



HAL
open science

Impedance Transmission Conditions for the Electric Potential across a Highly Conductive Casing

Hélène Barucq, Aralar Erdozain, David Pardo, Victor Péron

► **To cite this version:**

Hélène Barucq, Aralar Erdozain, David Pardo, Victor Péron. Impedance Transmission Conditions for the Electric Potential across a Highly Conductive Casing. V-MAD6 - Valparaíso's Mathematics and its Applications Days, Jan 2016, Valparaíso, Chile. hal-01692286

HAL Id: hal-01692286

<https://inria.hal.science/hal-01692286v1>

Submitted on 24 Jan 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Impedance Transmission Conditions for the Electric Potential across a Highly Conductive Casing

Aralar Erdozain¹, Victor Péron^{1,2}, Hélène Barucq^{1,2} and David Pardo^{3,4,5}

¹INRIA Bordeaux Sud-Ouest, Team magique 3D

²Université de Pau et des Pays de l'Adour

³University of the Basque Country (UPV/EHU), Leioa, Spain

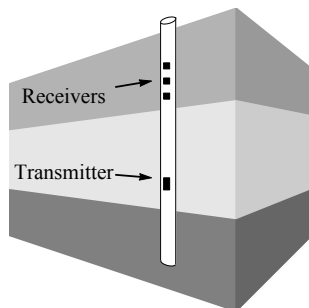
⁴Basque Center for Applied Mathematics (BCAM), Bilbao, Spain

⁵Ikerbasque (Basque Foundation for Sciences), Bilbao, Spain



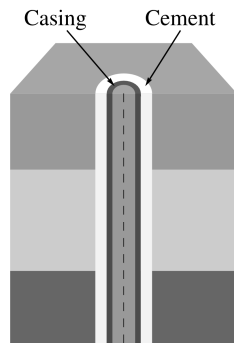
MOTIVATION

- **Main goal:** To obtain a better characterization of the Earth's subsurface
- **How:** Recording borehole resistivity measurements
- **Procedure:**
 - Well
 - Transmitters
 - Receivers



MOTIVATION

- **Practical Difficulties:**
 - It is not easy to drill a borehole
 - It may collapse
- **Practical Solutions:**
 - Use a metallic casing
 - Surround with a cement layer
- **Problem solved, but...**



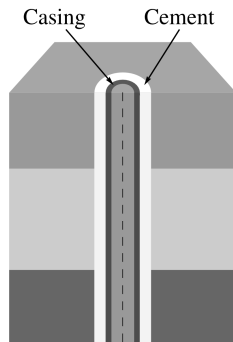
MOTIVATION

- **Practical Difficulties:**

- It is not easy to drill a borehole
- It may collapse

- **Practical Solutions:**

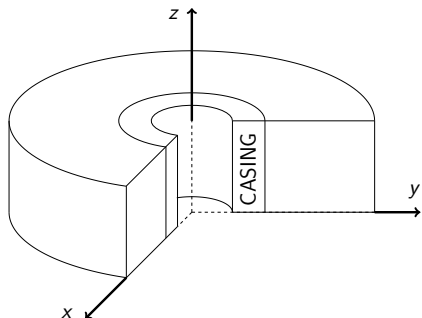
- Use a metallic casing
- Surround with a cement layer



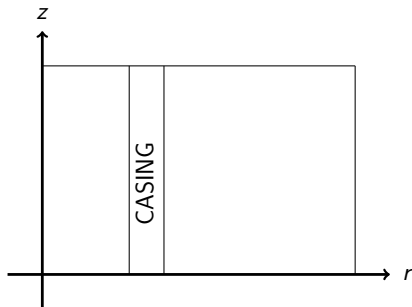
- **Problem solved, but... Numerical problems due to the high conductivity and thinness of the casing**

CONFIGURATION OF INTEREST

SECTIONED 3D DOMAIN



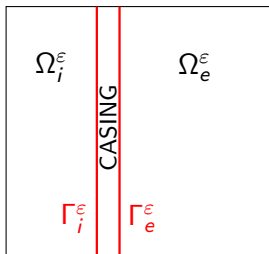
MERIDIAN DOMAIN



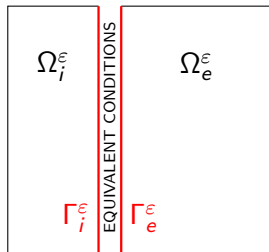
We reduce the 3D problem to a 2D problem due to the **axysimmetric** configuration.

