



Polynomial Space-Time Trefftz-DG Method for Elasto-Acoustics

Hélène Barucq, Henri Calandra, Julien Diaz, Elvira Shishenina

► To cite this version:

Hélène Barucq, Henri Calandra, Julien Diaz, Elvira Shishenina. Polynomial Space-Time Trefftz-DG Method for Elasto-Acoustics. MATHIAS 2018, Oct 2018, Serris, France. hal-01907784

HAL Id: hal-01907784

<https://inria.hal.science/hal-01907784>

Submitted on 29 Oct 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

HÉLÈNE BARUCQ¹ | HENRI CALANDRA² | JULIEN DIAZ¹ | ELVIRA SHISHENINA^{1,2}

Depth Imaging Partnership



¹ Magique-3D, Inria | ² TOTAL S.A.

POLYNOMIAL SPACE-TIME TREFFTZ-DG METHOD FOR ELASTO-ACOUSTICS

MATHIAS'18 | Computational Science Engineering & Data Science by TOTAL

Serris - France | 2018

CONTEXT

TREFFTZ-DG METHODS

BIBLIOGRAPHY

Frequency domain:

Farhat, Tezaur, Harari, Hetmaniuk (2003 - 2006), Gabard (2007), Badics (2014),
Hiptmair, Moiola, Pirugia (2011 - 2016), Barucq, Bendali, Diaz, Tordeux,
Fares, Mattesi (2017)

Time domain:

Tsukerman, Egger, Kretzschmar, Schnepf, Weiland (2014 - 2015), Wang,
Tezaur, Farhat (2014), Moiola, Pirugia (2016 - 2018), Banjai, Lijoka (2017)

DISCONTINUOUS GALERKIN METHODS

PROS AND CONS

Adapted to the complex geometries

High-order accuracy and hp-adaptivity

Explicit semi-discrete form

Conservation laws

Higher number of degrees of freedom, compared
to the methods with continuous approximation

TREFFTZ METHOD

E. TREFFTZ, 1926

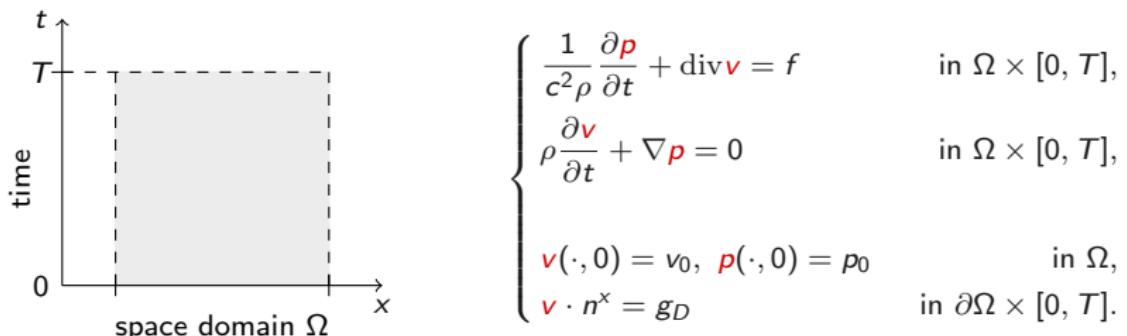
Basis functions are **local solutions of the initial PDEs**

MATHEMATICAL FORMULATION

ACOUSTIC SYSTEM

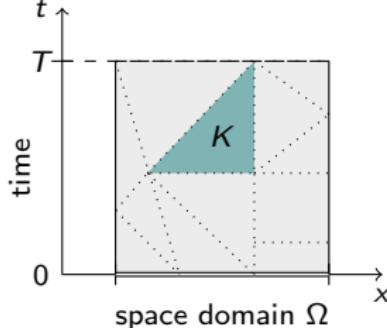
ACOUSTIC SYSTEM

DOMAIN DESCRIPTION



ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION

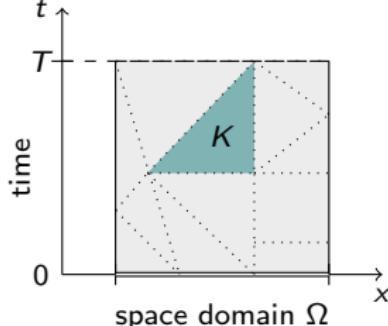


$$\mathcal{F}_h = \bigcup_{K \in \mathcal{T}_h} \partial K - \text{mesh skeleton}$$

- \mathcal{F}^I - internal
- \mathcal{F}^D - boundary
- == \mathcal{F}^0 - initial time
- \mathcal{F}^T - final time

ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION



$$\mathcal{F}_h = \bigcup_{K \in \mathcal{T}_h} \partial K - \text{mesh skeleton}$$

- \mathcal{F}^I - internal
- \mathcal{F}^D - boundary
- == \mathcal{F}^0 - initial time
- \mathcal{F}^T - final time

$$(\mathbf{v}_h, \mathbf{p}_h) \in V^h(\mathcal{T}_h)^d \times V^h(\mathcal{T}_h) \equiv \mathbf{V}^h(\mathcal{T}_h)$$

$$(\omega, q) \in V^h(\mathcal{T}_h)^d \times V^h(\mathcal{T}_h) \equiv \mathbf{V}^h(\mathcal{T}_h)$$

ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION

$$\begin{aligned} & - \sum_K \int_K \textcolor{red}{p_h} \left(\frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega \right) + \textcolor{red}{v_h} \cdot \left(\rho \frac{\partial \omega}{\partial t} + \nabla q \right) \\ & + \sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{p}_h q + \check{v}_h \cdot \omega \right) n_K^t + \left(q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q \end{aligned}$$

ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION

$$\begin{aligned} & - \sum_K \int_K \textcolor{red}{p_h} \left(\frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega \right) + \textcolor{red}{v_h} \cdot \left(\rho \frac{\partial \omega}{\partial t} + \nabla q \right) \\ & + \sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{p}_h q + \check{v}_h \cdot \omega \right) n_K^t + \left(q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q \end{aligned}$$

ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & -\sum_K \int_K \textcolor{red}{p_h} \left(\frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega \right) + \textcolor{red}{v_h} \cdot \left(\rho \frac{\partial \omega}{\partial t} + \nabla q \right) \\
 & + \sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{p}_h q + \check{v}_h \cdot \omega \right) n_K^t + \left(q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q
 \end{aligned}$$

TREFFTZ SPACE

$$\mathbf{T}(\mathcal{T}_h) \equiv \left\{ (\omega, q) \in \mathbf{V}^h(\mathcal{T}_h) \text{ s.t. } \frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega = \rho \frac{\partial \omega}{\partial t} + \nabla q = 0, \quad \forall K \in \mathcal{T}_h \right\}$$

ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & -\sum_K \int_K \textcolor{red}{p_h} \left(\frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega \right) + \textcolor{red}{v_h} \cdot \left(\rho \frac{\partial \omega}{\partial t} + \nabla q \right) \\
 & + \sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{p}_h q + \check{v}_h \cdot \omega \right) n_K^t + \left(q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q
 \end{aligned}$$

TREFFTZ SPACE

$$\mathbf{T}(\mathcal{T}_h) \equiv \left\{ (\omega, q) \in \mathbf{V}^h(\mathcal{T}_h) \text{ s.t. } \frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega = \rho \frac{\partial \omega}{\partial t} + \nabla q = 0, \quad \forall K \in \mathcal{T}_h \right\}$$

SPACE-TIME DISCRETIZATION IN \mathbf{T}

$$\sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{p}_h q + \check{v}_h \cdot \omega \right) n_K^t + \left(q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q$$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

$$\sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{\mathbf{p}}_h q + \check{\mathbf{v}}_h \cdot \omega \right) n_K^t + \left(q \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q$$

NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$$\begin{aligned} \begin{pmatrix} \hat{\mathbf{v}}_h \\ \hat{\mathbf{p}}_h \end{pmatrix} &\equiv \begin{pmatrix} \{\mathbf{v}_h\} + \beta [\![\mathbf{p}_h]\!]_x \\ \{\mathbf{p}_h\} + \alpha [\![\mathbf{v}_h]\!]_x \end{pmatrix} & \text{on } \mathcal{F}^I \\ \begin{pmatrix} \check{\mathbf{v}}_h \\ \check{\mathbf{p}}_h \end{pmatrix} &\equiv \begin{pmatrix} \{\mathbf{v}_h\} \\ \{\mathbf{p}_h\} \end{pmatrix} & \text{on } \mathcal{F}^I \end{aligned}$$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

Seek $(\mathbf{v}_h, p_h) \in \mathbf{V}^h(\mathcal{T}_h)$ s.t. $\forall (\omega, q) \in \mathbf{T}(\mathcal{T}_h)$ it holds true:

$$\mathcal{A}_{TDG}\left((\mathbf{v}_h, p_h); (\omega, q)\right) = \ell_{TDG}(\omega, q)$$

ACOUSTIC SYSTEM

WELL POSEDNESS

Trefftz-DG formulations for the first order Acoustic System^{1,2}, Elastodynamic System², and Elasto-Acoustic System² are well-posed.

¹ A. Moiola & I. Perugia.

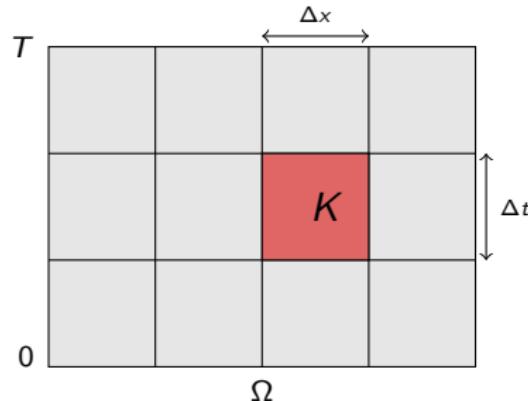
A space-time Trefftz-dG method for the acoustic wave equation in first-order formulation. (2018)
Numerische Mathematik, 138(2):389-435

² H. Barucq, H. Calandra, J. Diaz & Elvira Shishenina.

