



Linear response for spiking neuronal networks with unbounded memory

Bruno Cessac, Rodrigo Cofré, Ignacio Ampuero

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Linear response for spiking neuronal networks with unbounded memory

Bruno Cessac

Biovision Team, INRIA Sophia Antipolis, France.

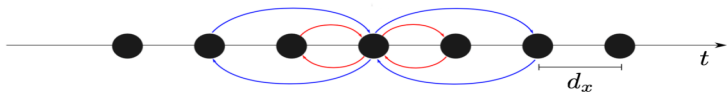
25-08-2020

with

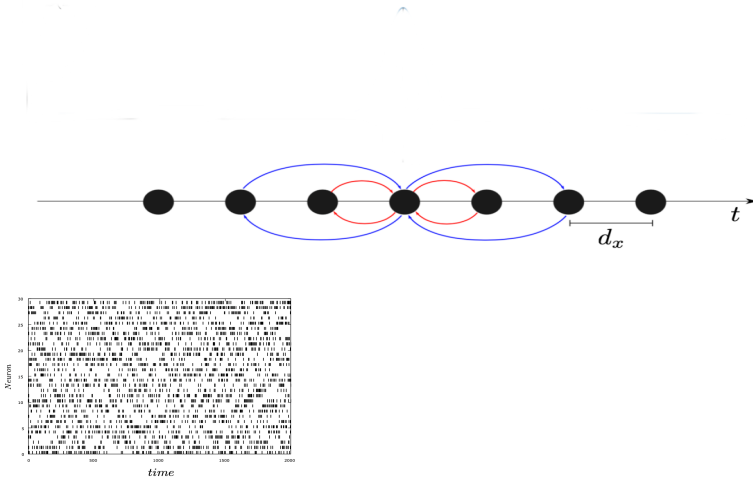
Rodrigo Cofré, CIMFAV, Facultad de Ingeniería, Universidad de Valparaíso,
Valparaíso, Chile. and

Ignacio Ampuero, Departamento de Informática, Universidad Técnica Federico Santa
María, Valparaíso, Chile

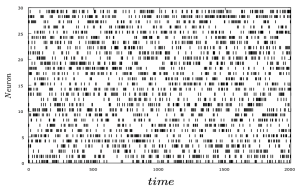
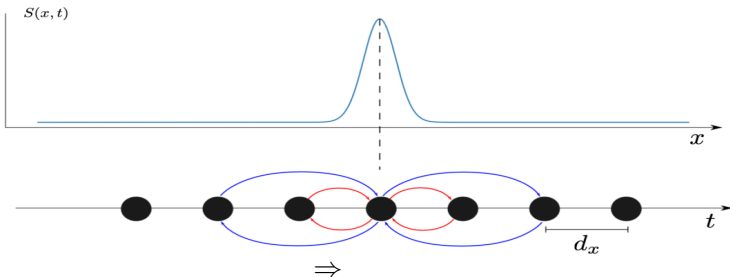
Spontaneous spiking activity



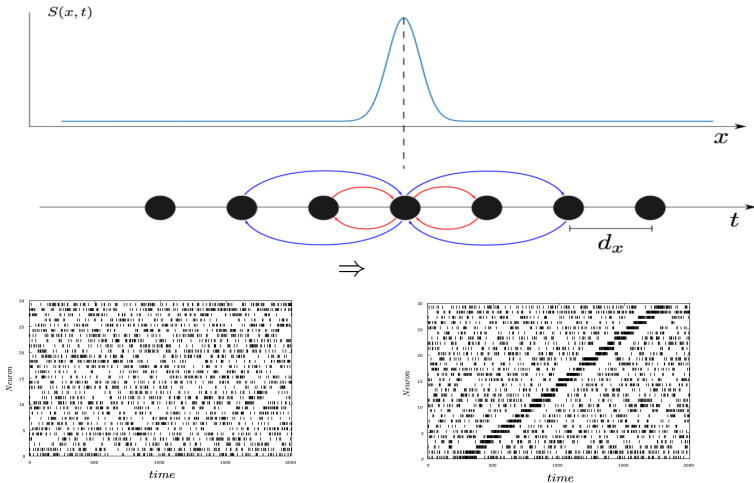
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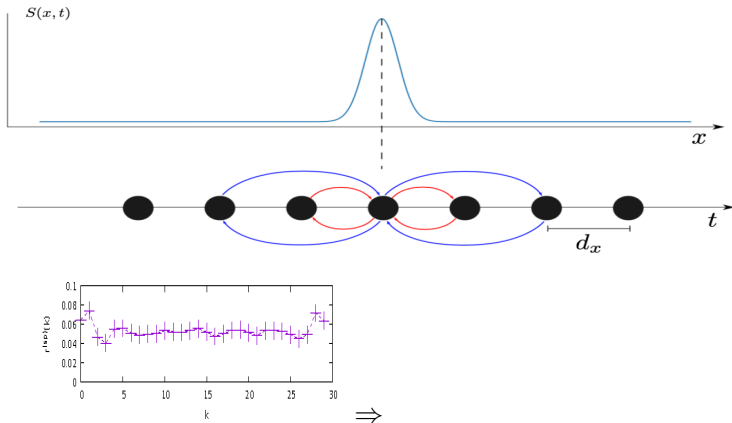
Spontaneous spiking activity



