



HAL
open science

Asymptotic Modelling for 3D Eddy Current Problems with a Conductive Thin Layer

Mohammad Issa, Victor Péron, Ronan Perrussel

► **To cite this version:**

Mohammad Issa, Victor Péron, Ronan Perrussel. Asymptotic Modelling for 3D Eddy Current Problems with a Conductive Thin Layer. ACOMEN 2017 - 7th International Conference on Advanced COmputational Methods in ENgineering, Sep 2017, Ghent, Belgium. hal-01679683

HAL Id: hal-01679683

<https://inria.hal.science/hal-01679683v1>

Submitted on 11 Jan 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Asymptotic Modelling for 3D Eddy Current Problems with a Conductive Thin Layer

Mohammad ISSA¹, Victor PÉRON² and Ronan PERRUSSEL¹

¹ *Laboratoire Plasma et Conversion d'Énergie, UMR 5213 CNRS/INPT/UPS, Université de
Toulouse, 31071 France*

² *LMAP CNRS UMR 5142 et Team MAGIQUE 3D INRIA, Université de Pau et des Pays de
l'Adour, 64013 France*

e-mails: mohammad.issa@laplace.univ-tlse.fr, victor.peron@univ-pau.fr,
perrussel@laplace.univ-tlse.fr

Abstract

In this work we derive and analyze an equivalent model for 3D Eddy Current problems with a conductive thin layer of small thickness ϵ . In our model, the conductive sheet is replaced by its mid-surface and their shielding behavior is satisfied by an equivalent transmission conditions on this interface. The transmission conditions are derived asymptotically for vanishing sheet thickness ϵ .

Key words: Asymptotic Expansions, Eddy-Current Problems, Thin Conducting Layers, Transmission Conditions

1 Introduction

We denote by $\Omega = \Omega_-^\epsilon \cup \overline{\Omega_0^\epsilon} \cup \Omega_+^\epsilon \subset \mathbb{R}^3$ the domain of study, where Ω_-^ϵ corresponds to a non-conductive linear material, Ω_+^ϵ the exterior of the structure domain, and Ω_0^ϵ a conductive thin layer of constant thickness ϵ (see figure 1). The discretisation of the conducting sheet by FEM needs a very fine mesh due to the rapid decay of the field under high conductivity. For this, we approximate a new model defined in ϵ -independent domains. Let Σ be a smooth surface, we denote by $[v]_\Sigma$ and $\{v\}_\Sigma$ the jump and mean of v respectively across Σ

$$[v]_\Sigma = v|_{\Sigma^+} - v|_{\Sigma^-}, \quad \{v\}_\Sigma = \frac{1}{2}(v|_{\Sigma^+} + v|_{\Sigma^-}), \quad \text{for } v \in (C^\infty(\Omega_\pm))^3.$$

We consider the eddy current problem as follows

$$\begin{cases} \operatorname{curl} H^\epsilon & = \sigma^\epsilon E^\epsilon + J_0 & \text{in } \Omega \\ \operatorname{curl} E^\epsilon & = iw\mu_0 H^\epsilon & \text{in } \Omega \\ \operatorname{div}(H^\epsilon) & = 0 & \text{in } \mathbb{R}^3 \\ [E \times n] = [H \times n] & = 0 & \text{on } \Gamma_\pm^\epsilon \end{cases}$$

$$\text{where } \sigma^\epsilon = \begin{cases} 0 & \text{in } \Omega_\pm^\epsilon \\ \sigma_0 = \epsilon^{-2} \bar{\sigma} & \text{in } \Omega_0^\epsilon \end{cases}$$

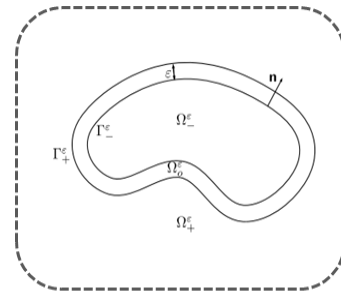


Figure 1: A cross Section of the domain Ω

Let u be a vector field on Γ , then we denote by $\gamma_D u = n \times (u \times n)$, and $\gamma_N u = \operatorname{curl} u \times n$, the Dirichlet and Neumann data, respectively.

