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Multilevel strategies for parametric shape optimization in aerodynamics

Badr Abou El Majd Jean-Antoine Désidéri
Régis Duvigneau

1 Introduction, hierarchical approaches in analysis and design

In recent years, the achievements in Computational Fluid Dynamics (CFD) have opened the way to optimization, mostly shape-optimization, and design in compressible aerodynamics and coupled disciplines. Aerodynamic design is a field at the interface between several classical disciplines and in which numerical simulations require assembling a number of technical elements either related to geometry (mesh and/or CAD), solvers for a detailed analysis of the physical situations, optimizers to devise a convergent loop permitting to optimize, or simply improve one or more performance indices, or software engineering. Thus, a number of acute topics in numerical analysis, multi-disciplinary engineering and software development are raised by these problems. These include non exhaustively the following:

- multi-disciplinary-optimization (MDO) or concurrent engineering from both technical and organizational viewpoints (design platforms);
- treatment of multi-criterion problems: identification of Pareto fronts, dynamic games;
- model reduction, meta-models and sensitivity analysis (response surface, Kriging, artificial neural networks);
- cost-efficient optimizers (BFGS, SQP, etc.);
- robust optimizers for multi-modal or non-differentiable optimization problems (evolutionary computing: genetic or particle-swarm algorithms, etc.); and hybridization with deterministic optimizers;
- software engineering: automatic-differentiation, code coupling; parallel and asynchronous algorithms;
- CAD and parameterization in shape optimization, and interface with mesh generation and adaptation.

Hence the field of optimum-shape design in aerodynamics is unsurprisingly very active, as the program of several recent courses and workshops demonstrate [15, 28, 22].

The present contribution relates to the development of cost-efficient optimization strategies relying on multilevel shape parameterization.

Our developments focus on the definition, mathematical analysis and experimentation of numerical methods for shape optimization for applications in which the cost functional evaluation relies on the prior solution of a complex set of partial-differential equations (PDEs), such as those governing compressible aerodynamics (e.g. the Euler equations), or related coupled disciplines such as structural mechanics (e.g. elasticity), or electromagnetics (e.g. the Maxwell equations). These PDEs are very commonly solved by Finite Elements or Volumes, by techniques that, although becoming increasingly standard, are still very costly when the accuracy requirement is high.

The important development of multigrid methods in recent years has demonstrated that such techniques not only permit to accelerate the iterative convergence of solution procedures, but also have the more general merit of a better control on grid dependency and convergence. In fact, a linear convergence rate can be demonstrated assuming adequate iterative termination criteria are devised, that is, when the grid-convergence control is enforced properly.

Thus, our efforts are mostly concentrated on improving the convergence rate of numerical procedures both from the viewpoint of cost-efficiency and accuracy, with the perspective of reducing the design cost, but also of mastering the election and control of the design parameters, geometrical ones in particular, in a more rational way, perhaps supported by error estimates.

Technically, our efforts tend to contribute to the following challenges:

- construct multilevel (multi-scale) shape-optimization algorithms;
- identify critical algorithmic ingredients (transfer operators, smoothers);
- evaluate efficiency, theorize convergence via error estimates or an appropriate *modal analysis*.

We note that prior to us, several other authors have applied multilevel principles in shape optimization. In particular, [16, 17] has used a multigrid method to solve both flow and continuous-adjoint equations with great success. Ta'asan [4] introduced the concept of one-shot methods in which these equations are solved in a simultaneous multigrid iteration. Lewis and Nash in [18, 19] have introduced a multigrid approach to differential systems in which different approximations to the optimization problem associated with a hierarchy of meshes are coordinated according to a multigrid strategy. Their algorithm is a nonlinear programming adaptation of the multigrid construction. We also point out the recent theoretical review of such methods by Borzi [5]. A somewhat different concept was introduced by A. Dervieux and collaborators who proposed a technique of *Hierarchical Preconditioning* [7] in which the multilevel geometrical data structure of agglomeration multigrid is exploited to define a hierarchical

optimization algorithm. The major concepts related to this approach have also been presented [6].

Besides, other types of hierarchical concepts have been introduced in the literature to enhance the efficiency. In particular, certain very efficient optimizers have been developed to employ several models to represent with different degrees of accuracy the same physical situation, the coarsest model having the merit to be very economical to evaluate. In this area, let us cite the concept of low/high-fidelity models in [3, 2]. [25, 26] has adapted a genetic algorithm to introduce a technique of *islands* in the population of individuals in order to perform a robust working-space *exploration* at the coarsest level combined with an accurate *exploitation* at the fine level.

