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# Designing Moore FSM with Transformation of State Codes

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**Abstract.** A design method is proposed for FPGA-based Moore FSMs. The method is based on transformation of state codes into codes of collections of output functions and classes of pseudoequivalent states. Example of design and results of investigations are given.

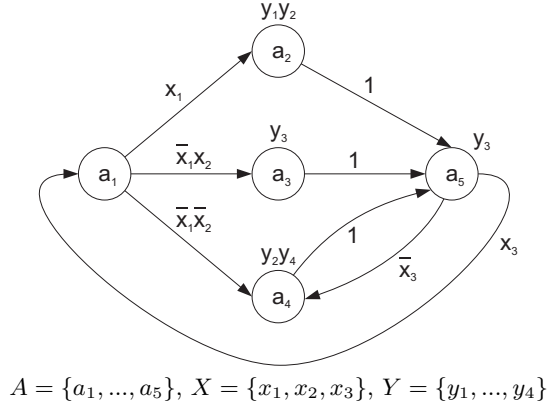
**Keywords:** Moore FSM, FPGA, look-up table, EMB, design, pseudoequivalent states

## 1 Introduction

The model of Moore finite state machine (FSM) is often used during the design of control units [2]. One of the important problems of FSM synthesis is need to reduce the hardware consumed by FSM logic circuit [3, 12]. The methods of solution of this problem depend strongly on features of logic elements used for designing the circuits [6, 8]. In this article we discuss the case when field-programmable gate arrays (FPGA) are used to implement the Moore FSM logic circuit.

To implement the FSM logic circuit, it is enough to use two components of FPGA fabric. These components are logic elements (LE) and a matrix of programmable interconnections [1, 21]. An LE consists of a look-up table (LUT) element, a programmable flip-flop and multiplexers. A LUT can be viewed as a random access memory block having  $S$  address inputs and a single output. One LUT can keep a truth table of an arbitrary Boolean function having up to  $S$  arguments. The flip-flop of LE could be bypassed. It means that the output of LE could be either combinational or registered.

A LUT has rather small number of inputs ( $S \leq 6$ ) [1, 21]. This peculiarity leads to necessity of functional decomposition [14, 16] for Boolean functions having more than  $S$  arguments. It results in multilevel logic circuits with complex interconnections. To reduce the hardware amount, it is necessary to diminish the numbers of arguments in Boolean functions representing an FSM logic circuit. We propose one of possible approaches for solution of this problem. Our approach is based on: 1) using the classes of pseudoequivalent states of Moore FSM [6] and 2) encoding of the collections of output functions [4].



**Fig. 1.** State transition graph for Moore FSM  $S_1$

## 2 Theoretical background

A Moore FSM is characterised by the following sets:  $A = \{a_1, \dots, a_M\}$  is a set of internal states;  $X = \{x_1, \dots, x_L\}$  is a set of input variables;  $Y = \{y_1, \dots, y_N\}$  is a set of output functions. Transitions between the states could be represented by a state transition graph (STG) [12]. There is an STG of Moore FSM  $S_1$  shown in Fig. 1.

The vertices of STG correspond to states  $a_m \in A$ , while the edges to transitions between the states. The transitions are determined by inputs  $X_h (h = \overline{1, H})$  equal to conjunctions of some elements  $x_e \in X$  (or their complements). There is a collection of output functions (COF)  $Y_q \subseteq Y$  marked above a vertex  $a_m \in A$ . It means that the COF  $Y_q ((q = \overline{1, a}))$  is generated while an FSM is in the state  $a_m \in A$ .

The states  $a_m \in A$  are encoded using state variables  $T_r \in T$ , where  $T = \{T_1, \dots, T_R\}$  is a set of state variables. State codes  $K(a_m)$  are assigned during the step of state assignment [12]. The number  $R$  is determined as

$$R = \lceil \log_2 M \rceil, \quad (1)$$

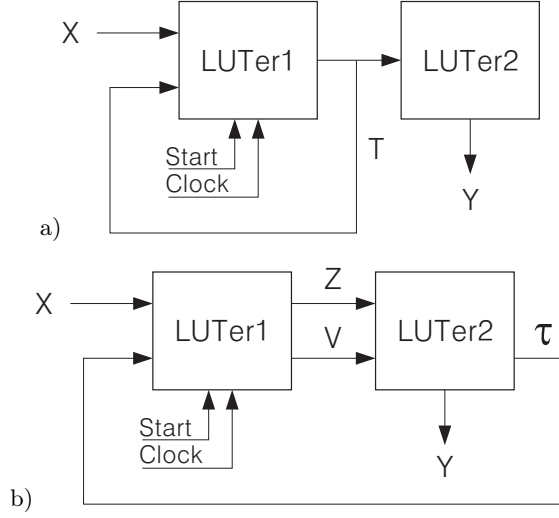
where  $\lceil a \rceil$  is a minimum integer not less than  $a$ . To change contents of flip-flops, input memory functions are used forming the set  $\Phi = \{D_1, \dots, D_R\}$ .