Space-Time Trefftz-Discontinuous Galerkin Approximation for Elasto-Acoustics. (2017)
<https://hal.inria.fr/hal-01614126/document>

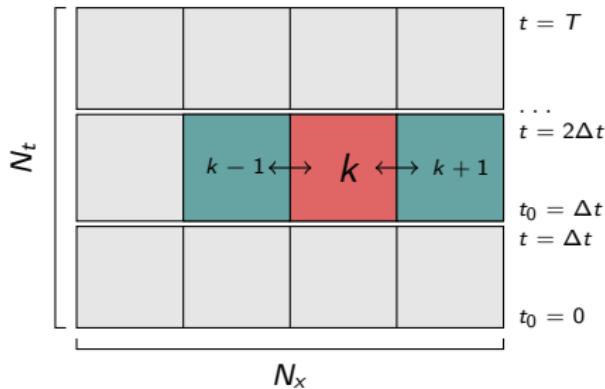
IMPLEMENTATION OF THE ALGORITHM

MESH AND ELEMENT NUMBERING



IMPLEMENTATION OF THE ALGORITHM

MESH AND ELEMENT NUMBERING



IMPLEMENTATION OF THE ALGORITHM

BILINEAR FORM

$$\mathcal{A}_{TDG} = \underbrace{\int_{\mathcal{F}^T} + \int_{\mathcal{F}^0}}_{\mathcal{A}_{TDG}^\Omega} + \underbrace{\int_{\mathcal{F}^I} + \int_{\mathcal{F}^D}}_{\mathcal{A}_{TDG}^I}$$

GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x \mathbf{M}_\Omega + \bar{\Delta}_t \mathbf{M}_I$$

IMPLEMENTATION OF THE ALGORITHM

BILINEAR FORM

$$\mathcal{A}_{TDG} = \underbrace{\int_{\mathcal{F}^T} + \int_{\mathcal{F}^0}}_{\mathcal{A}_{TDG}^\Omega} + \underbrace{\int_{\mathcal{F}^I} + \int_{\mathcal{F}^D}}_{\mathcal{A}_{TDG}^I}$$

GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x \mathbf{M}_\Omega + \bar{\Delta}_t \mathbf{M}_I$$

\mathbf{M}_Ω - block-diagonal

IMPLEMENTATION OF THE ALGORITHM

BILINEAR FORM

$$\mathcal{A}_{TDG} = \underbrace{\int_{\mathcal{F}^T} + \int_{\mathcal{F}^0}}_{\mathcal{A}_{TDG}^\Omega} + \underbrace{\int_{\mathcal{F}^I} + \int_{\mathcal{F}^D}}_{\mathcal{A}_{TDG}^I}$$

GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x \mathbf{M}_\Omega + \bar{\Delta}_t \mathbf{M}_I$$

\mathbf{M}_Ω - block-diagonal \mathbf{M}_I - sparse

IMPLEMENTATION OF THE ALGORITHM

GLOBAL MATRIX INVERSION

$$\mathbf{M}^{-1} \equiv \left[\bar{\Delta}_x \mathbf{M}_\Omega + \bar{\Delta}_t \mathbf{M}_I \right]^{-1}$$

$$= \\ \left[I + \frac{\bar{\Delta}_t}{\bar{\Delta}_x} (\mathbf{M}_\Omega^{-1} \mathbf{M}_I) \right]^{-1} \left[\bar{\Delta}_x \mathbf{M}_\Omega \right]^{-1}$$

TAYLOR EXPANSION (SMALL ENOUGH $\frac{\bar{\Delta}_t}{\bar{\Delta}_x} |\mathbf{M}_\Omega^{-1} \mathbf{M}_I| \propto \frac{\Delta t}{\Delta x} |\mathbf{M}_\Omega^{-1} \mathbf{M}_I|$)

$$\mathbf{M}^{-1} \approx \left[\sum_{i=0}^n (-1)^i \left(\frac{\bar{\Delta}_t}{\bar{\Delta}_x} \right)^i (\mathbf{M}_\Omega^{-1} \mathbf{M}_I)^i \right] \left[\bar{\Delta}_x \mathbf{M}_\Omega \right]^{-1}$$

IMPLEMENTATION OF THE ALGORITHM

GLOBAL MATRIX INVERSION

$$\mathbf{M}^{-1} \equiv \left[\bar{\Delta}_x \mathbf{M}_\Omega + \bar{\Delta}_t \mathbf{M}_I \right]^{-1}$$

=

$$\left[I + \frac{\bar{\Delta}_t}{\bar{\Delta}_x} (\mathbf{M}_\Omega^{-1} \mathbf{M}_I) \right]^{-1} \left[\bar{\Delta}_x \mathbf{M}_\Omega \right]^{-1}$$

TAYLOR EXPANSION (SMALL ENOUGH $\frac{\bar{\Delta}_t}{\bar{\Delta}_x} |\mathbf{M}_\Omega^{-1} \mathbf{M}_I| \propto \frac{\Delta t}{\Delta x} |\mathbf{M}_\Omega^{-1} \mathbf{M}_I|$)

$$\mathbf{M}^{-1} \approx \left[\sum_{i=0}^n (-1)^i \left(\frac{\bar{\Delta}_t}{\bar{\Delta}_x} \right)^i (\mathbf{M}_\Omega^{-1} \mathbf{M}_I)^i \right] \left[\bar{\Delta}_x \mathbf{M}_\Omega \right]^{-1}$$

block-diagonal

IMPLEMENTATION OF THE ALGORITHM

GLOBAL MATRIX INVERSION

$$\mathbf{M}^{-1} \equiv \left[\bar{\Delta}_x \mathbf{M}_\Omega + \bar{\Delta}_t \mathbf{M}_I \right]^{-1}$$

=

$$\left[I + \frac{\bar{\Delta}_t}{\bar{\Delta}_x} (\mathbf{M}_\Omega^{-1} \mathbf{M}_I) \right]^{-1} \left[\bar{\Delta}_x \mathbf{M}_\Omega \right]^{-1}$$