Here, as a sequel of [9, 12, 1, 11, 10, 8], we review the construction of multilevel algorithms, in the context of *parametric shape optimization*. Embedded search spaces are defined based on a geometrical hierarchy of nested shape parameterizations of Bézier type. We first consider two-dimensional geometries for which shapes are directly parameterized. Then, we recall the concept of *Free-Form Deformation* which allows us to extend the multilevel geometrical representation to three-dimensional cases. Then we provide some details on how the usual optimization algorithms (simplex, steepest-descent) can be formulated in the context of a multilevel shape parameterization. We then present results of optimum-shape design in 3D aerodynamics governed by the compressible Euler equations solved by a finite-volume method using unstructured grids. We finally conclude with some perspectives.

2 Nested Bézier parameterizations for multilevel shape representation

We begin with the simplest situation of a two-dimensional geometry for which we employ a Bézier shape representation:

$$x(t) = \sum_{k=0}^n B_n^k(t) x_k, \quad y(t) = \sum_{k=0}^n B_n^k(t) y_k \quad (1)$$

in which the parameter t varies from 0 to 1, n is the degree of the parameterization,

$$B_n^k(t) = C_n^k t^k (1-t)^{n-k} \quad (2)$$

is a Bernstein polynomial, $C_k^n = \frac{n!}{k!(n-k)!}$, and

$$P_k = \begin{pmatrix} x_k \\ y_k \end{pmatrix} \quad (k = 0, 1, \dots, n) \quad (3)$$

is the generic control point. The coordinates of these control points are split into two vectors

$$X = \{x_k\}, \quad Y = \{y_k\}, \quad k = 0, 1, \dots, n, \quad (4)$$

and we refer to the vector X as the *support* of the parameterization, and the vector Y as the *design* vector. Typically, we optimize the design vector for fixed support according to some physical criterion, such as drag reduction in aerodynamics. The somewhat unsymmetrical roles dispensed to the vectors X and Y are chosen to reduce (to n essentially) the dimension of the search space in the optimization phase, which is the most numerically costly and subject to numerical stiffness.

We also use the notation:

$$x(t) = B_n(t)^T X, \quad y(t) = B_n(t)^T Y, \quad (5)$$

in which the vector $B_n(t)^T = (B_n^0(t), B_n^1(t), \dots, B_n^n(t))$. In all this article, only supports for which the sequence $\{x_k\}$ is monotone increasing are said to be admissible and considered throughout. Thus, the function $x(t)$ is monotone-increasing and defines a one-to-one mapping of, say, $[0,1]$ onto itself. Recall also the simple formula for the derivative:

$$\frac{dx(t)}{dt} = n \sum_{k=0}^{n-1} B_{n-1}^k(t) (x_{k+1} - x_k) = n B_{n-1}(t)^T \Delta X \quad (6)$$

in which Δ denotes the forward-difference operator ($\Delta x_k = x_{k+1} - x_k$) as well as the associated $n \times (n+1)$ matrix.

In the prototypical case of an airfoil, we use such a parametric representation for both the upper and lower surfaces separately. The vertical slope at the leading edge is enforced by the conditions:

$$x_0 = x_1 = 0, \quad y_0 = 0 \quad (7)$$

for both surfaces which assures a smooth match; at the trailing edge, we simply have:

$$x_n = 1, \quad y_n = 0 \quad (8)$$

for a continuous match.

Our geometrical construction employs the *degree-elevation process*, well-known in the *Computer-Aided Design* literature (see for example [14]). This process permits to cast [1] into the following equivalent Bézier parameterization of degree $n+1$:

$$x(t) = \sum_{k=0}^{n+1} B_{n+1}^k(t) x'_k, \quad y(t) = \sum_{k=0}^{n+1} B_{n+1}^k(t) y'_k \quad (9)$$

in which the new control points $P'_k = (x'_k, y'_k)$ are obtained from the former by convex combinations:

$$P'_0 = P_0, P'_k = \frac{k}{n+1} P_{k-1} + \left(1 - \frac{k}{n+1}\right) P_k \quad (k = 1, 2, \dots, n), P'_{n+1} = P_n \quad (10)$$

obtained by multiplying [1] by $(1-t) + t$ and grouping together the monomials in $t^k(1-t)^{n+1-k}$, for each k .

Figure 1 represents the RAE2822 airfoil and the lower and upper control polygons of degree-16 Bézier least-squares curvefits. The RAE2822 airfoil is a classical geometry in computational aerodynamics, known for its low-drag performance in the transonic regime. This shape has been tabulated by the European Project ECARP [21].