To design the Moore FSM logic circuit, it is necessary to find the following systems of Boolean functions:

$$\Phi = \Phi(T, X), \quad (2)$$

$$Y = Y(T). \quad (3)$$

Systems (2)-(3) determine the structural diagram of Moore FSM  $S_1$  (Fig. 2a). This diagram targets the FPGA chips.



**Fig. 2.** Structural diagrams of Moore FSM  $U_1$  (a) and  $U_2$  (b)

We use the name “LUTer” to denote a logic circuit consisting from LUTs. If there are flip-flops in a circuit, they are controlled by pulses *Start* and *Clock*. The pulse *Start* loads the zero code into a register formed by these flip-flops. It corresponds to the initial state  $a_1 \in A$ . The pulse *Clock* allows changing content of flip-flops. It corresponds to a transition between the states.

The LUTer1 implements the system (2), the LUTer2 the system (3). Let the following condition take place:

$$R \leq S. \quad (4)$$

In this case, it is enough a single LUT to implement any function  $y_n \in Y$ . If the condition (4) is violated, then the number of LUTs exceeds  $N$  in the circuit of LUTer2. In this case, the circuit of LUTer2 is multilevel.

In this article, we discuss a design method for reducing the number of LUTs in LUTer2 if the condition (4) is violated. Also, we propose to use pseudoequivalent states [6] to decrease the hardware

### 3 Main idea of proposed method

The states  $a_m, a_s \in A$  are pseudoequivalent states (PES) if identical inputs result in identical next states for both states  $a_m, a_s \in A$ . For example, there are PES  $a_2, a_3, a_4 \in A$  for Moore FSM  $S_1$  (Fig. 1). Let  $\Pi_A = \{B_1, \dots, B_I\}$  be a partition of the set  $A$  by the classes of PES ( $I \leq M$ ).

In the case of  $S_1$ , there is  $I = 3$ . There are the following classes of PES:  $B_1 = \{a_1\}$ ,  $B_2 = \{a_2, a_3, a_4\}$ ,  $B_3 = \{a_5\}$ .

Let us encode the classes  $B_i \in \Pi_A$  by binary codes  $K(B_i)$  having  $R_I$  bits:

$$R_I = \lceil \log_2 I \rceil. \quad (5)$$

Let us use the variables  $\tau_r \in \mathcal{T}$  for encoding of the classes  $B_i \in \Pi_A$ , where  $\mathcal{T} = \{\tau_1, \dots, \tau_{RI}\}$ .

Let it be  $Q$  different COF  $Y_q \subseteq Y$  for a Moore FSM. Let us encode each COF  $Y_q$  by a binary code  $K(Y_q)$  having  $R_Q$  bits:

$$R_Q = \lceil \log_2 Q \rceil. \quad (6)$$

Let us use the variables  $z_r \in Z$  for encoding of the COF  $Y_q \subseteq Y$ , where  $|Z| = R_Q$ .

Analysis of Fig. 1 shows that there are the following COF in FSM  $S_1$ :  $Y_1 = \emptyset, Y_2 = \{y_2, y_3\}, Y_3 = \{y_3\}, Y_4 = \{y_2, y_4\}$ . So, there is  $Q = 4$ . It gives  $R_Q = 2$  and  $Z = \{z_1, z_2\}$ .

The COF  $Y_1$  corresponds to the class  $B_1$ . The collections  $Y_2$  and  $Y_3$  determine the class  $B_2$ . But the COF  $Y_4$  determines two classes,  $B_2$  and  $B_3$ . It means that it is necessary to have some identifiers to distinguish the classes  $B_2$  and  $B_3$  for the COF  $Y_4$ . In the discussed case, it is enough two identifiers ( $I_1$  and  $I_2$ ) to distinguish the classes. Let the pair  $\langle Y_4, I_2 \rangle$  determine the class  $B_2$ , whereas  $\langle Y_4, I_1 \rangle$  the class  $B_3$ .

In common case, it is enough  $K$  identifiers forming the set  $ID = \{I_1, \dots, I_K\}$ . Let us encode them using  $R_K$  variables where

$$R_K = \lceil \log_2 K \rceil. \quad (7)$$

Let us use variables  $v_r = V$  for encoding of identifiers where  $|V| = R_K$ .