TAYLOR EXPANSION (SMALL ENOUGH $\frac{\bar{\Delta}_t}{\bar{\Delta}_x} |\mathbf{M}_\Omega^{-1} \mathbf{M}_I| \propto \frac{\Delta t}{\Delta x} |\mathbf{M}_\Omega^{-1} \mathbf{M}_I|$)

$$\mathbf{M}^{-1} \approx \left[\sum_{i=0}^n (-1)^i \left(\frac{\bar{\Delta}_t}{\bar{\Delta}_x} \right)^i (\mathbf{M}_\Omega^{-1} \mathbf{M}_I)^i \right] \left[\bar{\Delta}_x \mathbf{M}_\Omega \right]^{-1}$$

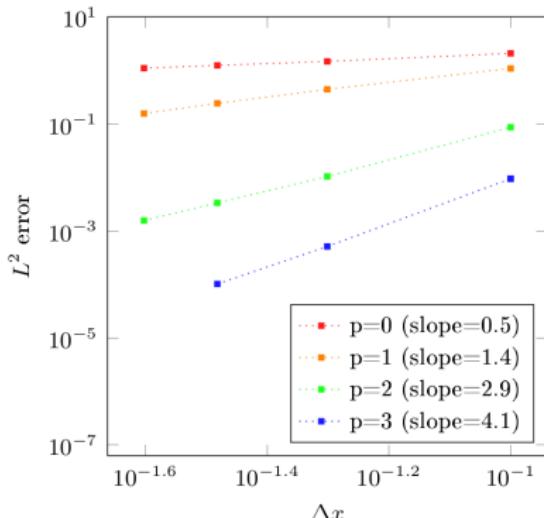
block-diagonal \times sparse

NUMERICAL RESULTS

2D ACOUSTIC AND ELASTODYNAMIC SYSTEMS

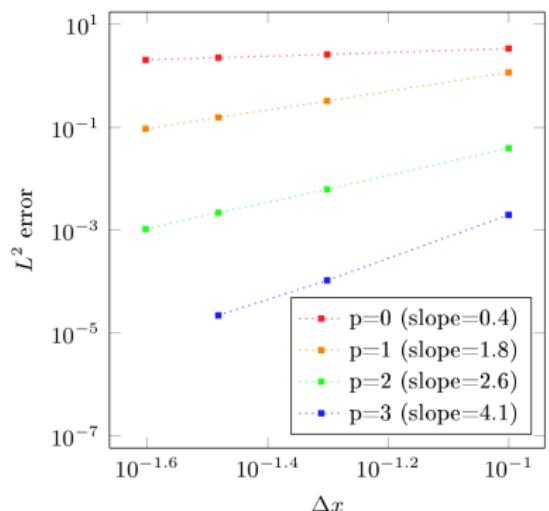
2D Acoustic domain.

Convergence of velocity v in function of cell size



2D Elastodynamic domain.

Convergence of velocity v in function of cell size



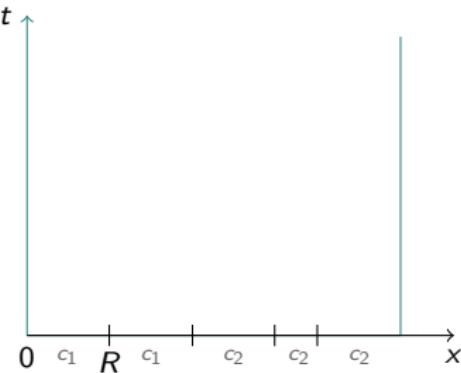
TENT PITCHING MESHES

APPLICATION TO TREFFTZ-DG METHOD

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



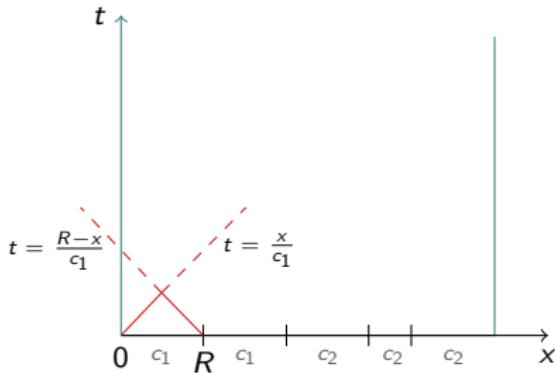
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



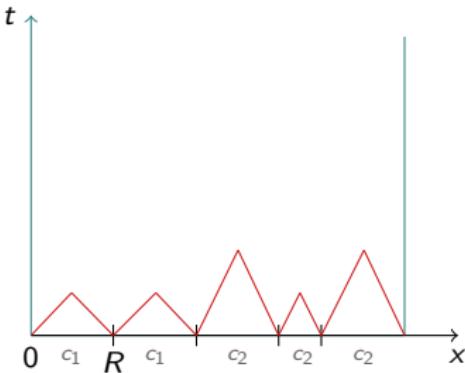
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



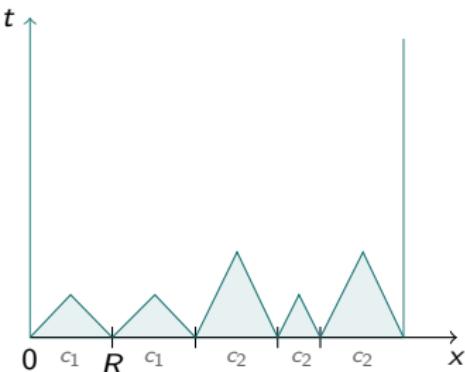
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