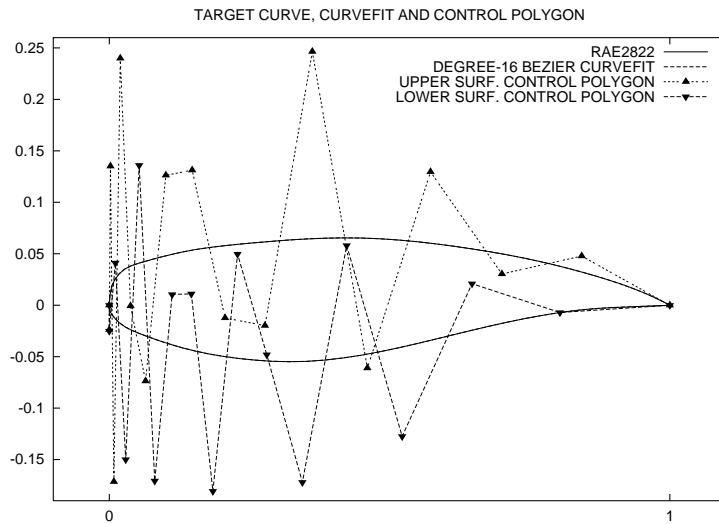


Figure 1: RAE2822 airfoil, examples of degree-16 Bézier curvefits of the upper and lower surfaces (superimposed), and corresponding control polygons

From a theoretical viewpoint, our construction guarantees rigorously nested search spaces, and exact upward transfer operators (from low to high-degree parameterization). This is illustrated on Figure 2 in which the supports of three nested parameterizations of the airfoil are sketched, without reference to the corresponding sets of ordinates.

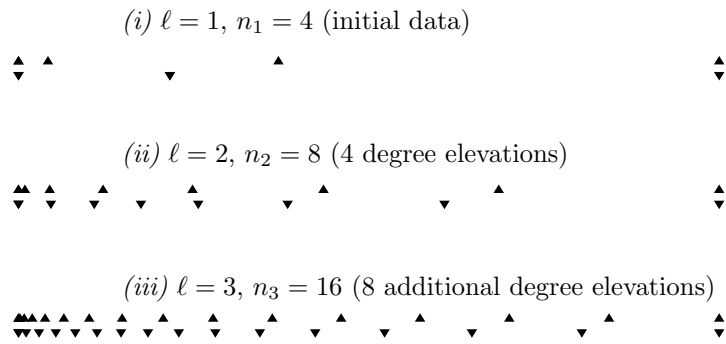


Figure 2: One-dimensional example of embedded parameterizations: the triangles represent the supports X of three nested Bézier parameterizations of degree 4, 8 and 16 of an RAE2822 airfoil obtained from the first by 4 and 12 successive degree elevations; the symbols pointing upward (resp. downward) are associated with the upper (resp. lower) surface; the degree-4 support has been optimized to regularize the control polygon associated with the degree-16 airfoil representation

Note that in the case of Figure 2, apart from the specified endpoints, the abscissas of the degree-4 support X are not a subset of the abscissas of any support of a higher degree parameterization. Nevertheless, any Bézier curve given on the degree-4 support can be expressed exactly on any other support of higher degree provided it results, as in this example, from the degree elevation process. The parameterizations are nested, or embedded in one another in this sense precisely.

3 FFD for multilevel three-dimensional shape deformation

A critical issue in aerodynamic design is the choice of the shape parameterization. Parameterization techniques for practical 3D aerodynamic shape optimization have to fulfill several criteria:

- the parameterization should be able to take into account complex geometries, possibly including constraints and singularities ;
- the number of parameters should be as small as possible, since the stiffness of the shape optimization numerical formulation increases abruptly with the number of parameters ;
- the parameterization should allow to control the smoothness of the resulting shapes.

A survey of shape parameterization techniques for multi-disciplinary optimization, which are analyzed according to the previous criteria, is proposed in [23]. Following his recommendation, conclusions, the Free-Form Deformation (FFD) technique [24] is adopted in the present study, since it provides an easy and powerful framework for the deformation of complex shapes, such as generic or elaborate aerodynamic configurations.

The FFD technique originates from the Computer Graphics field [24]. It allows the deformation of an object in a 2D or 3D space, regardless of the representation of this object. Instead of manipulating the surface of the object directly, by using classical B-Splines or Bézier parameterization of the surface, the FFD technique defines a deformation field over the space embedded in a lattice which is built around the object. By modifying the space coordinates inside the lattice, the FFD technique deforms the object, regardless of its geometrical description. In particular, the initial geometry, in our applications, is usually defined by a general, Finite-Element-type unstructured simplicial grid.

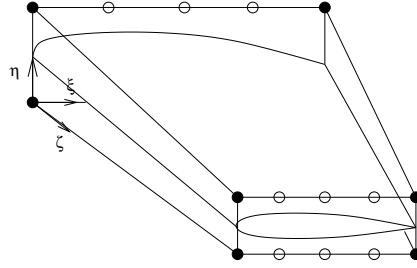
More precisely, consider a three-dimensional hexaedral lattice embedding the object to be deformed. Figure 3(a) shows an example of such a lattice built around a typical wing. A local coordinate system (ξ, η, ζ) is defined in the lattice, with $(\xi, \eta, \zeta) \in [0, 1] \times [0, 1] \times [0, 1]$. As a result of the deformation, the displacement Δq of each point q inside the lattice is here defined by a third-order

Bézier tensor product:

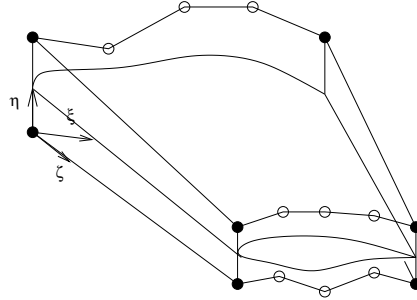
$$\Delta q = \sum_{i=0}^{n_i} \sum_{j=0}^{n_j} \sum_{k=0}^{n_k} B_{n_i}^i(\xi_q) B_{n_j}^j(\eta_q) B_{n_k}^k(\zeta_q) \Delta P_{ijk}. \quad (11)$$

$B_{n_i}^i$, $B_{n_j}^j$ and $B_{n_k}^k$ are again Bernstein polynomials of order n_i , n_j and n_k . $(\Delta P_{ijk})_{0 \leq i \leq n_i, 0 \leq j \leq n_j, 0 \leq k \leq n_k}$ are weighting coefficients, or control points displacements, which are used to monitor the deformation and are considered as design variables during the shape optimization procedure. The critical point is that only the *shape deformation* is represented, not the shape itself.

This technique is illustrated by Figure 3. A lattice is built around a wing and a Bzier tensor product of degree $n_i = 4$, $n_j = 1$ and $n_k = 1$ is defined over this lattice. Corner control points (filled markers) are supposed to be frozen in order to keep leading and trailing edges fixed during the deformation, whereas other control points (empty markers) are allowed to move vertically (Figure 3(a)). When these control points are moved, their displacements define a continuous deformation inside the lattice according to [11], yielding a shape deformation. The deformed lattice and shape can be seen in Figure 3(b)).



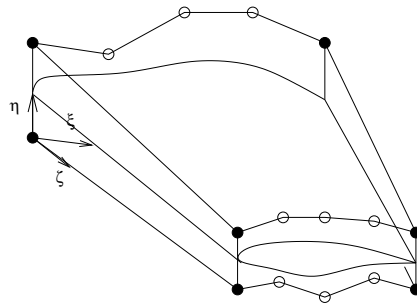
(a) Initial FFD lattice 4-1-1



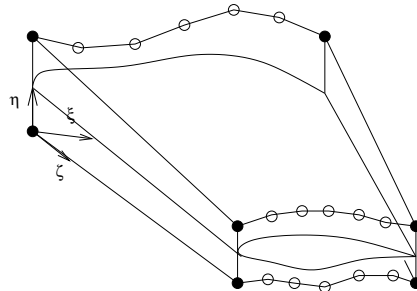
(b) Deformed FFD lattice 4-1-1

Figure 3: Example of Free-Form Deformation: by moving some control points of the lattice, a deformation field is defined continuously inside the lattice, yielding a shape deformation

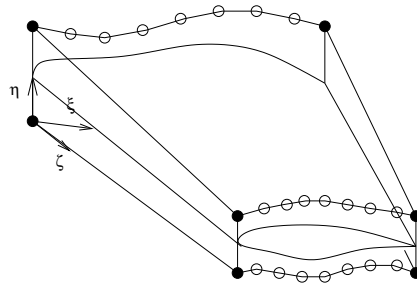
The FFD technique described above is well suited to complex shape optimization, thanks to the following properties:



(a) Deformed FFD lattice 4-1-1



(b) Deformed FFD lattice 6-1-1



(c) Deformed FFD lattice 8-1-1

Figure 4: Example of degree elevation process: by using the degree elevation process in the direction ξ , the number of control points is increased while the initial shape is not modified

- the initial shape can be exactly represented (no deformation occurs when all weighting coefficients are zero);
- the deformation is performed whatever the complexity of the shape (this is a free-form technique);
- geometric singularities can be taken into account (the initial shape including its singularities is deformed);

- the smoothness of the deformation is controlled (the deformation is ruled by Bernstein polynomials);
- the number of design variables depends on the user's choice (the deformation is independent of the shape itself);

This technique nicely deals with multilevel representation thanks to the Bézier degree elevation property. To apply the degree elevation process within the FFD framework, one should simply apply the standard degree elevation process for each direction of the tensor product. The process is illustrated by Figure 4. Degree elevation in the direction ξ is applied to the shape obtained previously using a deformation of degree 4-1-1 (Figure 4(a)). Figure 4(b) shows the result when the degree is elevated to 6-1-1 and Figure 4(c) when it is elevated to 8-1-1. One can notice that the shape is not modified, whereas the lattice counts an increasing number of control points.