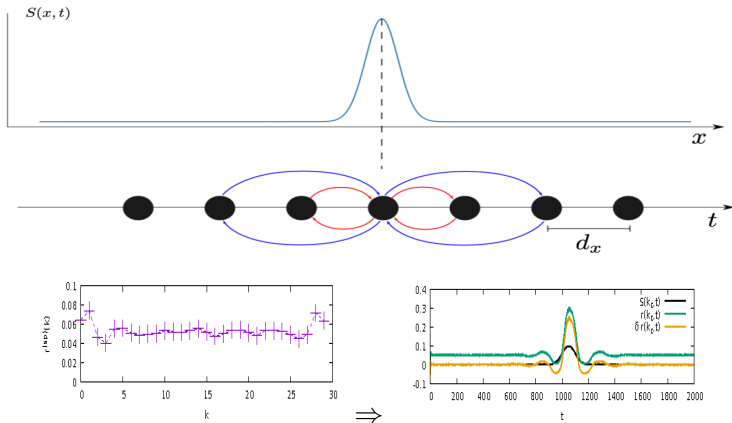
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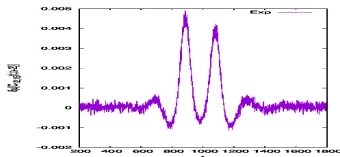
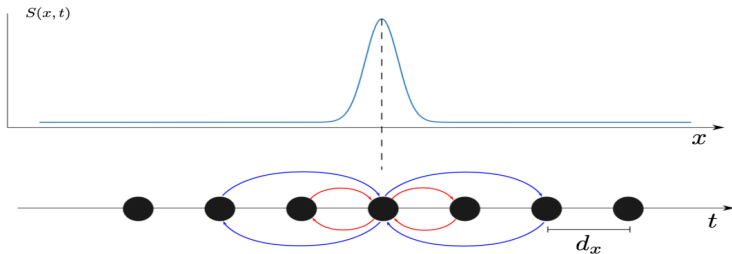


Spontaneous spiking activity



Sample averaging (non stationary)

Spontaneous spiking activity



Variation in the correlation $C_{12,15}(t, t + 2)$

Correlation variations are triggered by the stimulus, but they are constrained by the network dynamics.

Question

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How to compute the spatio-temporal response of a neuronal network to a time dependent stimulus ?

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Can we compute the time variation of spike-observables as a function of the stimulus and spontaneous dynamics ?

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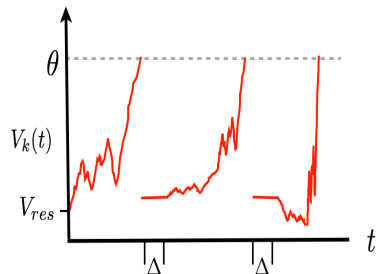
Linear response for spiking neuronal networks with unbounded memory,
Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré,

submitted to J. Math. Neuro, arXiv:1704.05344

From spiking neurons dynamics to linear response

An Integrate and Fire neural network model with unbounded memory

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013

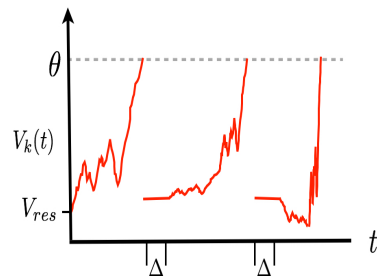


$\omega_k(n)$

0	0	1	0	0	0	0	1	0	0	1	0
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$\omega_k(n)$

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Spikes

- Voltage dynamics is time-continuous.
- Spikes are time-discrete events (time resolution $\delta > 0$).