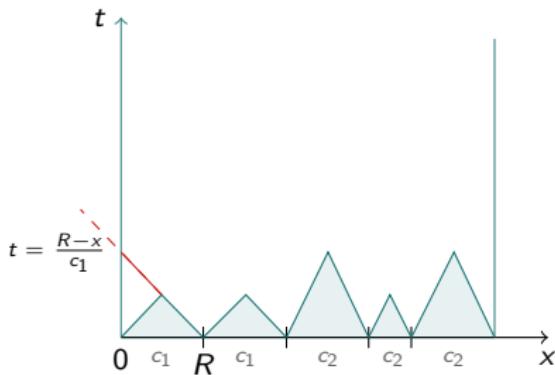
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



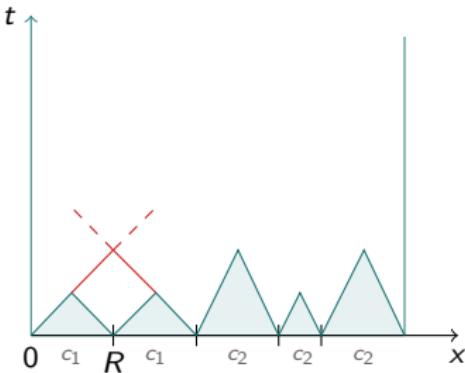
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



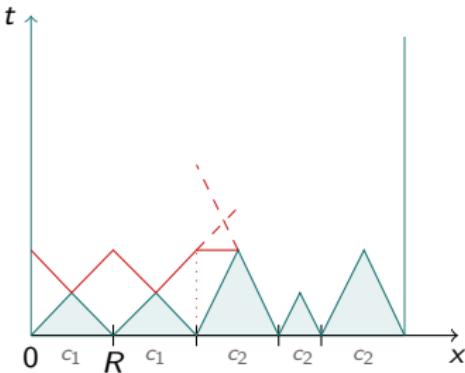
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



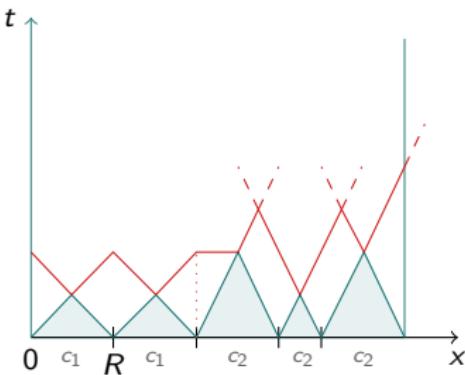
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



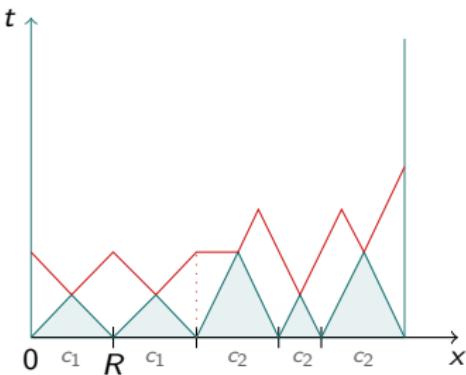
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



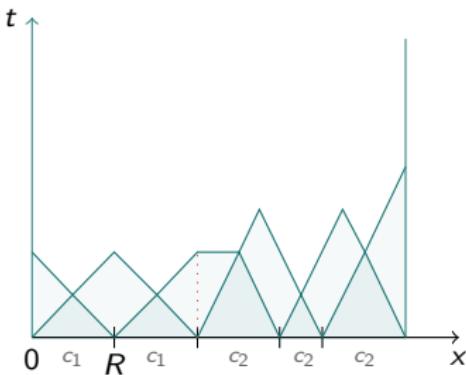
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



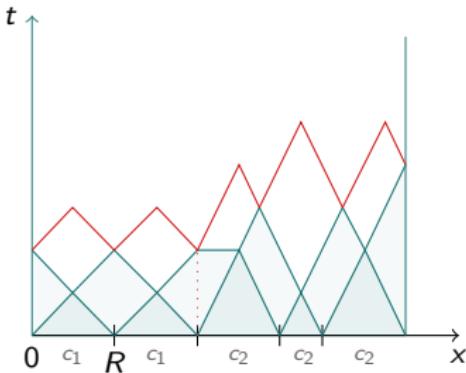
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



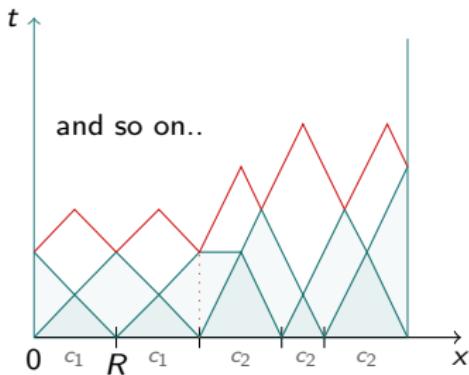
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



