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On deciding stability of high frequency amplifiers

L. Baratchart, S. Fueyo, G. Lebeau, J.-B. Pomet

September 10, 2019
15th IFAC Workshop on Time Delay Systems

The logo for Inria, consisting of the word "Inria" written in a red, cursive script font.

Motivation

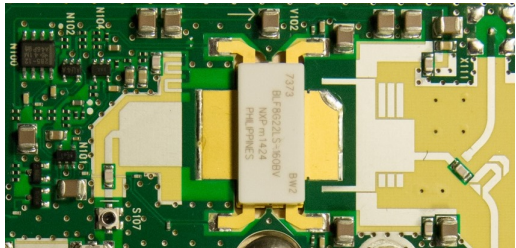
- Amplifiers at high frequency are ubiquitous (Cell phones, relays...). They need to be quick to design and produced in large quantities.
- Computer-assisted design (CAD) and simulation before production.
- Powerful “frequency simulation” tools give a reliable prediction of the response, but that response might be unstable.
- **Need** for a tool to predict stability/unstability in the frequency domain.

Inside

An amplifier is made of interconnected

- resistors, inductors, capacitors,
- diodes/transistors,
- lossless transmission lines which cannot be neglected at high frequency inducing delays.

Forcing periodic signal ►► periodic solution in the amplifier ►► amplified signal.



Harmonic Balance

The Harmonic Balance method, through Fourier development, Laplace transform and fixed point methods permits to :

- approximate the periodic solution of the circuit,
- linearize the circuit around the periodic solution,
- give a frequency response to a periodic signal which disturbs the linearized circuit.

Our focus : Structure of the harmonic transfer function, its singularities, links with stability.

Summary

- 1 Equations and stability
- 2 Harmonic transfer function
- 3 conclusion

General system, T -periodic :

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_{1,i}(t)y(t - \tau_i) \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i) + A_2(t)x(t), \quad t \geq s, \end{array} \right.$$

- $L^2 := \mathbb{R}^n \times L^2([-\tau_N, 0], \mathbb{R}^k)$.
- Solution operator $U(t, s) : L^2 \rightarrow L^2$
- Monodromy operator $U(T, 0)$
- L^2 exponential stability $\} \Leftrightarrow \left\{ \begin{array}{l} Sp(U(T, 0)) \text{ included in} \\ \text{disc of radius } r < 1 \end{array} \right.$

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- $L^2 := \mathbb{R}^n \times L^2([- \tau_N, 0], \mathbb{R}^k)$.
- Solution operator $U(t, s) : L^2 \rightarrow L^2$
- Monodromy operator $U(T, 0)$
- $\left. \begin{array}{l} L^2 \text{ exponential} \\ \text{stability} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} Sp(U(T, 0)) \text{ included in} \\ \text{disc of radius } r < 1 \end{array} \right.$

Behaviour at high frequency

High frequency system :

$$\begin{cases} x(t) = 0 \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i), \quad t \geq s, \end{cases}$$

- $\tilde{L}^2 := \{0_n\} \times L^2([-\tau_N, 0], \mathbb{R}^k)$.
- Solution operator $V(t, s) : \tilde{L}^2 \rightarrow \tilde{L}^2$.
- Monodromy operator $V(T, 0)$.

- $\left. \begin{array}{l} L^2 \text{ exponential} \\ \text{stability} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} Sp(V(T, 0)) \text{ included in} \\ \text{disc of radius } r < 1 \end{array} \right.$

Compact perturbation

Lemma

We have :

$$U(t, s) = V(t, s)P + K(t, s), \quad t \geq s$$

with $K(t, s)$ compact operator $L^2 \rightarrow L^2$ for all t, s and P the canonical projection $L^2 \rightarrow \tilde{L}^2$.

Theorem

If the high frequency system is exponentially stable then the monodromy operator $U(T, 0)$ possesses at most a finite number of eigenvalues ζ_1, \dots, ζ_n outside a disk of a radius strictly less than 1.

L^2 stability equivalent to C^0 stability

Proposition (Chitour and al 2016, BFLP 2019)

A periodic delay system is L^2 exponentially stable if and only if it is C^0 exponentially stable.

Theorem

Assuming the system at high frequency is L^2 exponentially stable. Then the general system is L^2 exponentially stable if and only if it is C^0 exponentially stable.

