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Numerical Approach to the Optimal Control and Efficiency of the Copepod Swimmer.

55th IEEE Conference on Decision and Control

Bernard Bonnard, Monique Chyba, Jérémy Rouot and Daisuke Takagi

- Geometric optimal control
 - . Maximum principle
 - . Second order optimality conditions (local, conjugate points)
- Swimming at low Reynolds number
 - . sub-Riemannian (SR) geometry framework
 - . Variational approach to find trajectories of a Hamiltonian system with periodic projections
 - . Find optimal stroke (efficiency concept)
 - . micro-device implementation (D. Takagi, M. Chyba)

$$\begin{array}{ll} \min & c(\tilde{q}(0), \tilde{q}(T)) \\ \text{subject to} & \dot{q} = F(q, u), \quad \dot{q}^0 = L(q, u) \\ & (\tilde{q}(0), \tilde{q}(T)) \in B \end{array}$$

- final time T is fixed
- $\tilde{q} = (q, q^0)$, q state, $q^0 \in \mathbb{R}$
- $u(\cdot) \in L^\infty([0, T], U)$, $U \subset \mathbb{R}^m$
- B : smooth closed set of $\mathbb{R}^n \times \mathbb{R}^n$, $(\tilde{q}(0), \tilde{q}(T)) \in B$ stands for **mixed** boundary conditions.

Ex. : $B = \{q(0) = q(T), q^0(0) = 0\}$,
 $B = \{q(0) = q_0, q(T) = q_f, q^0(0) = 0\} \dots$

Necessary condition. (\tilde{q}, u) optimal $\implies \exists \tilde{p} = (p, p^0) : [0, T] \rightarrow (\mathbb{R}^n \times \mathbb{R})$ abs. continuous and $\lambda \geq 0$ cst with $(\tilde{p}, \lambda) \neq (0, 0)$ and s.t.

$$\dot{\tilde{q}} = \frac{\partial H}{\partial \tilde{p}}, \quad \dot{\tilde{p}} = -\frac{\partial H}{\partial \tilde{q}}, \quad H(\tilde{q}, \tilde{p}, u) = \max_{v \in U} \mathcal{H}(\tilde{q}, \tilde{p}, v)$$

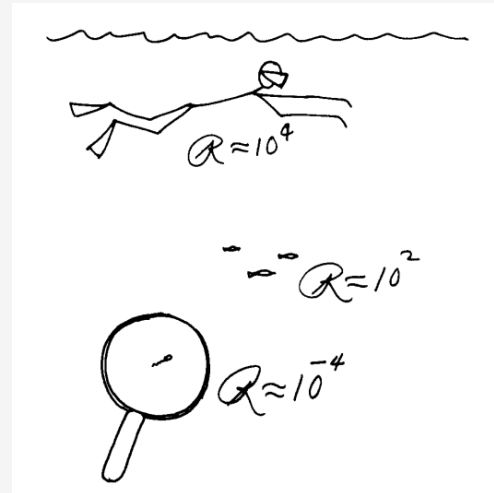
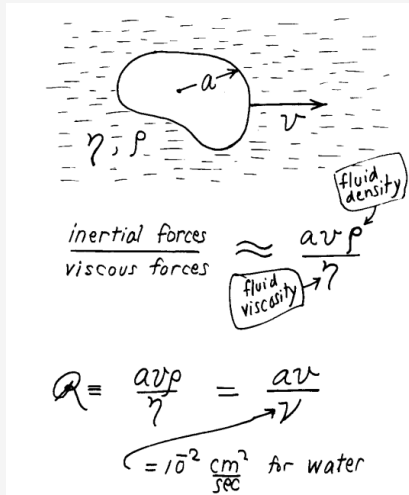
where $\mathcal{H}(\tilde{q}, \tilde{p}, u) = p \cdot F(q, u) + p^0 L(q, u)$.

