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# Polynomial Space-Time Trefftz-DG Method for Elasto-Acoustics

Hélène Barucq, Henri Calandra, Julien Diaz, Elvira Shishenina

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# POLYNOMIAL SPACE-TIME TREFFTZ-DG METHOD FOR ELASTO-ACOUSTICS

MATHIAS'18 | Computational Science Engineering & Data Science by TOTAL

# CONTEXT

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# TREFFTZ-DG METHODS

## BIBLIOGRAPHY

### Frequency domain:

Farhat, Tezaur, Harari, Hetmaniuk (2003 - 2006), Gabard (2007), Badics (2014), Hiptmair, Moiola, Pirugia (2011 - 2016), Barucq, Bendali, Diaz, Tordeux, Fares, Mattesi (2017)

### Time domain:

Tsukerman, Egger, Kretzschmar, Schnepf, Weiland (2014 - 2015), Wang, Tezaur, Farhat (2014), Moiola, Pirugia (2016 - 2018), Banjai, Lijoka (2017)

# DISCONTINUOUS GALERKIN METHODS

## PROS AND CONS

- Adapted to the complex geometries
- High-order accuracy and hp-adaptivity
- Explicit semi-discrete form
- Conservation laws

Higher number of degrees of freedom, compared to the methods with continuous approximation

# TREFFTZ METHOD

E. TREFFTZ, 1926

Basis functions are **local solutions of the initial PDEs**

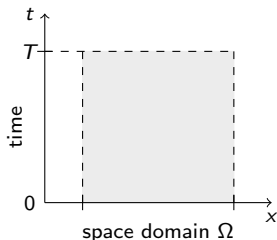
# MATHEMATICAL FORMULATION

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ACOUSTIC SYSTEM

# ACOUSTIC SYSTEM

## DOMAIN DESCRIPTION

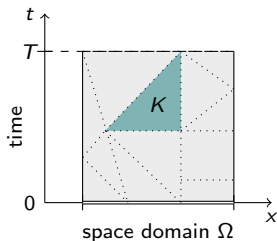


$$\left\{ \begin{array}{ll} \frac{1}{c^2 \rho} \frac{\partial p}{\partial t} + \operatorname{div} \mathbf{v} = f & \text{in } \Omega \times [0, T], \\ \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla p = 0 & \text{in } \Omega \times [0, T], \\ \mathbf{v}(\cdot, 0) = \mathbf{v}_0, \quad p(\cdot, 0) = p_0 & \text{in } \Omega, \\ \mathbf{v} \cdot \mathbf{n}^x = g_D & \text{in } \partial\Omega \times [0, T]. \end{array} \right.$$



# ACOUSTIC SYSTEM

## SPACE-TIME DISCRETIZATION

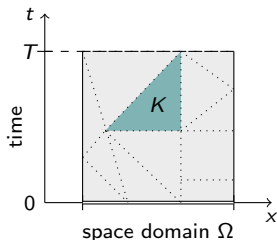


$$\mathcal{F}_h = \bigcup_{K \in \mathcal{T}_h} \partial K \text{ - mesh skeleton}$$

- .....  $\mathcal{F}^I$  - internal
- $\mathcal{F}^D$  - boundary
- ==  $\mathcal{F}^0$  - initial time
- $\mathcal{F}^T$  - final time

# ACOUSTIC SYSTEM

## SPACE-TIME DISCRETIZATION



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- .....  $\mathcal{F}^I$  - internal
- $\mathcal{F}^D$  - boundary
- ==  $\mathcal{F}^0$  - initial time
- $\mathcal{F}^T$  - final time

$$(\mathbf{v}_h, \mathbf{p}_h) \in \mathbf{V}^h(\mathcal{T}_h)^d \times V^h(\mathcal{T}_h) \equiv \mathbf{V}^h(\mathcal{T}_h)$$

$$(\omega, \mathbf{q}) \in V^h(\mathcal{T}_h)^d \times V^h(\mathcal{T}_h) \equiv \mathbf{V}^h(\mathcal{T}_h)$$

