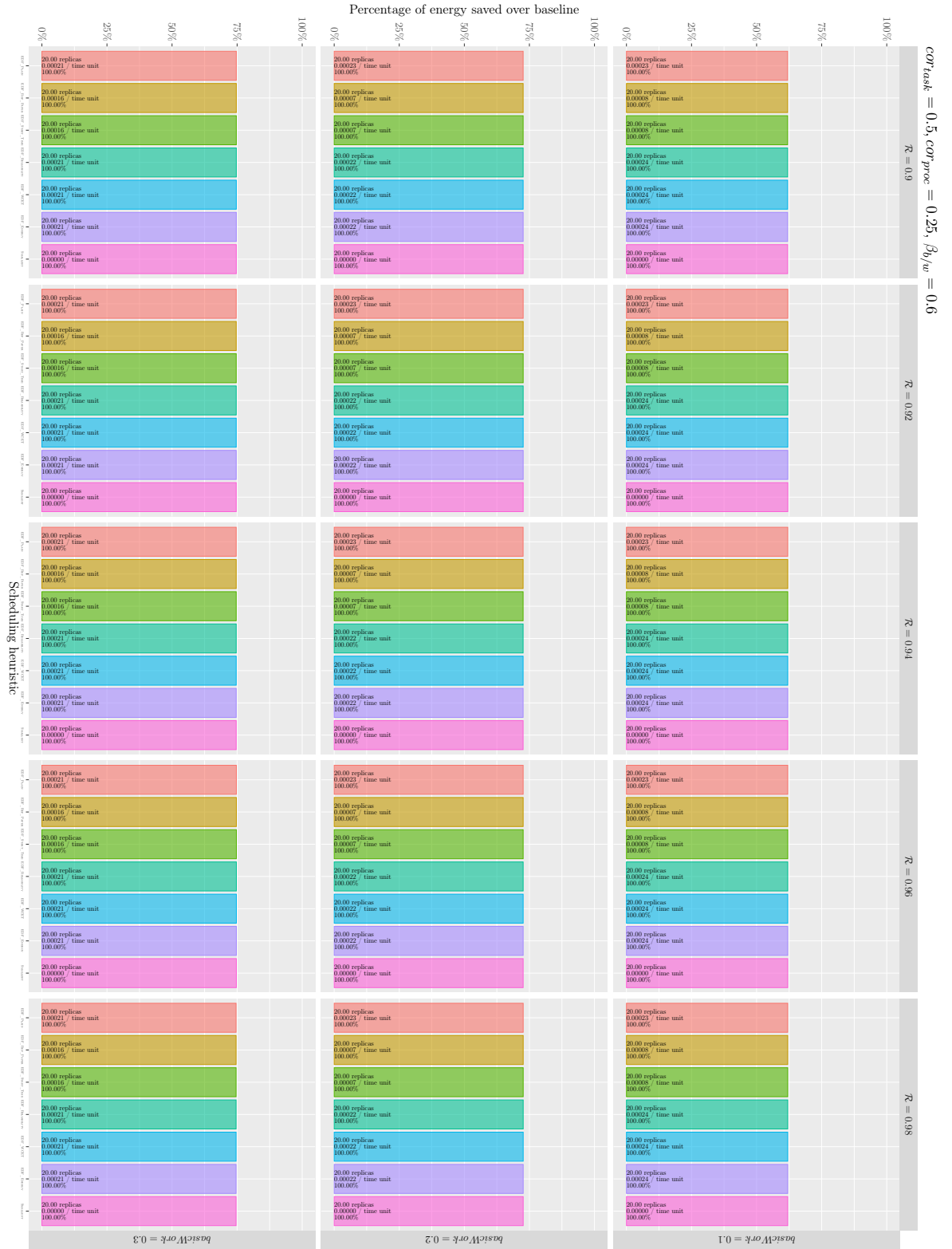


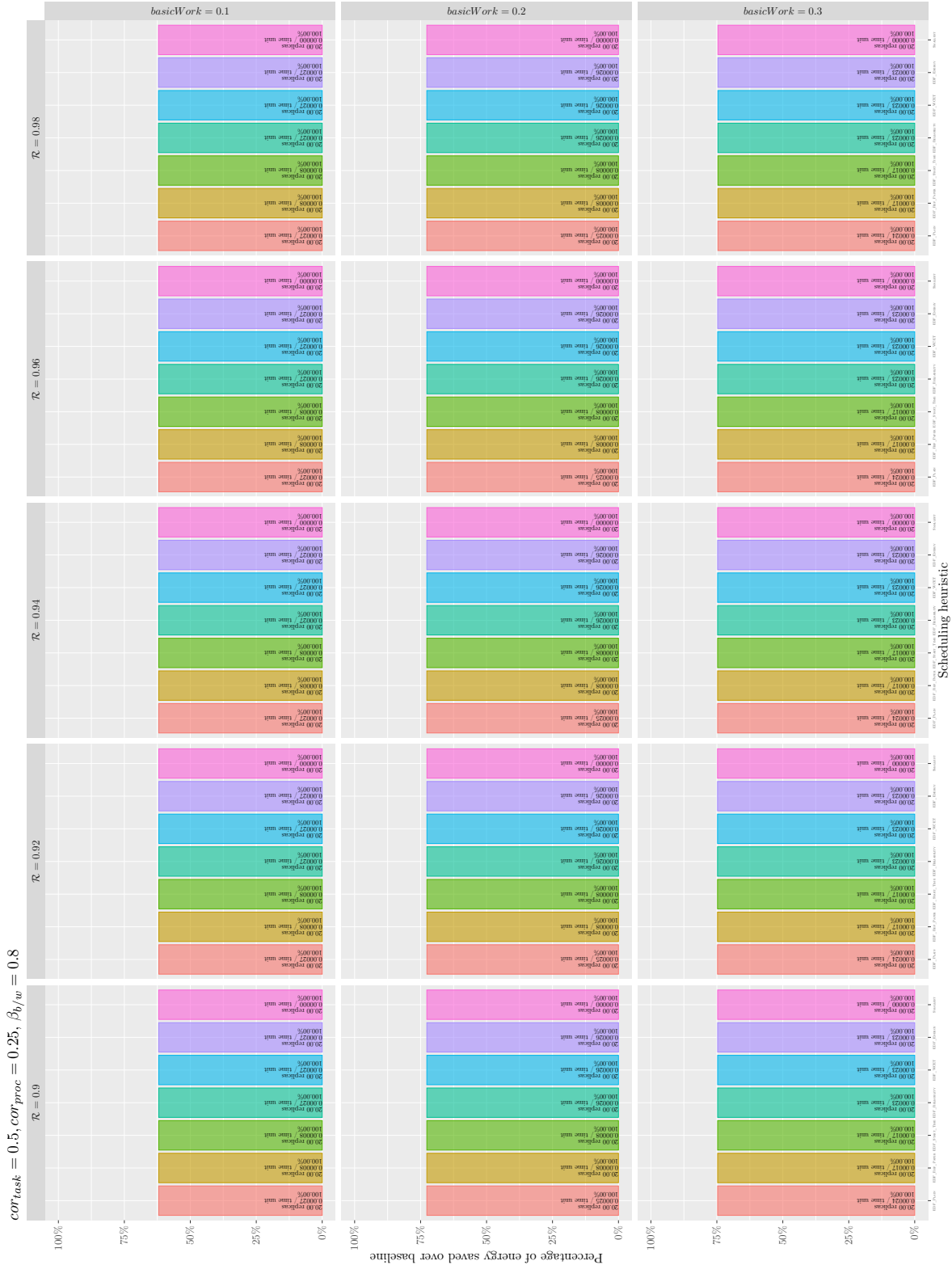
$COI_{task} = 0.25, COI_{proc} = 0.75, \beta_{hw} = 1$





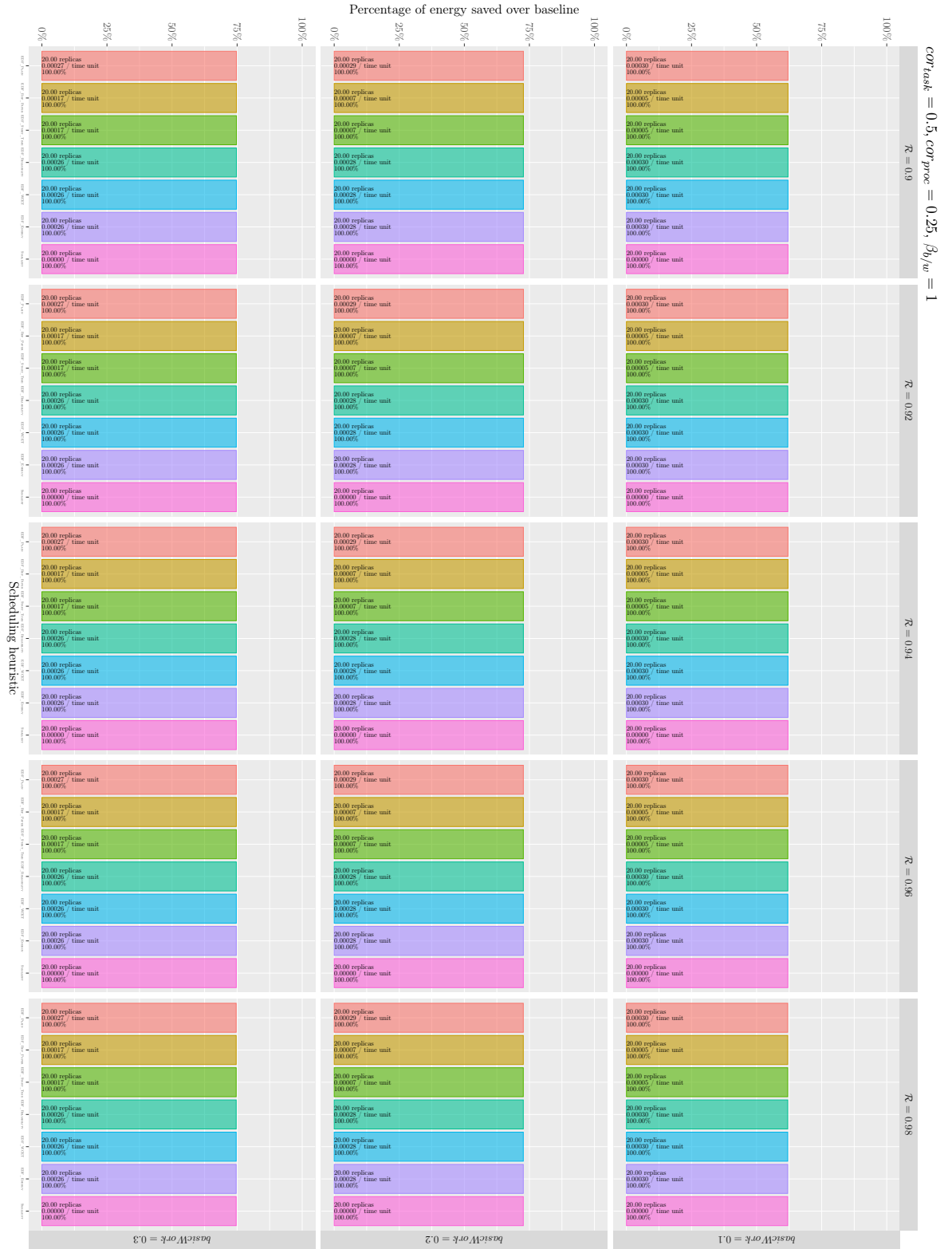
$cot_{task} = 0.5, cot_{proc} = 0.25, \beta_{hw} = 0.4$

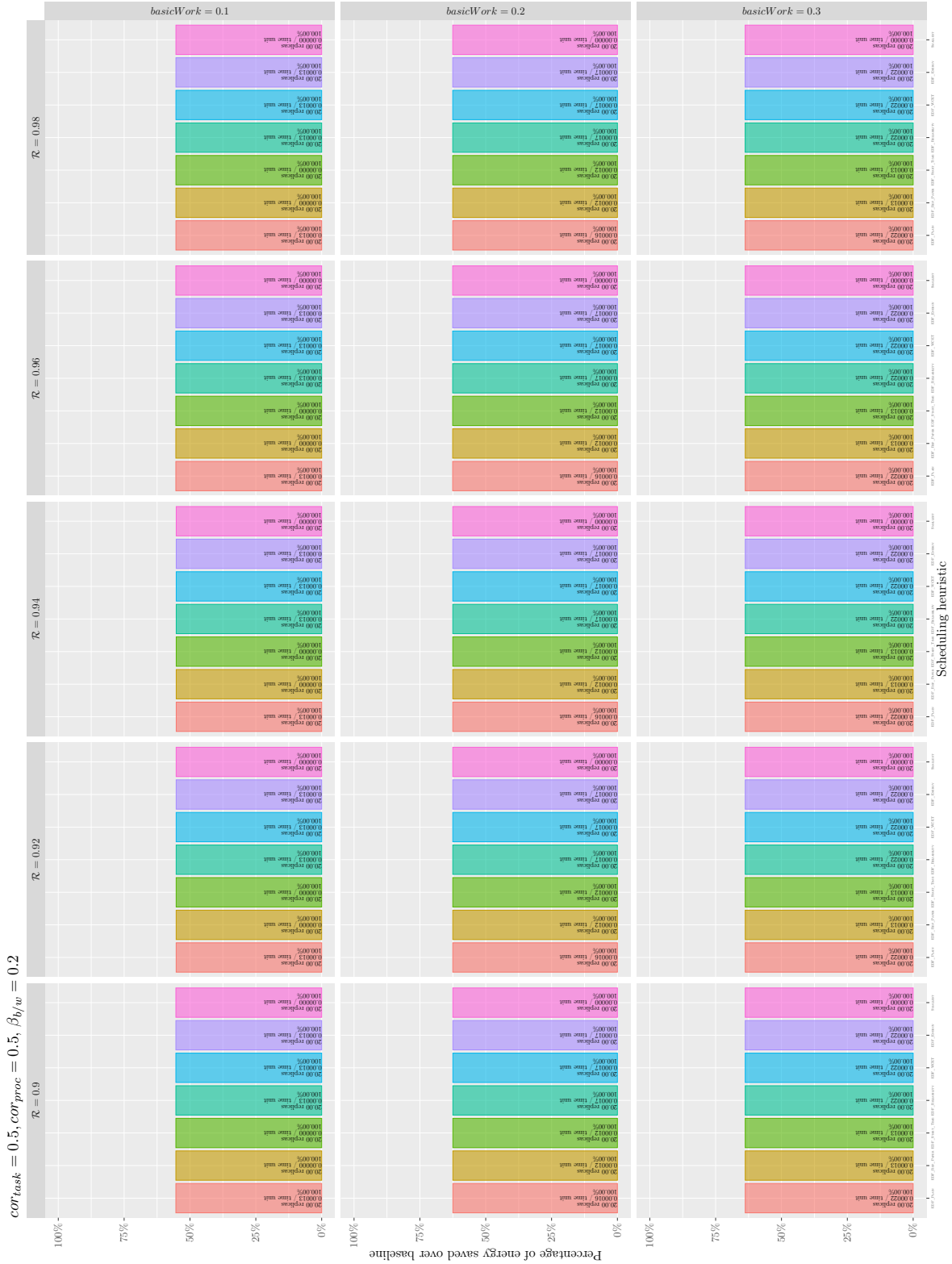




$cot_{task} = 0.5, cot_{proc} = 0.25, \beta_{hw} = 0.8$

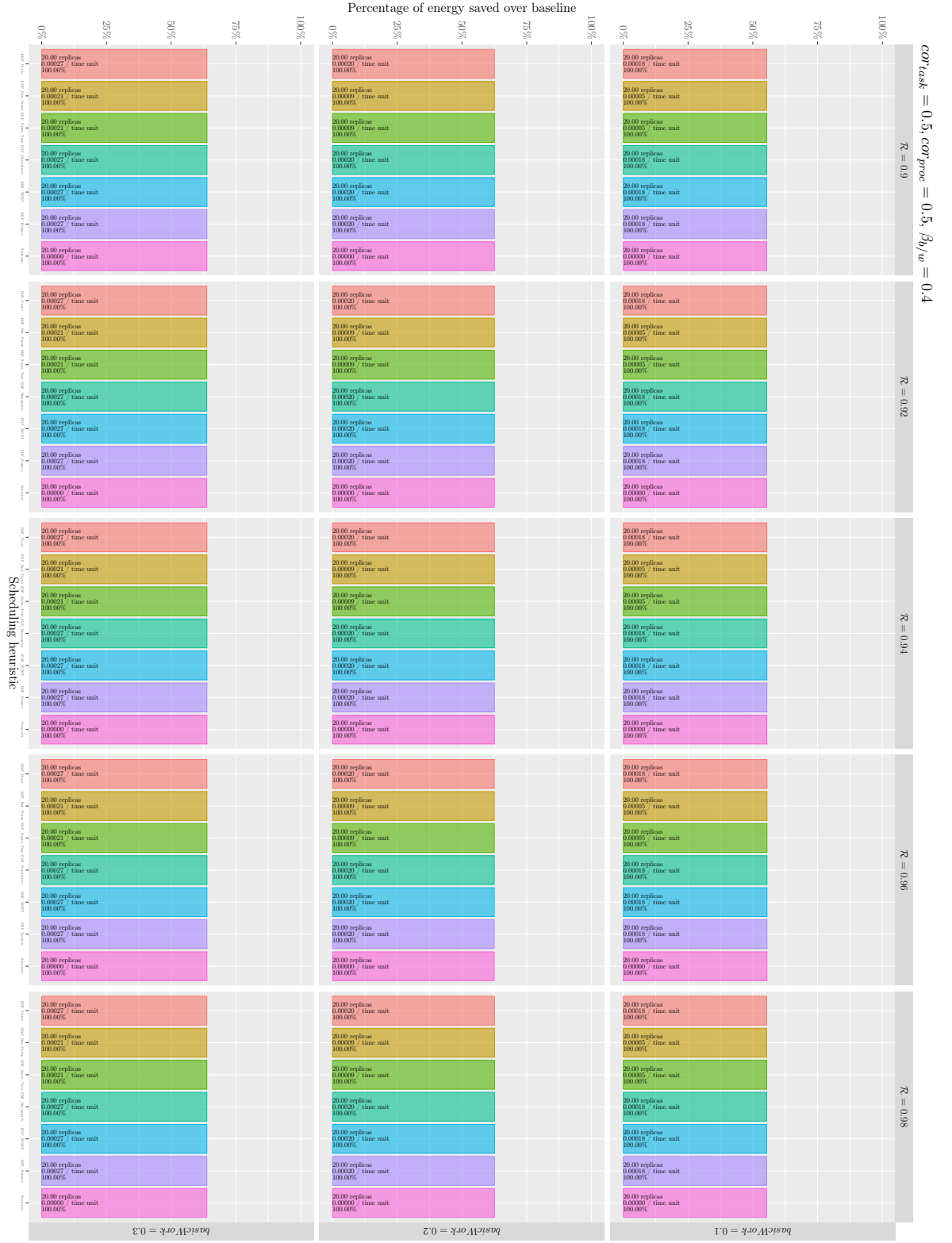
Scheduling heuristic



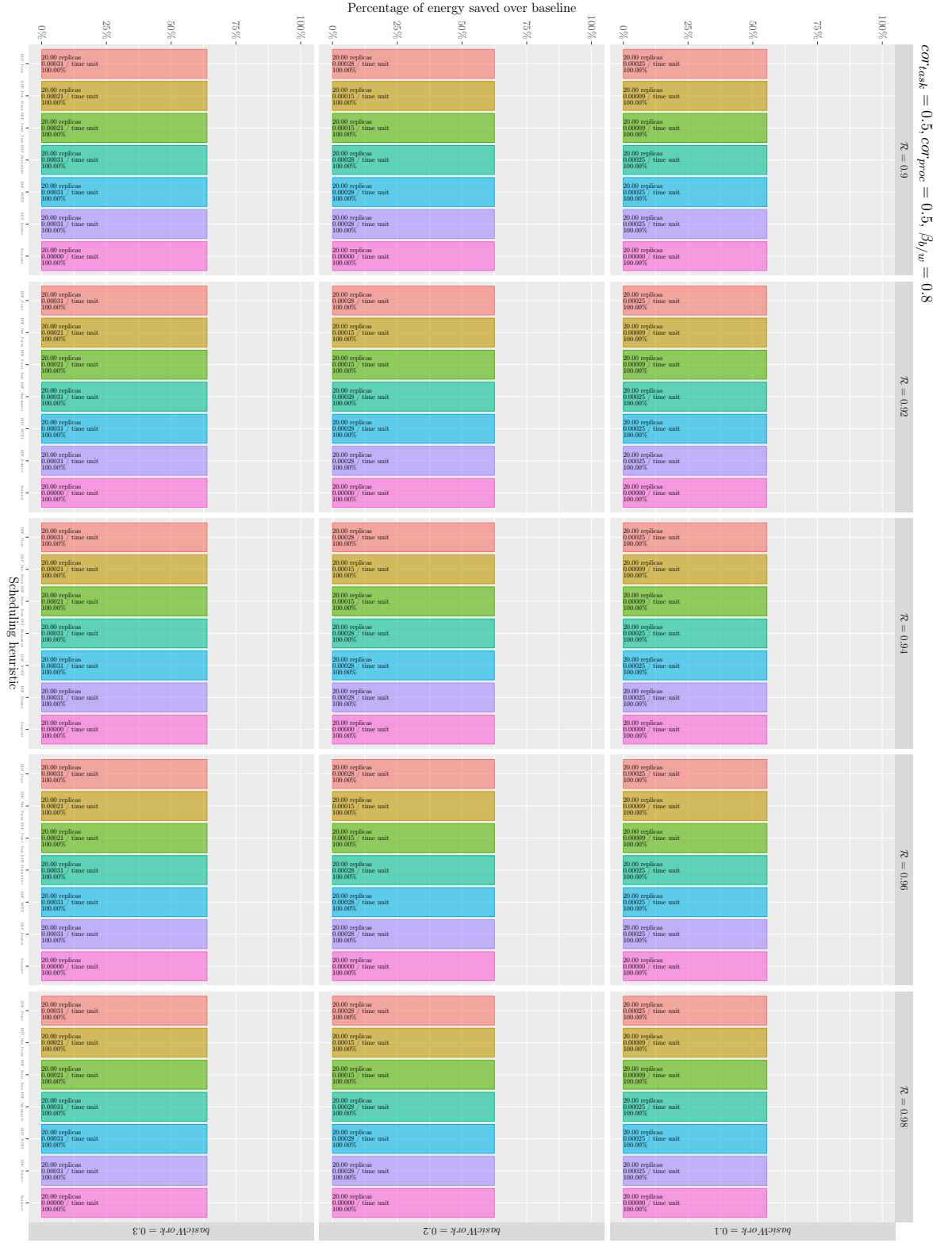


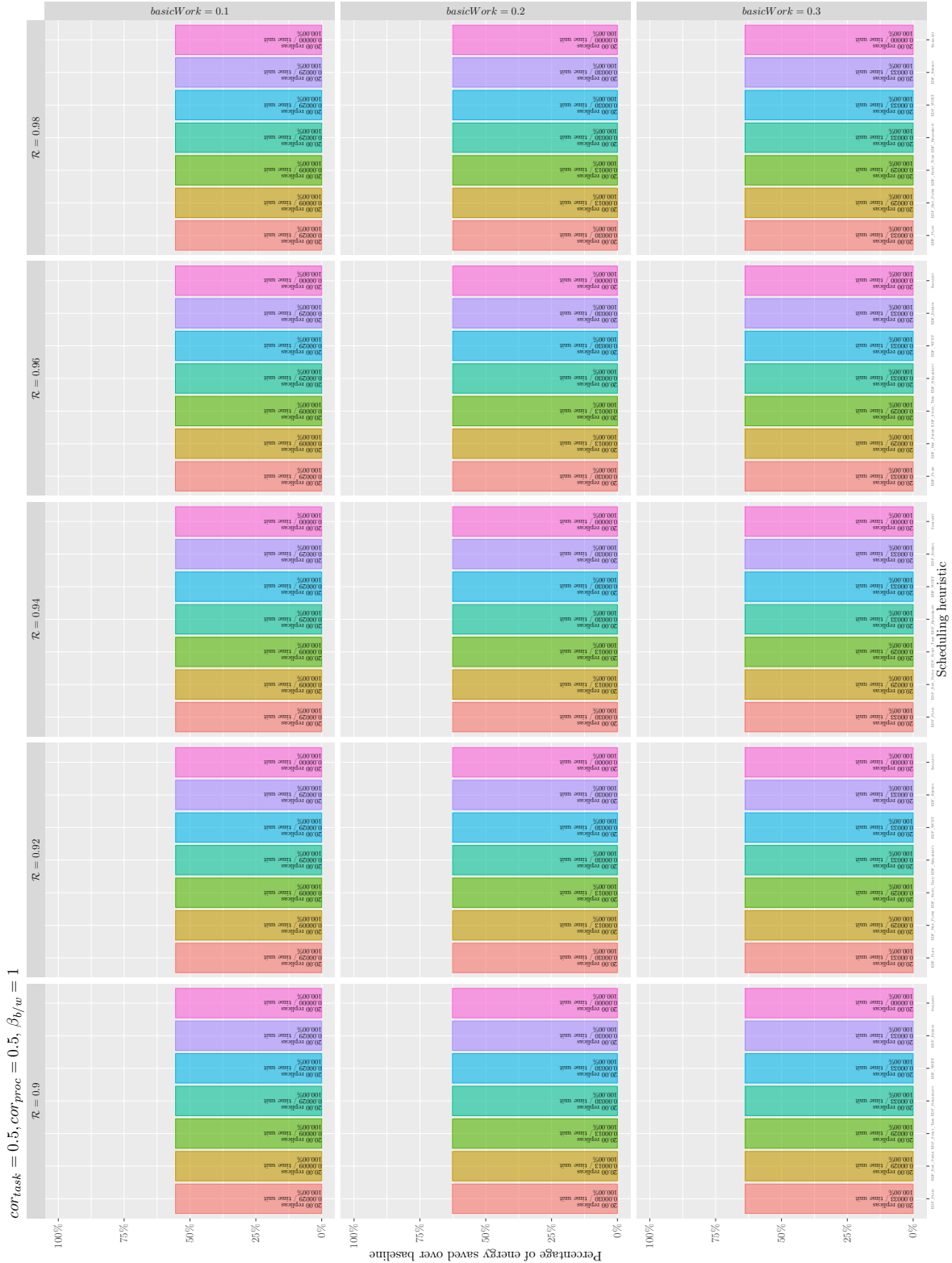
$cot_{task} = 0.5, cot_{proc} = 0.5, \beta_{by/w} = 0.2$