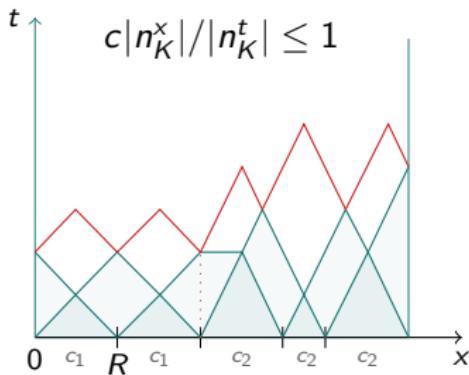
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 \mathbf{p}}{\partial t^2} - \Delta \mathbf{p} = 0, \\ \mathbf{p}(\cdot, 0) = p_0, \\ \frac{\partial \mathbf{p}}{\partial t}(\cdot, 0) = p_1. \end{cases}$$

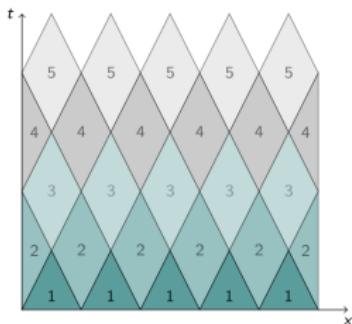


SOLUTION

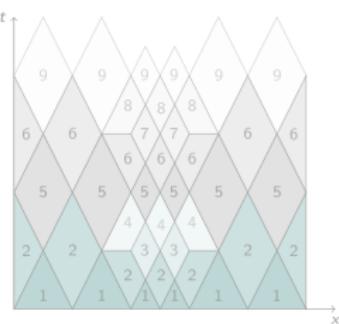
$$\mathbf{p}(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

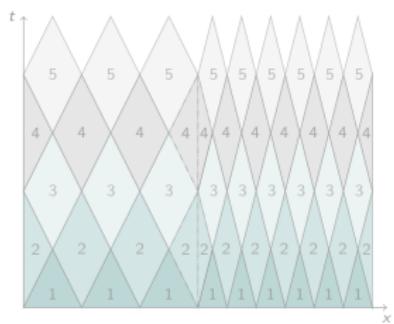
1D TENT PITCHING MESHES



Uniform tent mesh
(homogeneous medium)



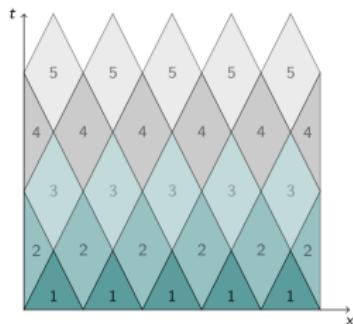
Non-uniform tent mesh
(homogeneous medium)



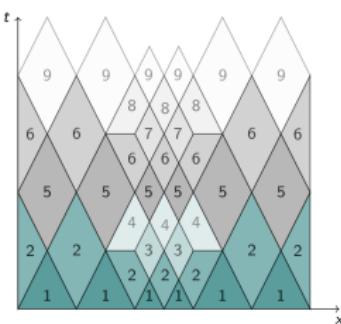
Non-uniform tent mesh
(heterogeneous medium)

TENT PITCHING MESHES

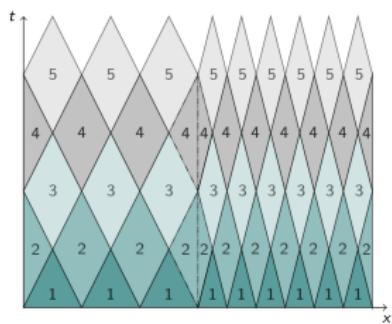
1D TENT PITCHING MESHES



Uniform tent mesh
(homogeneous medium)



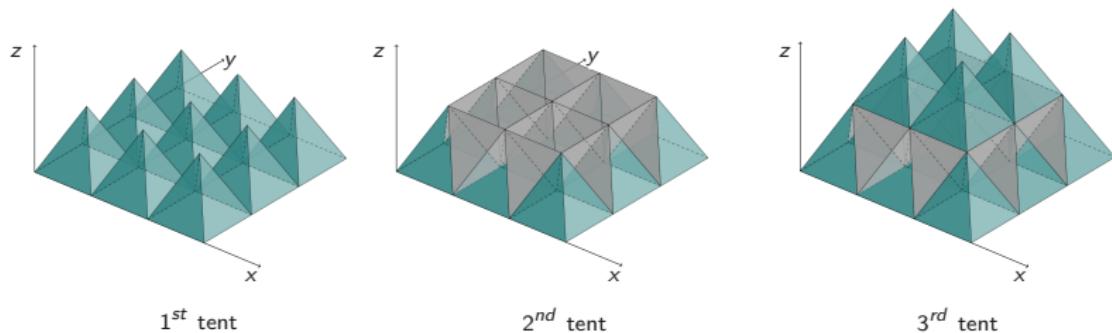
Non-uniform tent mesh
(homogeneous medium)



Non-uniform tent mesh
(heterogeneous medium)

TENT PITCHING MESHES

2D TENT PITCHING MESHES



TENT PITCHING MESHES

ANALYTICAL STABILITY CONDITIONS ¹

$$c|n_K^x|/|n_K^t| \leq 1 \quad - \text{TDG acoustic formulation}$$

$$\sqrt{|\mathbf{C}|}|n_K^x|/\sqrt{\rho}|n_K^t| \leq 1 \quad - \text{TDG elastodynamic formulation}$$

¹ H. Barucq, H. Calandra, J. Diaz & Elvira Shishenina.