# ACOUSTIC SYSTEM

## SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_K \int_K p_h \left( \frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega \right) + \mathbf{v}_h \cdot \left( \rho \frac{\partial \omega}{\partial t} + \nabla q \right) \\
 & + \sum_K \int_{\partial K} \left( \frac{1}{c^2 \rho} \check{p}_h q + \check{\mathbf{v}}_h \cdot \omega \right) n_K^t + \left( q \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q
 \end{aligned}$$

# ACOUSTIC SYSTEM

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## TREFFTZ SPACE

$$\mathbf{T}(\mathcal{T}_h) \equiv \left\{ (\omega, q) \in \mathbf{V}^h(\mathcal{T}_h) \text{ s.t. } \frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega = \rho \frac{\partial \omega}{\partial t} + \nabla q = 0, \forall K \in \mathcal{T}_h \right\}$$

# ACOUSTIC SYSTEM

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## SPACE-TIME DISCRETIZATION IN $\mathbf{T}$

$$\sum_K \int_{\partial K} \left( \frac{1}{c^2 \rho} \check{p}_h q + \check{\mathbf{v}}_h \cdot \omega \right) n_K^t + \left( q \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \omega \right) \cdot \mathbf{n}_K^x = \sum_K \int_K f q$$

# ACOUSTIC SYSTEM

## TREFFTZ-DG FORMULATION

$$\sum_K \int_{\partial K} \left( \frac{1}{c^2 \rho} \tilde{p}_h q + \tilde{v}_h \cdot \omega \right) n_K^t + \left( q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q$$

## NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$$\begin{aligned} \begin{pmatrix} \hat{v}_h \\ \hat{p}_h \end{pmatrix} &\equiv \begin{pmatrix} \{v_h\} + \beta \llbracket p_h \rrbracket_x \\ \{p_h\} + \alpha \llbracket v_h \rrbracket_x \end{pmatrix} && \text{on } \mathcal{F}^I \\ \begin{pmatrix} \tilde{v}_h \\ \tilde{p}_h \end{pmatrix} &\equiv \begin{pmatrix} \{v_h\} \\ \{p_h\} \end{pmatrix} && \text{on } \mathcal{F}^I \end{aligned}$$

# ACOUSTIC SYSTEM

## TREFFTZ-DG FORMULATION

Seek  $(\mathbf{v}_h, p_h) \in \mathbf{V}^h(\mathcal{T}_h)$  s.t.  $\forall (\omega, q) \in \mathbf{T}(\mathcal{T}_h)$  it holds true:

$$\mathcal{A}_{TDG} \left( (\mathbf{v}_h, p_h); (\omega, q) \right) = \ell_{TDG} (\omega, q)$$



# ACOUSTIC SYSTEM

## WELL POSEDNESS

**Trefftz-DG formulations for the first order Acoustic System<sup>1,2</sup>, Elastodynamic System<sup>2</sup>, and Elasto-Acoustic System<sup>2</sup> are **well-posed**.**

<sup>1</sup> A. Moiola & I. Perugia.

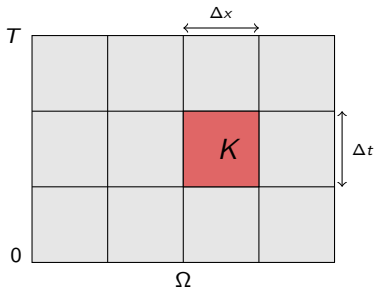
**A space-time Trefftz- dG method for the acoustic wave equation in first-order formulation.** (2018)  
Numerische Mathematik, 138(2):389-435

<sup>2</sup> H. Barucq, H. Calandra, J. Diaz & Elvira Shishenina.