Basing on these preliminaries, we propose the structural diagram of Moore FSM  $U_2$  (Fig. 2b). In  $U_2$ , the LUTer1 generates functions

$$Z = Z(\tau, X); \quad (8)$$

$$V = V(\mathcal{T}, X). \quad (9)$$

The LUTer2 implements the systems

$$Y = Y(Z); \quad (10)$$

$$\mathcal{T} = \mathcal{T}(Z, V). \quad (11)$$

Our analysis of the library of standard benchmarks [11] shows that the following conditions take places:

$$R_Q < R; \quad (12)$$

$$R_I < R; \quad (13)$$

$$H(U_2) < H(U_1). \quad (14)$$

We use the symbol  $H(U_i)$  to denote the number of terms in the system  $\Phi$  of  $U_i(\overline{1}, \overline{2})$ .

Basing on (12)–(14), we can expect that logic circuits obtained for FSM  $U_2$  consume less

Let us point out that a state of transition  $a_s \in A$  is represented by a pair  $\langle Y_q, I_k \rangle$ , where  $A_s \in B_i$  and  $Y_q \subseteq Y$  is generated in the state  $a_s \in A$ . It means that the code  $K(a_s)$  is represented as

$$K(a_s) = K(Y_q) * K(I_k). \quad (15)$$

In (15), the symbol  $K(Y_q)$  stands for the code of COF  $Y_q$ , the symbol  $K(I_k)$  for the code of identifier  $I_k \in ID$ . So, the code  $K(a_s)$  should be transformed into corresponding codes from (15).

## 4 Proposed design method

In this article, we propose a design method for  $U_2$ . It includes the following steps:

1. Constructing partition  $\Pi_A$  and collections  $Y_q \subseteq Y$  for a given STG.
2. Constructing the pairs  $\langle Y_q, I_k \rangle$  for states  $a_s \in A$ .
3. Encoding of classes  $B_i \in \Pi_A$ , collections  $Y_q \subseteq Y$  and identifiers  $I_k \in ID$ .
4. Transforming initial STG into class transition graph (CTG).
5. Finding systems (8)–(11).
6. Implementing FSM logic circuit using LUTs of a particular FPGA chip.

Let us apply this method for the Moore FSM  $S_1$ . As it is mentioned before, there are the partition  $\Pi_A = \{B_1, B_2, B_3\}$  with classes  $B_1 = \{a_1\}$ ,  $B_2 = \{a_2, a_3, a_4\}$  and  $B_3 = \{a_5\}$  and the collections  $Y_1 = \emptyset$ ,  $Y_2 = \{y_1, y_2\}$ ,  $Y_3 = \{y_3\}$  and  $Y_4 = \{y_2, y_4\}$ . It gives  $I = 3$  and  $Q = 4$ .

There are the following pairs  $\langle Y_q, I_k \rangle$  in the discussed case:  $\langle Y_1, \emptyset \rangle = a_1$ ,  $\langle Y_2, \emptyset \rangle = a_2$ ,  $\langle Y_3, \emptyset \rangle = a_3$ ,  $\langle Y_4, I_1 \rangle = a_4$  and  $\langle Y_4, I_2 \rangle = a_5$ . So, there is  $K = 2$  and  $I = \{I_1, I_2\}$ .

Using (1), (5), (6) and (7), we can find that  $R = 3$ ,  $R_I = 2$ ,  $R_Q = 2$  and  $R_K = 1$ . Using (15), we can find that the codes  $K(a_s)$  should include  $R_Q + R_K = 3$  bits. It gives the set  $\Phi = \{D_1, D_2, D_3\}$ . Functions  $D_1, D_2$  determine the variables  $z_1$  and  $z_2$ . Function  $D_3$  determines the variable  $v_1$ . So, there are the following sets:  $Z = \{z_1, z_2\}$ ,  $V = \{v_1\}$  and  $\mathcal{T} = \{\tau_1, \tau_2\}$ .

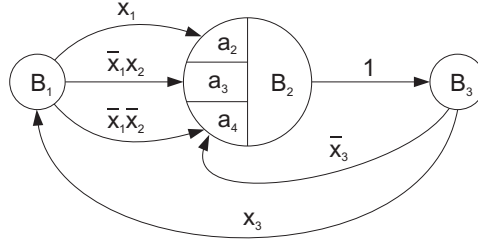
Let us encode in the trivial way the classes  $B_i \in \Pi_A$ , the collections  $Y_q \subseteq Y$  and identifiers  $I_k \in I$ . It gives the following codes:  $K(B_1) = K(Y_1) = 00$ ,  $K(B_2) = K(Y_2) = 01$ ,  $K(B_3) = K(Y_3) = 10$ ,  $K(Y_4) = 11$ ,  $K(I_1) = 0$  and  $K(I_2) = 1$ .