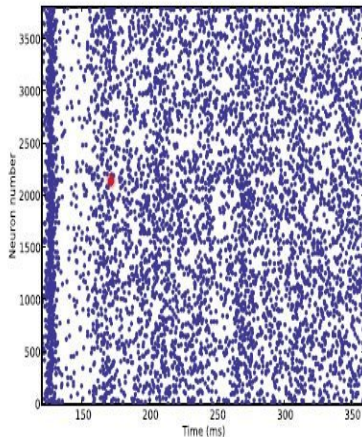
$$t_k^{(l)} \in [n\delta, (n+1)\delta[$$

$$\Rightarrow$$

$$\omega_k(n) = 1$$

An Integrate and Fire neural network model with unbounded memory

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013

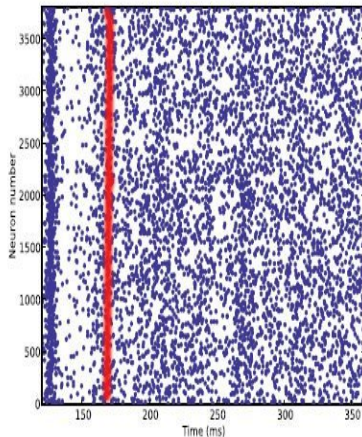


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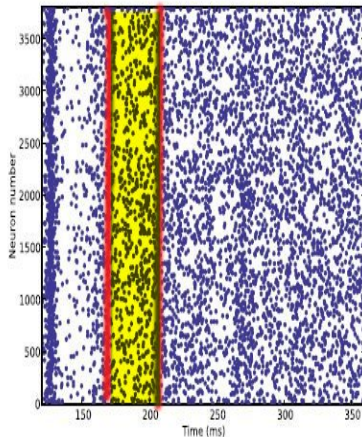


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- Spike state $\omega_k(n) \in \{0, 1\}$.
- Spike pattern $\omega(n)$.
- Spike block ω_m^n .

A conductance-based Integrate and Fire model

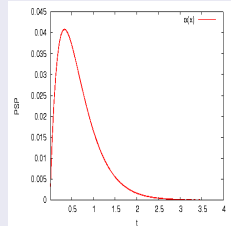
M. Rudolph, A. Destexhe, Neural Comput. 2006, (GIF model)

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} = -g_{L,k}(V_k - E_L) - \sum_j g_{kj}(t, \omega)(V_k - E_j)$$

Synapses



$$\alpha_{kj}(t) = \frac{t}{\tau} e^{-\frac{t}{\tau_{kj}}} H(t),$$

A conductance-based Integrate and Fire model

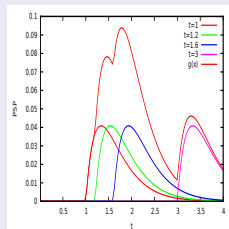
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Synapses



$$g_{kj}(t) = g_{kj}(t_j) + G_{kj}\alpha_{kj}(t - t_j)$$

$$t > t_j$$

A conductance-based Integrate and Fire model

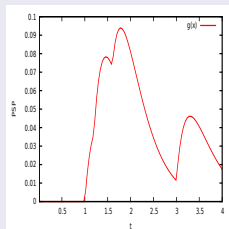
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$$g_{kj}(t) = G_{kj} \sum_{n \geq 0} \alpha_{kj}(t - t_j^{(n)})$$

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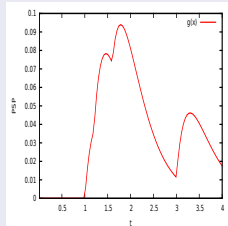
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$$g_{kj}(t, \omega) = G_{kj} \sum_{n \geq 0} \alpha_{kj}(t - n\delta) \omega_j(n)$$

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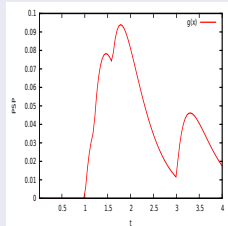
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$$+ S_k(t) + \sigma_B \xi_k(t)$$

Synapses



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A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

$$W_{kj} \stackrel{\text{def}}{=} G_{kj} E_j$$

$$\alpha_{kj}(t, \omega) = \sum_{n \geq 0} \alpha_{kj}(t - n\delta) \omega_j(n)$$

$$i_k(t, \omega) = g_{L,k} E_L + \sum_j W_{kj} \alpha_{kj}(t, \omega) + S_k(t) + \sigma_B \xi_k(t)$$

A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

- Linear in V .
- Spike history-dependent.