MAIN IDEA

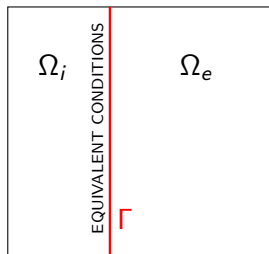
REFERENCE MODEL



1st Approach

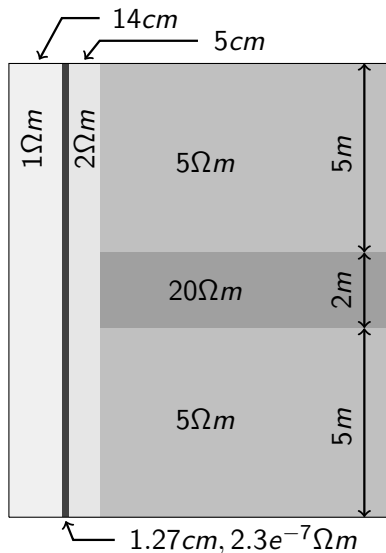


2nd Approach



Idea: Replace the casing by
equivalent conditions.

REALISTIC SCENARIO



- **Conductivity and casing width:**

$$\begin{cases} \varepsilon = 1.27e - 2m \\ \sigma_c = 4.34e6\ \Omega^{-1}m^{-1} \end{cases}$$

$$\Rightarrow \sigma_c \approx \varepsilon^{-3}$$

- **Classical approach:**

$$\sigma_c = \alpha \quad \alpha \in \mathbb{R}$$

- **High conductive case:**

$$\sigma_c = \alpha \varepsilon^{-3} \quad \alpha \in \mathbb{R}$$

REFERENCES

- [1] A.A. Kaufman. The electrical field in a borehole with a casing. *Geophysics*, Vol.55, Issue 1, pp. 29-38, 1990.
- [2] D.Pardo, C.Torres-Verdín and Z.Zhang. Sensitivity study of borehole-to-surface and crosswell electromagnetic measurements acquired with energized steel casing to water displacement in hydrocarbon-bearing layers. *Geophysics*, 73 No.6, F261-F268, 2008.
- [3] M. Duruflé, V. Péron and C. Poignard. Thin Layer Models for Electromagnetism. *Communications in Computational Physics* 16(1):213-238, 2014.
- [4] K. Schmidt, A. Chernov, Robust transmission conditions of high order for thin conducting sheets in two dimensions, *IEEE Trans. Magn.*, 50 (2014), pp. 41–44.

OUTLINE

- 1 Equivalent Conditions
- 2 Numerical Results
- 3 Second Approach
- 4 Application
- 5 Perspectives

OUTLINE

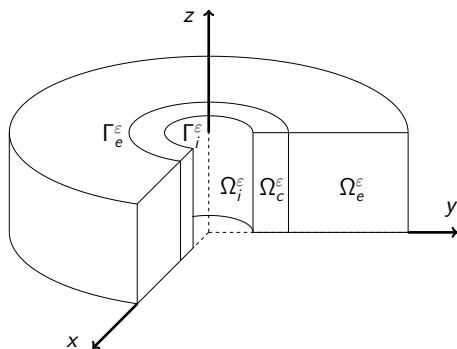
- 1 **Equivalent Conditions**
- 2 Numerical Results
- 3 Second Approach
- 4 Application
- 5 Perspectives

MODEL PROBLEM - 3D

STATIC ELECTRIC POTENTIAL

$$\operatorname{div}((\sigma - i\epsilon\omega) \nabla u) = f \text{ in } \Omega$$

$$\Omega = \Omega_i^\epsilon \cup \Omega_c^\epsilon \cup \Omega_e^\epsilon \cup \Gamma_i^\epsilon \cup \Gamma_e^\epsilon$$



Where f is a known data, σ is piecewise constant

$$\sigma = \begin{cases} \sigma_i & \text{in } \Omega_i^\epsilon \\ \sigma_c = \alpha\epsilon^{-3} & \text{in } \Omega_c^\epsilon \\ \sigma_e & \text{in } \Omega_e^\epsilon \end{cases}$$

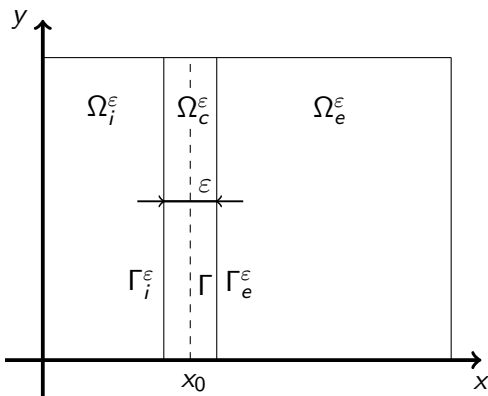
and the solution is expressed as

$$u = \begin{cases} u_i & \text{in } \Omega_i^\epsilon \\ u_c & \text{in } \Omega_c^\epsilon \\ u_e & \text{in } \Omega_e^\epsilon \end{cases}$$

MODEL PROBLEM - 2D

Reference problem (P) set in the domain $\Omega = \Omega_i^\varepsilon \cup \Omega_c^\varepsilon \cup \Omega_e^\varepsilon \cup \Gamma_i^\varepsilon \cup \Gamma_e^\varepsilon$.