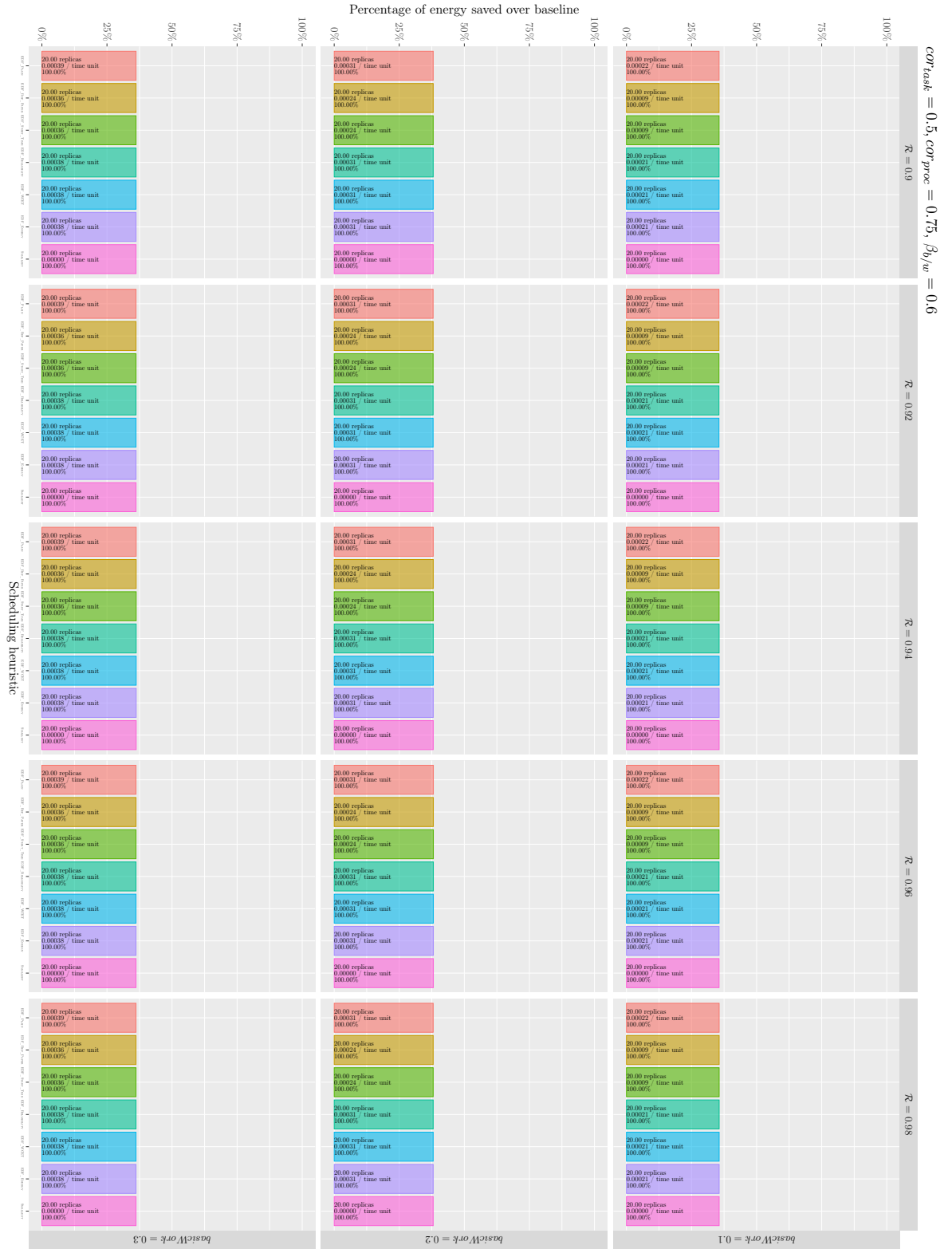


$cot_{task} = 0.5, cot_{proc} = 0.5, \beta_{by/w} = 1$





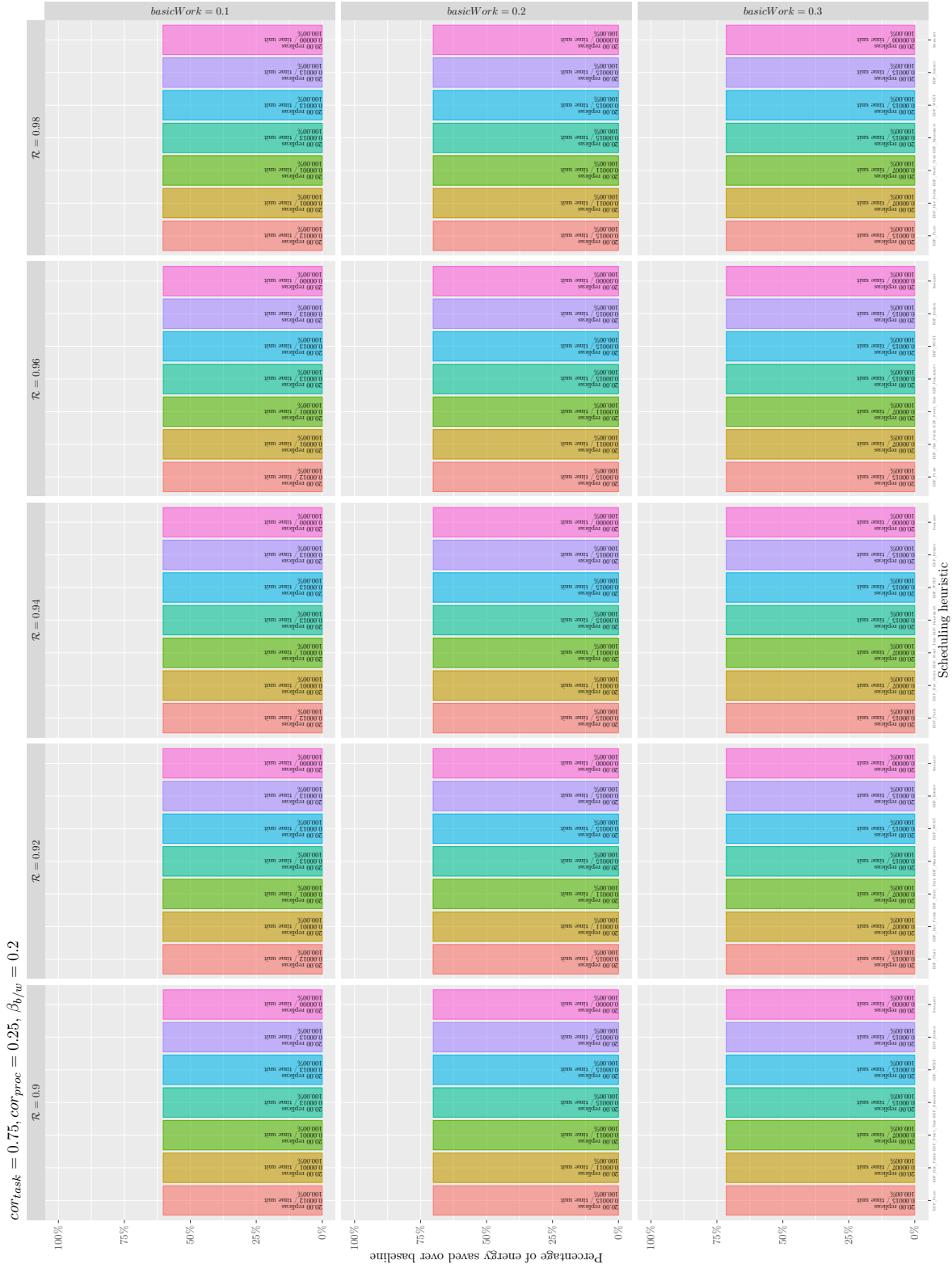
$cor_{task} = 0.5, cor_{proc} = 0.75, \beta_{hw} = 0.4$





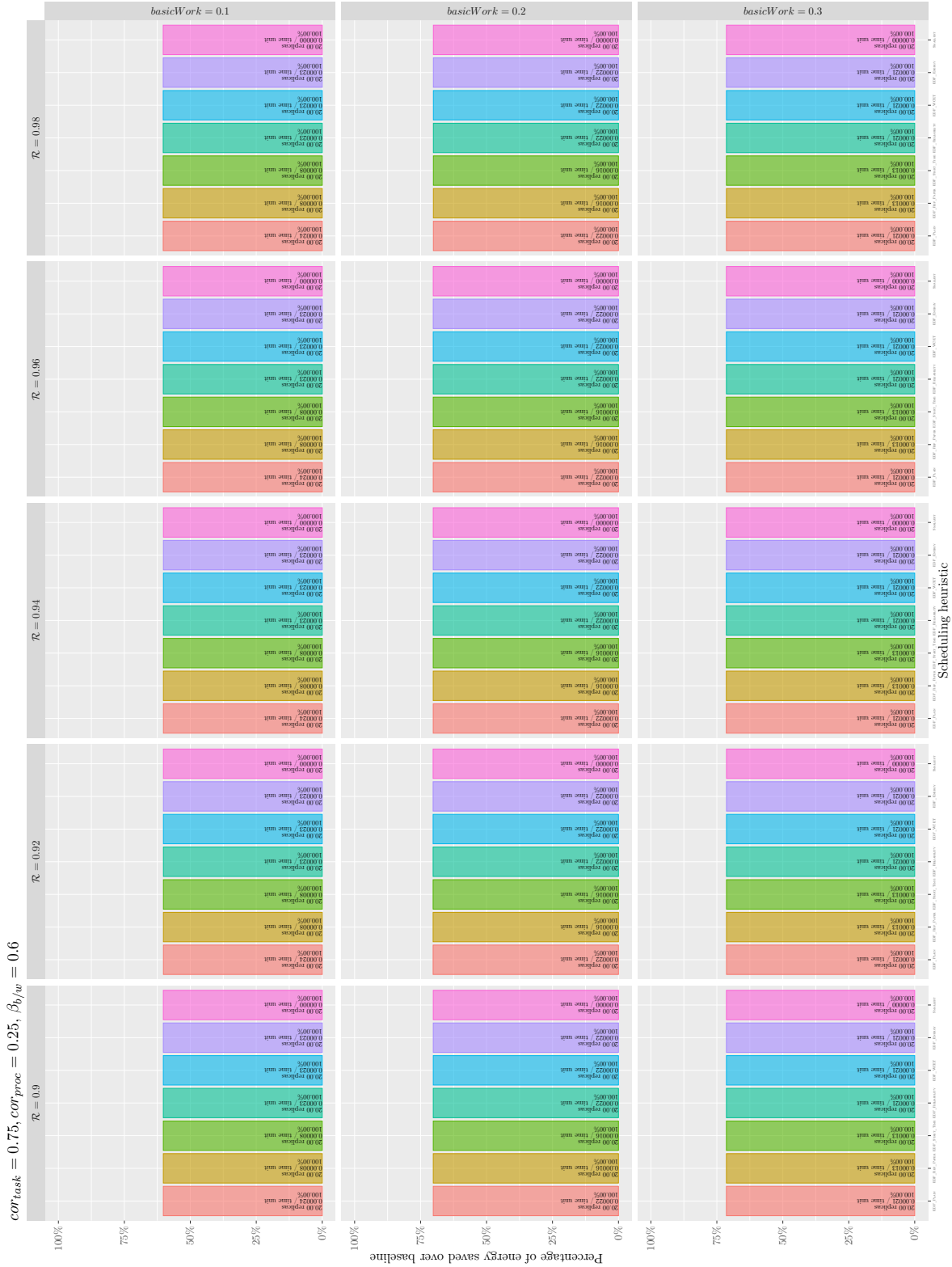






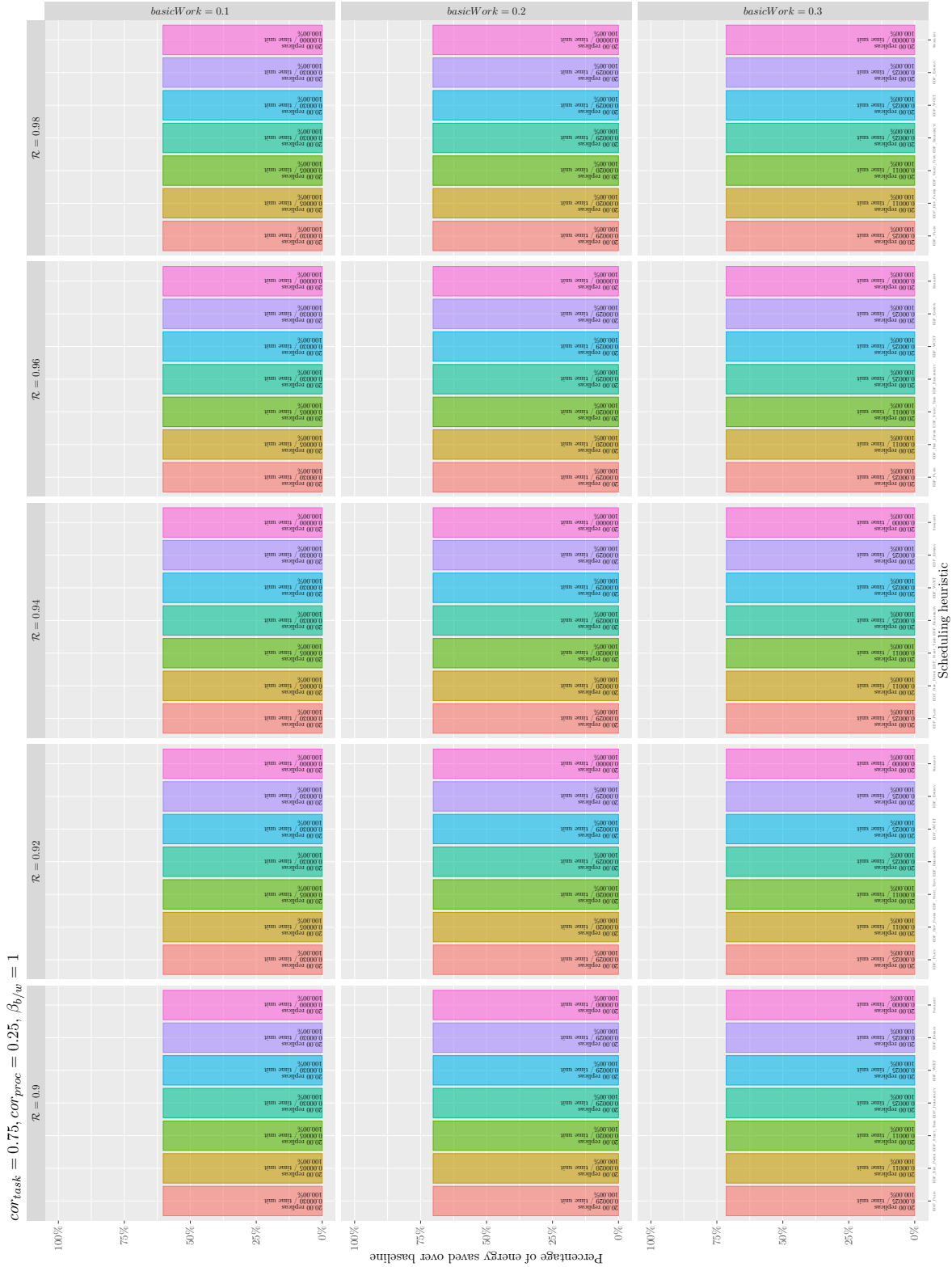
$COI_{task} = 0.75, COI_{proc} = 0.25, \beta_{h/w} = 0.2$





Scheduling heuristic















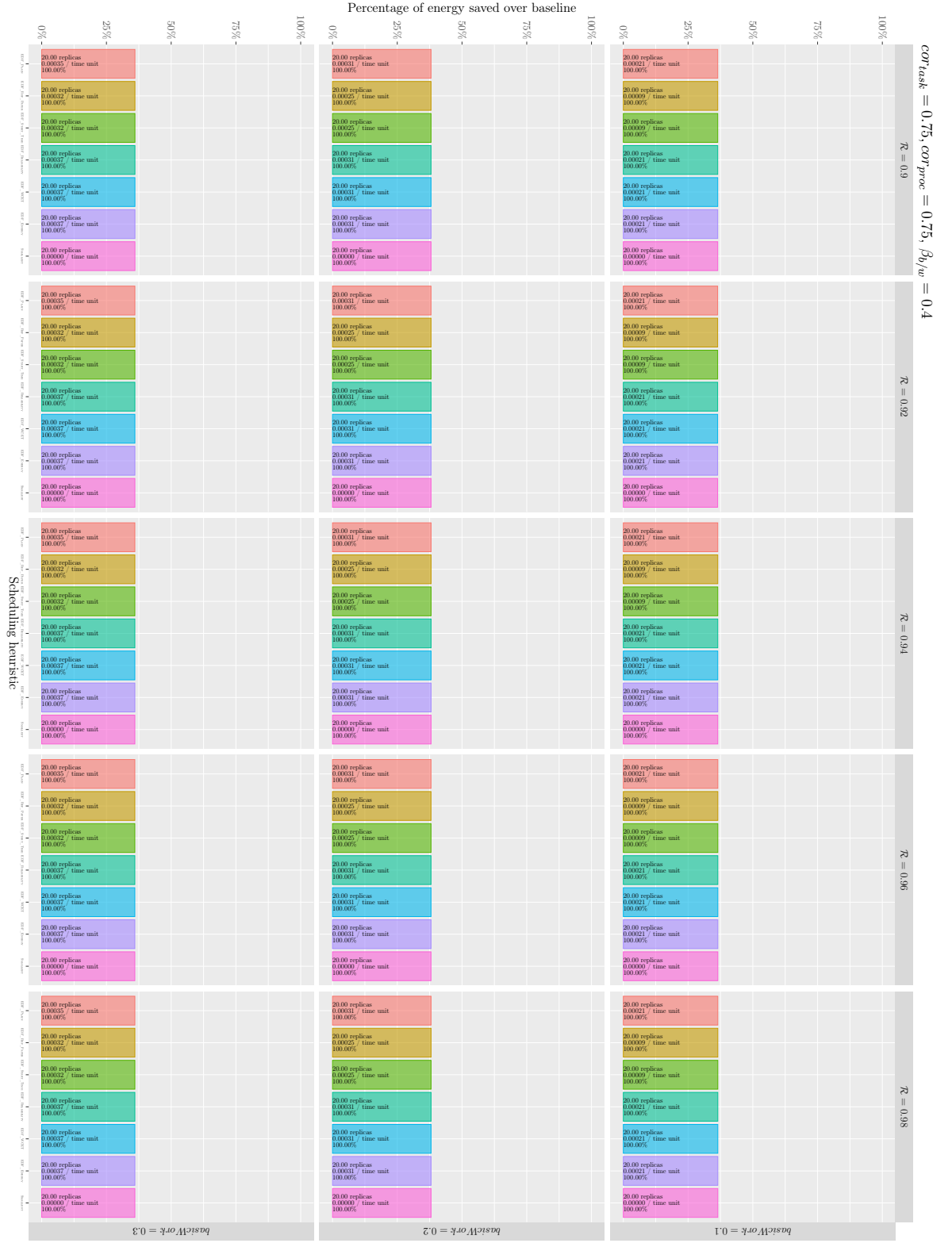
$COF_{task} = 0.75, COF_{proc} = 0.5, \beta_{hw} = 0.8$