Transversality conditions:

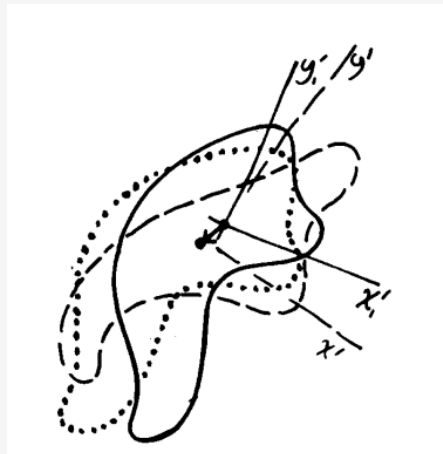
$$(\tilde{p}(0), -\tilde{p}(T)) - \lambda \left(\frac{\partial c}{\partial \tilde{q}(0)}(\tilde{q}(0), \tilde{q}(T)), \frac{\partial c}{\partial \tilde{q}(T)}(\tilde{q}(0), \tilde{q}(T)) \right) \in (T_{(\tilde{q}(0), \tilde{q}(T))} B)^\perp$$

Definition 1. $(\tilde{q}, \tilde{p}, \lambda, u)$ is a normal extremal if $\lambda > 0$, abnormal if $\lambda = 0$. An abnormal trajectory q which is not a projection of normal extremal is said strict.

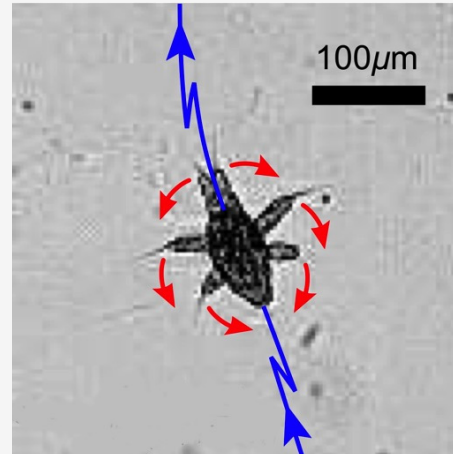
LIFE AT LOW REYNOLDS NUMBER - PURCELL, 1977



Reynolds number

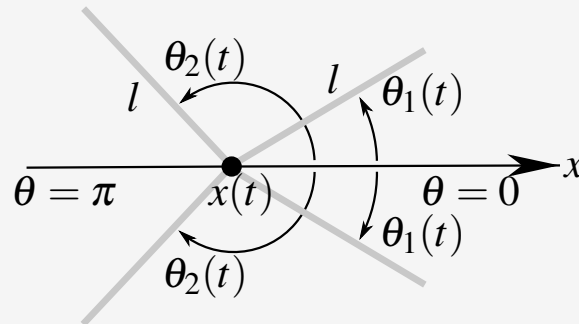


Swimmer deformations



Zooplankton

Symmetric 2-link swimmer

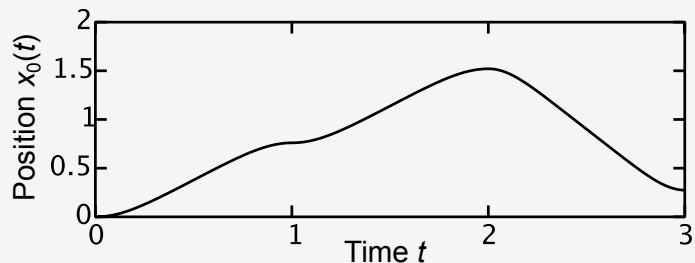
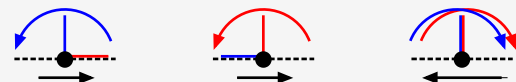
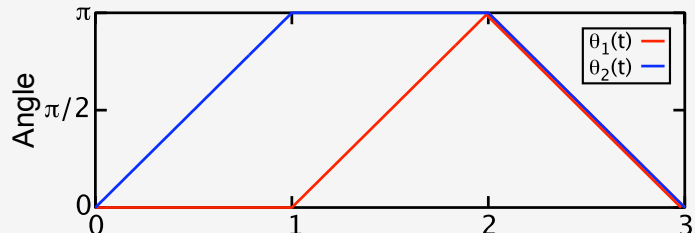
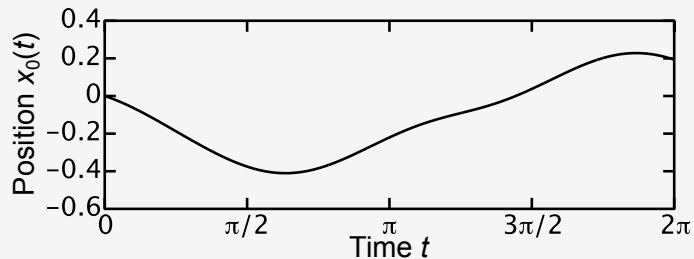
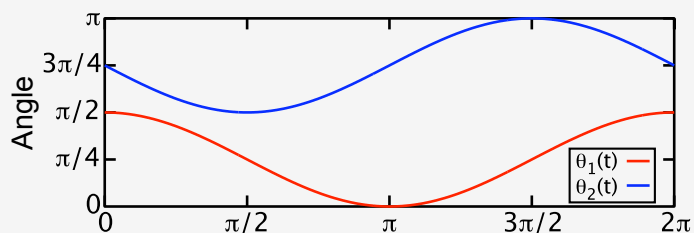