We denote the boundary $\Gamma_0 := (\overline{\partial\Omega_i^\varepsilon} - \Gamma_i^\varepsilon) \cup (\overline{\partial\Omega_e^\varepsilon} - \Gamma_e^\varepsilon)$.



$$\left\{ \begin{array}{ll} \sigma_i \Delta u_i = f_i & \text{in } \Omega_i^\varepsilon \\ \sigma_c \Delta u_c = 0 & \text{in } \Omega_c^\varepsilon \\ \sigma_e \Delta u_e = f_e & \text{in } \Omega_e^\varepsilon \\ u_i = u_c & \text{on } \Gamma_i^\varepsilon \\ u_c = u_e & \text{on } \Gamma_e^\varepsilon \\ \sigma_i \partial_n u_i = \sigma_c \partial_n u_c & \text{on } \Gamma_i^\varepsilon \\ \sigma_c \partial_n u_c = \sigma_e \partial_n u_e & \text{on } \Gamma_e^\varepsilon \\ u = 0 & \text{on } \partial\Omega \end{array} \right.$$

EXISTENCE AND UNIQUENESS

Theorem 1

There exists $\varepsilon_0 > 0$ s.t. for all $\varepsilon \in (0, \varepsilon_0)$, if $f_i \in L^2(\Omega_i^\varepsilon)$ and $f_e \in L^2(\Omega_e^\varepsilon)$, then $\exists! u \in H_0^1(\Omega)$ solution of (P) and

$$\|u\|_{1,\Omega} \leq C \left(\|f_i\|_{0,\Omega_i^\varepsilon} + \|f_e\|_{0,\Omega_e^\varepsilon} \right).$$

Proof:

- Derive **variational formulation**, for all $v \in H_0^1(\Omega)$, $a(u, v) = b(v)$.
- Prove that a **coercive** and **continuous**, b continuous in $H_0^1(\Omega)$.
- Apply the **Lax-Milgram** Lema.

DEFINITIONS

Definition 1

We define the **jump** and **mean value** of the solution u across the casing as

$$[u] = u|_{\Gamma_e^\varepsilon} - u|_{\Gamma_i^\varepsilon}$$

$$\{u\} = \frac{1}{2} \left(u|_{\Gamma_e^\varepsilon} + u|_{\Gamma_i^\varepsilon} \right)$$

Definition 2

Let u be the reference solution. We say an asymptotic model is of **Order $k+1$** , if its solution $u^{[k]}$ satisfies

$$\|u - u^{[k]}\|_{H^1} \leq C\varepsilon^{k+1} \quad \text{when} \quad \varepsilon \rightarrow 0$$

METHODOLOGY

- **Step1:** Derive an **Asymptotic Expansion** for u when $\varepsilon \rightarrow 0$

- In the casing:
$$u_c(x, y) = \sum_{n \in \mathbb{N}} \varepsilon^n U^n \left(\frac{x - x_0}{\varepsilon}, y \right)$$

- Outside the casing:
$$u(x, y) = \sum_{n \in \mathbb{N}} \varepsilon^n u^n(x, y)$$

- **Step2:** Obtain **Equivalent Conditions** of order $k + 1$ by identifying a simpler problem satisfied by the truncated expansion and neglecting the higher order terms in ε

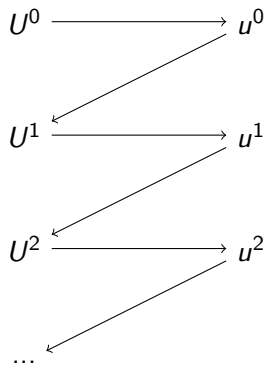
- $$u_{k,\varepsilon} := u^0 + \varepsilon u^1 + \varepsilon^2 u^2 + \dots + \varepsilon^k u^k$$

- **Step3:** **Prove Convergence** rates for asymptotic models

MULTISCALE EXPANSION

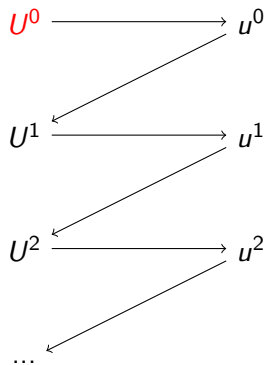
$$\left\{ \begin{array}{ll}
 \partial_X^2 U^k + \partial_y^2 U^{k-2} = 0 & \text{in } \Omega_c^\varepsilon \\
 \sigma_i \Delta u_i^k = f_i & \text{in } \Omega_i^\varepsilon \\
 \sigma_e \Delta u_e^k = f_e & \text{in } \Omega_e^\varepsilon \\
 u_i^k = U^k & \text{on } \Gamma_i^\varepsilon \\
 U^k = u_e^k & \text{on } \Gamma_e^\varepsilon \\
 \sigma_i \partial_x u_i^{k-4} = \alpha \partial_X U^k & \text{on } \Gamma_i^\varepsilon \\
 \alpha \partial_X U^k = \sigma_e \partial_x u_e^{k-4} & \text{on } \Gamma_e^\varepsilon \\
 U^k = 0 & \text{on } \partial\Omega
 \end{array} \right.$$