Scheduling heuristic



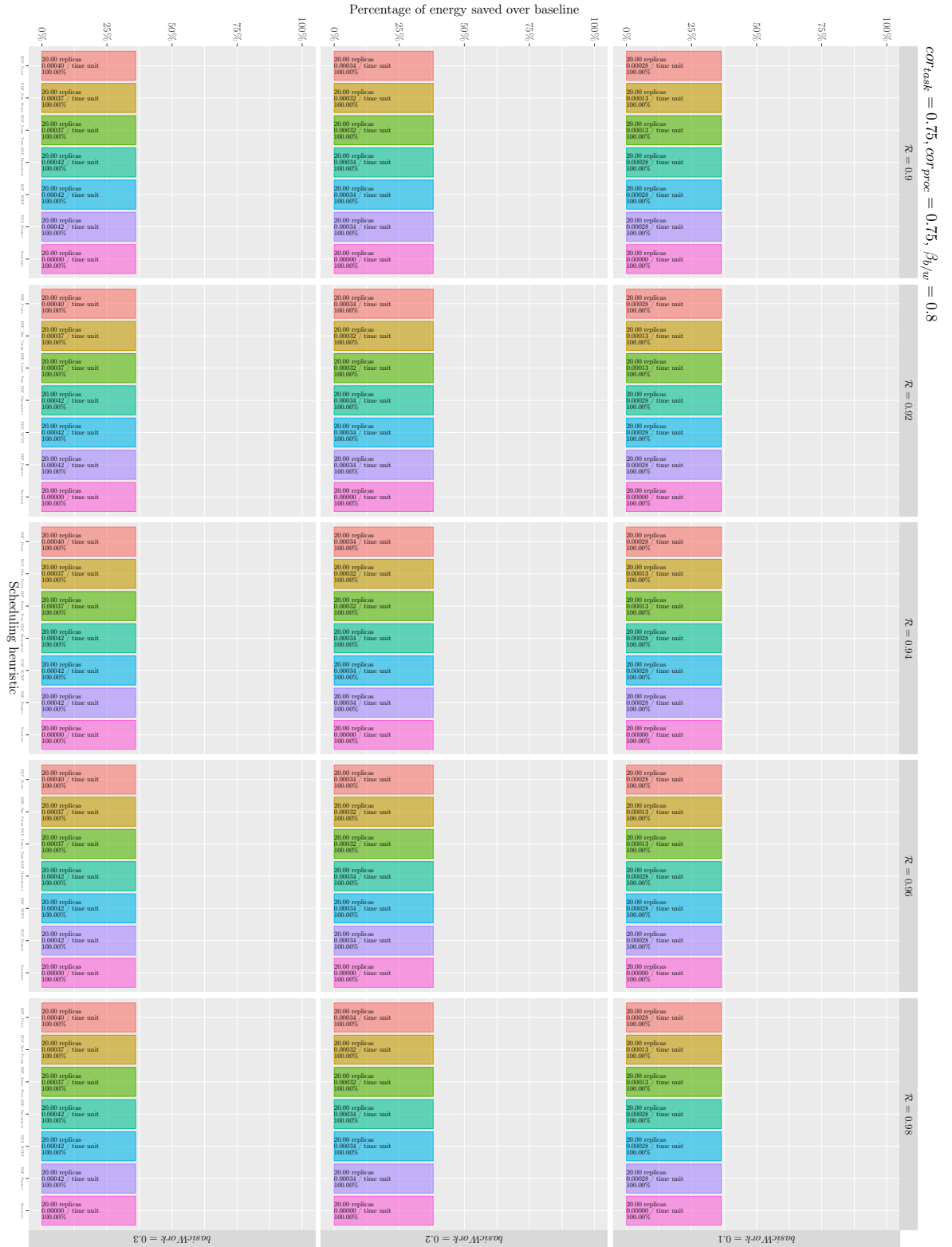


$COI_{task} = 0.75, COI_{proc} = 0.75, \beta_{i,w} = 0.2$





Scheduling heuristic

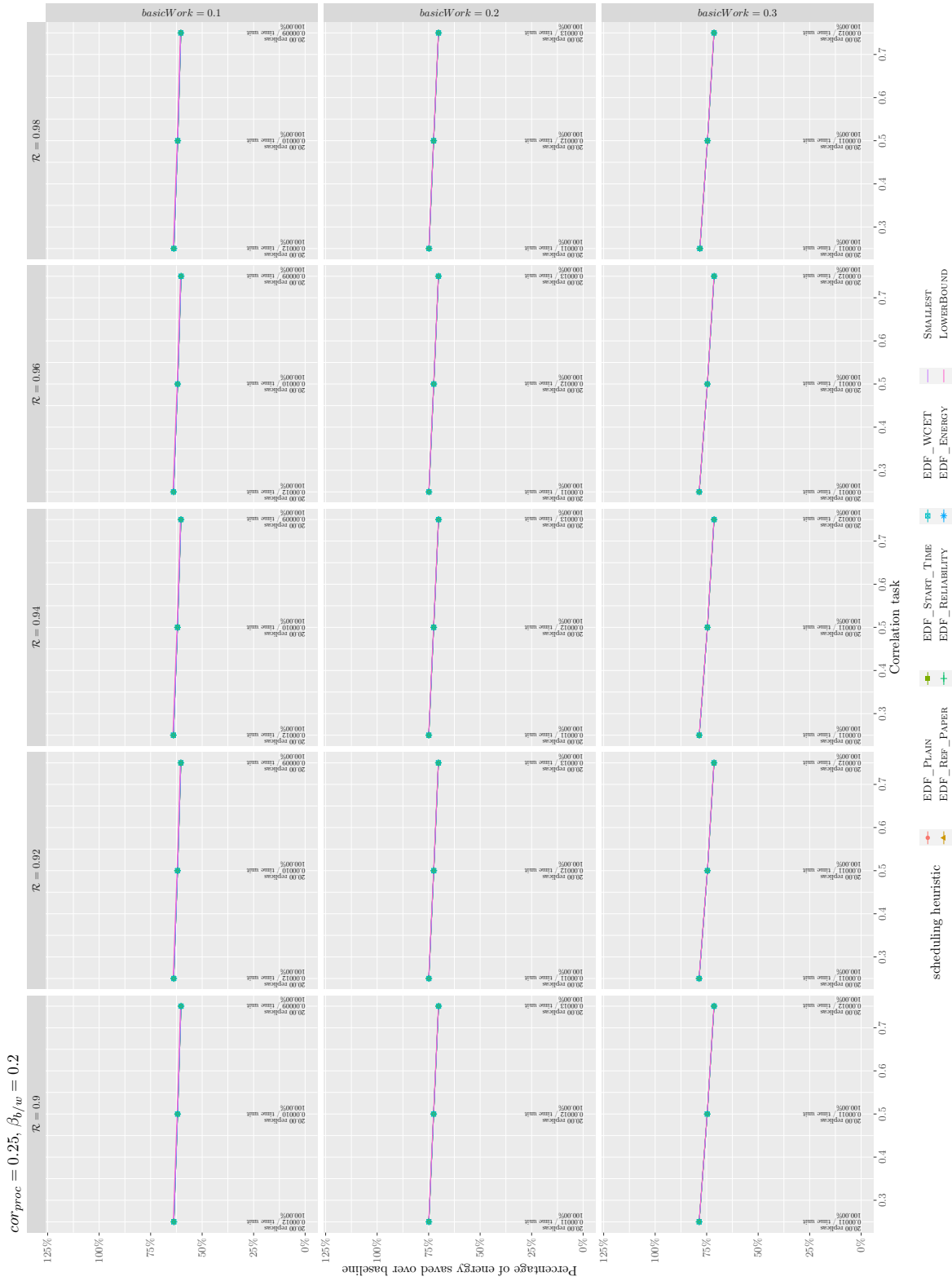




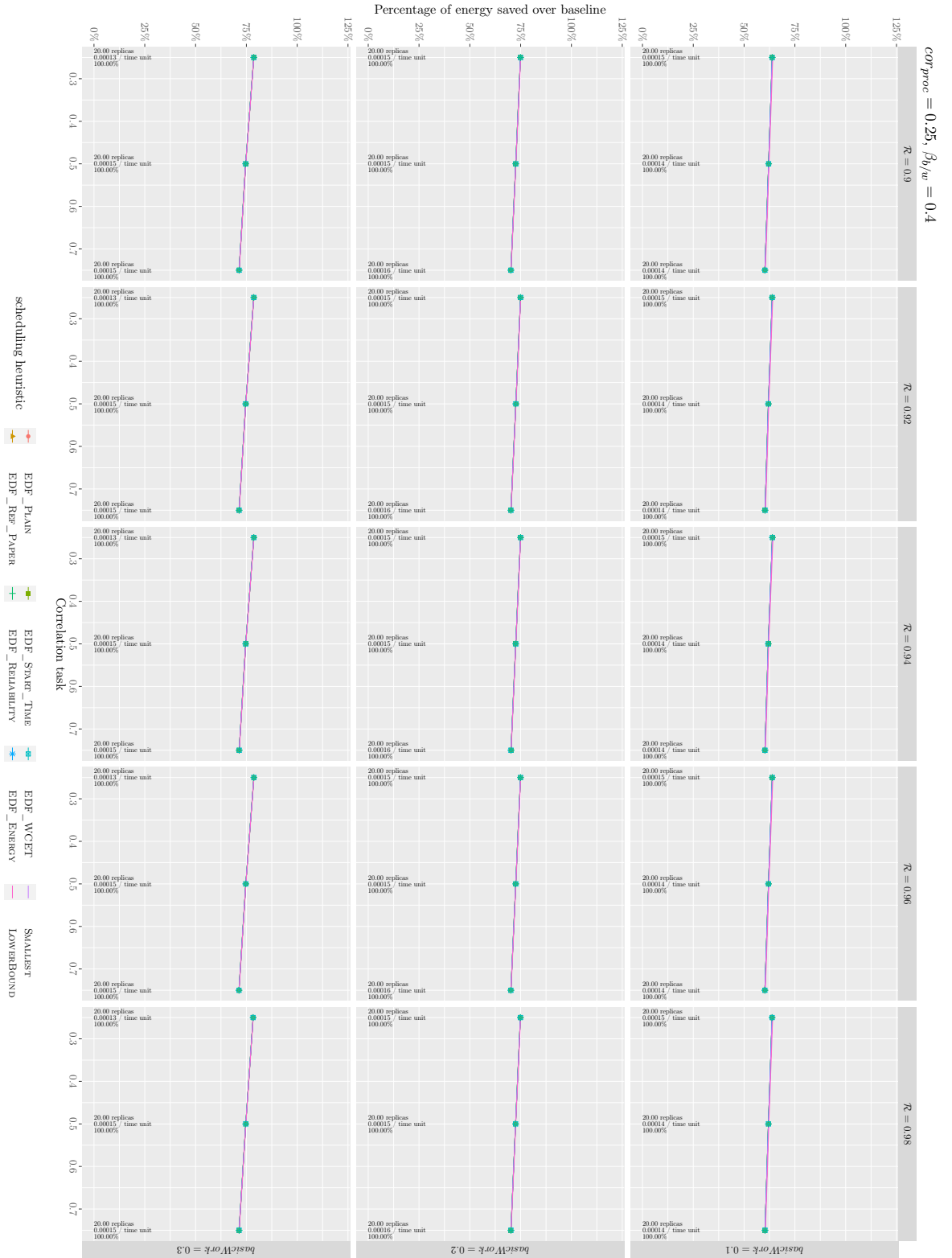


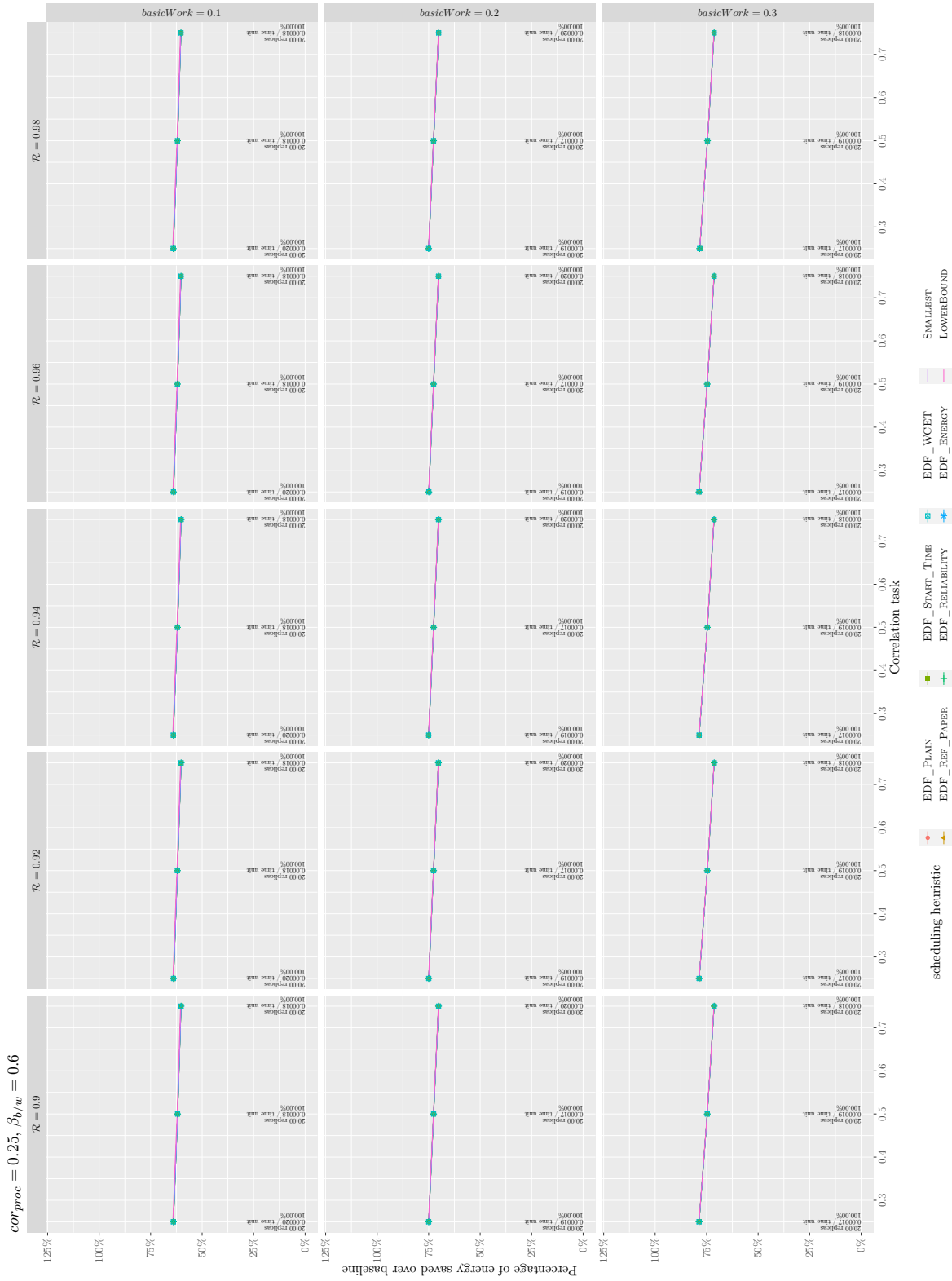
## **Task correlation**

The following section presents percentage of energy saved over baseline using different scheduling strategies under small failure rate, when varying  $cor_{task}$ .