Controlled dynamic (Takagi, 2014)

$$\dot{x} = \frac{\sum_{i=1}^2 l^2 \dot{\theta}_i \sin(\theta_i)}{2 + \sum_{i=1}^2 \sin^2(\theta_i)}, \quad \dot{\theta}_i = u_i, \quad i = 1, 2, \quad \text{state constraints: } 0 \leq \theta_1 \leq \theta_2 \leq \pi.$$

Criterion : min of the dissipative energy: $\dot{q}S(q)\dot{q}^\top$ where $q = (x, \theta_1, \theta_2)$ and S is a positive-definite matrix \implies quadratic form in (u_1, u_2) .

Definition 2. An admissible stroke is a periodic motion of the shape variables (θ_1, θ_2) associated with a periodic control and producing a displacement of the position variable after one period T (we can fix $T = 2\pi$).



(Left) Trigonometric controls. (Right) Piecewise controls.

$$\mathcal{E} = \text{Geometric Efficiency} = x(T)/l_{SR}(q)$$

SR length:

$$l_{SR}(q) = \int_0^T \sqrt{L(q, u)} dt, \quad L(q, u) = a(q) u_1^2 + 2b(q) u_1 u_2 + c(q) u_2^2$$

$$\min l_{SR}(q) \iff \min \int_0^T |\dot{q}|_L^2 dt = \min \int_0^T L(q, u) dt$$

With $c(\tilde{q}(0), \tilde{q}(T)) = q^0(T)$:

$$\dot{q} = \sum_{i=1}^2 u_i F_i \leftarrow \min \int_0^T L(q, u) dt,$$

$$\max \mathcal{E} \iff \left(\min \int_0^T L(q, u) dt \quad \text{with } \mathbf{x(T)} \text{ fixed, then } \underbrace{\text{select } \max_{x(T)} \mathcal{E}}_{\text{transversality cond.}} \right)$$

Compute points on the SR-sphere \rightarrow
provide candidates to maximize the efficiency.

$$\dot{q} = \sum_{i=1}^2 u_i F_i(q), \quad q = (x, \theta_1, \theta_2)$$

$$F_i = \frac{\sin(\theta_i)}{\Delta} \frac{\partial}{\partial x} + \frac{\partial}{\partial \theta_i}, \quad \Delta = 2 + \sin^2(\theta_1) + \sin^2(\theta_2)$$

PMP:

$$\mathcal{H}(z, u) = u_1 H_1(z) + u_2 H_2(z) + p^0 L(q, u), \quad z = (q, p)$$

with $H_i = p \cdot F_i$, $i = 1, 2$.