**Space-Time Trefftz-Discontinuous Galerkin Approximation for Elasto-Acoustics.** (2017)  
<https://hal.inria.fr/hal-01614126/document>

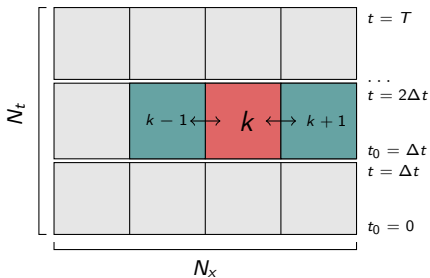
# IMPLEMENTATION OF THE ALGORITHM

## MESH AND ELEMENT NUMBERING



# IMPLEMENTATION OF THE ALGORITHM

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# IMPLEMENTATION OF THE ALGORITHM

## BILINEAR FORM

$$\mathcal{A}_{TDG} = \underbrace{\int_{\mathcal{F}^T} + \int_{\mathcal{F}^0}}_{\mathcal{A}_{TDG}^{\Omega}} + \underbrace{\int_{\mathcal{F}^I} + \int_{\mathcal{F}^D}}_{\mathcal{A}_{TDG}^I}$$

## GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x M_{\Omega} + \bar{\Delta}_t M_I$$

# IMPLEMENTATION OF THE ALGORITHM

## BILINEAR FORM

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## GLOBAL SPACE-TIME MATRIX

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$M_{\Omega}$  - block-diagonal

# IMPLEMENTATION OF THE ALGORITHM

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## GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x M_{\Omega} + \bar{\Delta}_t M_I$$

$M_{\Omega}$  - block-diagonal

$M_I$  - sparse

# IMPLEMENTATION OF THE ALGORITHM

## GLOBAL MATRIX INVERSION

$$\begin{aligned} \mathbf{M}^{-1} &\equiv \left[ \bar{\Delta}_x M_\Omega + \bar{\Delta}_t M_I \right]^{-1} \\ &= \\ &\left[ I + \frac{\bar{\Delta}_t}{\bar{\Delta}_x} (M_\Omega^{-1} M_I) \right]^{-1} \left[ \bar{\Delta}_x M_\Omega \right]^{-1} \end{aligned}$$

TAYLOR EXPANSION (SMALL ENOUGH  $\frac{\bar{\Delta}_t}{\bar{\Delta}_x} |M_\Omega^{-1} M_I| \propto \frac{\Delta t}{\Delta x} |M_\Omega^{-1} M_I|$ )

$$\mathbf{M}^{-1} \approx \left[ \sum_{i=0}^n (-1)^i \left( \frac{\bar{\Delta}_t}{\bar{\Delta}_x} \right)^i (M_\Omega^{-1} M_I)^i \right] \left[ \bar{\Delta}_x M_\Omega \right]^{-1}$$

# IMPLEMENTATION OF THE ALGORITHM

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block-diagonal



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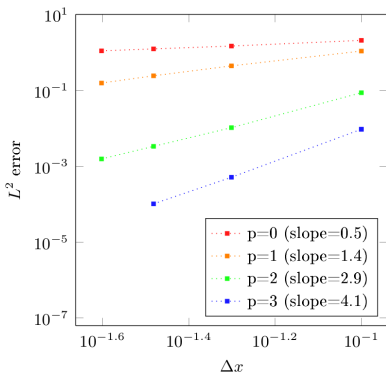
block-diagonal x sparse

# NUMERICAL RESULTS

## 2D ACOUSTIC AND ELASTODYNAMIC SYSTEMS

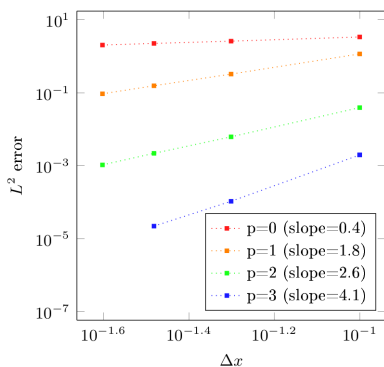
2D Acoustic domain.

Convergence of velocity  $v$  in function of cell size



2D Elastodynamic domain.