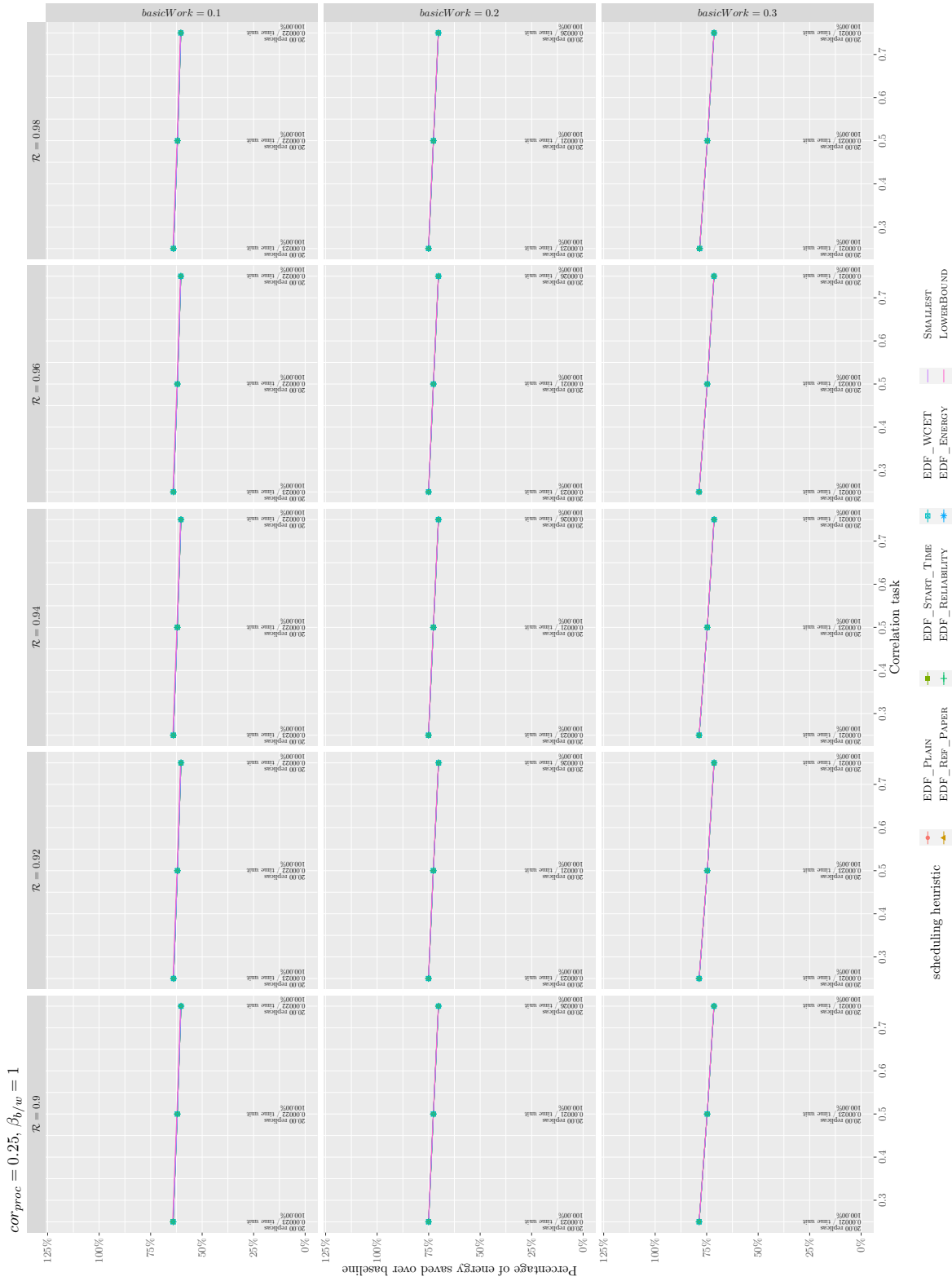
$col_{proc} = 0.25, \beta_{t/w} = 0.2$



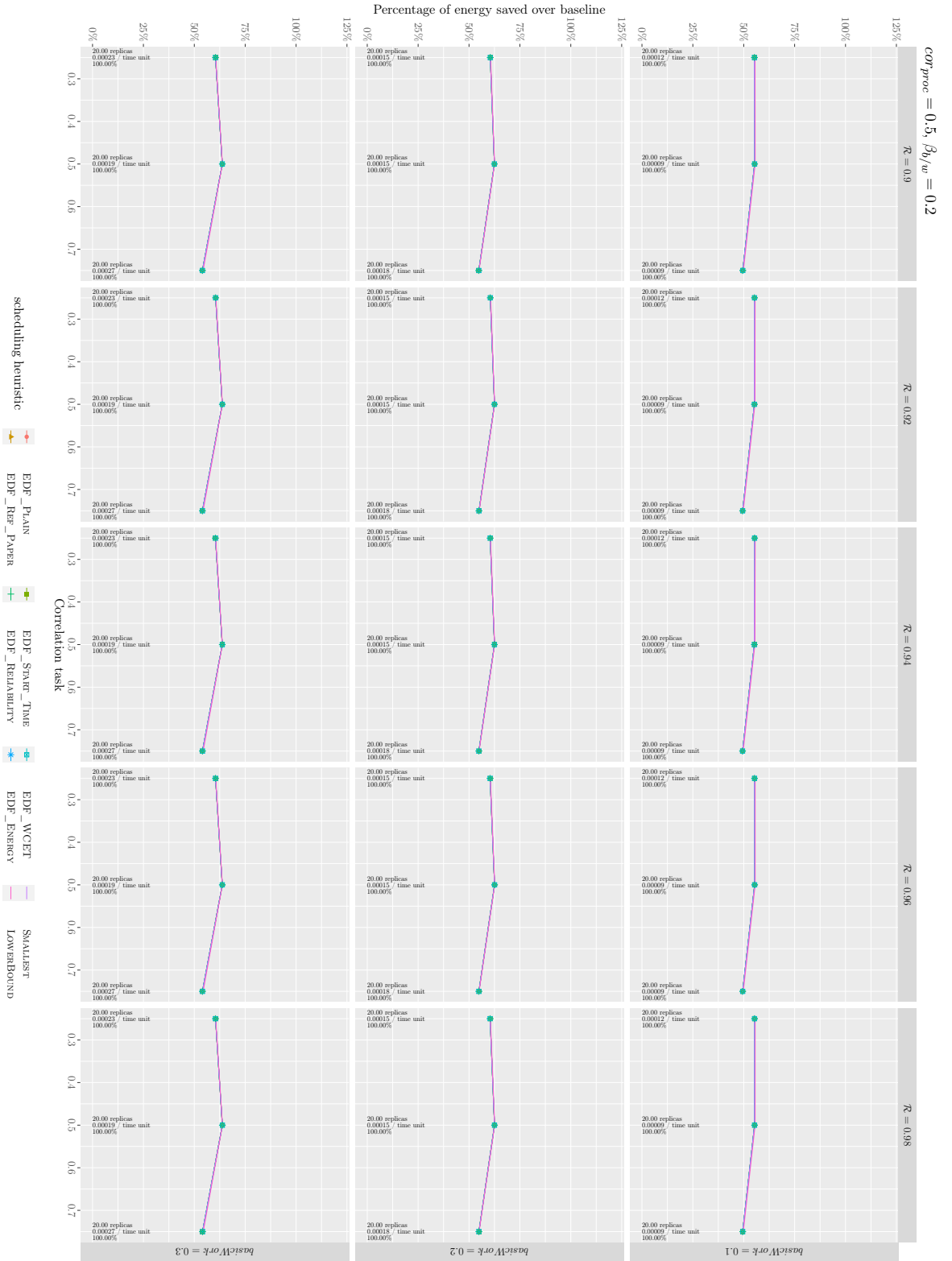


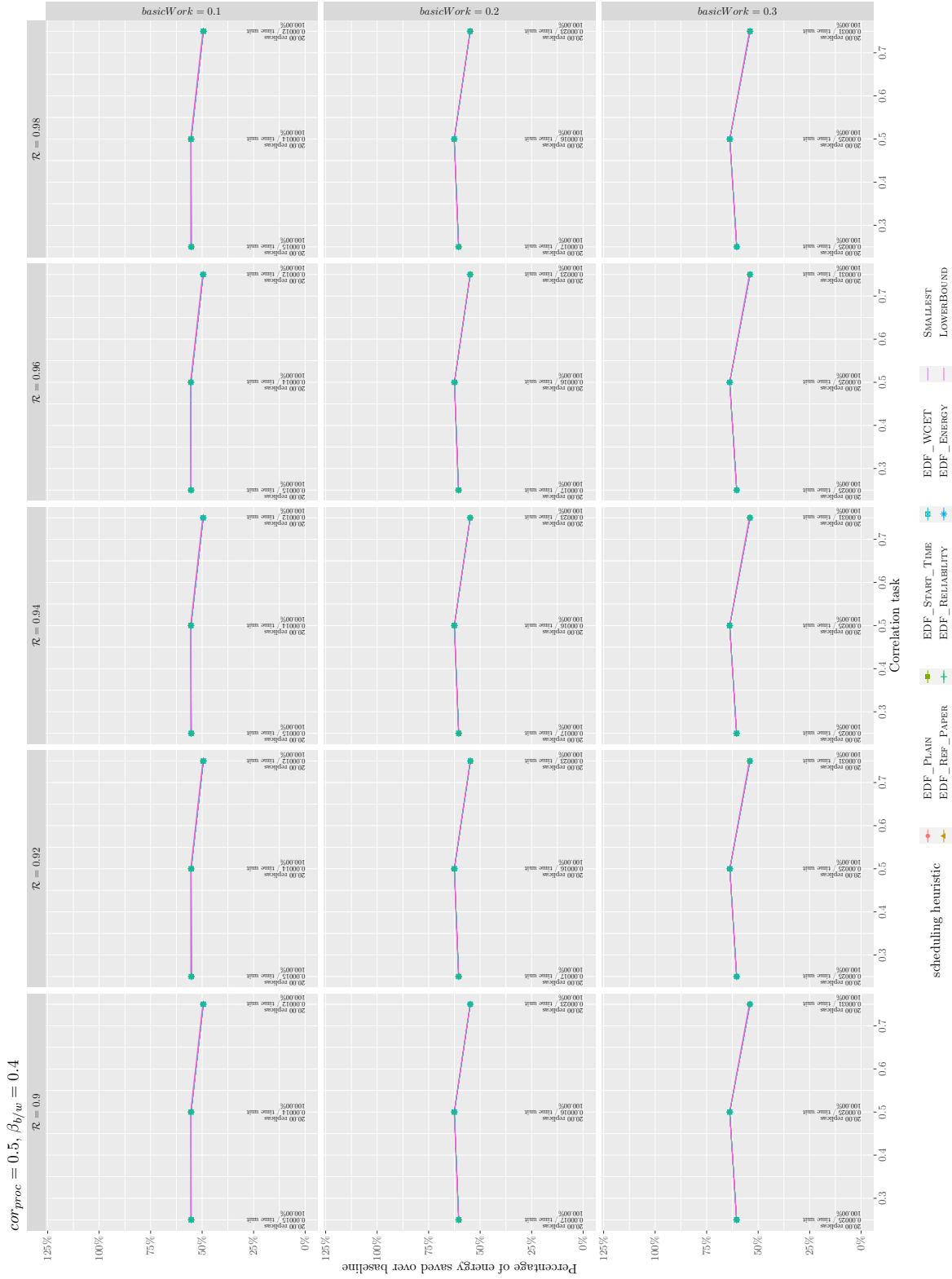
$col_{proc} = 0.25, \beta_{t/w} = 0.6$





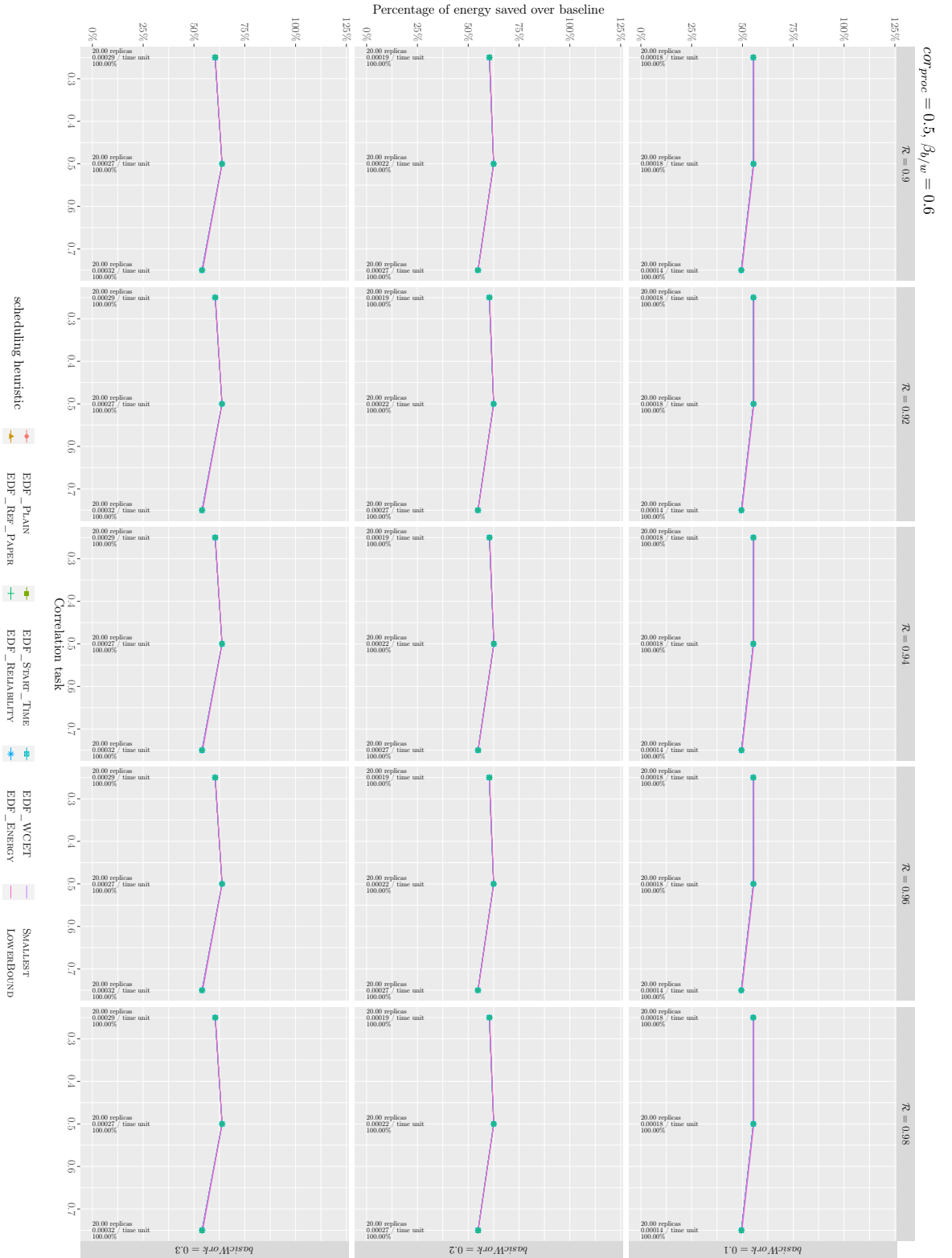
$col_{proc} = 0.25, \beta_{t/w} = 1$

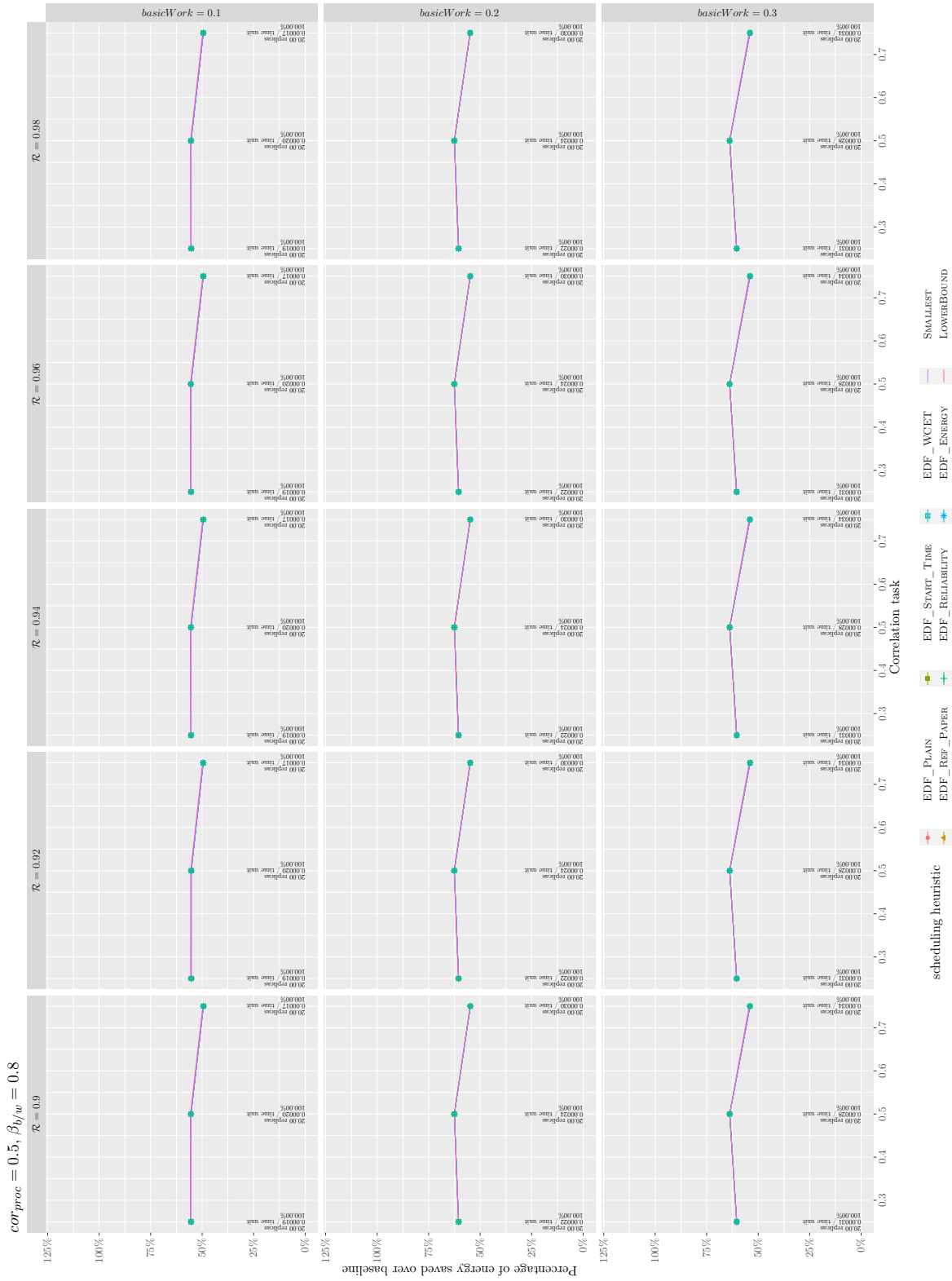




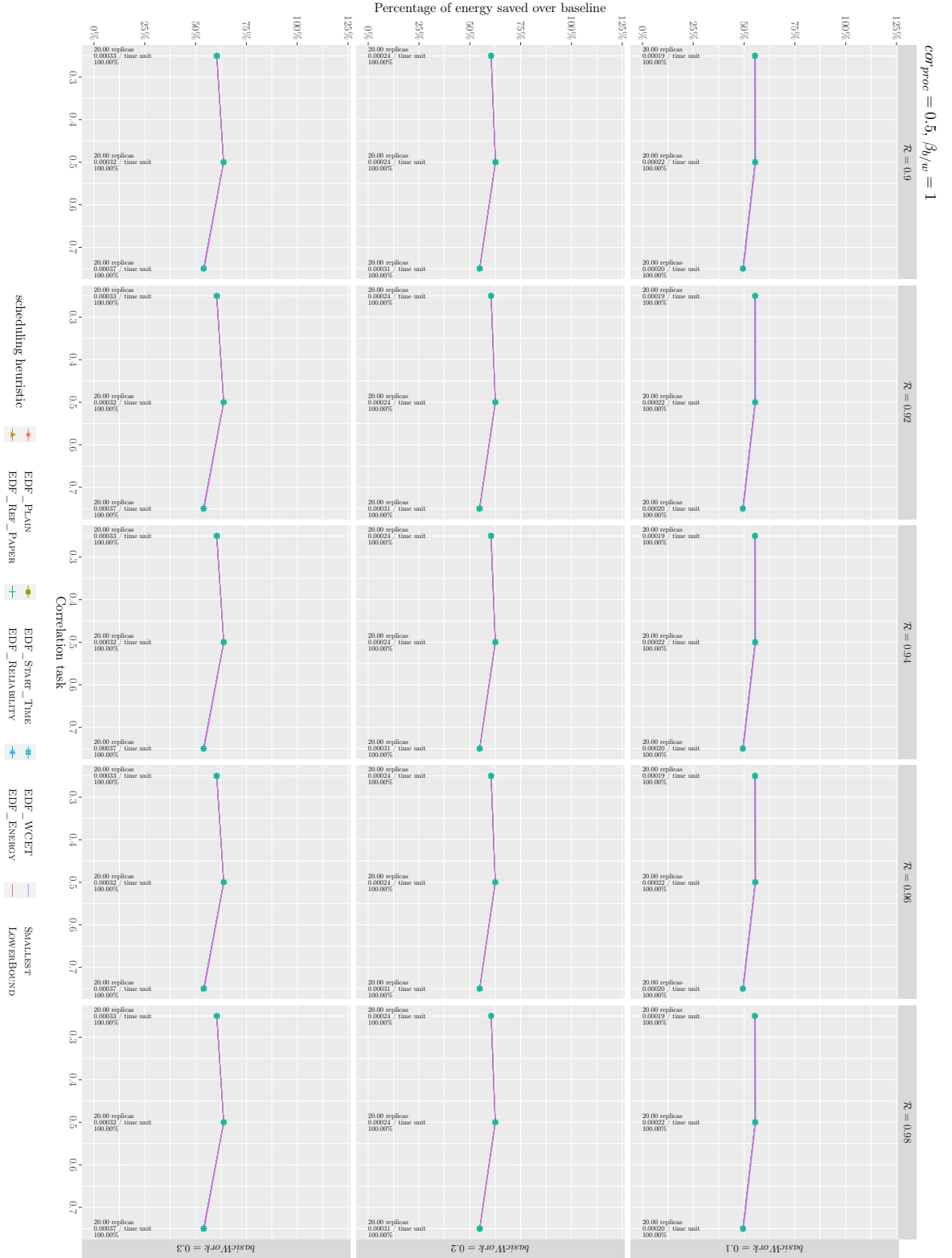
$col_{proc} = 0.5, \beta_{b/w} = 0.4$

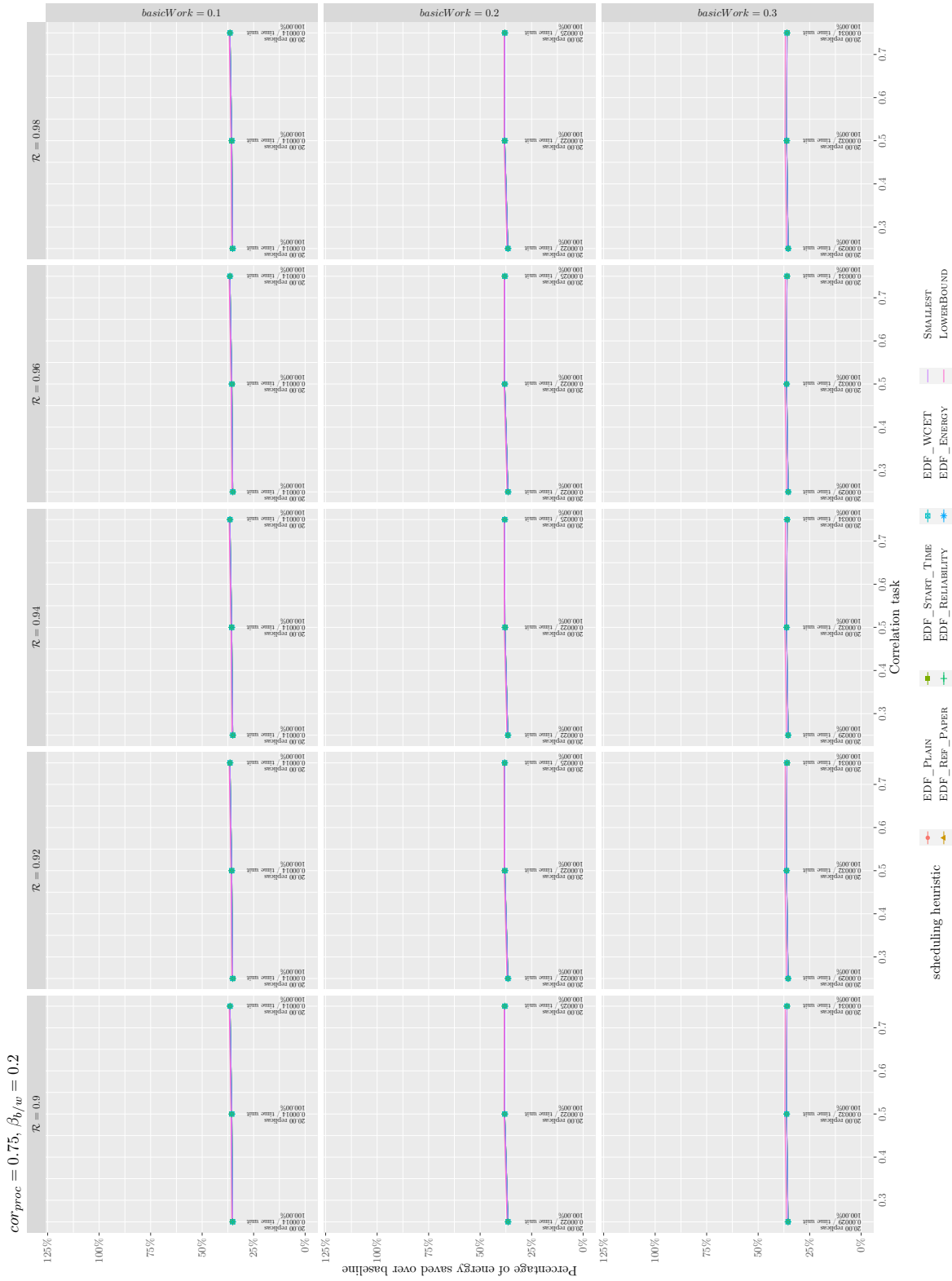


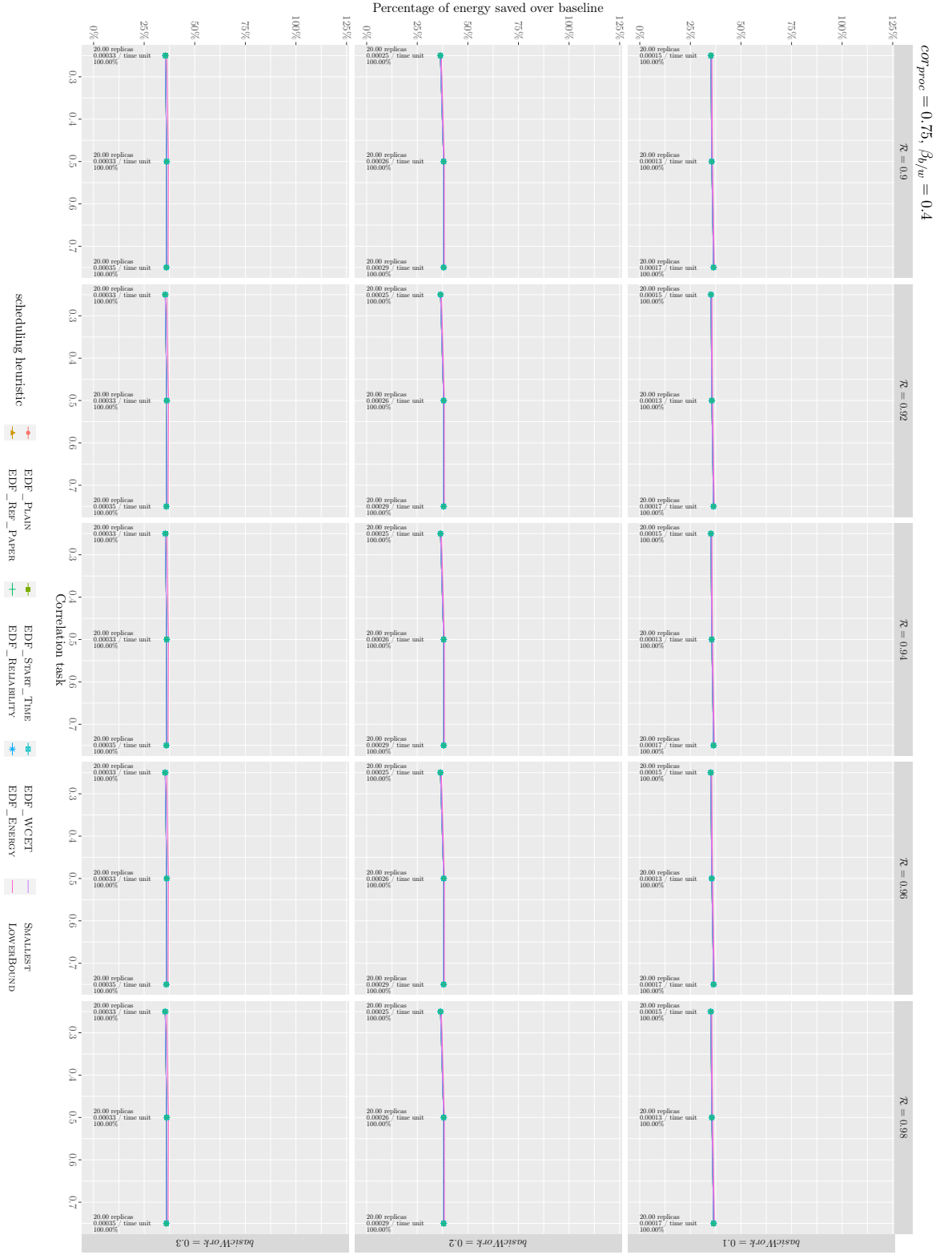


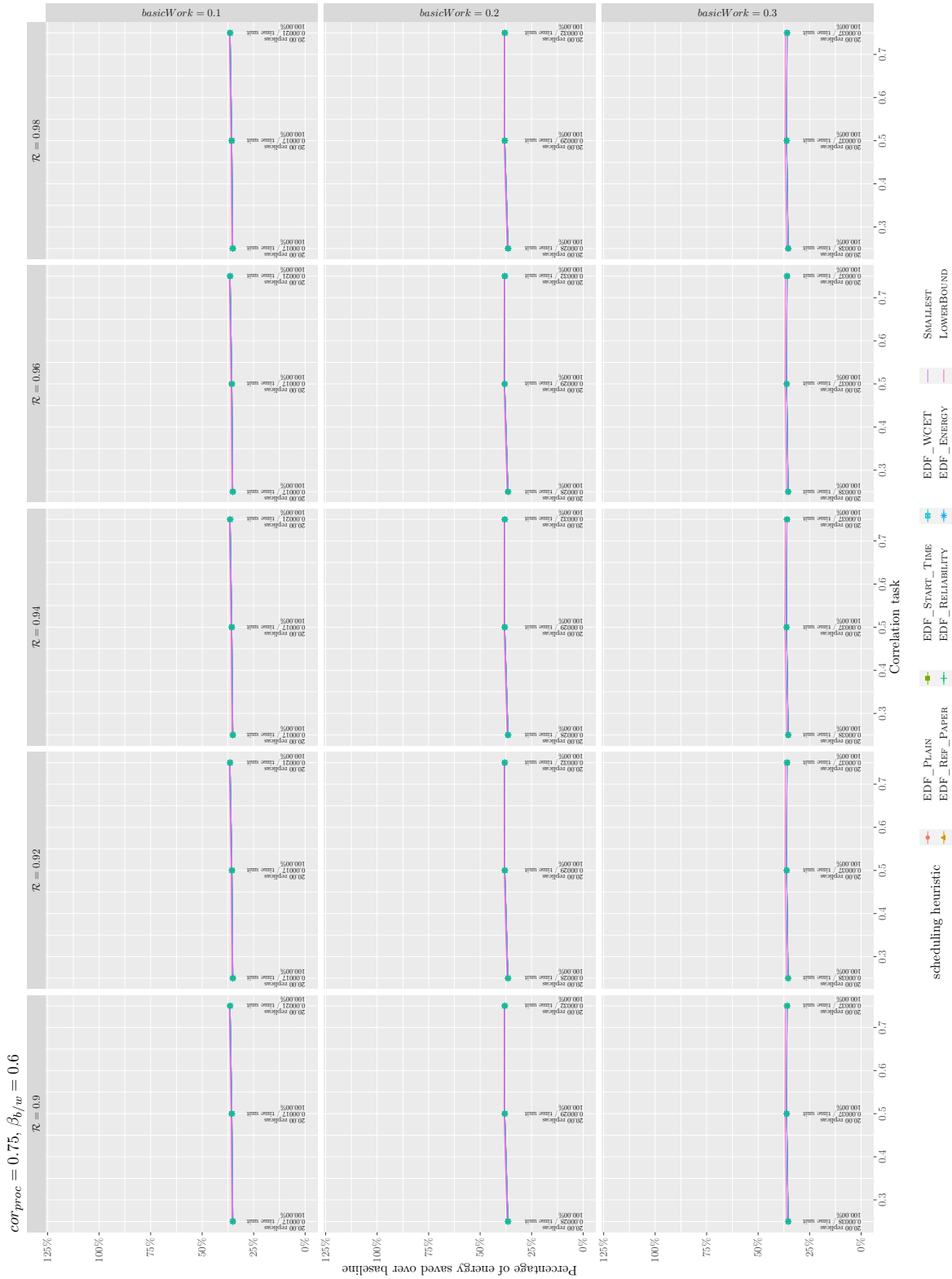