- normal case: $p^0 = -1/2$ and u_i , $i = 1, 2$ given by $\partial \mathcal{H} / \partial u = 0$

$$H_n = 1/2 (a(q) H_1^2 + 2b(q) H_1 H_2 + c(q) H_2^2)$$

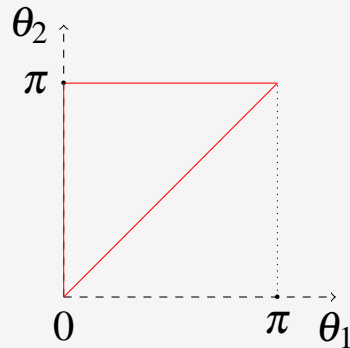
- abnormal case: $p^0 = 0$ hence $H_1 = H_2 = \{H_1, H_2\} = 0$, then

$$u_1 \{ \{H_1, H_2\}, H_1 \} + u_2 \{ \{H_1, H_2\}, H_2 \} = 0.$$

Proposition 3. *The surface $\Sigma : \{q; \det(F_1(q), F_2(q), [F_1, F_2](q)) = 0\}$ contains abnormal trajectories and is characterized by*

- $\theta_i = 0$ or π , for $i = 1, 2$,
- $\theta_1 = \theta_2$.

→ **physical boundary of the state domain.**: $\theta_i \in [0, \pi]$, $i = 1, 2$, $\theta_1 \leq \theta_2$.



: abnormal trajectories
in the plane (θ_1, θ_2)

Boundary conditions defined by

$$x(0) = 0, \quad x(T) = x_f, \quad \theta_i(0) = \theta_i(T), \quad i = 1, 2$$

Transversality conditions

$$p_{\theta_i}(0) = p_{\theta_i}(T), \quad i = 1, 2$$

Shooting problem

$$\begin{cases} \dot{z} = \vec{H}_n(z) \\ x(0) = 0, \quad x(T) = x_f \text{ (fixed) } , \\ \theta_i(0) = \theta_i(T), \quad i = 1, 2 \\ p_{\theta_i}(0) = p_{\theta_i}(T), \quad i = 1, 2 \end{cases}$$

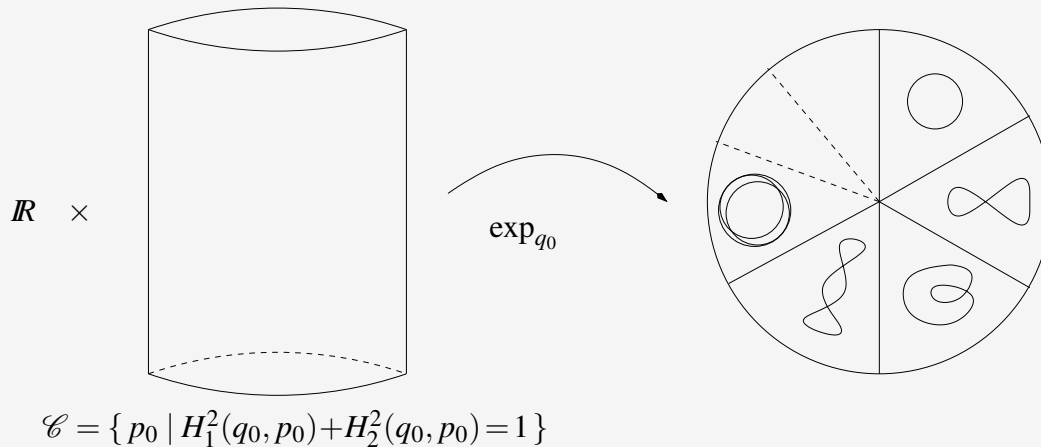
- HamPath *indirect method*: based on the PMP, *Newton* type algorithm and compute **necessary second order optimality conditions**, not robust w.r.t. initialization.
- Bocop *direct method*: give an initialization of the shooting algorithm of the HamPath code.