PROCEDURE



MULTISCALE EXPANSION

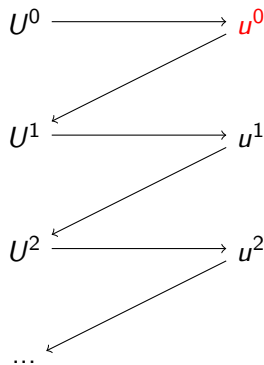
$$\left\{ \begin{array}{ll}
 \partial_X^2 U^k + \partial_y^2 U^{k-2} = 0 & \text{in } \Omega_c^\varepsilon \\
 \sigma_i \Delta u_i^k = f_i & \text{in } \Omega_i^\varepsilon \\
 \sigma_e \Delta u_e^k = f_e & \text{in } \Omega_e^\varepsilon \\
 u_i^k = U^k & \text{on } \Gamma_i^\varepsilon \\
 U^k = u_e^k & \text{on } \Gamma_e^\varepsilon \\
 \sigma_i \partial_x u_i^{k-4} = \alpha \partial_X U^k & \text{on } \Gamma_i^\varepsilon \\
 \alpha \partial_X U^k = \sigma_e \partial_x u_e^{k-4} & \text{on } \Gamma_e^\varepsilon \\
 U^k = 0 & \text{on } \partial\Omega
 \end{array} \right.$$

PROCEDURE

MULTISCALE EXPANSION

$$\left\{ \begin{array}{ll}
 \partial_X^2 U^k + \partial_y^2 U^{k-2} = 0 & \text{in } \Omega_c^\varepsilon \\
 \sigma_i \Delta u_i^k = f_i & \text{in } \Omega_i^\varepsilon \\
 \sigma_e \Delta u_e^k = f_e & \text{in } \Omega_e^\varepsilon \\
 u_i^k = U^k & \text{on } \Gamma_i^\varepsilon \\
 U^k = u_e^k & \text{on } \Gamma_e^\varepsilon \\
 \sigma_i \partial_x u_i^{k-4} = \alpha \partial_X U^k & \text{on } \Gamma_i^\varepsilon \\
 \alpha \partial_X U^k = \sigma_e \partial_x u_e^{k-4} & \text{on } \Gamma_e^\varepsilon \\
 U^k = 0 & \text{on } \partial\Omega
 \end{array} \right.$$

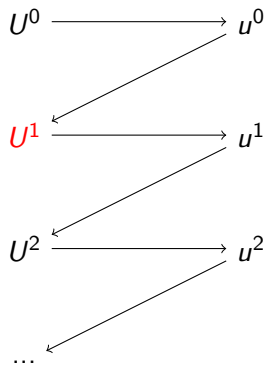
PROCEDURE



MULTISCALE EXPANSION

$$\left\{ \begin{array}{ll}
 \partial_X^2 U^k + \partial_y^2 U^{k-2} = 0 & \text{in } \Omega_c^\varepsilon \\
 \sigma_i \Delta u_i^k = f_i & \text{in } \Omega_i^\varepsilon \\
 \sigma_e \Delta u_e^k = f_e & \text{in } \Omega_e^\varepsilon \\
 u_i^k = U^k & \text{on } \Gamma_i^\varepsilon \\
 U^k = u_e^k & \text{on } \Gamma_e^\varepsilon \\
 \sigma_i \partial_x u_i^{k-4} = \alpha \partial_X U^k & \text{on } \Gamma_i^\varepsilon \\
 \alpha \partial_X U^k = \sigma_e \partial_x u_e^{k-4} & \text{on } \Gamma_e^\varepsilon \\
 U^k = 0 & \text{on } \partial\Omega
 \end{array} \right.$$

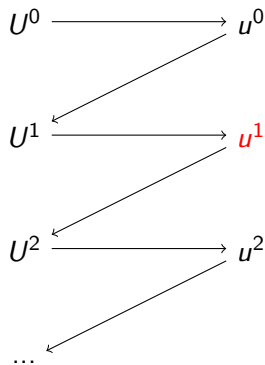
PROCEDURE



MULTISCALE EXPANSION

$$\left\{ \begin{array}{ll}
 \partial_X^2 U^k + \partial_y^2 U^{k-2} = 0 & \text{in } \Omega_c^\varepsilon \\
 \sigma_i \Delta u_i^k = f_i & \text{in } \Omega_i^\varepsilon \\
 \sigma_e \Delta u_e^k = f_e & \text{in } \Omega_e^\varepsilon \\
 u_i^k = U^k & \text{on } \Gamma_i^\varepsilon \\
 U^k = u_e^k & \text{on } \Gamma_e^\varepsilon \\
 \sigma_i \partial_x u_i^{k-4} = \alpha \partial_X U^k & \text{on } \Gamma_i^\varepsilon \\
 \alpha \partial_X U^k = \sigma_e \partial_x u_e^{k-4} & \text{on } \Gamma_e^\varepsilon \\
 U^k = 0 & \text{on } \partial\Omega
 \end{array} \right.$$