Convergence of velocity  $v$  in function of cell size



# TENT PITCHING MESHES

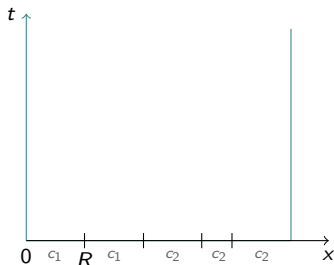
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APPLICATION TO TREFFTZ-DG METHOD

# TENT PITCHING MESHES

## 1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



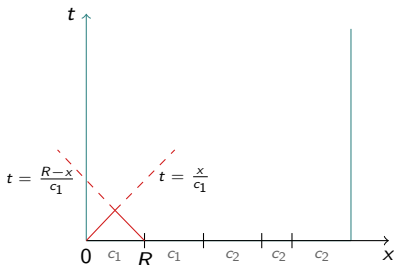
## SOLUTION

$$p(x, t) = \frac{1}{2} \left( p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left( p_1(x - ct) + p_1(x + ct) \right)$$

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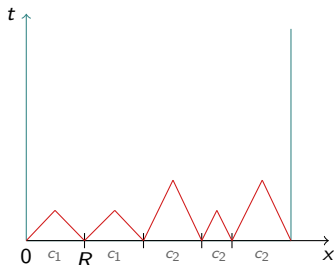
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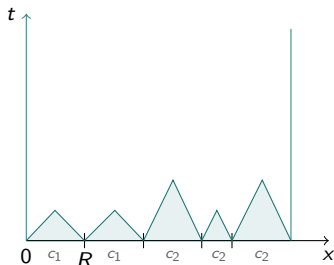
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$$p(x, t) = \frac{1}{2} \left( p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left( p_1(x - ct) + p_1(x + ct) \right)$$

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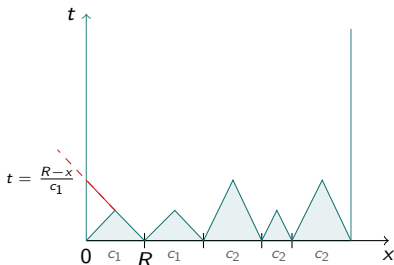
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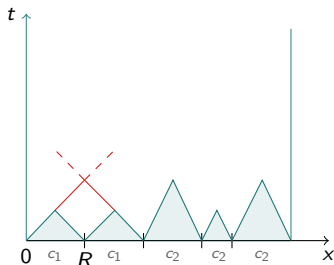
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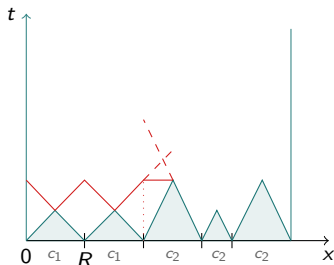
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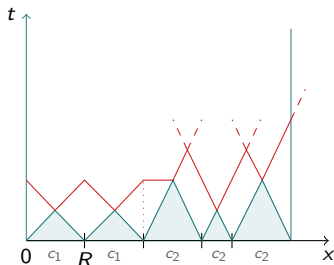
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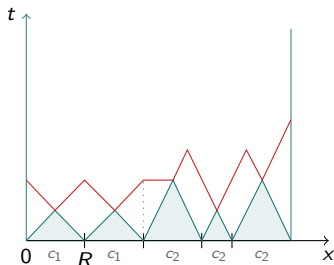
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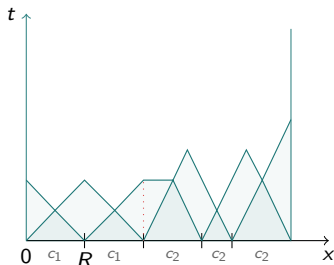
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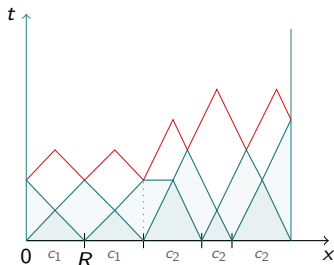
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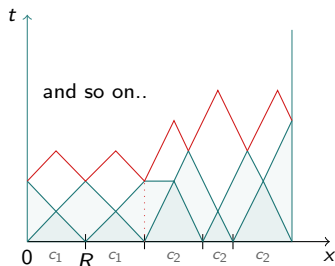
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$$p(x, t) = \frac{1}{2} (p_0(x - ct) + p_0(x + ct)) + \frac{1}{2} (p_1(x - ct) + p_1(x + ct))$$