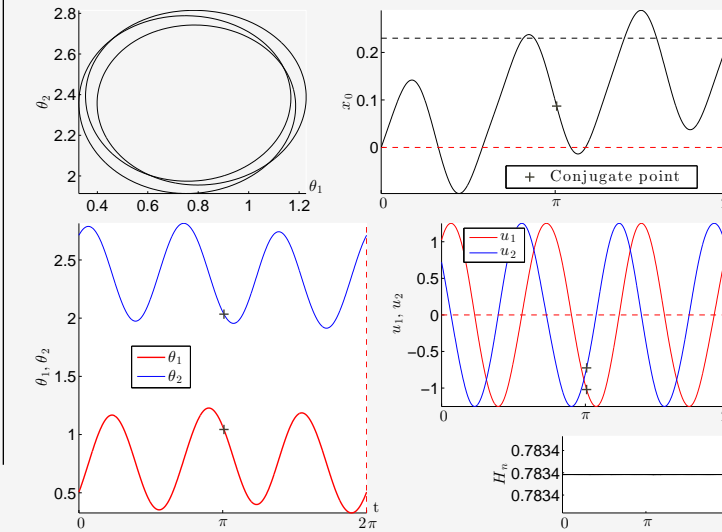
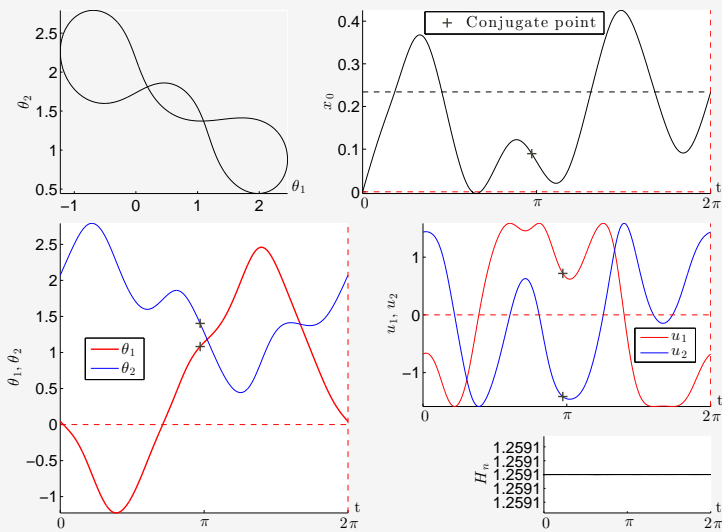
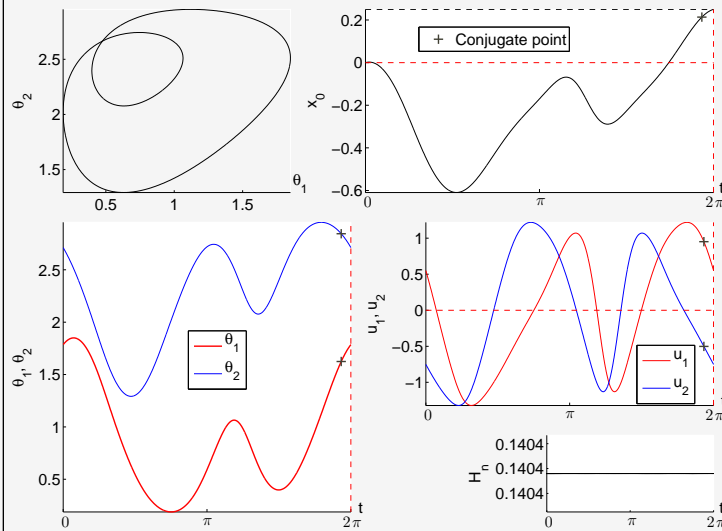
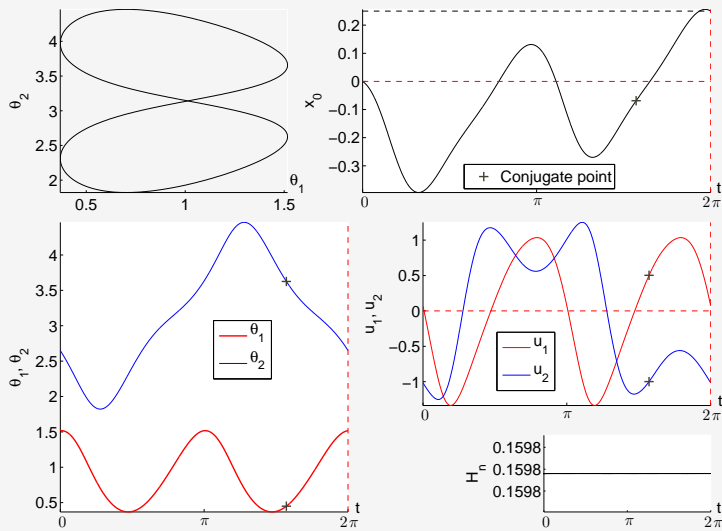
First conjugate time t_c : the exponential mapping