PROCEDURE



EQUIVALENT MODELS

FIRST APPROACH

(Transmission conditions across the casing)

- Order 2:

$$\begin{cases} \sigma_i \Delta u_i = f_i & \text{in } \Omega_i^\varepsilon \\ u_i = 0 & \text{on } \partial\Omega_i^\varepsilon \end{cases} \quad (P_2)$$

$$\begin{cases} \sigma_e \Delta u_e = f_e & \text{in } \Omega_e^\varepsilon \\ u_e = 0 & \text{on } \partial\Omega_e^\varepsilon \end{cases}$$

- Order 4:

$$\begin{cases} \sigma_i \Delta u_i = f_i & \text{in } \Omega_i^\varepsilon \\ \sigma_e \Delta u_e = f_e & \text{in } \Omega_e^\varepsilon \\ [u] = 0 \\ [\sigma \partial_n u] = -\frac{\alpha}{\varepsilon^2} \Delta_{\Gamma^\varepsilon} \{u\} \\ u = 0 & \text{on } \Gamma_0 \end{cases} \quad (P_4)$$

CONVERGENCE

Theorem 2

There exists $\varepsilon_0 > 0$ s.t. for all $\varepsilon \in (0, \varepsilon_0)$, if $f_i \in L^2(\Omega_i^\varepsilon)$ and $f_e \in L^2(\Omega_e^\varepsilon)$, then $\exists! u^{[k]} \in V_{k+1}$ solution of P_{k+1} , $k = 1, 3$, and

$$\|u^{[k]}\|_{1, \Omega_i^\varepsilon \cup \Omega_e^\varepsilon} \leq C \left(\|f_i\|_{0, \Omega_i^\varepsilon} + \|f_e\|_{0, \Omega_e^\varepsilon} \right)$$

$$\|u - u^{[k]}\|_{1, \Omega_i^\varepsilon \cup \Omega_e^\varepsilon} \leq C\varepsilon^{k+1}$$

Where

$$V_2 = \{v : v_i \in H_0^1(\Omega_i^\varepsilon), v_e \in H_0^1(\Omega_e^\varepsilon)\}$$

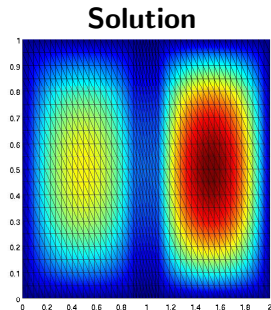
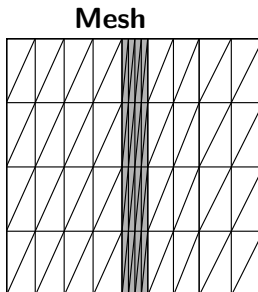
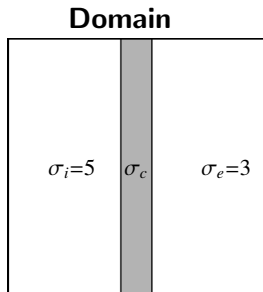
$$V_4 = \{v : v_i \in H^1(\Omega_i^\varepsilon), v_e \in H^1(\Omega_e^\varepsilon), \nabla_{\Gamma^\varepsilon} \{v\} \in L^2(\Gamma^\varepsilon), v|_{\Gamma_i^\varepsilon} = v|_{\Gamma_e^\varepsilon}, v|_{\Gamma_0} = 0\}$$

OUTLINE

- 1 Equivalent Conditions
- 2 Numerical Results**
- 3 Second Approach
- 4 Application
- 5 Perspectives

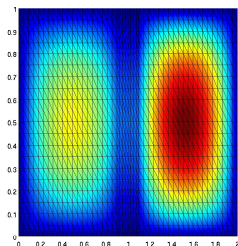
FEM CODE

- Classic Finite Element Method code
- Straight triangular elements (h refinement)
- Lagrange shape functions of any degree (p refinement)

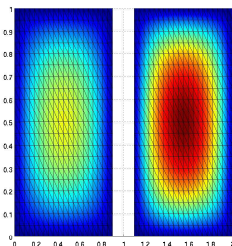


QUALITATIVE COMPARISON

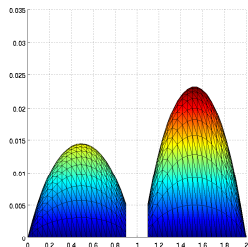
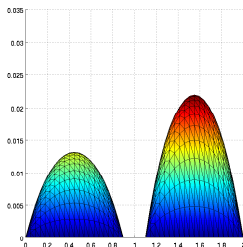
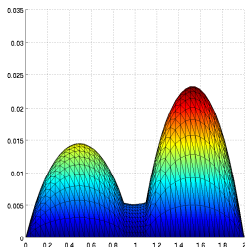
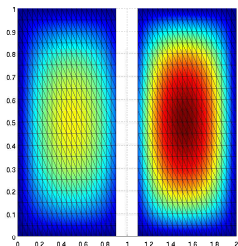
Reference Model



