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A CONVEX OPTIMISATION APPROACH TO YOULA'S BROADBAND MATCHING THEORY

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INTRODUCTION

UNMATCHED LOAD
Reflection of the load goes through the filter

UNWANTED REFLECTIONS

MATCHING FILTER
Filter input signal & reject reflections

NO REFLECTION

CLASSICAL APPROACH

matching filter optimisation → non convex techniques → no optimality guaranteed

Belevitch model

$$F = \frac{1}{q} \begin{pmatrix} \epsilon p^* & -\epsilon r^* \\ r & p \end{pmatrix}$$

$$qq^* = pp^* + rr^*$$

CONCEPT: OVERALL DESIGN + DE-EMBEDDING

YOULA'S DE-EMBEDDING CONDITIONS
Synthesize global system instead of matching filter

CONCEPT
Darlington equivalent + design: global system + load de-embedding

Interpolation conditions at load transmission zeros allow **de-embedding**

$$L_{21}(\alpha_i)L_{12}(\alpha_i) = 0 \quad 1 \leq i \leq M$$

$$S_{22} = L_{22} + \frac{L_{12}F_{22}L_{21}}{1 - L_{11}F_{22}} \Rightarrow S_{22}(\alpha_i) = L_{22}(\alpha_i)$$

BRINGING OPTIMISATION INTO CONTEXT

$S_{22} = UB$: U outer factor, B inner factor

Belevitch form of U :

$$|U_P(\omega)|^2 = \frac{P(\omega)}{P(\omega) + R(\omega)} \quad \begin{matrix} P = pp^* \\ R = rr^* \end{matrix}$$

$P, R \in \mathbb{P}_{2N}^+$ R fixed, $R(\alpha_i) = 0$

Optimisation problem:

$$\min_{P \in \mathbb{P}_{2N}^+} \max_{\omega \in \mathbb{I}} |U_P(\omega)| \quad \text{s.t.}$$

$$\exists B : U_P(\alpha_i)B(\alpha_i) = L_{22}(\alpha_i)$$

MINIMISE REFLECTION

relaxed set of S_{22} functions: degree can increase by $\deg(B)$

PRACTICAL USE
Choice of global response: **Butterworth / Tchebyshev**

Interpolation conditions on S_{22} :
Rigid approach: Not optimisation friendly

EXISTENCE OF FACTOR B

Given U_P , does B inner exist s.t. $B(\alpha_i) = \frac{L_{22}(\alpha_i)}{U_P(\alpha_i)}$?

THEORETICAL RESULTS & NUMERICAL IMPLEMENTATION

NEVANLINNA-PICK INTERPOLATION
Existence of inner function B satisfying interpolation conditions
Practical implementation of Youla's characterisation

$$\exists B \text{ s.t. } B(\alpha_i) = \gamma_i \Leftrightarrow \Delta(P) \succeq 0$$

$$\Delta(P)_{i,k} = \frac{1 - \gamma_i \bar{\gamma}_k}{j(\alpha_i - \bar{\alpha}_k)} \quad \gamma_i = \frac{L_{22}(\alpha_i)}{U_P(\alpha_i)}$$

$\Delta(P)$: **CONCAVE OPERATOR** $\Rightarrow \{P : \Delta(P) \succeq 0\}$ **CONVEX SET**

$$\Delta((1 - \lambda)P_1 + \lambda P_2) \succeq (1 - \lambda)\Delta(P_1) + \lambda\Delta(P_2)$$

Allow handling of Pick matrix by augmented lagrangian techniques

CONVEX OPTIMISATION PROBLEM
Modulus of U_P is obtained from the filtering function P/R
SDP with constrains on positive polynomials

define $\Gamma \geq \frac{P(\omega)}{R(\omega)}, \omega \in \mathbb{I}$

$$\min_{P \in \mathbb{P}_{2N}^+} \Gamma : \begin{matrix} \Delta(P) \succeq 0 \\ P(\omega) \leq \Gamma R(\omega) \quad \omega \in \mathbb{I} \end{matrix}$$

POSITIVITY: GRAM MATRIX PARAMETRISATION

$$P = P(\Theta_P), \quad P \in \mathbb{P}_{2N}^+ \Leftrightarrow \Theta_P \succeq 0$$

$$Q(\omega)_{a \leq \omega \leq b} \geq 0 \Leftrightarrow Q = F - (\omega - a)(\omega - b)G, \quad F, G \in \mathbb{P}_{2N}^+$$

AT THE OPTIMUM $\Delta(P)$ IS SINGULAR AND $\deg(B) < M$

$$\deg(U_P^{opt}) \leq \deg(S_{22}^{opt}) \leq \deg(U_P^{opt}) + M - 1$$

A NON-LINEAR SEMI-DEFINITE PROGRAM

$$\min_{\Theta_P, \Theta_F, \Theta_G, \Gamma} \Gamma \quad \text{s.t.} \quad \begin{matrix} \Theta_P, \Theta_F, \Theta_G, \Delta[P(\Theta_P)] \succeq 0 \\ Q(\Theta_F, \Theta_G) = \Gamma R - P(\Theta_P) \end{matrix}$$

RESULTS