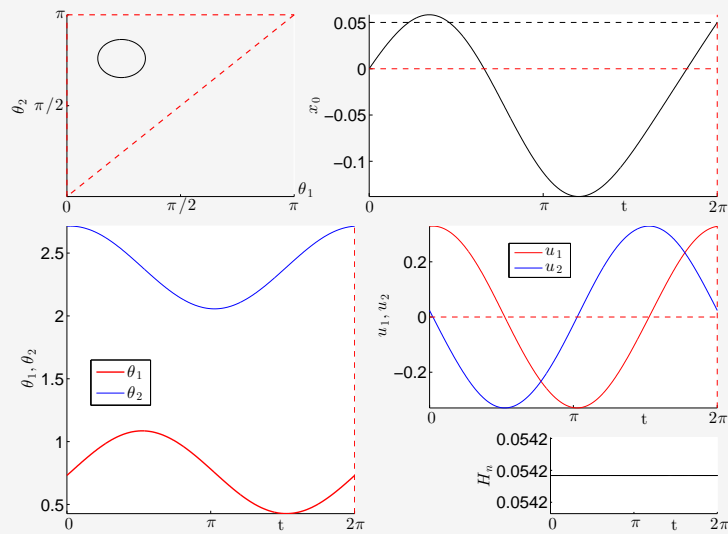
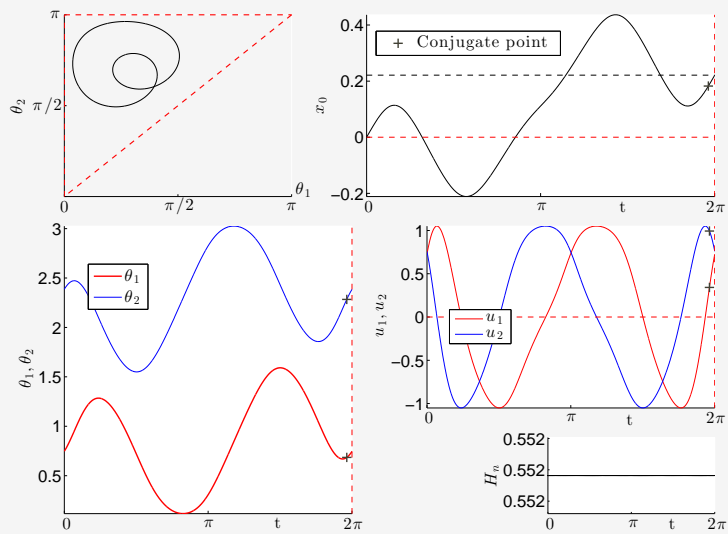
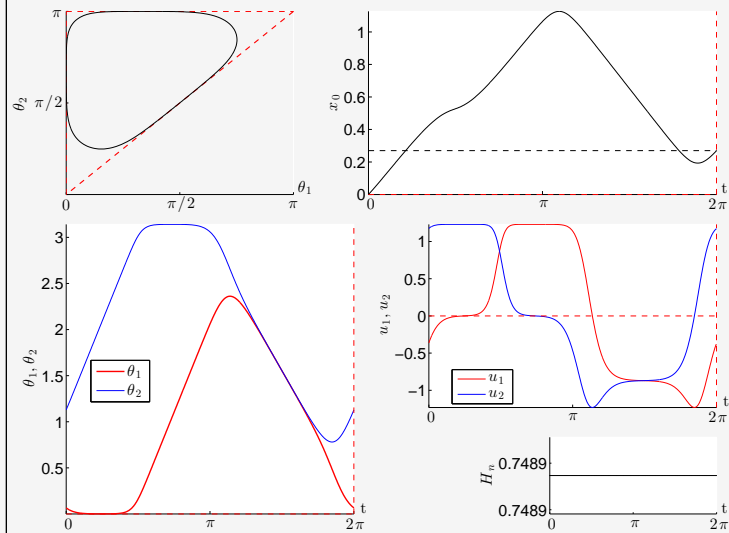
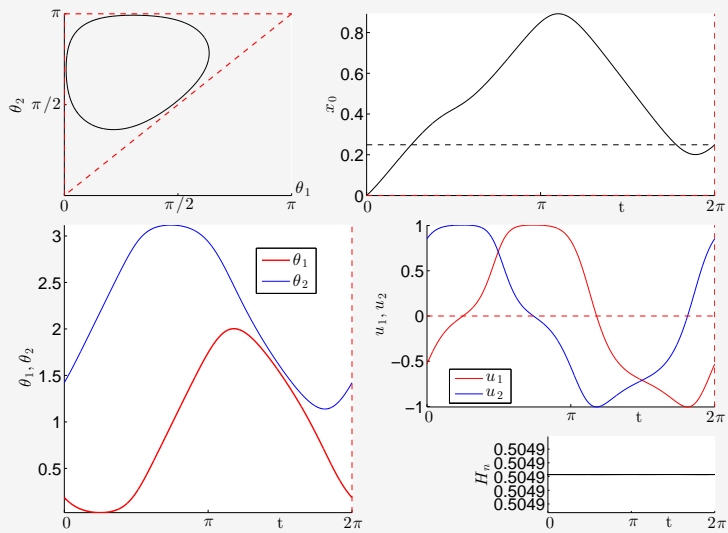
$$\exp_{q_0} : \mathbb{R} \times \mathcal{C} \rightarrow M, \quad (t, p_0) \mapsto q(t, q_0, p_0)$$