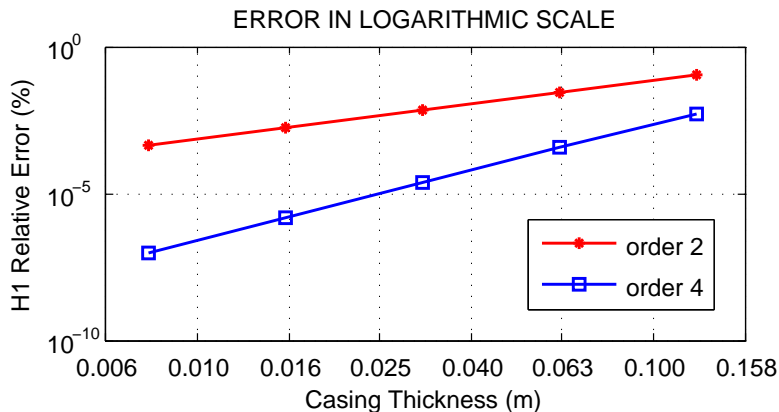
Order 2 Model



Order 4 Model



CONVERGENCE RATES



Casing Thickness (ε)	0.0117	0.0234	0.0469	0.0938	Expected ($\varepsilon \rightarrow 0$)
Order 2 Slopes	1.9969	1.9899	1.9638	1.8624	2
Order 4 Slopes	4.0046	4.0039	3.9879	3.8992	4

OUTLINE

- 1 Equivalent Conditions
- 2 Numerical Results
- 3 Second Approach**
- 4 Application
- 5 Perspectives

EQUIVALENT MODELS

SECOND APPROACH

(Transmission conditions across the interface Γ)

• Order 1:

$$\begin{cases} \sigma_i \Delta u_i = f_i & \text{in } \Omega_i \\ u_i = 0 & \text{on } \partial\Omega_i \\ \sigma_e \Delta u_e = f_e & \text{in } \Omega_e \\ u_e = 0 & \text{on } \partial\Omega_e \end{cases} \quad (P_1)$$

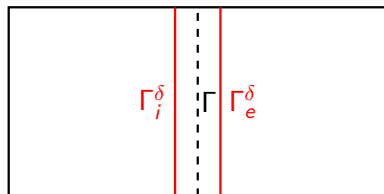
• Order 2:

$$\begin{cases} \sigma_i \Delta u_i = f_i & \text{in } \Omega_i \\ \sigma_e \Delta u_e = f_e & \text{in } \Omega_e \\ [u] = -\varepsilon \{\partial_n u\} \\ [\partial_n u] = -\frac{4}{\varepsilon} \{u\} \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

Stability

Problem

Stability problems for the order 2 model of the second approach due to a non coercive term.



Solution

Use of **Artificial Boundaries** to move the transmission conditions and recover stability. For $\delta > 0.5$

$$\Gamma_i^\delta = \{(x, y) : x = x_0 - \delta\varepsilon, y \in (0, y_0)\}$$

$$\Gamma_e^\delta = \{(x, y) : x = x_0 + \delta\varepsilon, y \in (0, y_0)\}$$

EQUIVALENT MODELS

SECOND APPROACH

(Transmission conditions across the artificial interfaces)

$$\bullet \text{ Order 2: } \left\{ \begin{array}{ll} \sigma_i \Delta u_i = f_i & \text{in } \Omega_i \\ \sigma_e \Delta u_e = f_e & \text{in } \Omega_e \\ [u] = -\varepsilon (1 - 2\delta) \{\partial_n u\} & \\ [\partial_n u] = -\frac{4(1 - 2\delta)}{\varepsilon} \{u\} & \\ u = 0 & \text{on } \partial\Omega \end{array} \right. \quad (P_2)$$

Stable if $\delta > 0.5$

CONVERGENCE

Theorem 3

There exists $\varepsilon_0 > 0$ s.t. for all $\varepsilon \in (0, \varepsilon_0)$, if $f_i \in L^2(\Omega_i^\varepsilon)$ and $f_e \in L^2(\Omega_e^\varepsilon)$, then $\exists! u^{[k]} \in V_{k+1}$ solution of P_{k+1} , $k = 1, 2$, and

$$\|u^{[k]}\|_{1, \Omega_i^\varepsilon \cup \Omega_e^\varepsilon} \leq C \left(\|f_i\|_{0, \Omega_i^\varepsilon} + \|f_e\|_{0, \Omega_e^\varepsilon} \right)$$

$$\|u - u^{[k]}\|_{1, \Omega_i^\varepsilon \cup \Omega_e^\varepsilon} \leq C \varepsilon^{k+1}$$