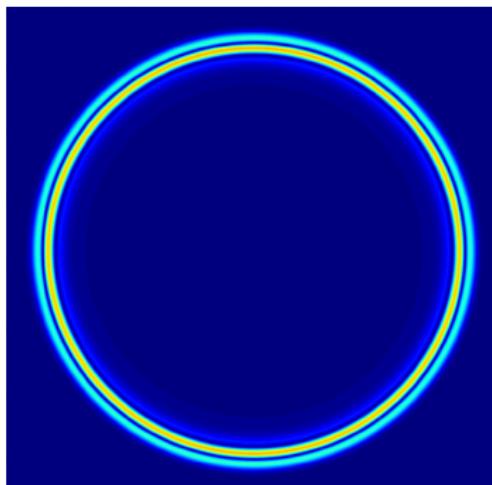
Space-Time Discretization of Elasto-Acoustic Wave Equation in Polynomial Trefftz-DG Bases. (2018)
PhD thesis [under review]

TENT PITCHING MESHES

PROPAGATION OF NUMERICAL VELOCITY

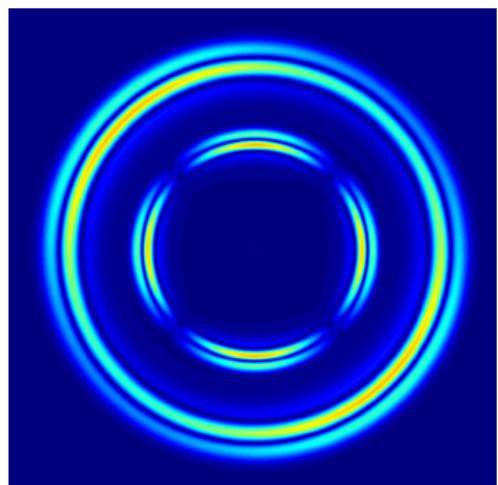
2D Acoustic domain.

L^2 norm of numerical velocity v



2D Elastodynamic domain.

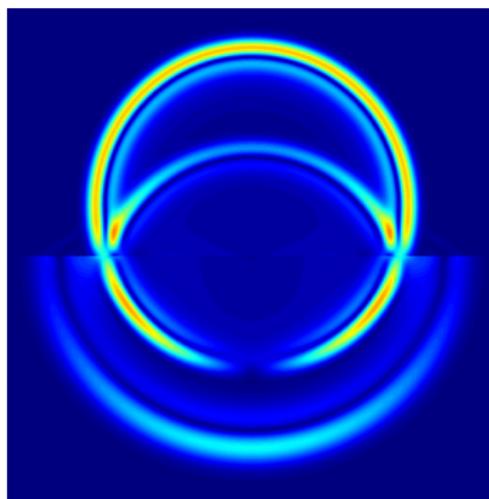
L^2 norm of numerical velocity v



TENT PITCHING MESHES

PROPAGATION OF NUMERICAL VELOCITY

2D Elasto-Acoustic domain.
 L^2 norm of numerical velocity v

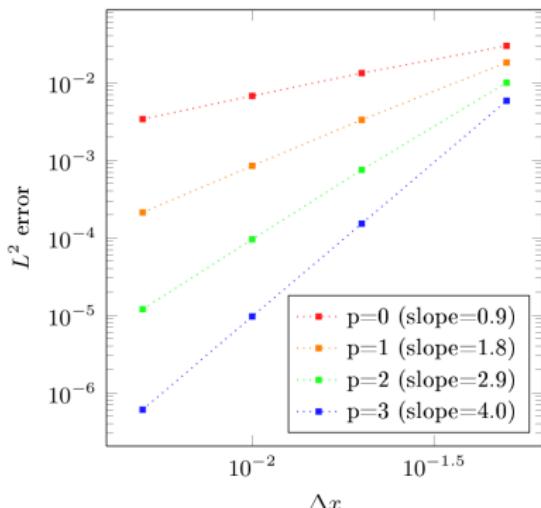


TENT PITCHING MESHES

CONVERGENCE CURVES

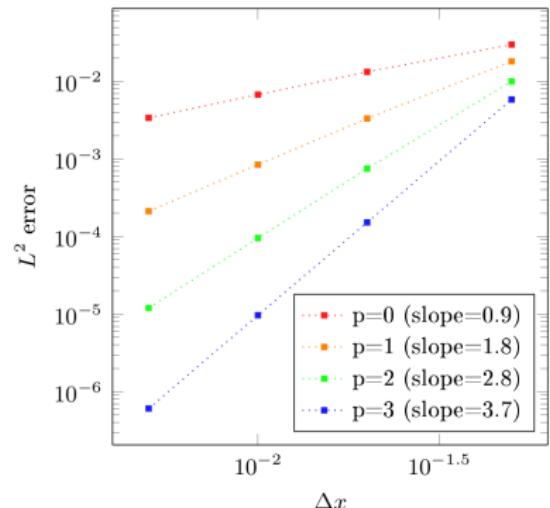
2D Acoustic domain.