4 Multilevel variants of classical optimization algorithms

With the support of the nested geometrical representations constructed in the previous sections, we can now define precisely our algorithms which combine standard iterations (simplex method, particle-swarm optimizer or steepest-descent) with a multilevel geometrical treatment. To be specific, we consider for simplicity the two uniform and nested parameterization supports of Figure 5 associated with representations of shape, or shape-deformation of degree 4 and 8 of a two-dimensional problem. Let γ denote the shape to be optimized, and $J(\gamma)$ the *intrinsic cost function* to be minimized. Let $B_n(X, Y)$ denote the Bézier curve of degree n associated with the control points (X, Y) . Then, the *parametric cost function* is taken to be

$$j_n(Y) := J(B_n(X, Y)) \quad (12)$$

where X is fixed (and possibly subsequently adapted).

Then the classical steepest descent method can be combined with a two-level parameterization in the following algorithm, in which E_4^8 is the rectangular matrix representing the degree-elevation process from $n = 4$ to 8:

1. partially solve coarse-level parametric optimization problem :

$$\text{For } i = 1, 2, \dots, K \text{ do : } Y^i = Y^{i-1} - \rho_i \nabla j_{COARSE}(Y^{i-1}) \quad (13)$$

Here, $j_{COARSE} = j_4$ and $Y^i \in \mathbb{R}^5$ (but with only 3 d.o.f.'s since $y_0 = y_4 = 0$)

2. transfer optimum-shape design vector onto upper level

$$Y^K = E_4^8 Y'^K \implies Y^K \in \mathbb{R}^9 \quad (14)$$

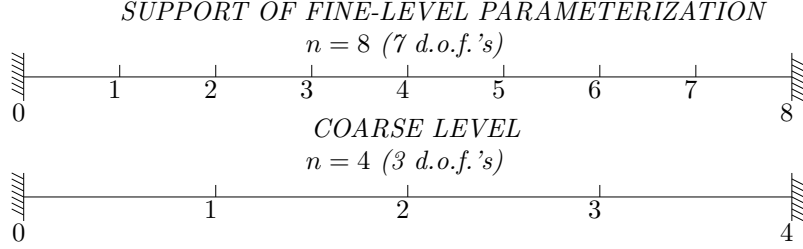


Figure 5: Nested supports of parameterizations of degree 4 and 8; the degrees of freedom associated with both endpoints ($k = 0$ or n) are fixed

3. solve fine-level parametric optimization problem:

Reset $Y^0 = Y^K$ and for $i = 1, 2, \dots$ until convergence:

$$Y^i = Y^{i-1} - \rho_i \nabla j_{FINE}(Y^{i-1}) \quad (15)$$

Here, $j_{FINE} = j_8$ and $Y^i \in \mathbb{R}^9$ (but with only 7 d.o.f.'s since $y_0 = y_8 = 0$)
 \implies final solution $Y^* \in \mathbb{R}^9$.

In fact, in the above algorithm, all the evaluations are performed on the upper level. Now, since the supports are nested (embedded),

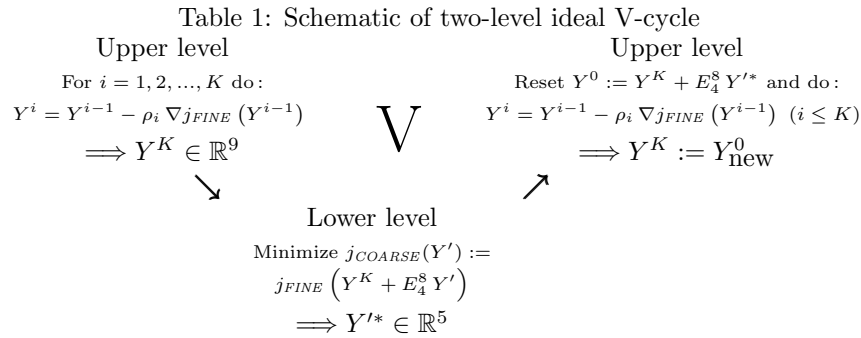
$$X^{(8)} = E_4^8 X^{(4)} \quad (16)$$

the above transfer (item 2) is exact :

$$B_8(X^{(8)}, Y^K) = B_4(X^{(4)}, Y'^K) \quad (17)$$

and in terms of shape, the upper-level iteration begins with the last update coming from the previous lower-level iteration.

Similarly, the *two-level "ideal V-cycle"* based on steepest descent is defined by the schematic of Table 1.



Note that above algorithm is said to be *ideal* if the coarse-level problem is solved to complete convergence regardless the method (evolutionary algorithm, simplex method, steepest-descent, (P)CG, BFGS, etc). Additionally, the iteration is formulated as a *correction algorithm* since the coarse-level problem defined in terms of a correction vector to be applied to the unknown (upper-level) design vector.

The V-cycle being defined, more elaborate strategies mimicking classical multigrid iterations can be devised. Once the nested supports have been constructed from a coarse parameterization, all multigrid-type strategies (sawtooth, V or W-cycle, Full-Multi-Grid (FMG), [27]) can be implemented; at any point in the graph, the current approximate optimum shape admits an exact representation on the fine level.