Dynamics integration

$$\Gamma_k(t_1, t, \omega) = e^{-\frac{1}{C_k} \int_{t_1 \vee \tau_k(t, \omega)}^t g_k(u, \omega) du}$$

A conductance-based Integrate and Fire model

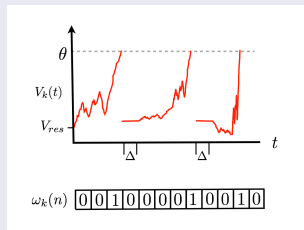
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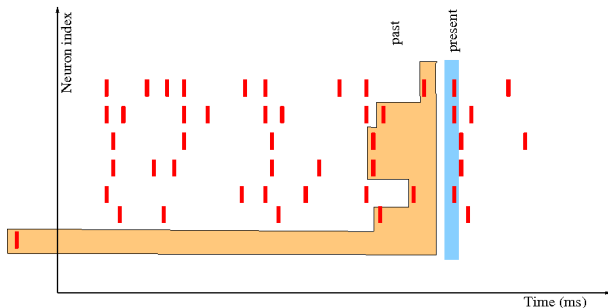
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A conductance-based Integrate and Fire model

Variable length Markov chain with unbounded memory

$$P_n [\omega(n) \mid \omega_{-\infty}^{n-1}] \equiv \Pi \left(\omega(n), \frac{\theta - V_k^{(det)}(n-1, \omega)}{\sigma_k(n-1, \omega)} \right)$$



Markov chains and Gibbs distributions

Markov chain: $P \left[\omega(n) \mid \omega_{n-D}^{n-1} \right] > 0 \Rightarrow \exists$ a unique, invariant, ergodic distribution μ .

Markov chains and Gibbs distributions

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$$\mu[\omega_m^n] = \prod_{l=m+D}^n P[\omega(l) \mid \omega_{l-D}^{l-1}] \mu[\omega_m^{m+D-1}], \quad \forall m < n \in \mathbb{Z}$$

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$$\phi \left(\omega_{l-D}^l \right) = \log P \left[\omega(l) \mid \omega_{l-D}^{l-1} \right]$$

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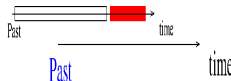
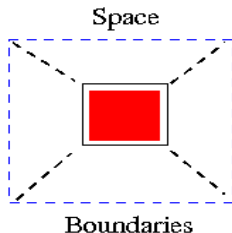
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Markov chains and Gibbs distributions

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$$\mu \left[\omega_m^n \mid \omega_m^{m+D-1} \right] = \exp \sum_{l=m+D}^n \phi_l \left(\omega_{l-D}^l \right), \quad \forall m < n \in \mathbb{Z}$$

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- Equilibrium stat. phys. (Max. Entropy Principle).
- Non equ. stat. phys. Onsager theory.

$$P[\omega] = \frac{1}{Z} e^{-\beta H\{\omega\}}$$

$$H\{\omega\} = \sum_{\alpha} \lambda_{\alpha} X_{\alpha}\{\omega\}$$

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$X_{\alpha}(\omega)$ = Product of spike events

Hammersley, Clifford, 1971

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- Chains with complete connections - infinite memory (Left Interval Specification).

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Hammersley, Clifford, 1971

O. Onicescu and G. Mihoc. CRAS
Paris, 1935

R. Fernandez, G. Maillard, A. Le Ny,
J.R. Chazottes, ...

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

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$$\phi(n, \omega) = \log P_n [\omega(n) \mid \omega_{-\infty}^{n-1}] \equiv \log II \left(\omega(n), \frac{\theta - V_k^{(sp)}(t, \omega) - V_k^{(S)}(t, \omega)}{\sigma_k(n-1, \omega)} \right)$$

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$$\phi(n, \omega) = \log P_n [\omega(n) \mid \omega_{-\infty}^{n-1}] \equiv \log II \left(\omega(n), \frac{\theta - V_k^{(sp)}(t, \omega) - V_k^{(S)}(t, \omega)}{\sigma_k(n-1, \omega)} \right)$$

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History dependence.

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History dependence, observable

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History dependence, observable, network dynamics

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Spontaneous correlation between observable and network dynamics

An illustrative example

The simplest example of the glF model is the ... discrete time
leaky-integrate and fire model

$$V_k(n+1) = \gamma V_k(n) + \sum_j W_{kj} \omega_j(n) + I_0 + S_k(t) + \sigma_B \xi_k(n), \quad \text{if } V_k(n) < \theta$$

Linear response

$$\delta\mu^{(1)}[f(n)] \sim - \sum_{k=1}^N \sum_{m=1}^{D+1} \sum_{l=0}^{\frac{1}{\nu_k}} \gamma^l \mathcal{K}_{km}^{(1)} S_k(n-m-l)$$

$$\mathcal{K}_{k,m}^{(1)} = \mathcal{C}^{(sp)}[f(m, \cdot), \zeta_k(0, \cdot)]$$

$$\zeta_k(r-1, \omega) = \frac{\mathcal{H}_k^{(1)}(r, \omega)}{\sigma_k(r-1, \omega)}$$

Firing rates

Computations have been done using the INRIA Pranas software, B. Cessac et al, Frontiers in Neuroinformatics, Vol 11, page 49, (2017).