is not an immersion at (t_c, p_0) .

Theorem 4. *Let $q : [0, T] \rightarrow \mathbb{R}^n$ a strict normal stroke. If $q(\cdot)$ has at least one conjugate point on $]0, T[$, then q is not a local minimizer for the L^∞ topology on controls, considering the fixed extremities problem.*



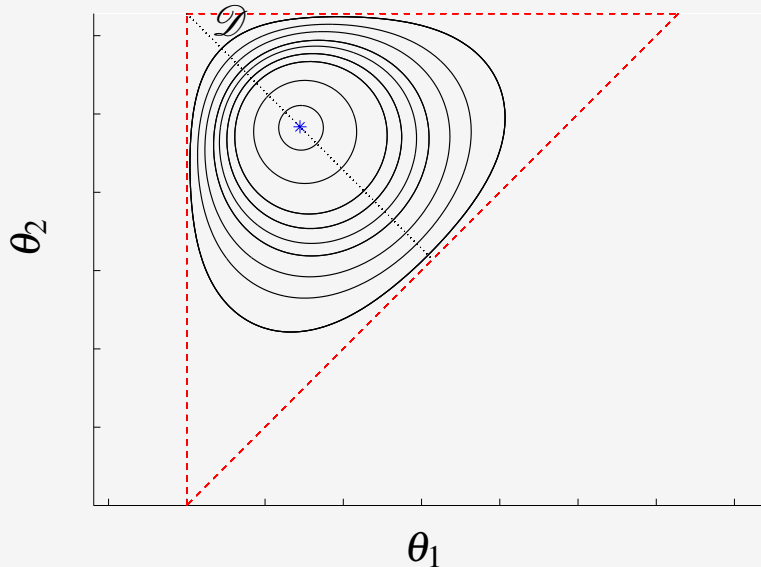




Numerical fact:

Only simple loops doesn't have a conjugate point on $]0, T[$: they are the only candidates for optimality.