# TENT PITCHING MESHES

## 1D WAVE EQUATION

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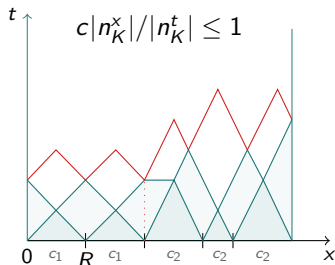
## SOLUTION

$$p(x, t) = \frac{1}{2} \left( p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2} \left( p_1(x - ct) + p_1(x + ct) \right)$$

# TENT PITCHING MESHES

## 1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



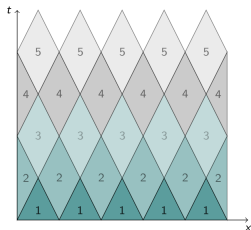
## SOLUTION

$$p(x, t) = \frac{1}{2} (p_0(x - ct) + p_0(x + ct)) + \frac{1}{2} (p_1(x - ct) + p_1(x + ct))$$

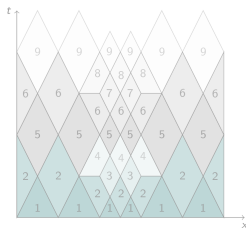


# TENT PITCHING MESHES

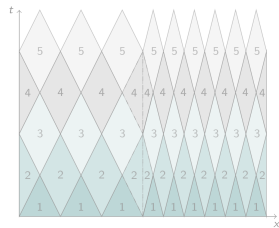
## 1D TENT PITCHING MESHES



Uniform tent mesh  
(homogeneous medium)



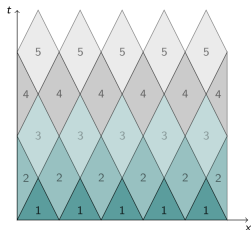
Non-uniform tent mesh  
(homogeneous medium)



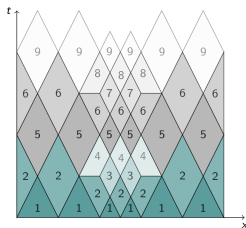
Non-uniform tent mesh  
(heterogeneous medium)

# TENT PITCHING MESHES

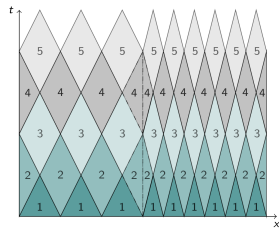
## 1D TENT PITCHING MESHES



Uniform tent mesh  
(homogeneous medium)



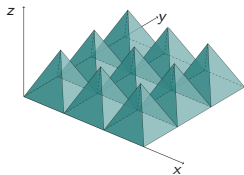
Non-uniform tent mesh  
(homogeneous medium)



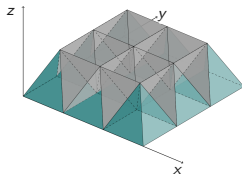
Non-uniform tent mesh  
(heterogeneous medium)

# TENT PITCHING MESHES

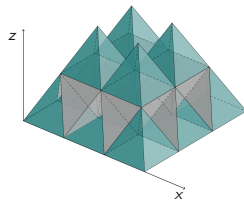
## 2D TENT PITCHING MESHES



1<sup>st</sup> tent



2<sup>nd</sup> tent



3<sup>rd</sup> tent

# TENT PITCHING MESHES

## ANALYTICAL STABILITY CONDITIONS <sup>1</sup>

$$c|n_K^x|/|n_K^t| \leq 1 \quad - \text{TDG acoustic formulation}$$

$$\sqrt{|\mathbf{C}|}|n_K^x|/\sqrt{\rho}|n_K^t| \leq 1 \quad - \text{TDG elastodynamic formulation}$$

<sup>1</sup> H. Barucq, H. Calandra, J. Diaz & Elvira Shishenina.