$col_{proc} = 0.5, \beta_{b/w} = 0.8$



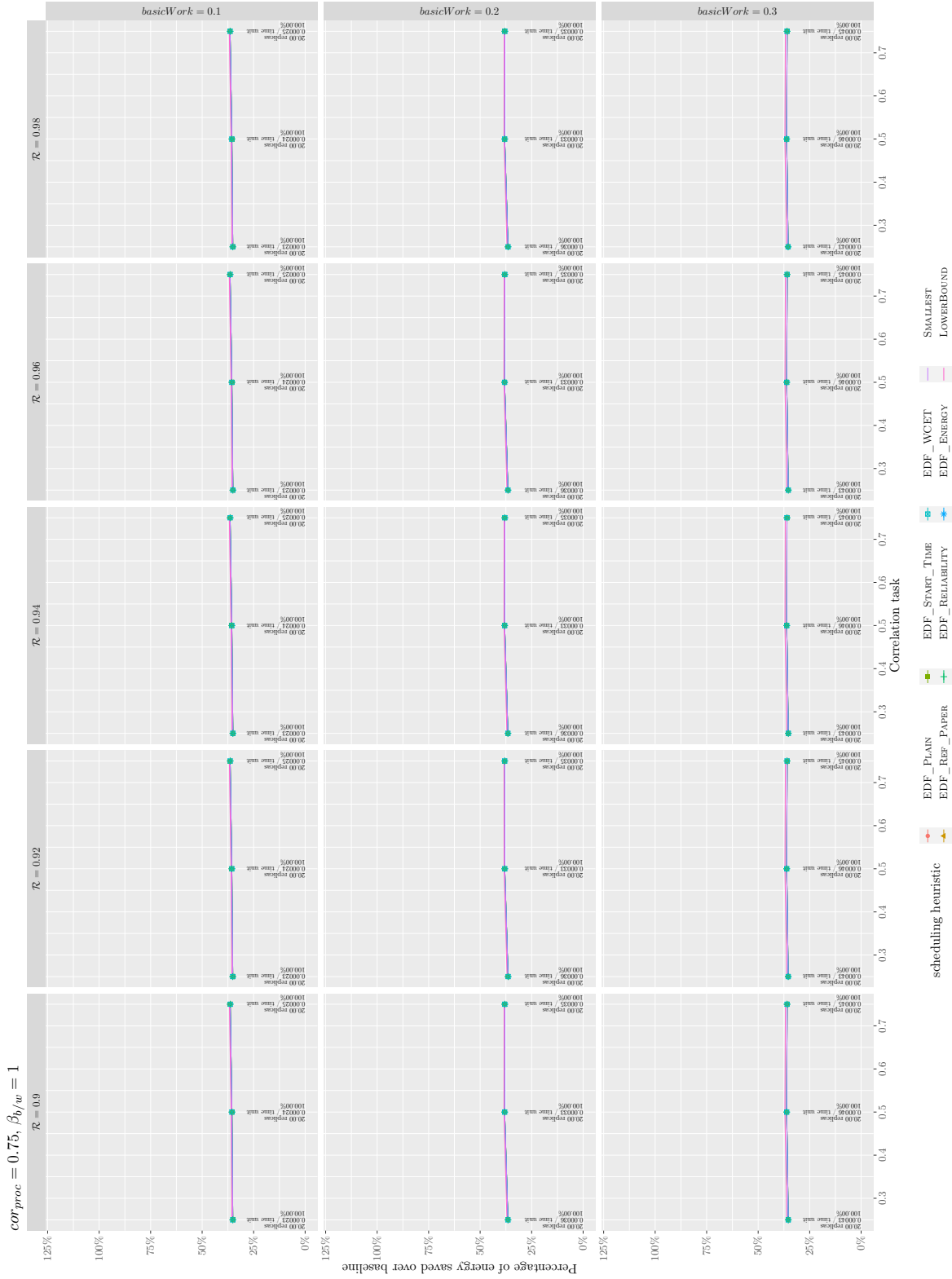






$col_{proc} = 0.75, \beta_{0/w} = 0.6$



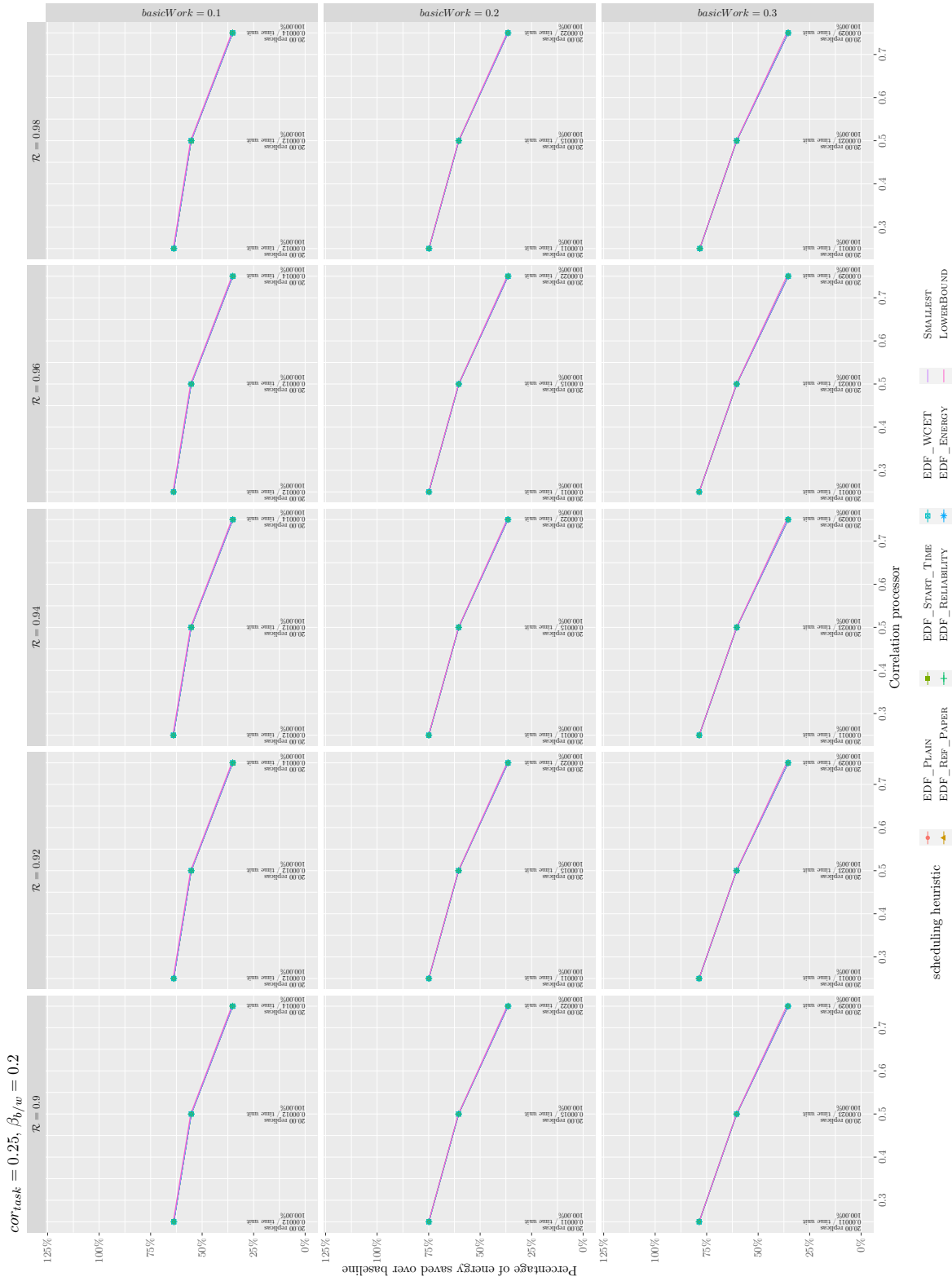


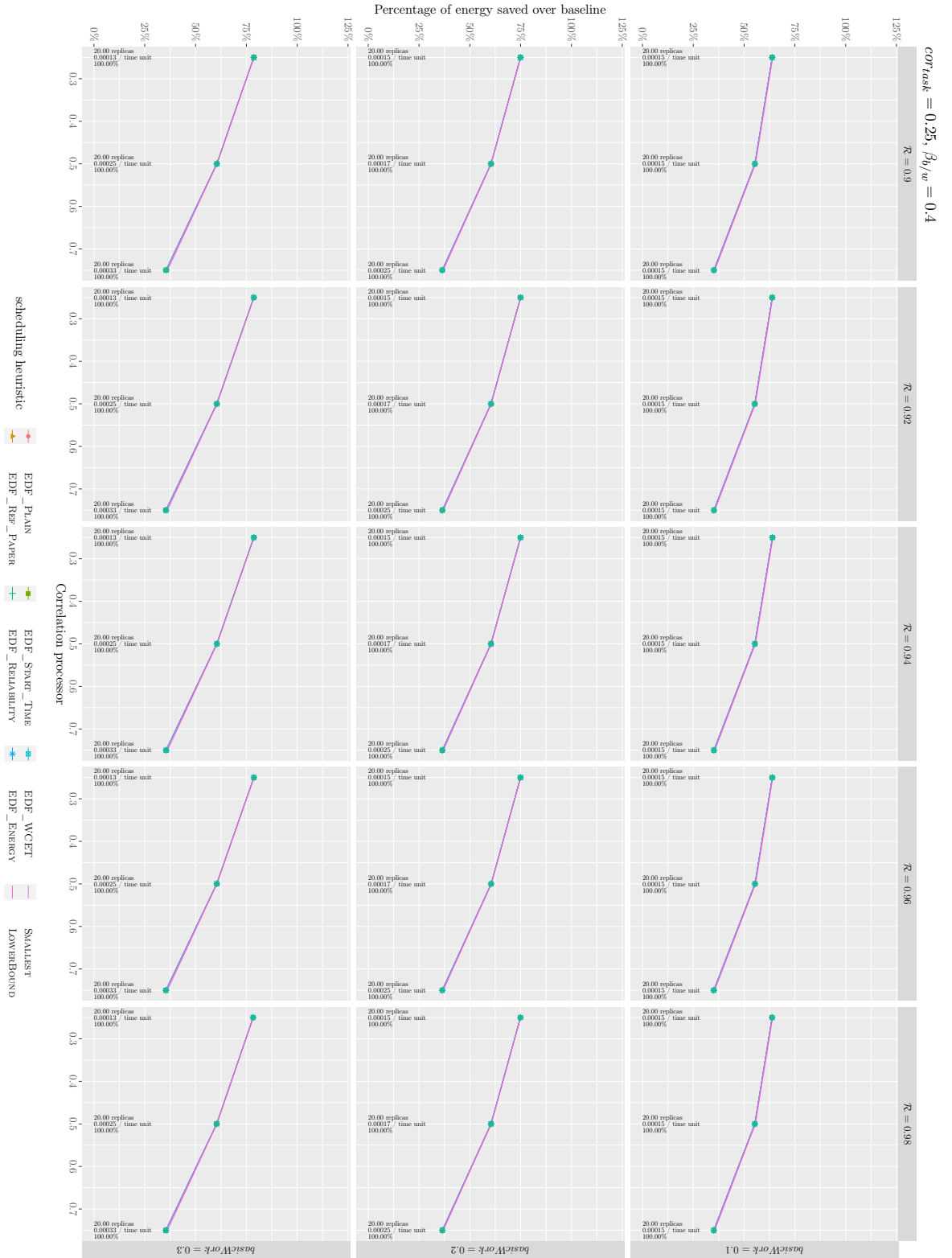
$col_{proc} = 0.75, \beta_0/w = 1$

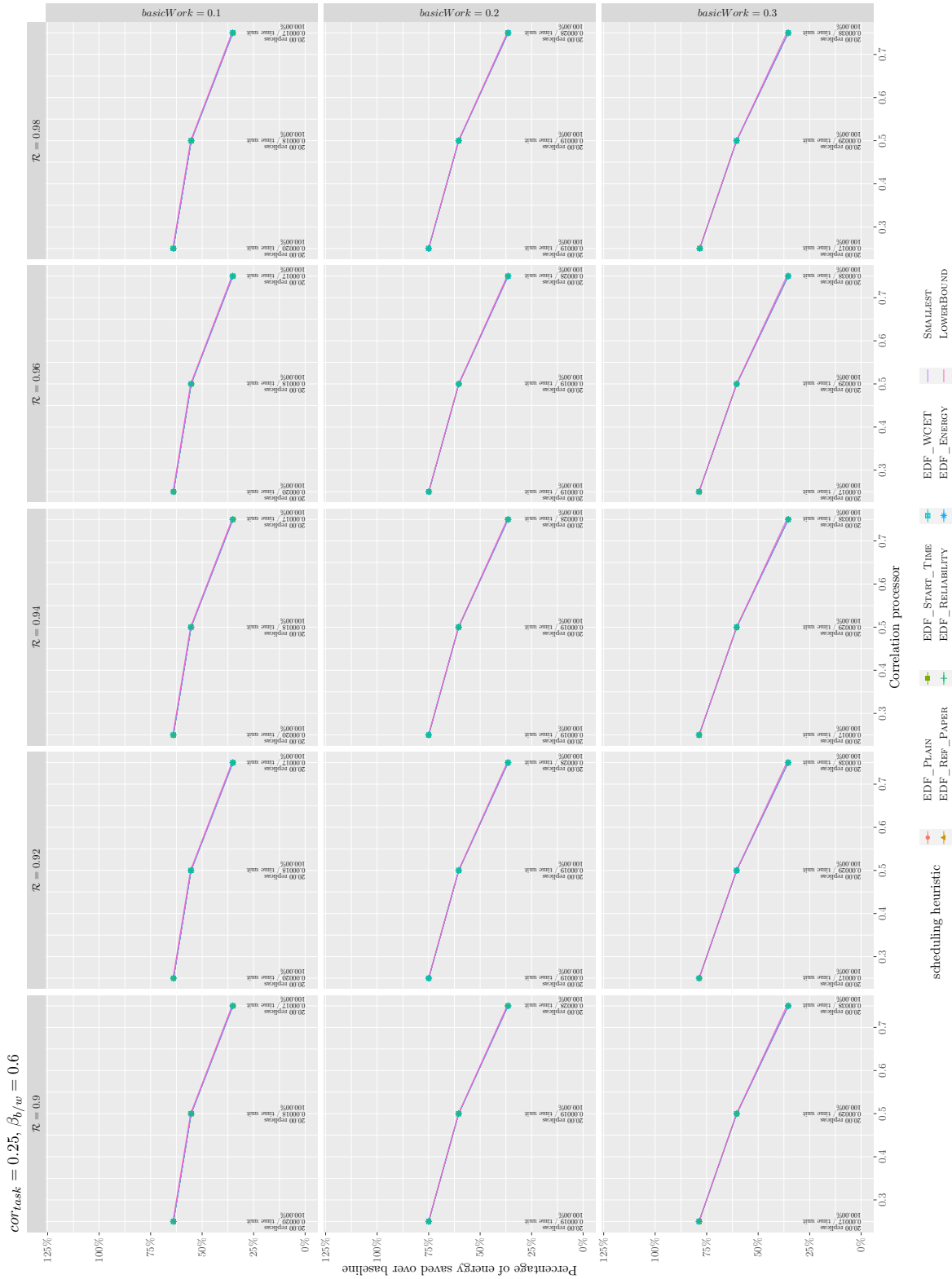


## Processor correlation

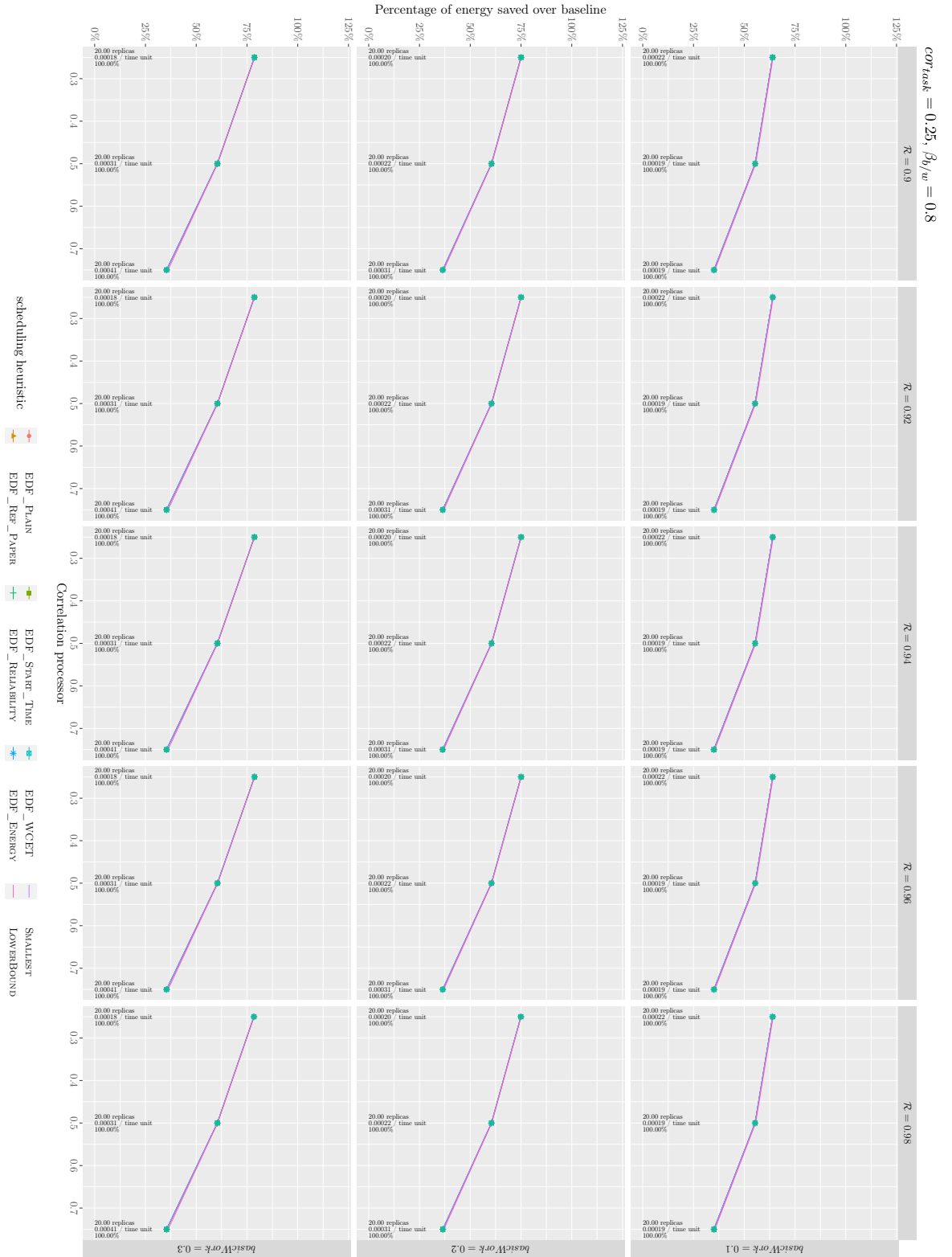
The following section presents percentage of energy saved over baseline using different scheduling strategies under small failure rate, when varying  $cor_{proc}$ .