Note that nested supports of parameterization of shape-deformation are far simpler to construct and handle than nested grids (particularly in the unstructured-grid finite-volume formulation). Consequently, the present multilevel parametric shape optimization algorithms are easy to implement.

In particular, iterations involving all intermediate levels could easily all be considered without particular implementation difficulties.

5 Application to aerodynamics design

5.1 Test-case description

The test-case considered here corresponds to the optimization of the wing shape of a business aircraft (courtesy of Piaggio Aero Industries) in a transonic regime. The free-stream Mach number is $M_\infty = 0.83$ and the incidence $\alpha = 2^\circ$. Initially, the wing section corresponds to the NACA 0012 airfoil. An unstructured mesh, composed of 31 124 nodes and 173 445 elements, is generated around the wing, including a refined area in the vicinity of the shock (Figure 6). Flow fields are obtained by solving compressible Euler equations using a finite-volume method.

The goal of the optimization is to reduce the drag coefficient C_D subject to the constraint that the lift coefficient C_L should not decrease more than 0.1%. The constraint is taken into account using a penalization approach. Then, the resulting cost function is:

$$\mathcal{J}_{OPT} = \frac{C_D}{C_{D0}} + 10^4 \max(0, 0.999 - \frac{C_L}{C_{L0}}). \quad (18)$$

C_{D0} and C_{L0} are respectively the drag and lift coefficients corresponding to the initial shape (NACA 0012 section).

The FFD lattice is built around the wing with ξ , η and ζ in the chord-wise, span-wise and thickness directions respectively. The lattice is chosen in order to fit the planform of the wing. Then, the leading and trailing edges are kept fixed during the optimization by freezing the control points that correspond to $i = 0$ and $i = n_i$. Moreover, control points are only moved vertically. Results are presented for three parameterizations. The coarsest one corresponds to $n_i = 3$,

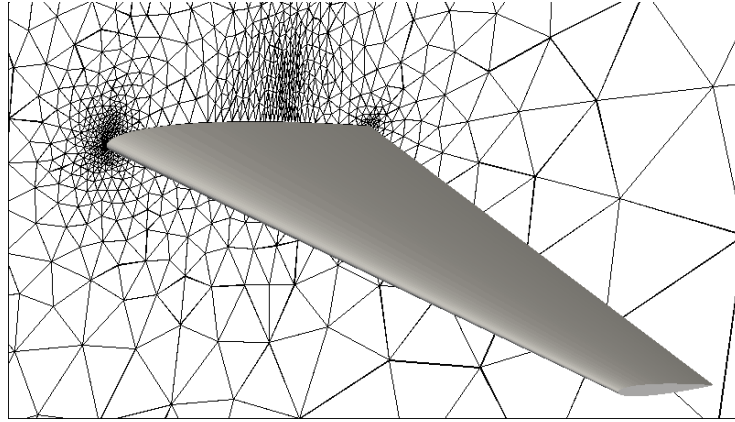


Figure 6: Initial wing shape and mesh in the symmetry plane

$n_j = 1$ and $n_k = 1$. Therefore, $(4 - 2) \times 2 \times 2 = 8$ degrees of freedom are taken into account in the optimization. The medium parameterization corresponds to $n_i = 6$, $n_j = 1$ and $n_k = 1$ and counts $(7 - 2) \times 2 \times 2 = 20$ degrees of freedom. Finally, the finest parameterization corresponds to $n_i = 9$, $n_j = 1$ and $n_k = 1$ and counts $(10 - 2) \times 2 \times 2 = 32$ degrees of freedom. In this study, the Nelder-Mead simplex method [20] is used as optimization algorithm. Three strategies corresponding to different ways of handling the geometrical parameterization are compared:

- basic method (test A): single parameterization until full convergence;
- progressive degree elevation (test B): 3 levels (coarse, medium and fine) are considered successively using the degree elevation transfer;
- FMOSA (test C): using the same 3 levels of parameterization as the ones of test B in a strategy including a FMG-like approach defined in Table 2.

5.2 Aerodynamic coefficients

The aerodynamic coefficients obtained for each method are compared in Table 3. The lift coefficient is approximately maintained or slightly increased by the shape optimization process. Important reductions of the drag coefficient are reported. As observed, the multilevel strategies improve significantly the aerodynamic performance.

5.3 Convergence history plots

Figure 7 depicts the convergence history of the basic method (single parameterization) for three different parameterizations. With a coarse parameterization,

6 Conclusions and perspectives

We have shown in this study that multilevel strategies are better equipped to alleviate the numerical stiffness in numerical shape optimization, which in the parametric formulation increases very rapidly with the number of degrees of freedom [8].