Assumption : High frequency system exponentially stable
 \Leftarrow always true for "realistic" circuit.

Input-Output system

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_i^1(t)y(t - \tau_i) + C_1(t)u(t) \\ y(t) = \sum_{i=1}^N B_i^2(t)y(t - \tau_i) + A_2(t)x(t) + C_2(t)u(t) \\ z(t) = \sum_{i=1}^N B_i^3(t)y(t - \tau_i) + A_3(t)x(t) + C_3(t)u(t), \quad t \geq 0, \end{array} \right.$$

- $x(t), y(t), z(t) = 0$ for $t < 0$,
- Input $u \in L^2_{loc}([0, +\infty), \mathbb{R})$ current perturbation, output z the voltage,
- All coefficients are T -periodic.

- $X(t, \alpha)$ response at time t to an impulse at time α
 $z(t) = \int_0^t X(t, \alpha) u(\alpha) d\alpha$
- $G(s, t) = \int_0^{+\infty} X(t, t - \alpha) e^{-s\alpha} d\alpha$: Laplace Transform
- $G_k(s) = \frac{1}{T} \int_0^T G(s, t) e^{ik\omega_0 t} dt$ with $\omega_0 := \frac{2\pi}{T}$

Definition (Harmonic Transfer Function HTF)

The infinite matrix $H(s)$ defined by $H_{m,n}(s) := G_{n-m}(s + \frac{2i\pi m}{T})$ for $s \in \mathbb{C}$ is called the harmonic transfer function.

$$Z(s) := \int_0^{+\infty} z(t)e^{-st} dt \text{ and } U(s) := \int_0^{+\infty} u(t)e^{-st} dt.$$

$$\begin{pmatrix} \vdots \\ Z(s+i\omega_0) \\ Z(s) \\ Z(s-i\omega_0) \\ \vdots \end{pmatrix} = \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \ddots \\ \cdots & G_2(s-i\omega_0) & G_1(s) & G_0(s+i\omega_0) & \cdots \\ \cdots & G_1(s-i\omega_0) & G_0(s) & G_{-1}(s+i\omega_0) & \cdots \\ \cdots & G_0(s-i\omega_0) & G_{-1}(s) & G_{-2}(s+i\omega_0) & \cdots \\ \ddots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ U(s+i\omega_0) \\ U(s) \\ U(s-i\omega_0) \\ \vdots \end{pmatrix}$$

- HTF is an operator valued analytic map
 (values: continuous ops $l^2(\mathbb{Z}) \rightarrow l^2(\mathbb{Z})$)
- its entries $\{G_n\}$ are complex valued analytic maps

Structure of the Harmonic Transfer Function

Define $z_{j,k} = \frac{\ln(\zeta_j) + 2ik\pi}{T}$ for j in $\{1\dots n\}$, k in \mathbb{Z} .

Theorem

In $\{s \in \mathbb{C}, \Re(s) \geq \gamma\}$ for some $\gamma < 0$,

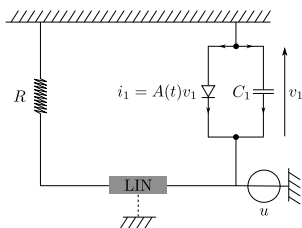
- H is a meromorphic operator $l^2(\mathbb{Z}) \rightarrow l^2(\mathbb{Z})$ with possibly poles at $\{z_{j,k}, j \in \{1\dots n\}, k \in \mathbb{Z}\}$.

Under observability/controllability assumptions,

- for all j , there is at least a k such that $z_{j,k}$ is a pole of H , and also a pole of one G_n .

If no **pole** in right half plane, exponential C^0 stability.

Example : Brayton 1976



Theorem

If $T/\tau_1 \notin \mathbb{Q}$, all points of a certain vertical line in the left-half plane are essential singularities of the HTF, as an operator valued analytic map.

But are they singularities for some G_n ?

Contribution

- Math. foundation of HF amplifiers stability decision in CAD
- Projection on the unstable part and rational approximation to find the poles
- Advances in stability of periodic delay systems

Open questions

- For fixed j , which $z_{j,k}$ is a pole of which G_n ?
(In practice, few G_n are computed.)
- Bound on the number of unstable poles?
- May the (stable) singularities of the G_n 's be other than poles ?