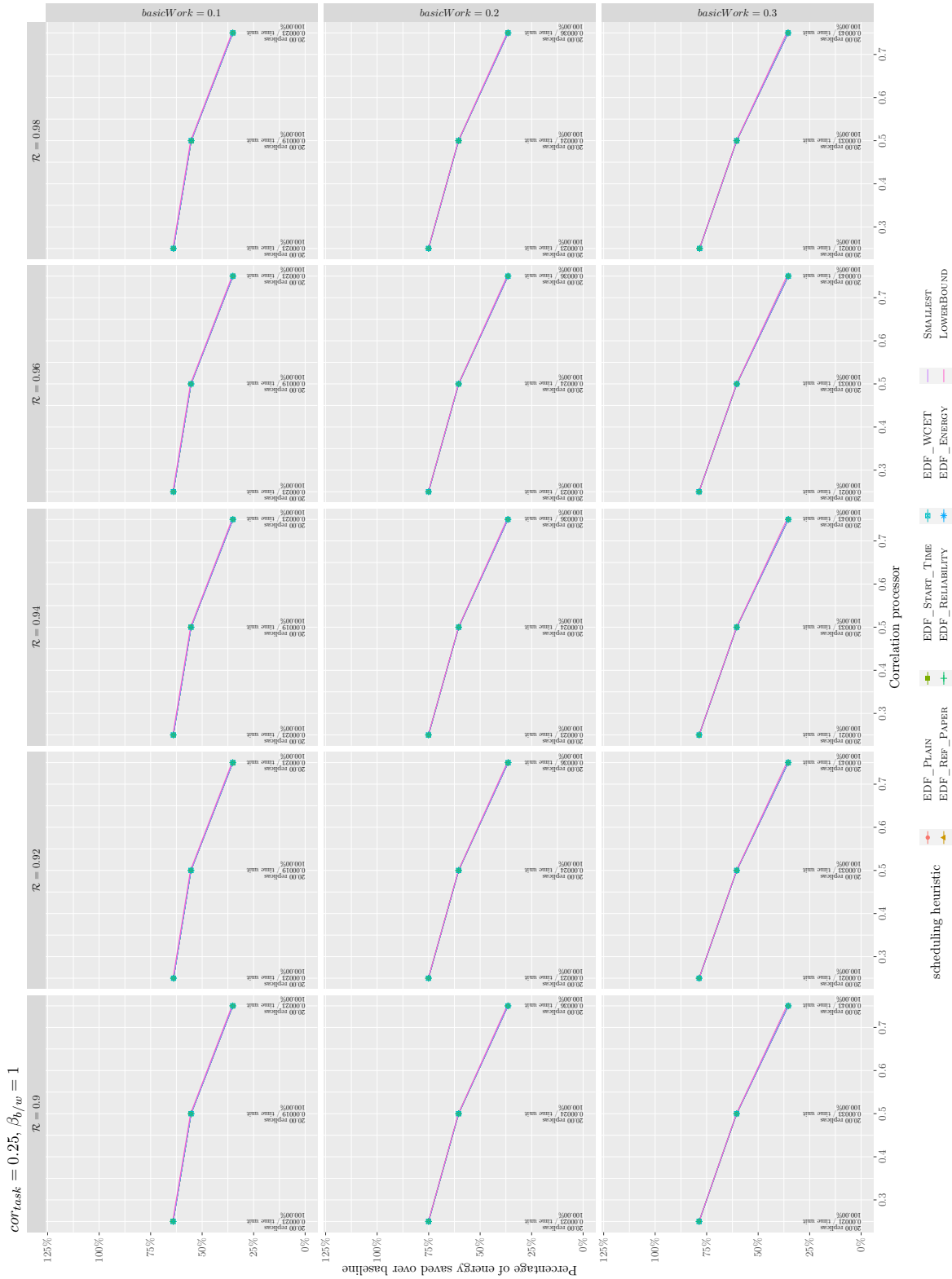


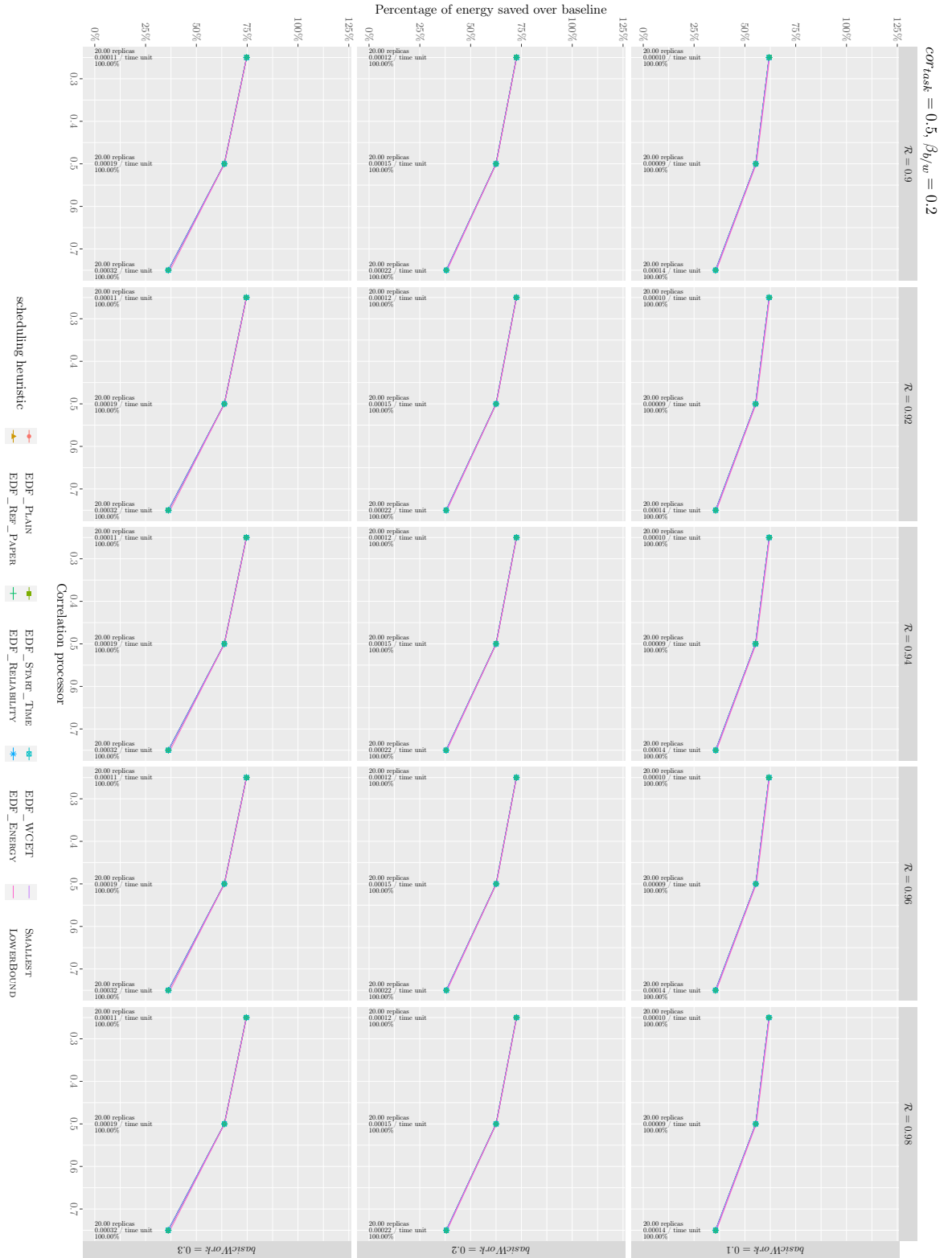


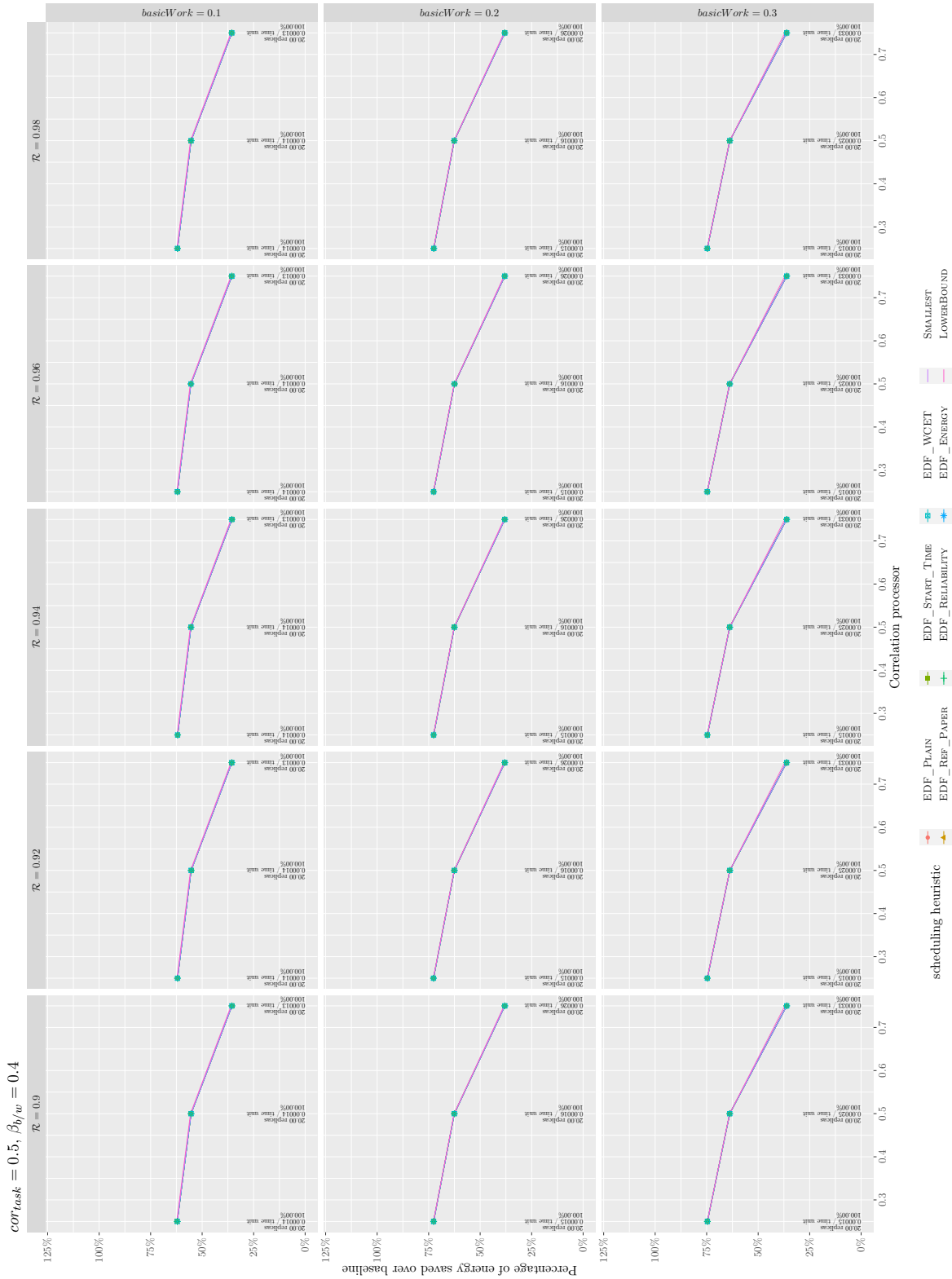


$cop_{task} = 0.25, \beta_{t/w} = 0.6$



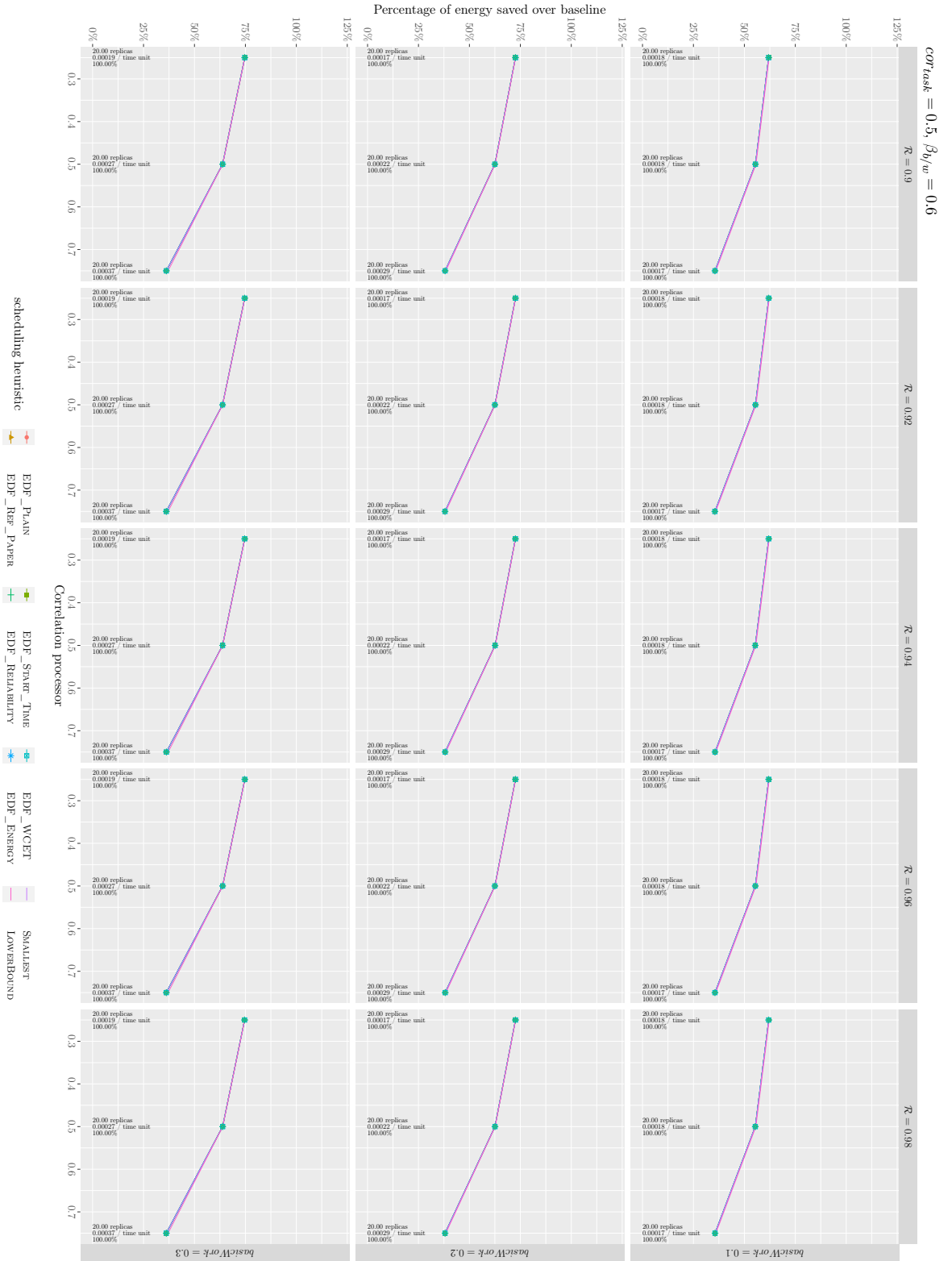


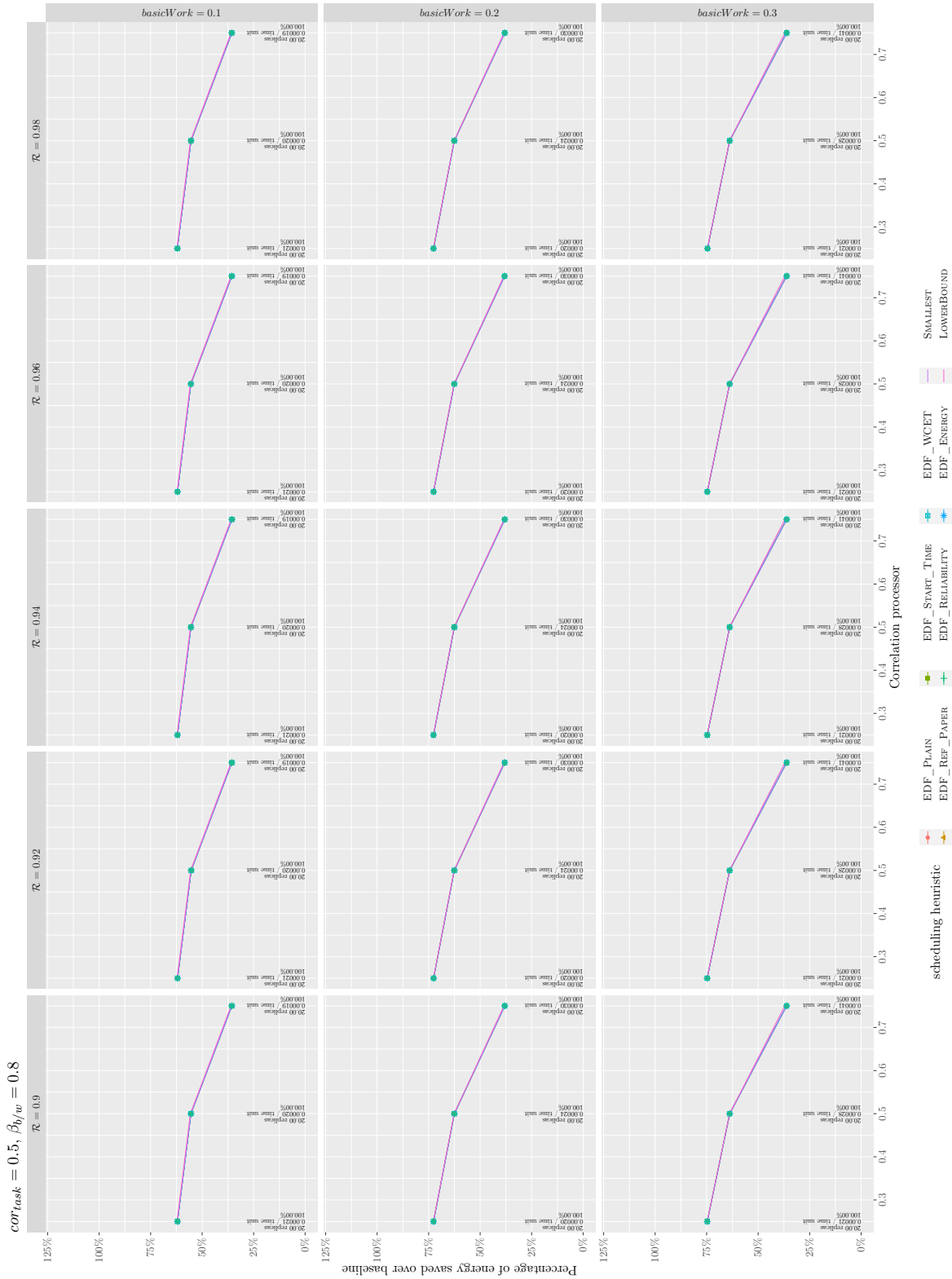




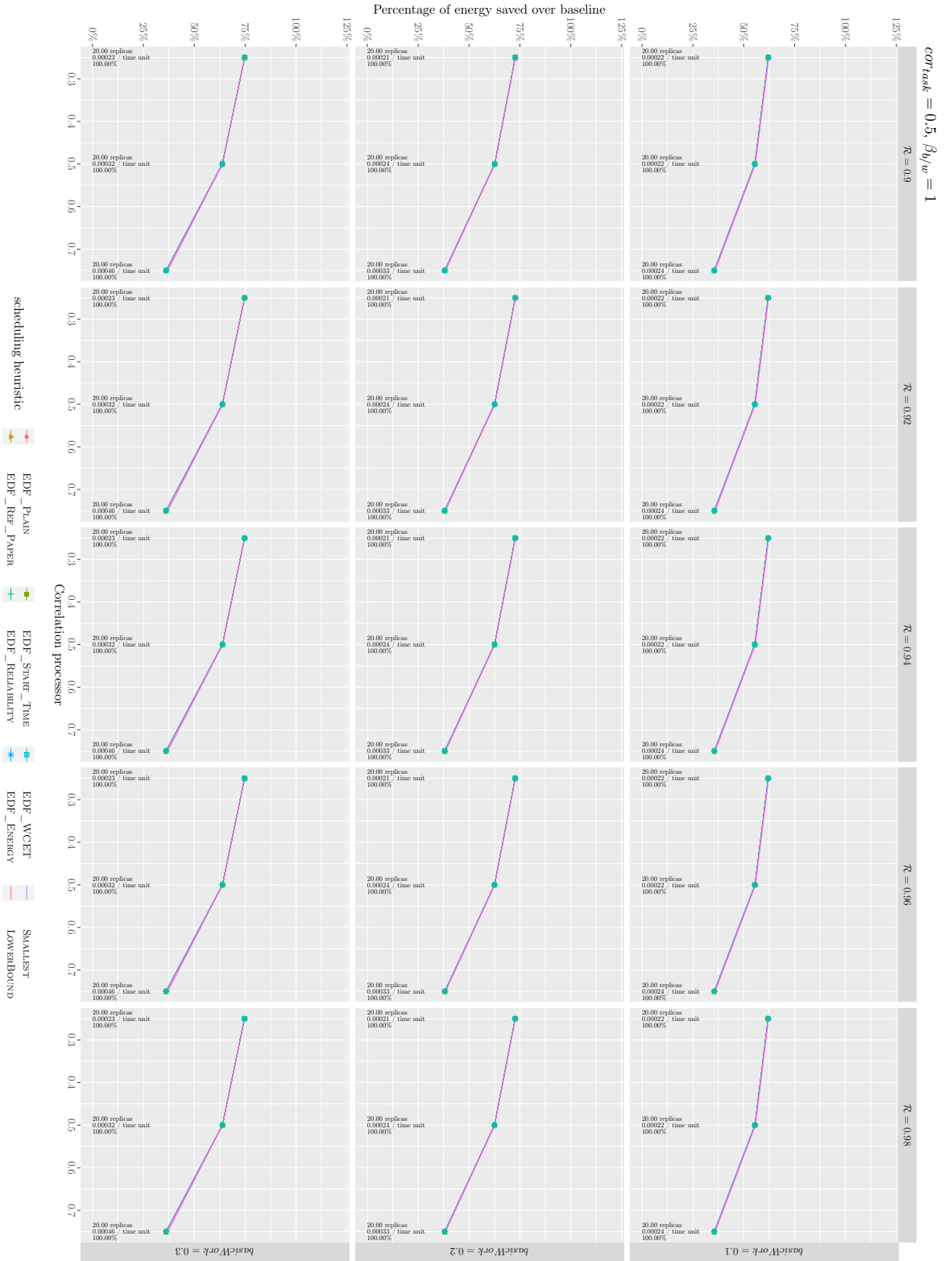
$cop_{task} = 0.5, \beta_{hw} = 0.4$

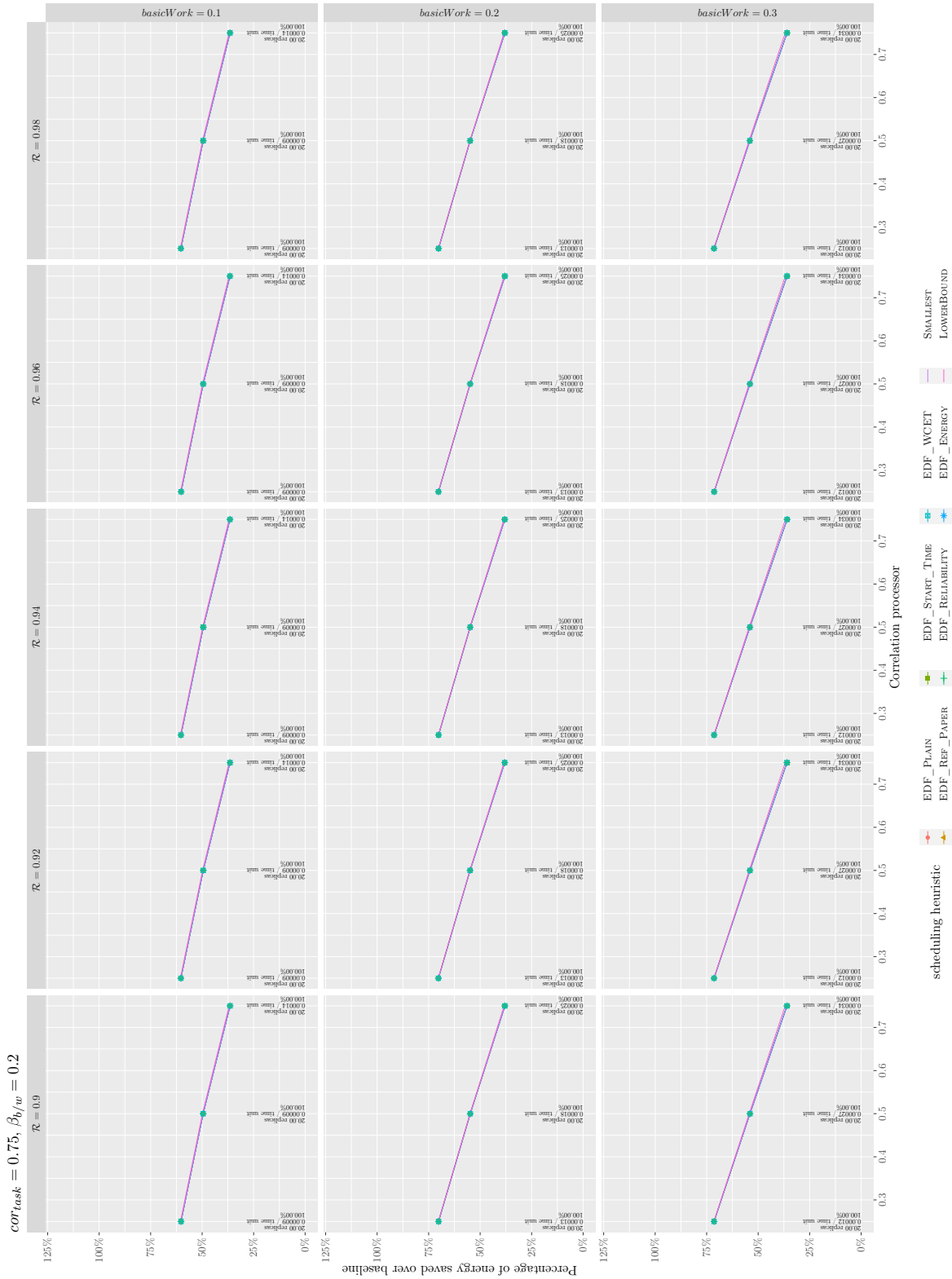




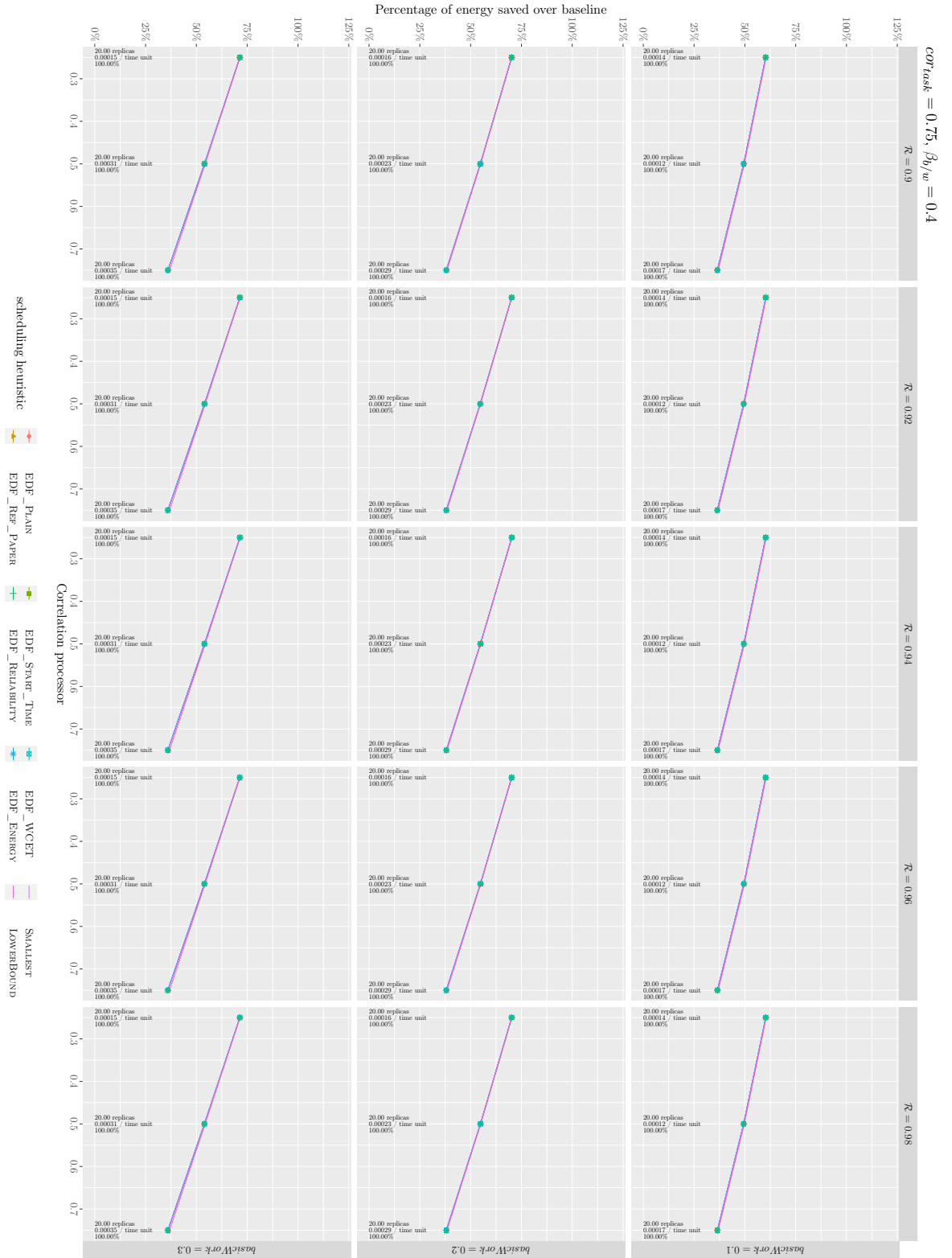


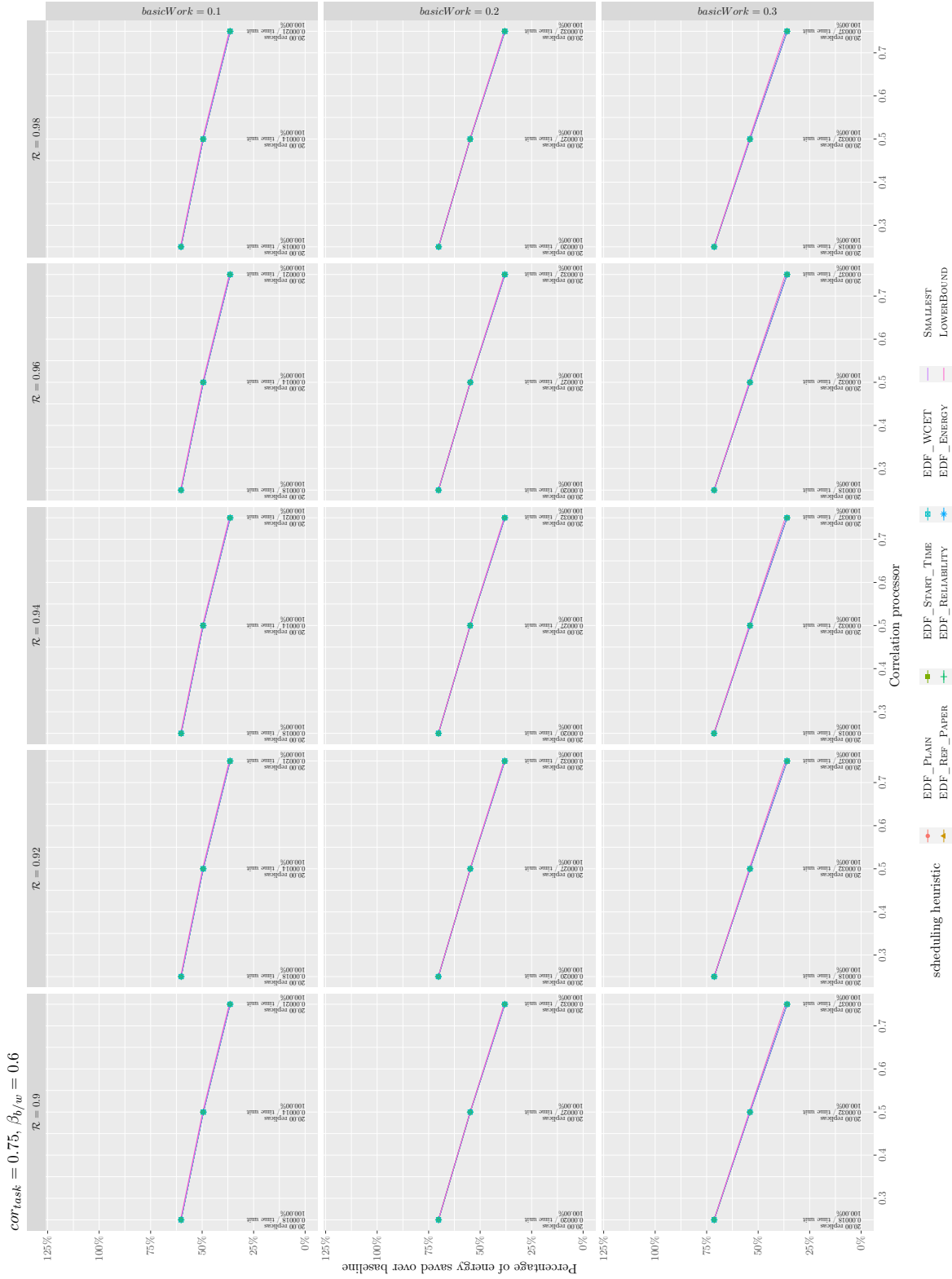
$cOP_{task} = 0.5, \beta_{h/w} = 0.8$