Besides, adequate parameterization adaption techniques have been devised and tested in model problems [10] as well as in three-dimensional flow problems [13]. These procedures which readjust the parameterization support to regularize the shape representation, were also found to be very effective in accelerating the convergence process. From a theoretical point of view, we have also proposed [8, 11] a shape reconstruction or inverse problem as a simple model to analyze the convergence mechanism of our parametric shape optimization method. There, we have identified a pertinent eigensystem associated with a linear iteration. This analysis comforts us in the analogy with multigrid methods supporting our algorithms, but at the same time, raises a number of differences and theoretical questions requiring further investigation, currently being carried out.

Ultimately, we observe that the multilevel geometrical structure could also be used to support a hierarchical method in which *reduced models* are used on lower levels. These reduced models can be based on:

- *variable physics*: e.g. Euler on coarse levels, Navier-Stokes on upper;
- *variable numerics*: use of a hierarchy of meshes, and/or simplified state-cost functional dependency (response surface, artificial neural networks, etc), and/or alternate optimizer (e.g., evolutionary on coarsest level for robustness, simplex method on intermediate level, deterministic on fine level for accuracy; hybridization), etc.

Thus, a great number of promising algorithmic variants can be devised.

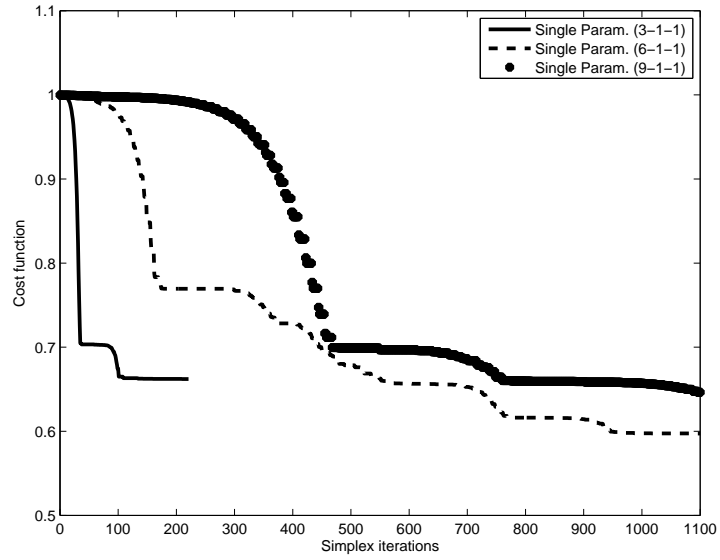


Figure 7: Basic method: convergence history plot for three different single parameterizations

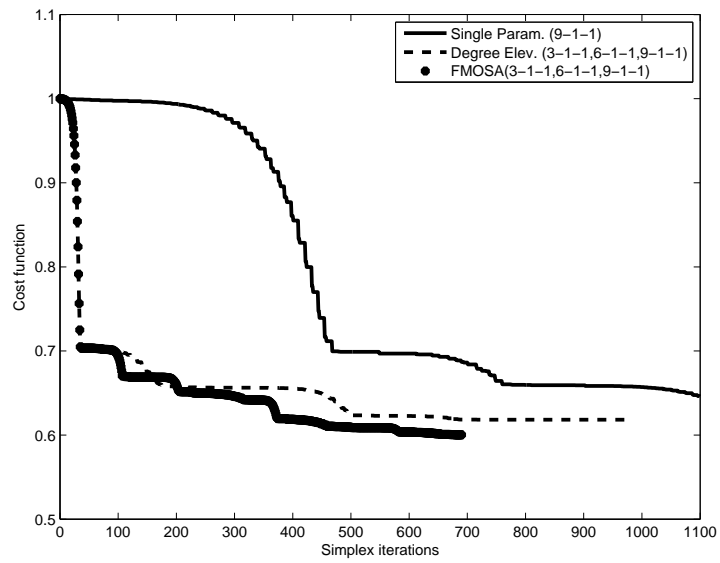


Figure 8: Comparison of the convergence history for the three strategies

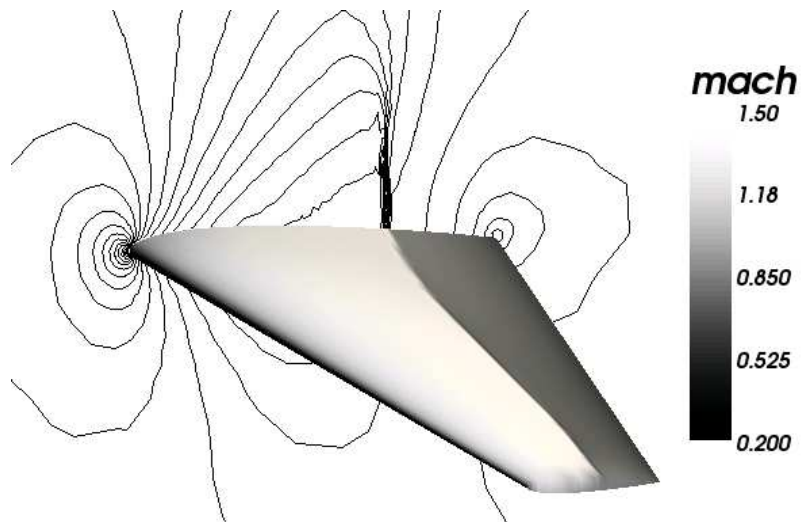


Figure 9: Mach number field on the wing and Mach number contours in the symmetry plane: initial shape