Theorem 5 (Numerical result). *For each fixed displacement $x(T)$, one can find a simple loop associated with an energy level \implies one parameter family of strokes.*

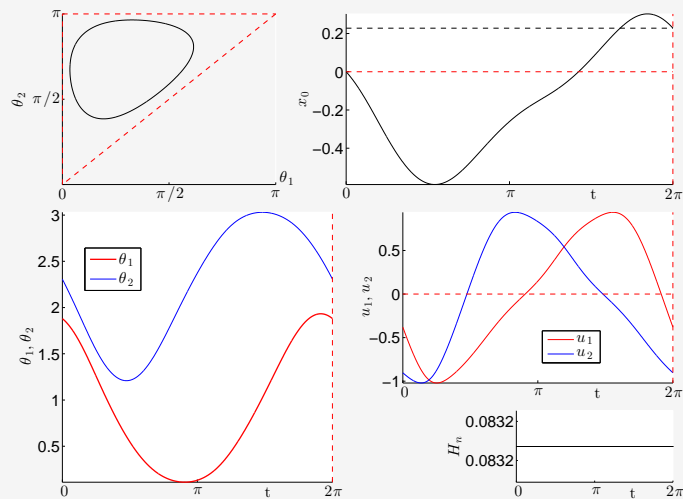
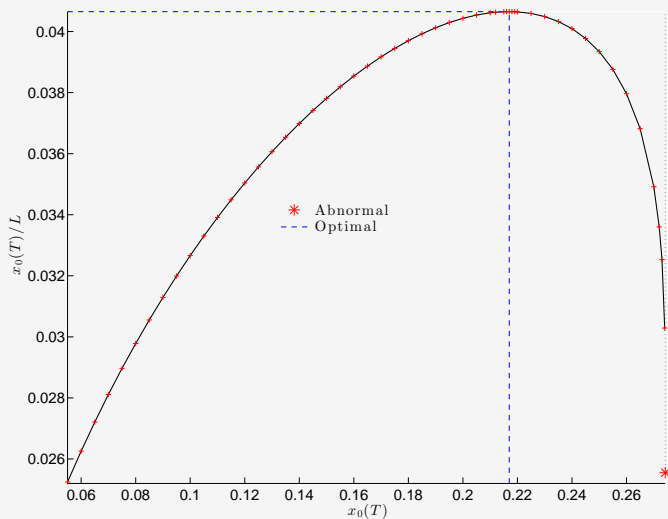


SELECT A STROKE WITH TRANSVERSALITY CONDITIONS.

Geometric efficiency, $c(\tilde{q}(0), \tilde{q}(T)) = -x(T) / l_{SR}(\tilde{q})$

Transversality conditions.

$$p_x(T) = q^0(T)/x(T), \quad p^0(T) = -1/2$$



Efficiency curve with continuation method on $x(T)$. Corresponding optimal stroke

Theorem 6. *The abnormal stroke is not minimizing for the two costs (length with fixed displacement and geometric efficiency).*

- More general models,
- Local geometric study

Nilpotent model

$$\dot{x} = u_1 \hat{F}_1 + u_2 \hat{F}_2, \quad \min \int_0^T (u_1^2 + u_2^2) dt$$

- **Interior point of the triangle:** weights $x_1, x_2 \leftarrow 1, x_3 \leftarrow 2$

$$\hat{F}_1 = \frac{\partial}{\partial x_1} + x_2 (1 + Q(x_1, x_2)) \frac{\partial}{\partial x_3}, \quad \hat{F}_2 = \frac{\partial}{\partial x_2} - x_1 (1 + Q(x_1, x_2)) \frac{\partial}{\partial x_3}$$

\implies simple loops and limaçons families

- **Point on the edges and \neq vertices of the triangle:** weights $x_1, x_2 \leftarrow 1, x_3 \leftarrow 3$

$$\hat{F}_1 = \frac{\partial}{\partial x_1} + \frac{x_2^2}{2} \frac{\partial}{\partial x_3}, \quad \hat{F}_2 = \frac{\partial}{\partial x_2}.$$

metric : $g = (1 + \alpha x_2)^2 dx_1 + (1 + \beta x_1 + \gamma x_2)^2 dx_2.$

\implies eight strokes