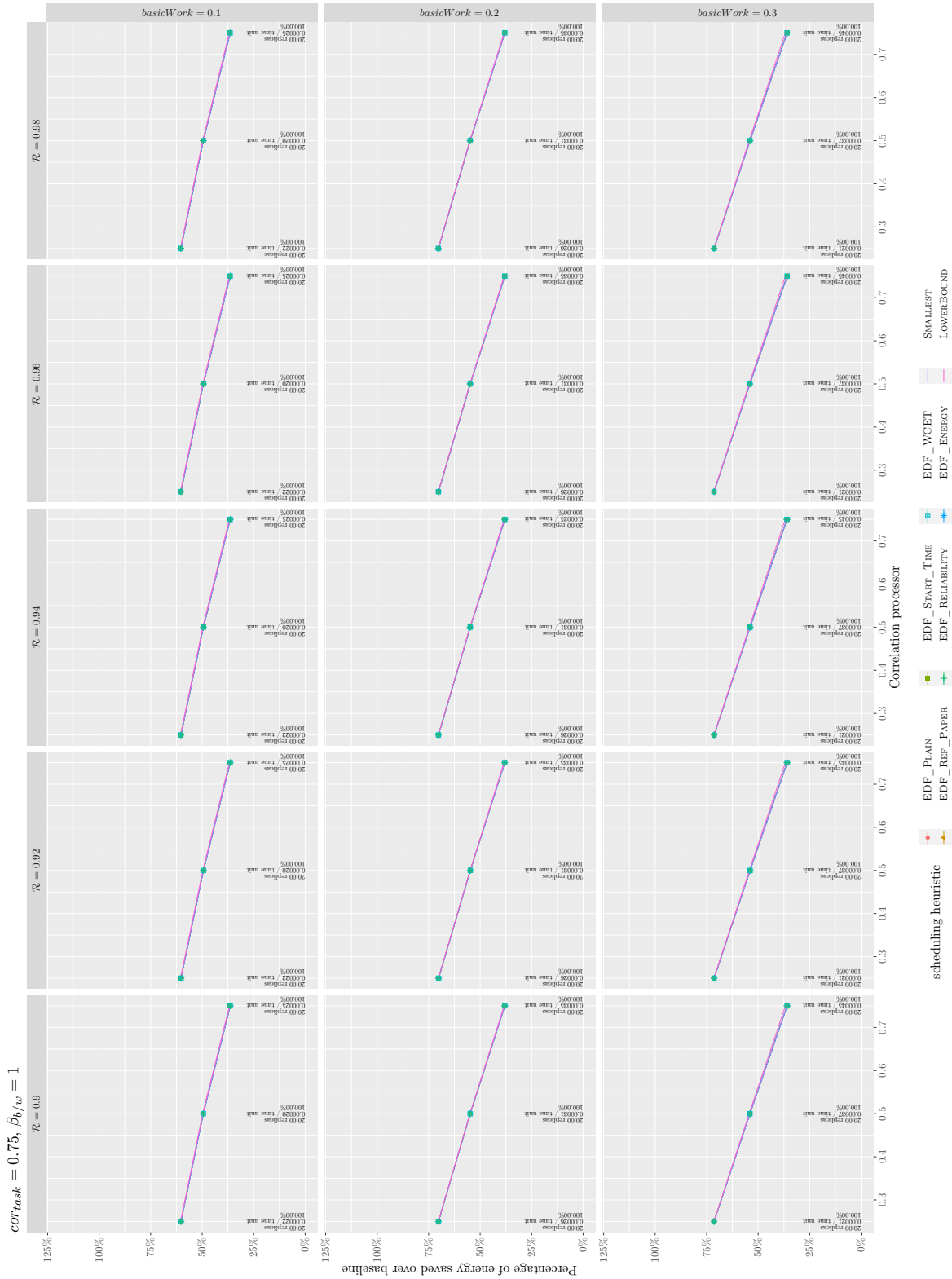
$cop_{task} = 0.75, \beta_{h/w} = 0.2$





$COP_{task} = 0.75, \beta_{H/w} = 0.6$



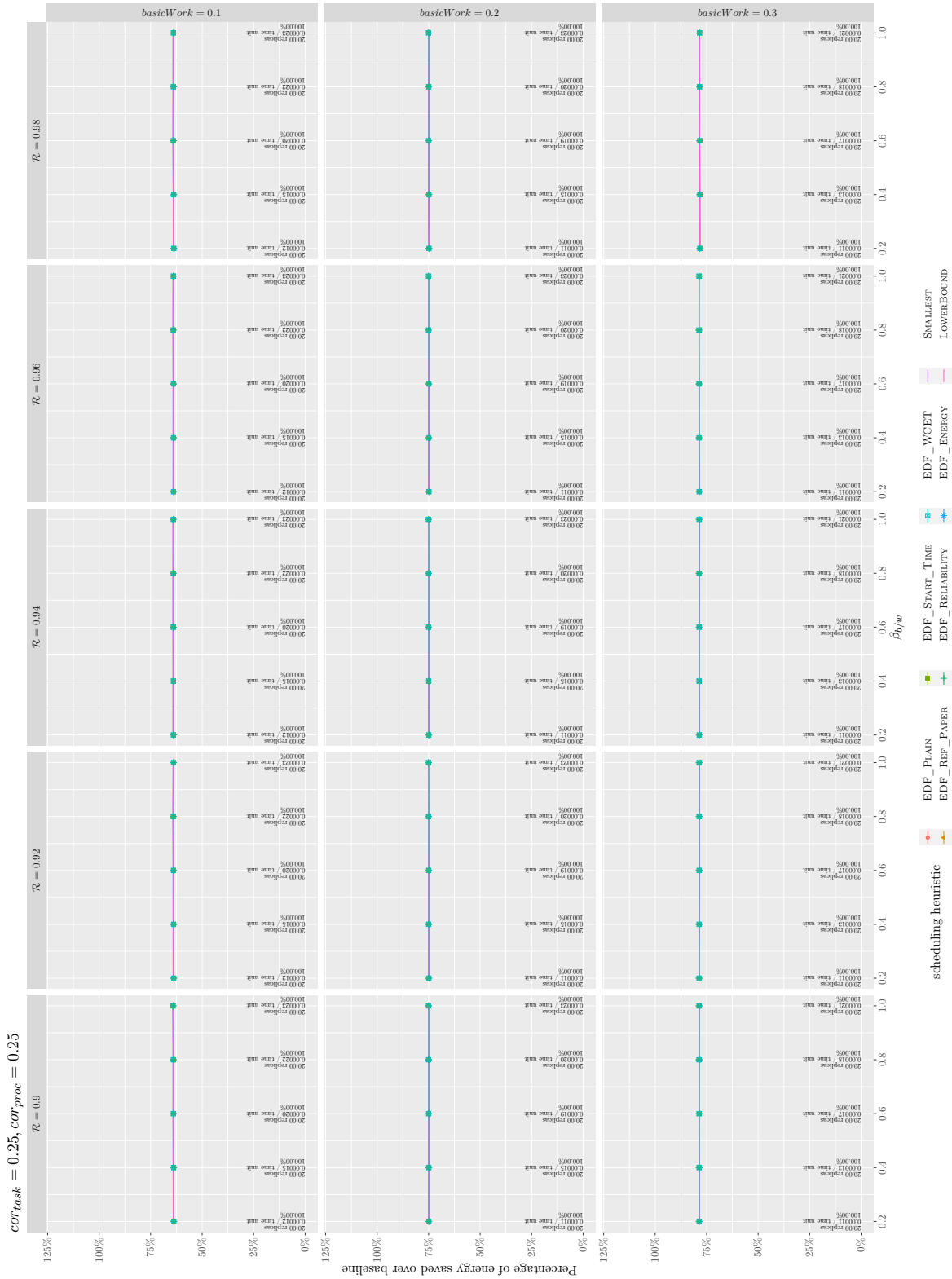


$COP_{task} = 0.75, \beta_H/w = 1$

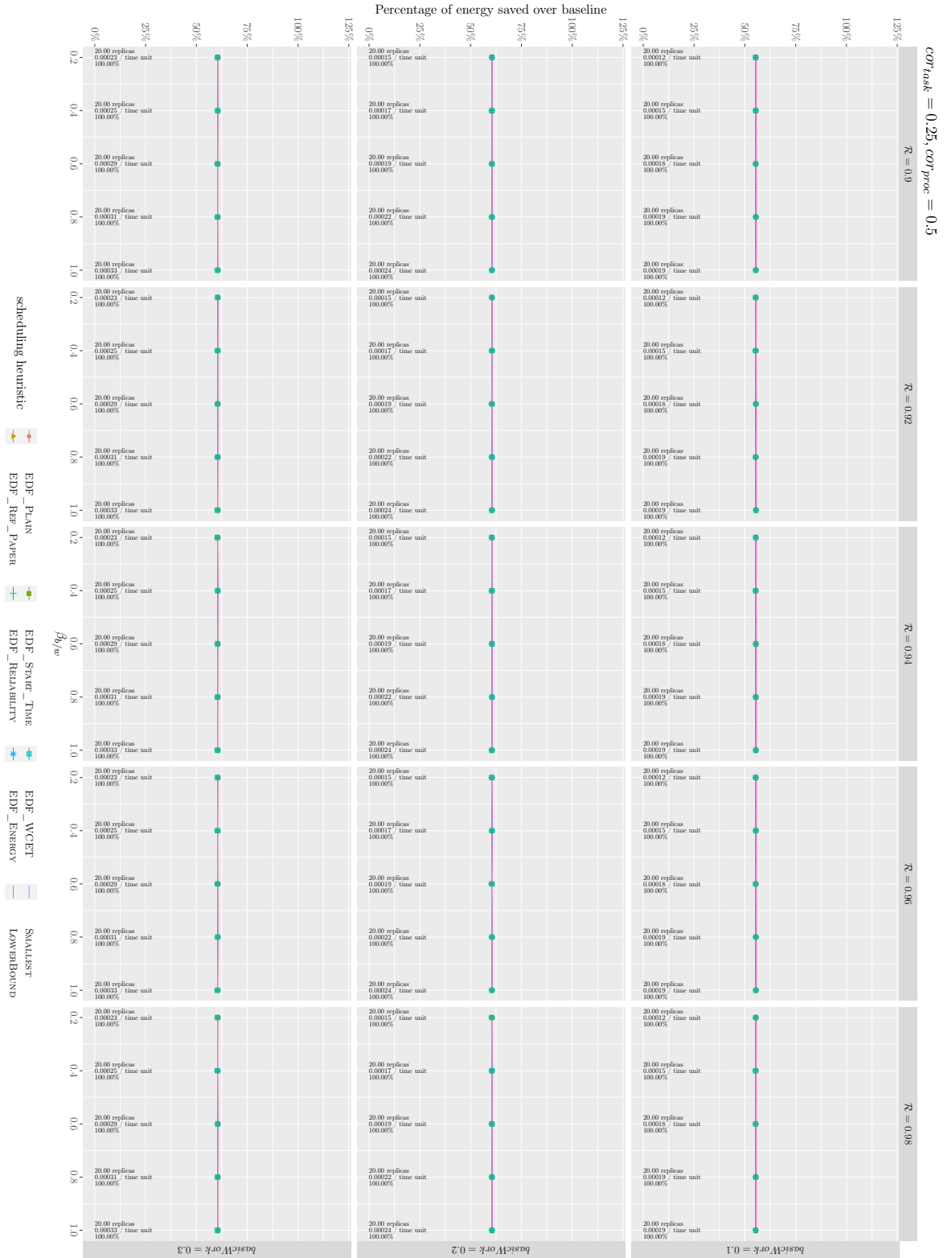


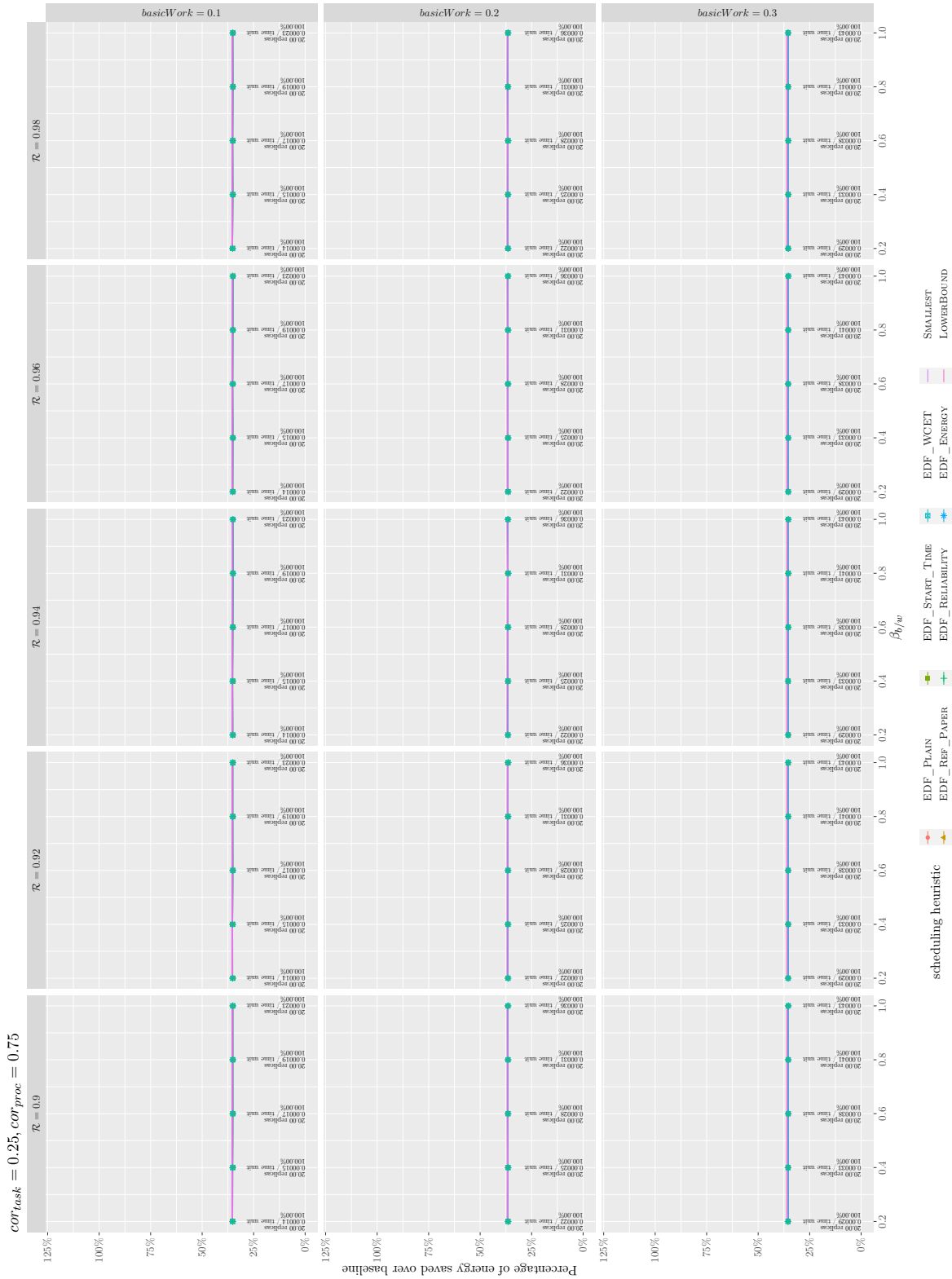
## **Task variability**

The following section presents percentage of energy saved over baseline using different scheduling strategies under small failure rate, when varying  $\beta_{b/w}$ .