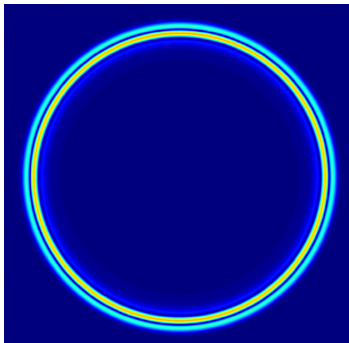
**Space-Time Discretization of Elasto-Acoustic Wave Equation in Polynomial Trefftz-DG Bases.** (2018)  
PhD thesis [under review]

# TENT PITCHING MESHES

## PROPAGATION OF NUMERICAL VELOCITY

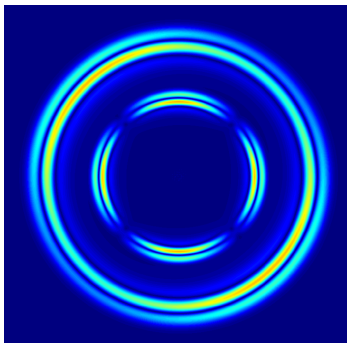
2D Acoustic domain.

$L^2$  norm of numerical velocity  $v$



2D Elastodynamic domain.

$L^2$  norm of numerical velocity  $v$

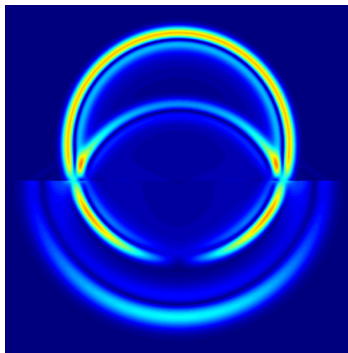


# TENT PITCHING MESHES

## PROPAGATION OF NUMERICAL VELOCITY

2D Elasto-Acoustic domain.

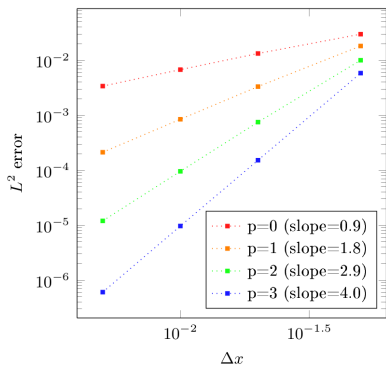
$L^2$  norm of numerical velocity  $v$



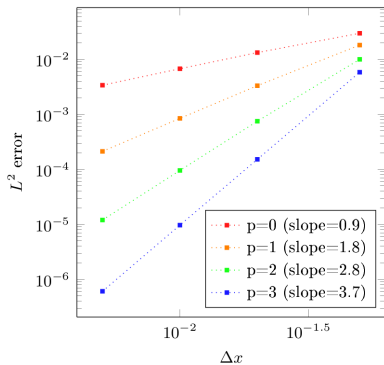
# TENT PITCHING MESHES

## CONVERGENCE CURVES

2D Acoustic domain.  
Convergence of velocity  $v$  in function of cell size

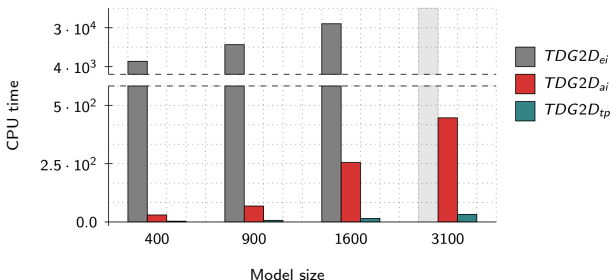


2D Elastodynamic domain.  
Convergence of velocity  $v$  in function of cell size



# TENT PITCHING MESHES

## SPEEDUP FOR DIFFERENT MODEL SIZES





# CONCLUSION

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# CONCLUSION

## ON-GOING WORK AND PERSPECTIVES

### **Standard Trefftz-DG method:**

- Development of 2D-3D  $hp$  propagators
- Analytical study of convergence, comparison with numerical results
- Study of CFL condition, comparison with standard DG method

### **Tent pitching Trefftz-DG method:**

- Mesh generation study
- Implementation of the boundary conditions
- 3D case

THANK YOU!