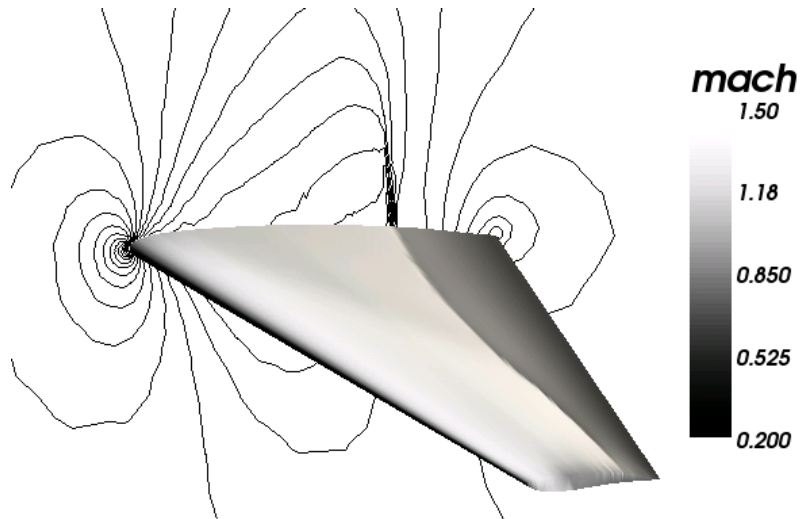


Figure 10: Mach number field on the wing and Mach number contours in the symmetry plane: single fine parameterization

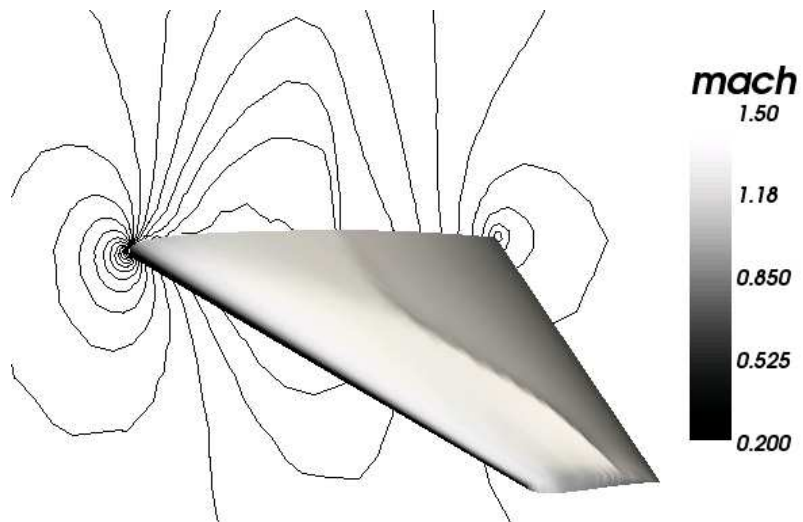


Figure 11: Mach number field on the wing and Mach number contours in the symmetry plane: progressive degree elevation strategy

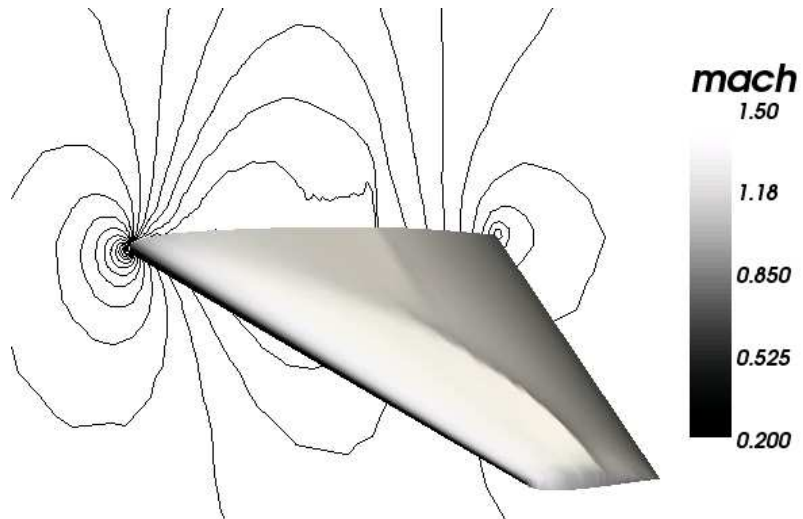


Figure 12: Mach number field on the wing and Mach number contours in the symmetry plane: FMOSA strategy

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