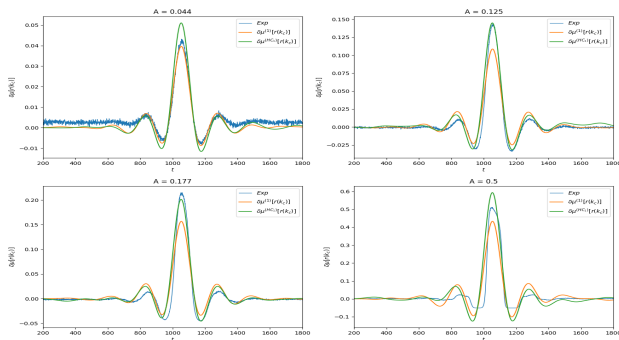


Figure: Linear response of $f(\omega, n) = \omega_{k_c}(n)$ for different values of stimulus amplitude A . From <https://arxiv.org/abs/1704.05344>

Pairwise correlations

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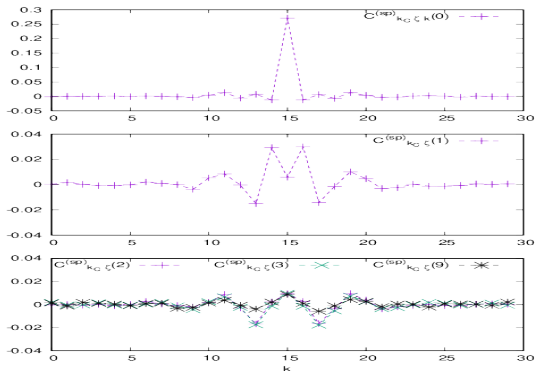


Figure: Correlation functions corresponding to the firing rate of the neuron $k_C = \frac{N}{2}$ as a function of the neuron index k (abscissa), for different values of the time delay m . From <https://arxiv.org/abs/1704.05344>

Higher order observable

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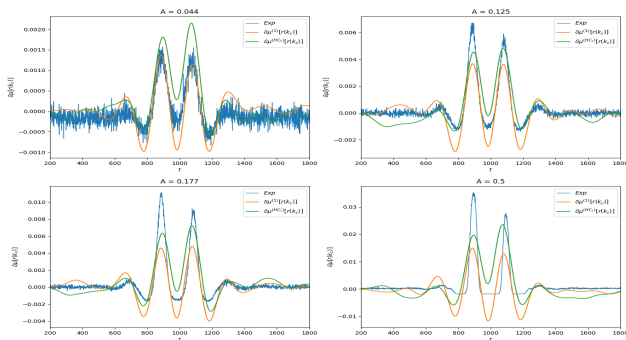


Figure: Linear response of the observable $f(n, \omega) = \omega_{k_C-2}(n-3)\omega_{k_C}(n)$.

Range of validity

Computations have been done using the INRIA Pranas software, B. Cessac et al, *Frontiers in Neuroinformatics*, Vol 11, page 49, (2017).

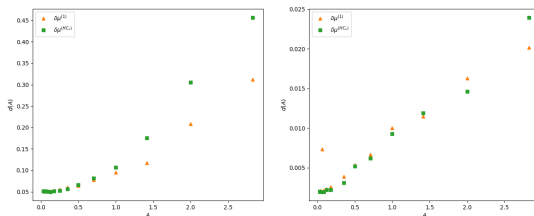


Figure: L^2 distance between the curves $\delta\mu^{(1)} [f(n)]$, $\delta\mu^{(HC1)} [f(n)]$ and the empirical curve, as a function of the stimulus amplitude A . Left panel show distance between rate curves and right panel distance between pairwise observable with delay.

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Which ones are relevant ? $\Rightarrow$ Requires a method to reduce the "dimensionality of the potential"

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 - Similar to Volterra-expansion.
 - Higher order terms computed by D. Ruelle (Nonlinearity, 11(1):518, 1998.) for hyperbolic dynamical systems $\Rightarrow$ Complex expansion hard to handle from data.
 - Sigmoid are "hard" to approximate with Taylor expansions.

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- Toward an extension of the concept of receptive-field ?