2 Multiscale Expansion

Assuming that Γ is a smooth surface, then E^ϵ and H^ϵ can be expanded with an asymptotic expansion in power series of the small parameter ϵ . [1]

$$\begin{aligned} E^\epsilon(x) &\approx E_0(x) + \epsilon E_1(x) + \epsilon^2 E_2(x) + \dots + \mathcal{O}(\epsilon^k) \quad \text{in } \Omega_\pm^\epsilon \\ H^\epsilon(x) &\approx \mathcal{H}_0(y_\alpha, \frac{h}{\epsilon}) + \epsilon \mathcal{H}_1(y_\alpha, \frac{h}{\epsilon}) + \dots + \mathcal{O}(\epsilon^k) \quad \text{in } \Omega_0^\epsilon \end{aligned}$$

Here, $x \in \mathbb{R}^3$ are the cartesian coordinated, and (y_α, h) is the local normal coordinate system, $h \in (-\frac{\epsilon}{2}, \frac{\epsilon}{2})$ is the normal coordinate to Γ . The term \mathcal{H}_j is a profile defined on $\Gamma \times (-\frac{1}{2}, \frac{1}{2})$. The derivation is based on the expansion of the differential operators inside the thin layer Ω_0^ϵ , and the Taylor expansion of $E_j|_{\Gamma_\pm^\epsilon}$ around the mid-surface Γ .

3 Equivalent Model of Order 2

We introduce a problem satisfied by an approximation E_ϵ^k of the expression $E_0(x) + \epsilon E_1(x) + \epsilon^2 E_2(x) + \dots + \epsilon^k E_k(x)$ up to a residual term $\mathcal{O}(\epsilon^{k+1})$.

The second order approximate solution E_ϵ^1 , solves

$$\begin{aligned} \text{curl}(\text{curl}E_\epsilon^1) &= i\omega\mu J_0 && \text{in } \Omega_\pm \\ \begin{pmatrix} [\mathcal{Y}_D E_\epsilon^1]_\Gamma \\ \{\mathcal{Y}_D E_\epsilon^1\}_\Gamma \end{pmatrix} &= \epsilon \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix} \begin{pmatrix} \{\mathcal{Y}_N E_\epsilon^1\}_\Gamma \\ [\mathcal{Y}_N E_\epsilon^1]_\Gamma \end{pmatrix} && \text{on } \Gamma \end{aligned}$$

where

$$\begin{aligned} C_1 &= -1 + \frac{2 \tanh(\frac{\gamma}{2})}{\gamma} & , & \quad C_2 = -\frac{1}{4} + \frac{\coth(\frac{\gamma}{2})}{2\gamma} \\ \gamma &= \exp(\frac{3i\pi}{4}) \sqrt{\omega\mu_0\sigma}. \end{aligned}$$

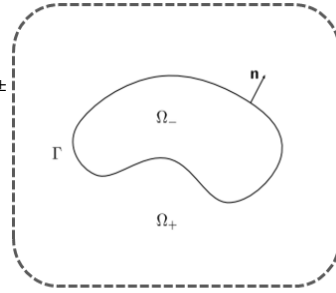


Figure 2: A cross section of the ϵ -independent subdomains

4 Numerical Results

Numerical experiments are performed to assess the accuracy of our model. The results are in particular compared to the model given in [2]. Complementary simulations will be conducted to study the robustness with respect to the sheet conductivity and the convergence of the modelling error.

Acknowledgements

We acknowledge the collaboration of B. BANNWARTH, O. CHADEBEC, and G. MEUNIER who participated in the implementation of the model, and the GDR SEEDS for the financial support.

References

- [1] Victor Péron , *Impedance transmission conditions for eddy current problems*. 2017. <https://hal.inria.fr/hal-01505612>.
- [2] Thanh-Trung Nguyen, Gérard Meunier, Jean-Michel Guichon, and Olivier Chadebec, *3-D Integral Formulation Using Facet Elements for Thin Conductive Shells Coupled With an External Circuit*. IEEE Transactions on Magnetics, Vol. 51, Issue 3, march 2015.