Convergence of velocity v in function of cell size



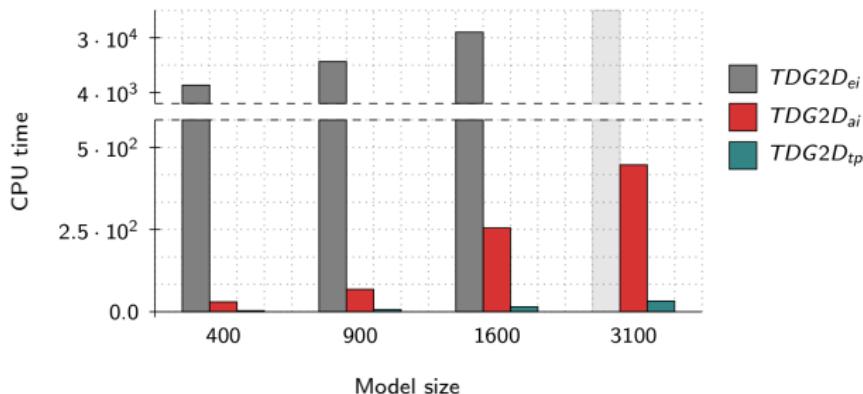
2D Elastodynamic domain.

Convergence of velocity v in function of cell size



TENT PITCHING MESHES

SPEEDUP FOR DIFFERENT MODEL SIZES



CONCLUSION

CONCLUSION

ON-GOING WORK AND PERSPECTIVES

Standard Trefftz-DG method:

- Development of 2D-3D hp propagators
- Analytical study of convergence, comparison with numerical results
- Study of CFL condition, comparison with standard DG method

Tent pitching Trefftz-DG method:

- Mesh generation study
- Implementation of the boundary conditions
- 3D case

THANK YOU!

IMPLEMENTATION OF THE ALGORITHM

POLYNOMIAL BASIS (ORDER 3)

p=0, ndof=2

$$\phi_1^V = 0$$

$$\phi_2^V = 1$$

$$\phi_1^P = -c_F$$

$$\phi_2^P = 0$$

p=1, ndof=4

$$\phi_3^V = x$$

$$\phi_4^V = c_F t$$

$$\phi_3^P = -c_F^2 t$$

$$\phi_4^P = -c_F x$$

p=2, ndof=6

$$\phi_5^V = -\frac{x^2}{2} - \frac{c_F^2 t^2}{2}$$

$$\phi_6^V = -c_F x t$$

$$\phi_5^P = c_F^2 x t$$

$$\phi_6^P = c_F \left(\frac{x^2}{2} + \frac{c_F^2 t^2}{2} \right)$$

p=3, ndof=8

$$\phi_7^V = -\frac{x^3}{6} - \frac{x c_F^2 t^2}{2}$$

$$\phi_8^V = -\frac{c_F^3 t^3}{6} - \frac{x^2 c_F t}{2}$$

$$\phi_7^P = c_F \left(\frac{c_F^3 t^3}{6} + \frac{x^2 c_F t}{2} \right)$$

$$\phi_8^P = c_F \left(\frac{x^3}{6} + \frac{x c_F^2 t^2}{2} \right)$$

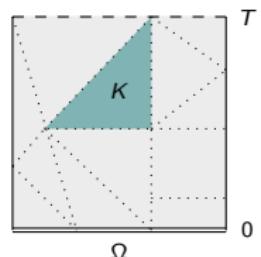
ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

$$\sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{\mathbf{p}}_h q + \check{\mathbf{v}}_h \cdot \omega \right) n_K^t + \left(q \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q$$

NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$$\begin{aligned}
 \begin{pmatrix} \hat{\mathbf{v}}_h \\ \hat{\mathbf{p}}_h \end{pmatrix} &\equiv \begin{pmatrix} \{\mathbf{v}_h\} + \beta [\mathbf{p}_h]_x \\ \{\mathbf{p}_h\} + \alpha [\mathbf{v}_h]_x \end{pmatrix} & \text{on } \mathcal{F}^I & \dots \\
 \begin{pmatrix} \check{\mathbf{v}}_h \\ \check{\mathbf{p}}_h \end{pmatrix} &\equiv \begin{pmatrix} \{\mathbf{v}_h\} \\ \{\mathbf{p}_h\} \end{pmatrix} & \text{on } \mathcal{F}^I & \dots \\
 \begin{pmatrix} \hat{\mathbf{v}}_h \cdot n_K^x \\ \hat{\mathbf{p}}_h \end{pmatrix} &\equiv \begin{pmatrix} g_D \\ \mathbf{p}_h + \alpha(\mathbf{v}_h \cdot n_K^x - g_D) \end{pmatrix} & \text{on } \mathcal{F}^D & — \\
 \begin{pmatrix} \check{\mathbf{v}}_h \\ \check{\mathbf{p}}_h \end{pmatrix} &\equiv \begin{pmatrix} \mathbf{v}_h \\ \mathbf{p}_h \end{pmatrix} & \text{on } \mathcal{F}^T & — \\
 \begin{pmatrix} \check{\mathbf{v}}_h \\ \check{\mathbf{p}}_h \end{pmatrix} &\equiv \begin{pmatrix} v_0 \\ p_0 \end{pmatrix} & \text{on } \mathcal{F}^0 & —
 \end{aligned}$$



REFERENCES

J.S.Hesthaven T.Warburton. *Nodal Discontinuous Galerkin Methods. Algorithms, analysis, and applications.* 2007.

F.Kretzschmar A.Moiola I.Perugia. *A Priori Error Analysis of Space-Time Trefftz Discontinuous Galerkin Methods for Wave Problems.* 2016.

O.C.Zienkiewicz. *Trefftz Type Approximation and the Generalized Finite Element Method - History and Developement.* 1997.

I.Herrera. *Trefftz Method: a General Theory.* 2000

A.Maciag. *Wave Polynomials in Elasticity Problems.* 2006