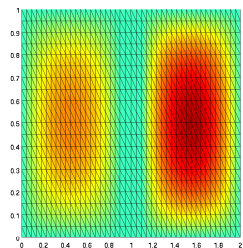
Where

$$V_1 = \{v : v_i \in H_0^1(\Omega_i), v_e \in H_0^1(\Omega_e)\}$$

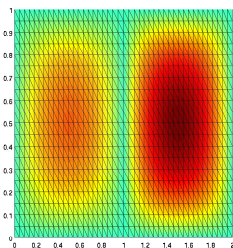
$$V_2 = \{v : v_i \in H^1(\Omega_i), v_e \in H^1(\Omega_e), v_i|_\Gamma = v_e|_\Gamma, v|_{\Gamma_0} = 0\}$$

QUALITATIVE COMPARISON

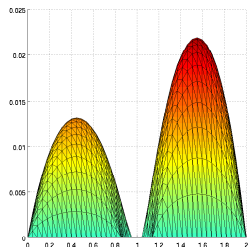
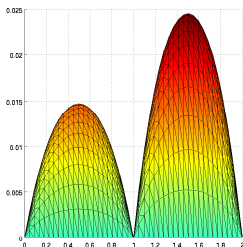
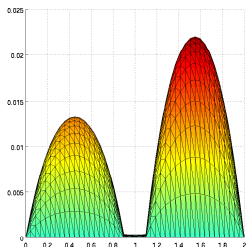
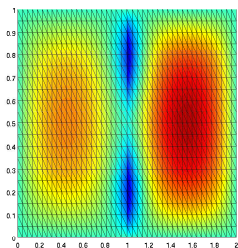
Reference Model



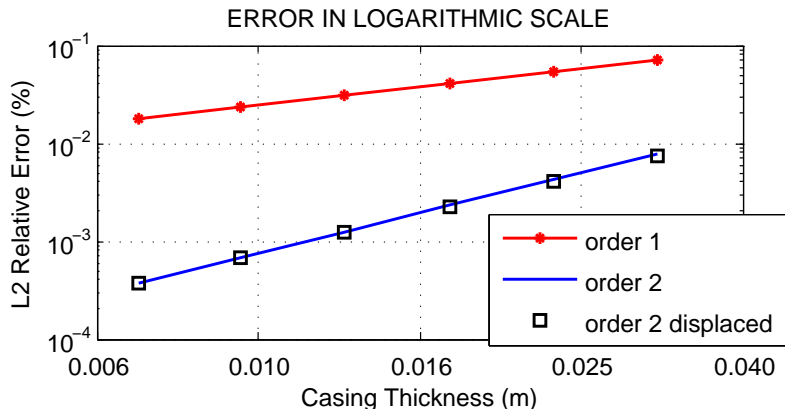
Order 1 Model



Order 2 Model

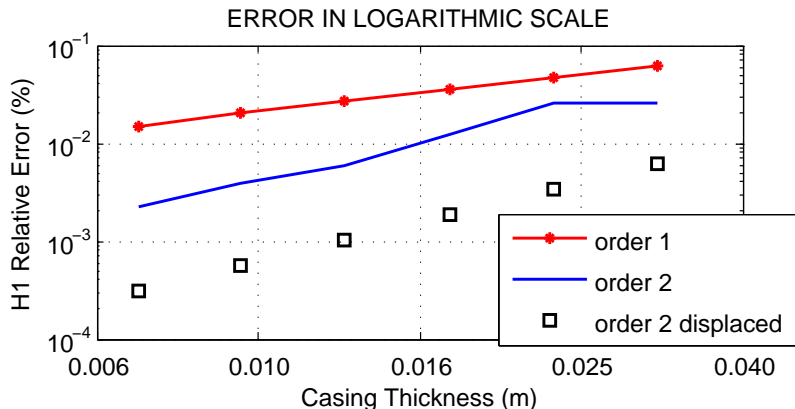


CONVERGENCE RATES L^2



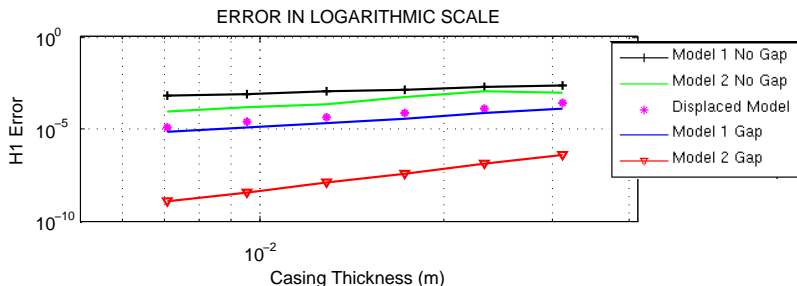
Casing Thickness (ε)	0.008	0.011	0.015	0.020	0.027	Expected ($\varepsilon \rightarrow 0$)
Order 1 Slopes	0.972	0.962	0.948	0.928	0.899	1
Order 2 Slopes	2.032	2.011	2.062	2.090	1.9636	2
Order 2 displaced	2.005	2.006	2.007	2.008	2.007	2

CONVERGENCE RATES H^1



Casing Thickness (ε)	0.008	0.011	0.015	0.020	0.027	Expected ($\varepsilon \rightarrow 0$)
Order 1 Slopes	0.982	0.975	0.966	0.953	0.935	1
Order 2 Slopes	1.915	1.302	2.463	2.569	-0.125	2
Order 2 displaced	2.004	2.005	2.006	2.008	2.007	2

