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# Adaptive Image Assimilation for 2D Velocity Reconstruction

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## Introduction

- 4D-Var is formulated as a nonlinear least-squares problem combining all the available heterogeneous information (i.e. mathematical models, observation, and *priori* knowledge) for calibrating the state of dynamical equations.
- Insufficient estimation of background, error covariance matrix and scarce observation will lead to **unreliable prediction**.
- **A regularizer based on structure dictionary sparsity would be a tool of choice to improve the performance.**

## Dictionary Learning and Regularization

### Dictionary Learning

- In the data-driven approach (CAI et al., 2014), a dictionary  $W$  is learned adaptively from dataset  $f$ ,

$$\min_{\rho, W} \|\rho - Wf\|_2^2 + \eta^2 \|\rho\|_0, \quad \text{s.t. } W^T W = I \quad (1)$$

$\rho$ : sparse coefficients,  $\|\cdot\|_0$ : number of nonzero elements

**Remark:** The updated dictionary (Fig.5) contains more directional structures from the dataset (Fig.4) than fixed ones (Fig.1-3).

Figure 1 – Multi-level Haar wavelet

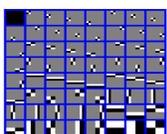


Figure 2 – Linear spline framelet

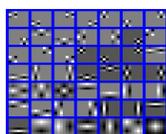
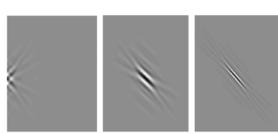


Figure 3 – Curvelet (at three scales)



### Regularization

- In view of the fact that features in images contribute to optimizing the flow (TITAUD et al., 2010), **we introduce a “vorticity structure dictionary” sparsity regularizer in the framework of 4D-Var for 2D velocity reconstruction.**

- In the case that the velocity field is indirectly accessible from the observation of passive tracers, the object function is expressed as

$$\min_{(u_0, v_0)} \frac{\delta}{2} \sum_{i=1}^N \|H_i C(t_i) - C^{obs}(t_i)\|_{\mathbf{R}_i}^2 + \frac{\alpha}{2} \|(u_0, v_0) - (u_0^b, v_0^b)\|_{\mathbf{B}_i}^2 + \underbrace{\mu \|W \zeta_0\|_1}_{\text{regularizer}} \quad (2)$$

$\zeta_0 = \frac{\partial v_0}{\partial x} - \frac{\partial u_0}{\partial y}$ : relative vorticity,  $W$ : learned dictionary

Figure 4 – Data from a vorticity model

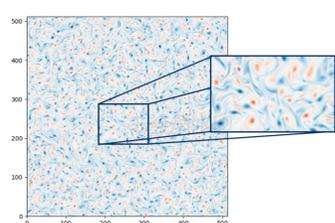
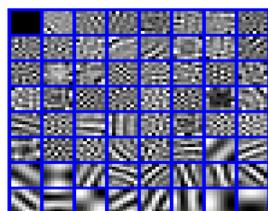


Figure 5 – Learned Dictionary from the data



## Numerical Implementation

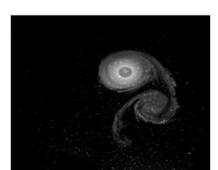
- **Models:** 2D shallow water equation and concentration transport equation

$$\begin{cases} \frac{\partial u}{\partial t} - (f + \zeta)v + \frac{\partial B}{\partial x} = -ru + v\Delta u \\ \frac{\partial v}{\partial t} + (f + \zeta)u + \frac{\partial B}{\partial y} = -rv + v\Delta v \\ \frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} + \frac{\partial hv}{\partial y} = 0 \\ \frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = 0 \end{cases} \quad (5)$$

$B = gh + \frac{1}{2}(u^2 + v^2)$ : the Bernoulli's potential,  $g$ : the gravity,  $f = f_0 + \beta y$ : the Coriolis parameter on the  $\beta$ -plane

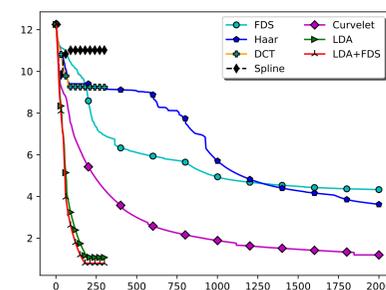
- **Settings**

- Test 1: Observation is generated by models and missing. Velocity is optimized via following different approaches. (First-order derivative smooth(FDS), Haar wavelet(Haar), discrete cosine framelet(DCT), linear spline framelet(Spline), Curvelet, **learned dictionary(LDA)** and the **hybrid one(LDA+FDS)**)
- Test 2: 400 images (Middle: initial moment; Right: final moment) are acquired from **Grenoble CORIOLIS platform** (Left). Velocity is reconstructed by the proposed method.

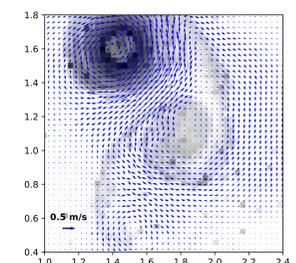
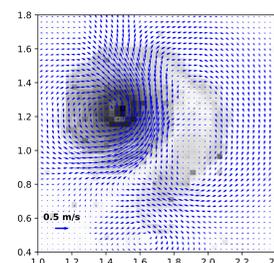


## Result

- Reconstruction errors comparison among seven approaches in test 1:



- The analyzed  $(u, v)$  at initial (Left) and final (Right) moment by the proposed method in test 2:



## Split Bregman Iteration for Adaptive 4D-Var

- Due to fast convergence, the split Bregman iteration is widely applied to solve the  $l_2 + l_1$ -norm non-smooth convex problem:

$$\min_{w, d} |d| + R(w) + \frac{\lambda}{2} \|d - \Phi(w)\|_2^2 \quad (3)$$

- Regarding to the adaptive 4D-Var problem, we redefine variables

$$w := (u_0, v_0), \quad \Phi(w) := W \zeta_0,$$

$$R(w) := \frac{\delta}{2} \sum_{i=1}^N \|H_i C(t_i) - C^{obs}(t_i)\|_{\mathbf{R}_i}^2 + \frac{\alpha}{2} \|w - w^b\|_{\mathbf{B}_i}^2$$

- **The split Bregman iteration (GOLDSTEIN et al., 2009)**

$$\begin{aligned} (w^{k+1}, d^{k+1}) &= \min_{w, d} |d| + R(w) + \frac{\lambda}{2} \|d - \Phi(w) - b^k\|_2^2 \\ b^{k+1} &= b^k + c(\Phi(w^{k+1}) - d^{k+1}) \end{aligned} \quad (4)$$

## Conclusion

- Owing to the sparsity penalty and the directional structure in the learned dictionary, promising performance is shown on 2D velocity reconstruction, such as **fast convergence and more consistency with the true state**.
- In the real data test, the proposed approach achieves to **reconstruct the anticyclonic secondary vortex**.

## Reference

- CAI, J.-F. et al. Data-driven tight frame construction and image denoising. **Applied and Computational Harmonic Analysis**, v. 37, n. 1, p. 89–105, 2014.
- GOLDSTEIN, T. et al. The split Bregman method for L1-regularized problems. **SIAM Journal on Imaging Sciences**, v. 2, n. 2, p. 323–343, 2009.
- TITAUD, O. et al. Assimilation of image sequences in numerical models. **Tellus A: Dynamic Meteorology and Oceanography**, v. 62, n. 1, p. 30–47, 2010.