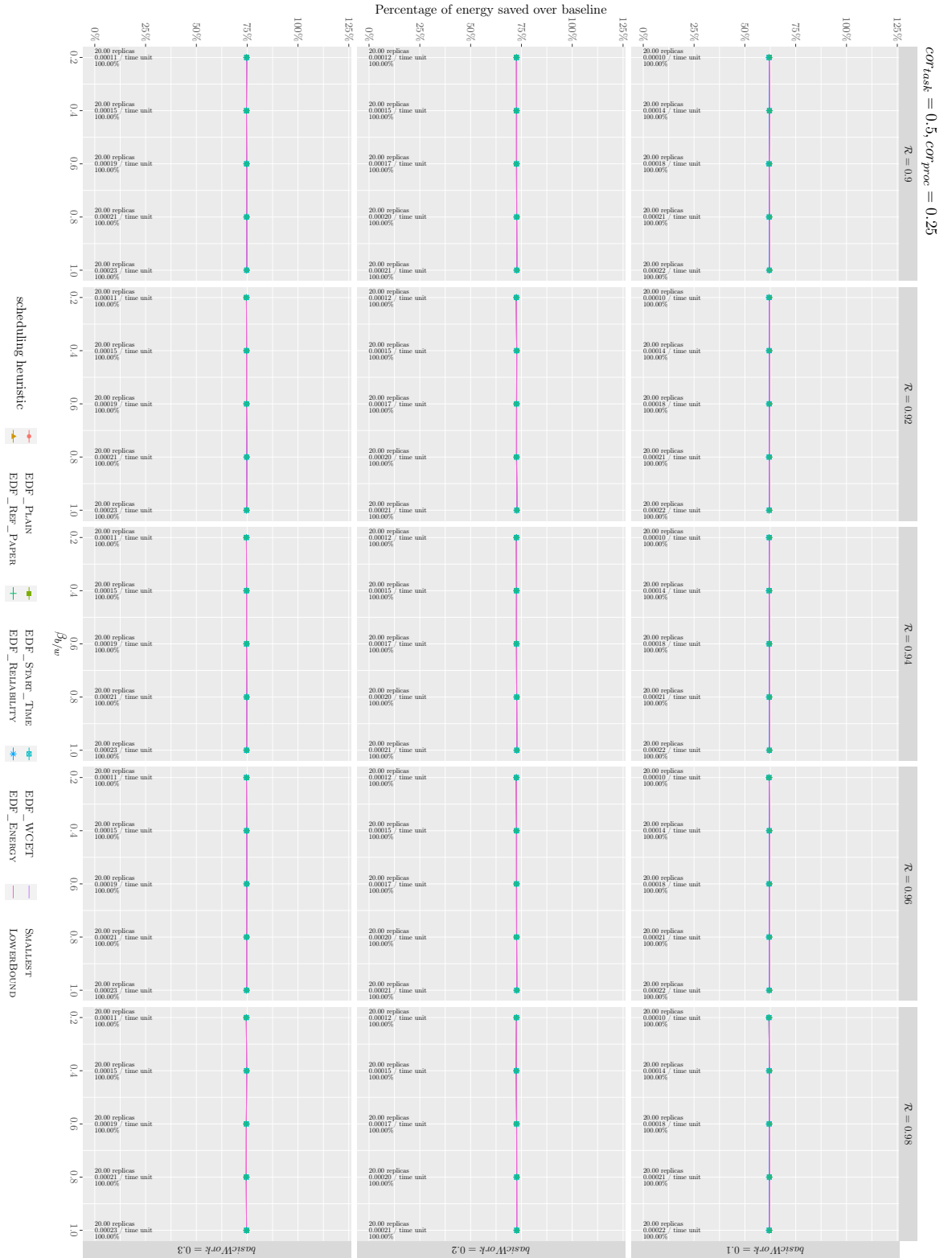


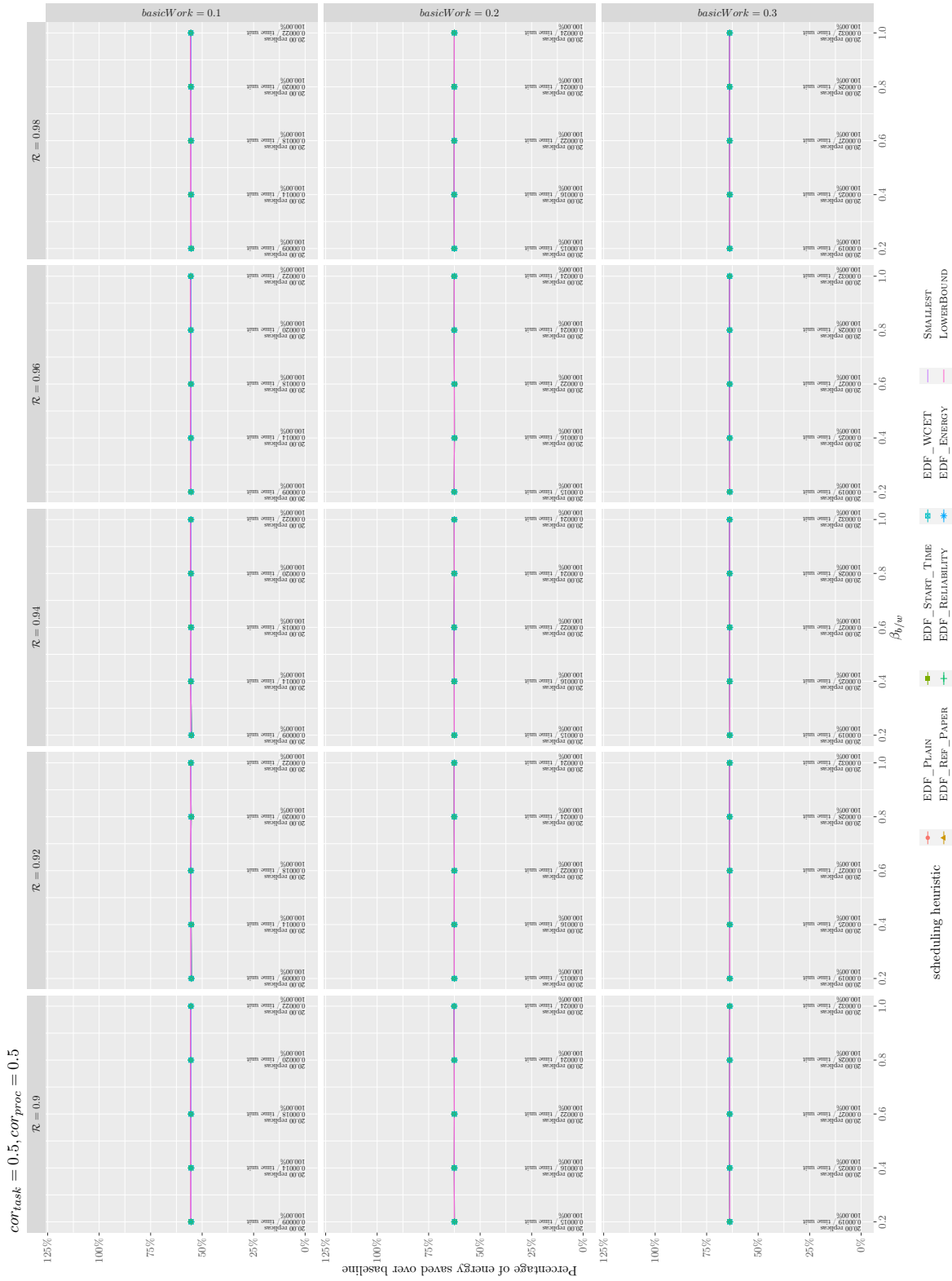
$cot_{task} = 0.25, cot_{proc} = 0.25$





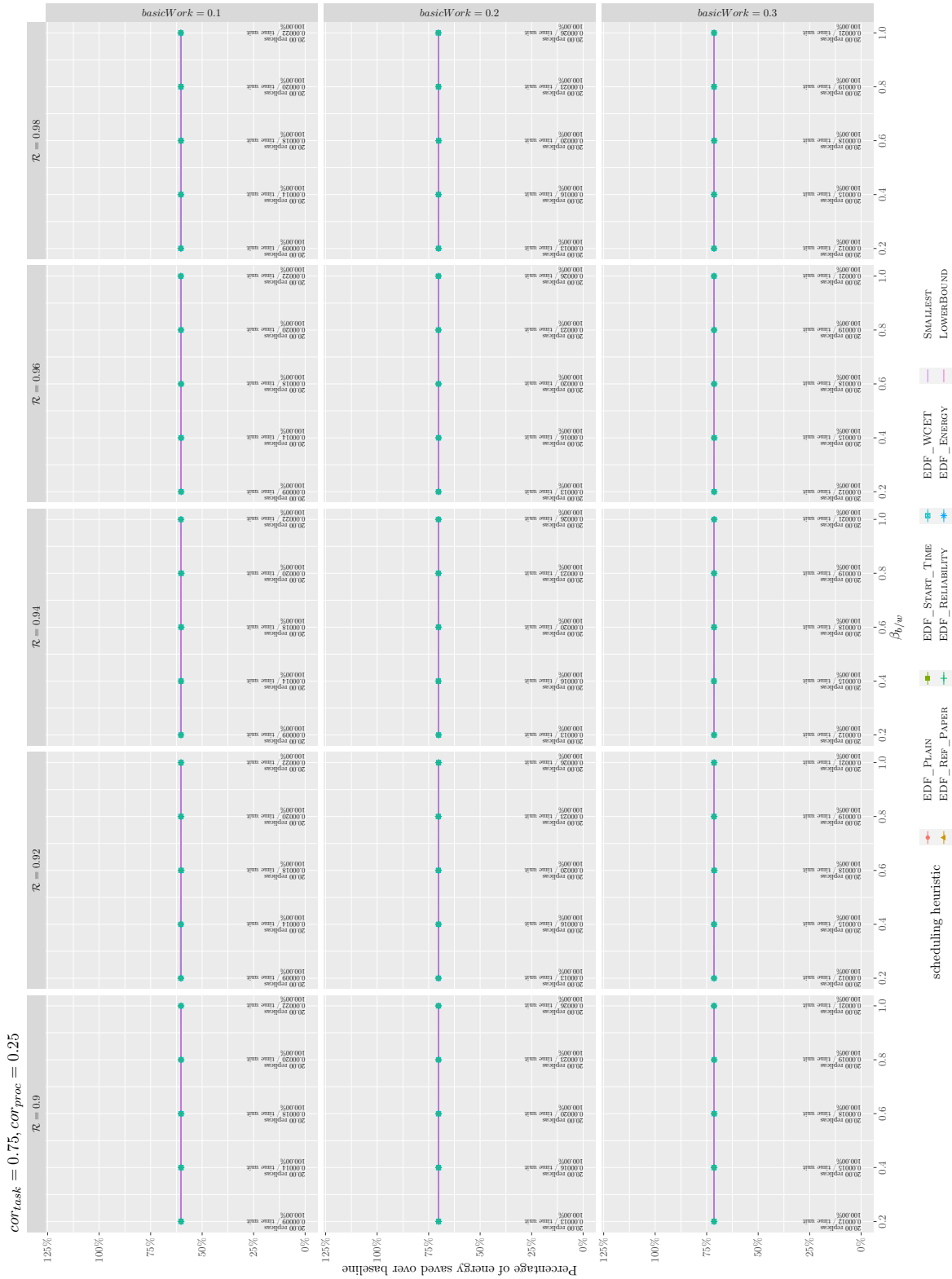
$cop_{task} = 0.25, cop_{proc} = 0.75$





$cop_{task} = 0.5, cop_{proc} = 0.5$

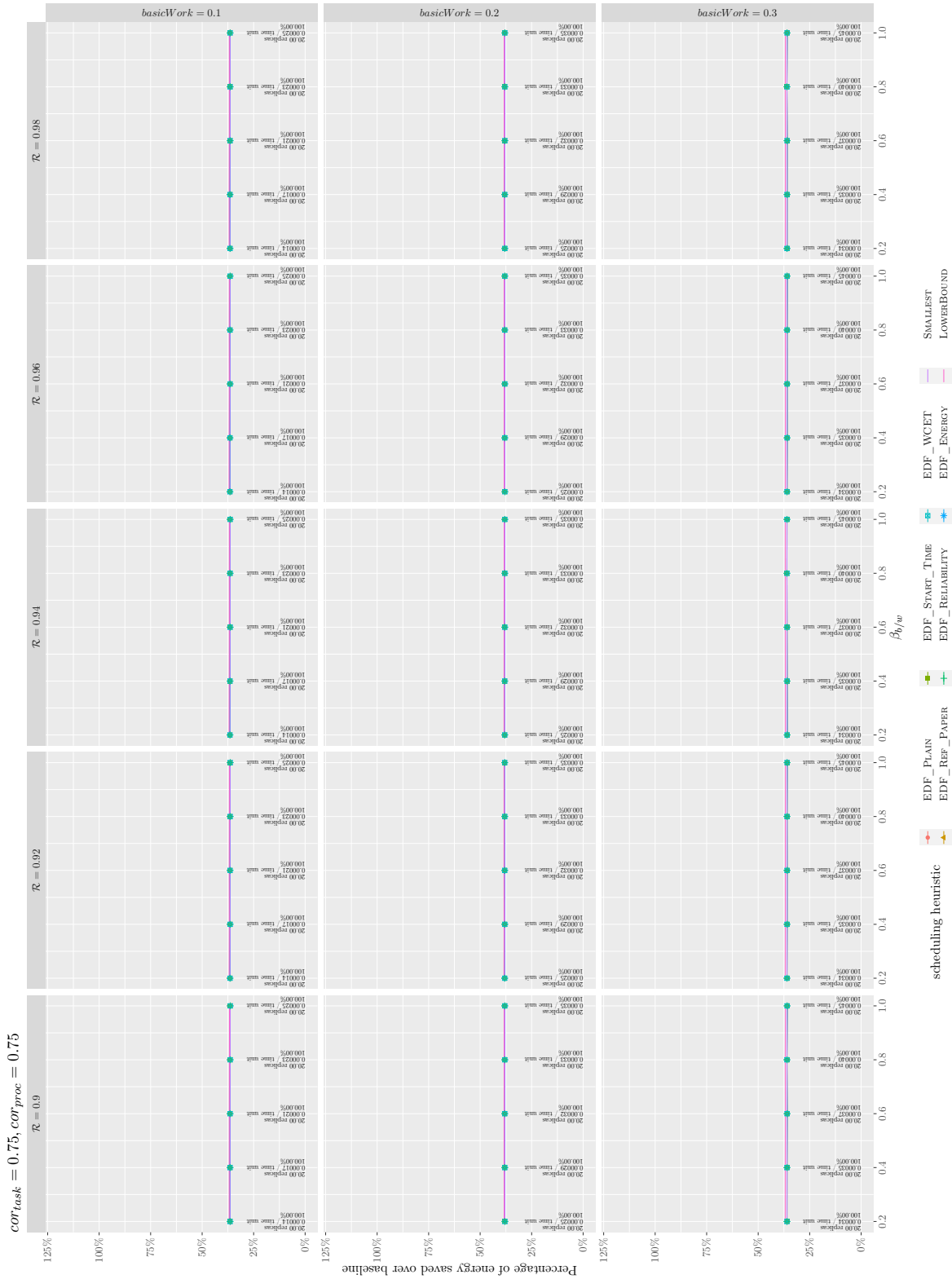




$cot_{task} = 0.75, cot_{proc} = 0.25$







$cot_{task} = 0.75, cot_{proc} = 0.75$

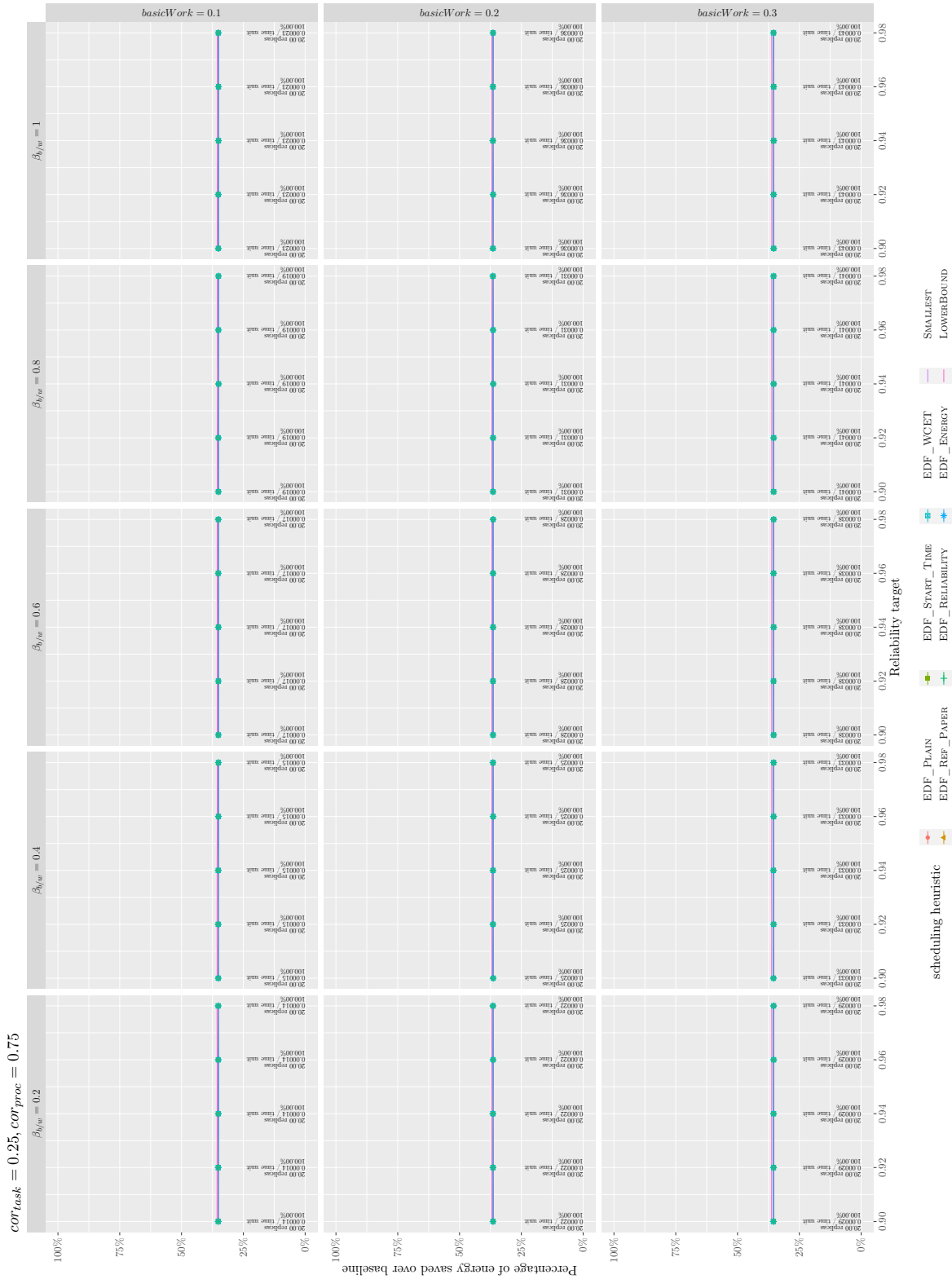
## Utilization and reliability threshold

The following section presents percentage of energy saved over baseline using different scheduling strategies under small failure rate, when varying *basicWork* and  $\mathcal{R}$ .



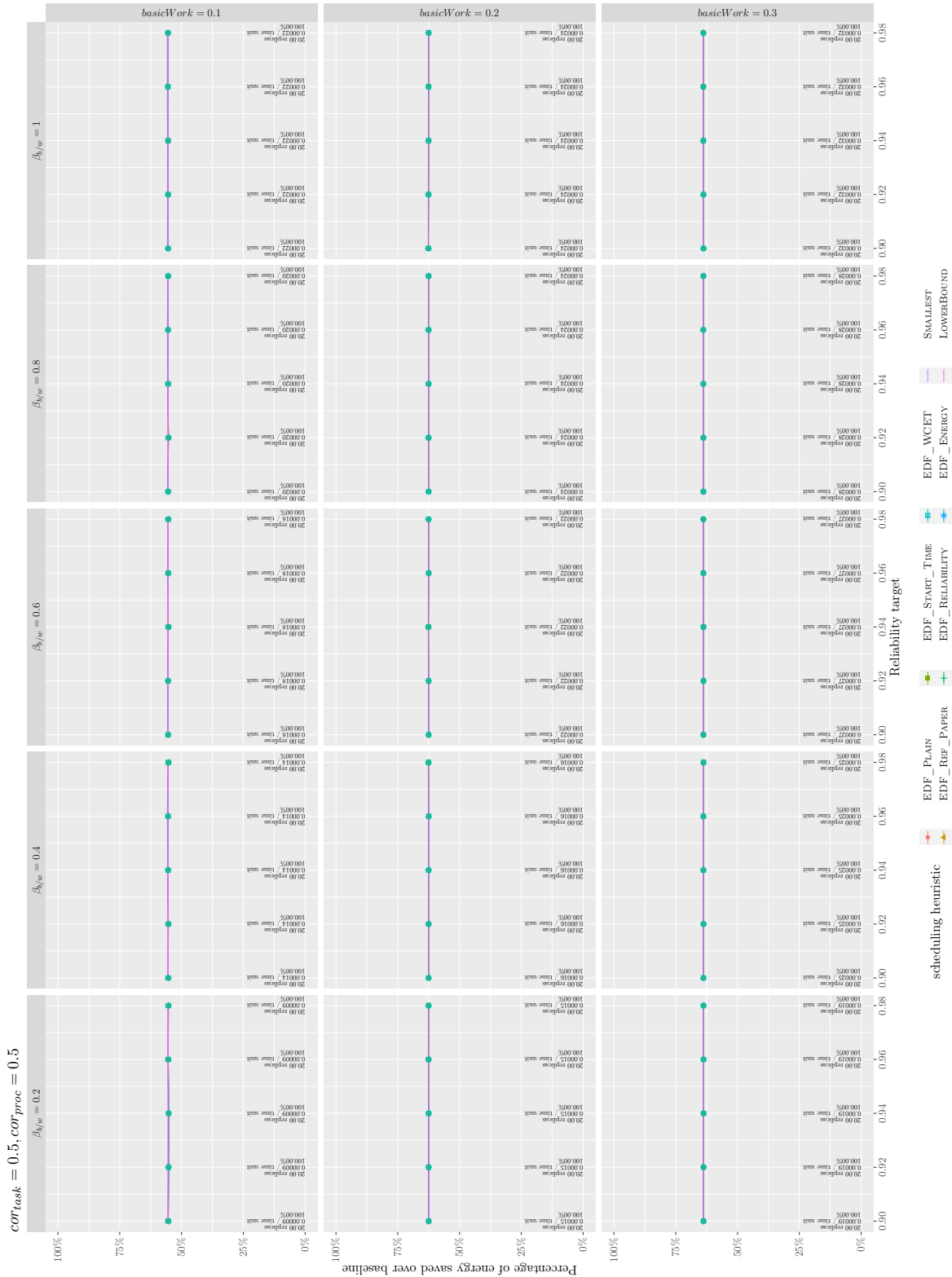
$cof_{task} = 0.25, cof_{proc} = 0.25$



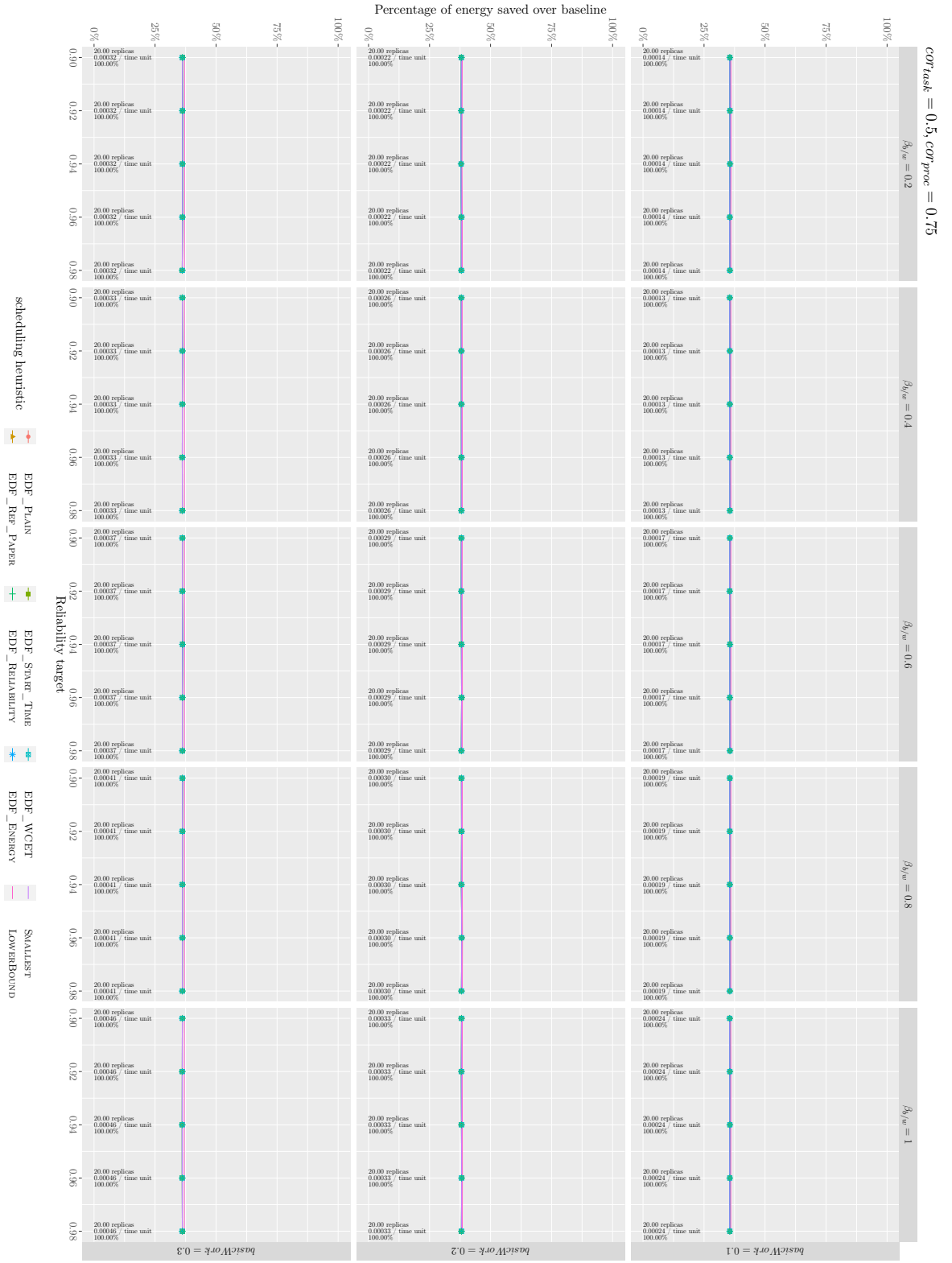


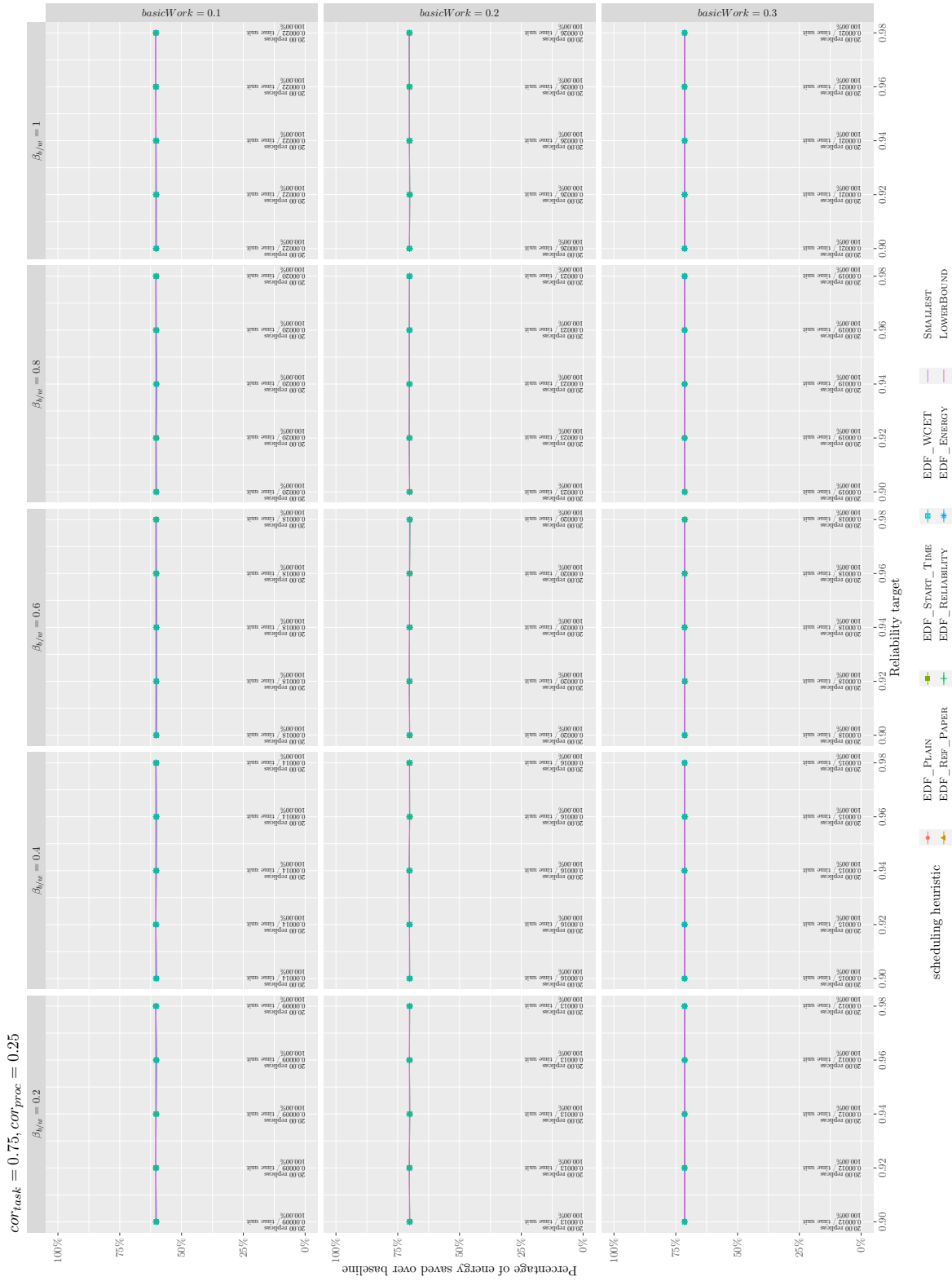
$COI_{task} = 0.25, COI_{proc} = 0.75$











$cof_{task} = 0.75, cof_{proc} = 0.25$





$COI_{task} = 0.75, COI_{proc} = 0.75$



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