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# IMPLEMENTATION OF THE ALGORITHM

## POLYNOMIAL BASIS (ORDER 3)

**p=0, ndof=2**

$$\phi_1^v = 0$$

$$\phi_2^v = 1$$

$$\phi_1^p = -c_F$$

$$\phi_2^p = 0$$

**p=1, ndof=4**

$$\phi_3^v = x$$

$$\phi_4^v = c_F t$$

$$\phi_3^p = -c_F^2 t$$

$$\phi_4^p = -c_F x$$

**p=2, ndof=6**

$$\phi_5^v = -\frac{x^2}{2} - \frac{c_F^2 t^2}{2}$$

$$\phi_6^v = -c_F x t$$

$$\phi_5^p = c_F^2 x t$$

$$\phi_6^p = c_F \left( \frac{x^2}{2} + \frac{c_F^2 t^2}{2} \right)$$

**p=3, ndof=8**

$$\phi_7^v = -\frac{x^3}{6} - \frac{x c_F^2 t^2}{2}$$

$$\phi_8^v = -\frac{c_F^3 t^3}{6} - \frac{x^2 c_F t}{2}$$

$$\phi_7^p = c_F \left( \frac{c_F^3 t^3}{6} + \frac{x^2 c_F t}{2} \right)$$

$$\phi_8^p = c_F \left( \frac{x^3}{6} + \frac{x c_F^2 t^2}{2} \right)$$

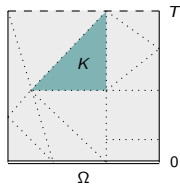
# ACOUSTIC SYSTEM

## TREFFTZ-DG FORMULATION

$$\sum_K \int_{\partial K} \left( \frac{1}{c^2 \rho} \check{p}_h q + \check{v}_h \cdot \omega \right) n_K^t + \left( q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q$$

## NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$$\begin{aligned} \begin{pmatrix} \hat{v}_h \\ \hat{p}_h \end{pmatrix} &\equiv \begin{pmatrix} \{v_h\} + \beta [p_h]_x \\ \{p_h\} + \alpha [v_h]_x \end{pmatrix} && \text{on } \mathcal{F}^I & \dots \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} &\equiv \begin{pmatrix} \{v_h\} \\ \{p_h\} \end{pmatrix} && \text{on } \mathcal{F}^I & \dots \\ \begin{pmatrix} \hat{v}_h \cdot n_K^x \\ \hat{p}_h \end{pmatrix} &\equiv \begin{pmatrix} g_D \\ p_h + \alpha (v_h \cdot n_K^x - g_D) \end{pmatrix} && \text{on } \mathcal{F}^D & \text{---} \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} &\equiv \begin{pmatrix} v_h \\ p_h \end{pmatrix} && \text{on } \mathcal{F}^T & \text{-- --} \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} &\equiv \begin{pmatrix} v_0 \\ p_0 \end{pmatrix} && \text{on } \mathcal{F}^0 & \equiv \end{aligned}$$



# REFERENCES

J.S.Hesthaven T.Warburton. *Nodal Discontinuous Galerkin Methods. Algorithms, analysis, and applications.* 2007.

F.Kretzschmar A.Moiola I.Perugia. *A Priori Error Analysis of Space-Time Trefftz Discontinuous Galerkin Methods for Wave Problems.* 2016.

O.C.Zienkiewicz. *Trefftz Type Approximation and the Generalized Finite Element Method - History and Development.* 1997.

I.Herrera. *Trefftz Method: a General Theory.* 2000

A.Maciag. *Wave Polynomials in Elasticity Problems.* 2006