COMPARISON

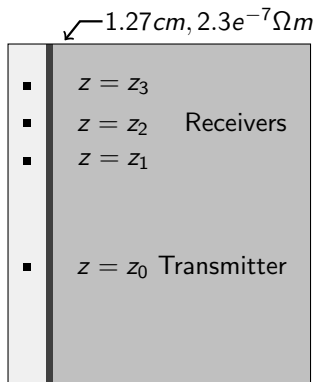


	Order	Stability	ε -independent Domain
Model 1 No Gap	1	✓	✓
Model 2 No Gap	2	✗	✓
Displaced Model	2	✓	✗
Model 1 Gap	2	✓	✗
Model 2 Gap	4	✓	✗

OUTLINE

- 1 Equivalent Conditions
- 2 Numerical Results
- 3 Second Approach
- 4 Application**
- 5 Perspectives

APPLICATION



- **Equation:** $\operatorname{div}(\sigma \nabla u) = f$

- **Right hand side:**

$$f = \begin{cases} 1 & \text{In the transmitter} \\ 0 & \text{Outside the transmitter} \end{cases}$$

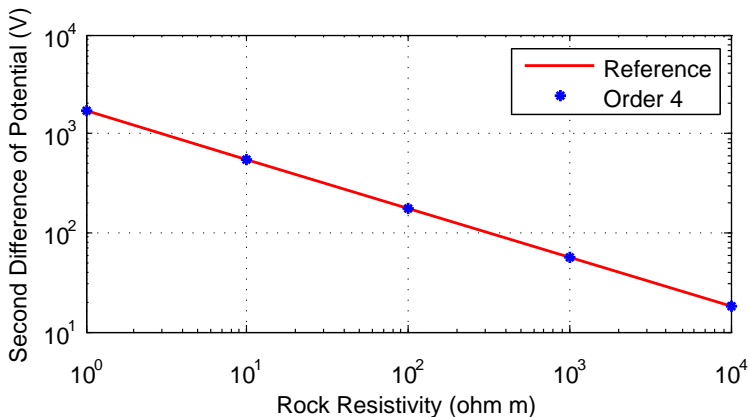
- **Objective:** Measure the second difference of potential on the Receivers

$$U_2 = u(z_1) - 2u(z_2) + u(z_3)$$

- **Expected Result:** (Model of Kaufman) Relation between second difference of potential and rock resistivity of the form

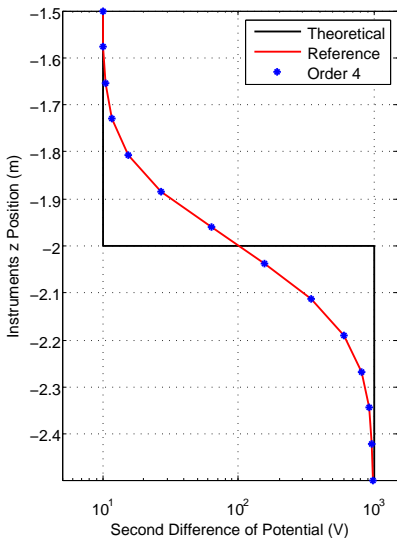
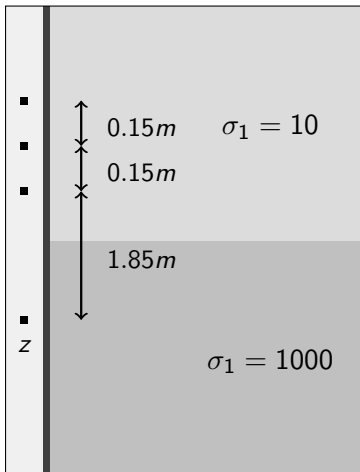
$$U_2 = k \cdot \rho_{\text{rock}}^{-\frac{1}{2}} \quad k \in \mathbb{R}$$

VARYING ROCK CONDUCTIVITY



Resistivity	[1,10]	[10,10 ²]	[10 ² ,10 ³]	[10 ³ ,10 ⁴]	Expected
Reference Model Slopes	-0.4914	-0.4924	-0.4978	-0.4991	-0.5
Order 4 Slopes	-0.4914	-0.4924	-0.4977	-0.4993	-0.5

TWO ROCK LAYERS



OUTLINE

- 1 Equivalent Conditions
- 2 Numerical Results
- 3 Second Approach
- 4 Application
- 5 Perspectives**

Perspectives

- Obtain semianalytical solutions to reduce the computational cost.
- Consider physically more realistic scenarios.
- Develop 3D electromagnetic models.
- Study highly deviated boreholes.

Perspectives

- Obtain semianalytical solutions to reduce the computational cost.
- Consider physically more realistic scenarios.
- Develop 3D electromagnetic models.
- Study highly deviated boreholes.

**THANK YOU FOR YOUR
ATTENTION**