Using these codes, we can find the codes  $K(a_s)$  corresponding to (15). There are the following codes:  $K(a_1) = 00*$ ,  $K(a_2) = 01*$ ,  $K(a_3) = 10*$ ,  $K(a_4) = 110$  and  $K(a_5) = 111$ . The symbol “\*” corresponds to the element  $\emptyset$  in the pairs  $\langle Y_q, I_k \rangle$ .

To construct a CTG, let us replace the states  $a_m \in B_i$  by a single vertex  $B_i \in \Pi_A$ . In the discussed case, the CTG is shown in Fig. 3.

If some class  $B_i \in \Pi_A$  has more than one state, then these states are shown inside a particular vertex of CTG. We did it for the class  $B_2$ .

There are the following columns in the structure table of  $U_2$ :  $B_i$ ,  $K(B_i)$ ,  $a_s$ ,  $K(a_s)$ ,  $X_h$ ,  $\Phi_h$ ,  $h$ . This table is constructed on the base of CTG. The column

**Fig. 3.** Class transition graph for Moore FSM  $S_1$ **Table 1.** Structure table of Moore FSM  $S_1$ 

$B_i$	$K(B_i)$	$a_s$	$K(a_s)$	$X_h$	$\Phi_h$	$h$
		$a_2$	010	$x_1$	$D_2$	1
$B_1$	00	$a_3$	100	$\bar{x}_1\bar{x}_2$	$D_1$	2
		$a_4$	110	$\bar{x}_1\bar{x}_2$	$D_1D_2$	3
$B_2$	*1	$a_5$	111	1	$D_1D_2D_3$	4
$B_3$	1*	$a_1$	000	$x_3$	–	5
		$a_4$	110	$\bar{x}_3$	$D_1D_2$	6

$K(a_s)$  includes a code of the state of transition (12). The column  $X_h$  contains an input signal determining a transition from a state  $a_m \in B_i$  into a state  $a_s \in A$ . The column  $\Phi_h$  contains input memory functions  $D_r \in \Phi$  equal to 1 to load the codes  $K(Y_q)$  and  $K(I_K)$  into the distributed state register.

The column  $h$  contains the number of transition ( $h = \overline{1, H(U_2)}$ ). In the case of  $S_1$ , this table has  $H(U_2) = 6$  rows (Table 1). We use the combination 11 to optimize the codes of  $B_2$  and  $B_3$ . We replace all “\*” in state codes by zeros.

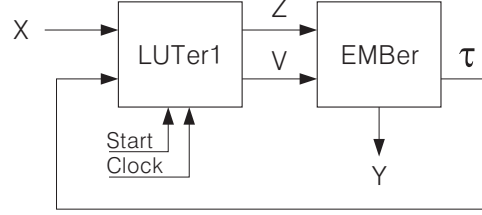
The ST is used to derive the functions (8) and (9). In the discussed case, there are the following functions:

$$\begin{aligned}
 z_1 &= D1 = \bar{\tau}_1\bar{\tau}_2\bar{x}_1 \vee \tau_2 \vee \tau_1\bar{x}_3; \\
 z_2 &= D2 = \bar{\tau}_1\bar{\tau}_2x_1 \vee \bar{\tau}_1\bar{\tau}_2\bar{x}_2 \vee \tau_2 \vee \tau_1\bar{x}_3; \\
 v_1 &= D3 = \tau_2.
 \end{aligned} \tag{16}$$

To find the functions (10), we should find disjunctions of COF  $Y_q \subseteq Y$  for each function  $y_n \in Y$ . In the discussed case, there are the following functions:

$$\begin{aligned}
 y_1 &= Y_2 = \bar{z}_1z_2; \\
 y_2 &= Y_2 \vee Y_4 = z_2; \\
 y_3 &= Y_3 = z_1\bar{z}_2; \\
 y_4 &= Y_4 = z_1z_2.
 \end{aligned} \tag{17}$$

To find the functions (11), we should find disjunctions of state codes  $a_s \in B_i$  for each class  $B_i$ :  $B_1 = A_1$ ,  $B_2 = A_2 \vee A_3 \vee A_4$  and  $B_3 = A_5$ . It gives the following functions:  $B_1 = \bar{z}_1\bar{z}_2$ ,  $B_2 = \tau_2 = \bar{z}_1z_2 \vee z_1\bar{z}_2 \vee z_1z_2v_1$ ;  $B_3 = \tau_1 = z_1z_2v_1$ .



**Fig. 4.** Structural diagram of Moore FSM  $U_3$

To execute the last step of the proposed design method, such problems should be solved as the mapping, placing and routing [10]. To do it, some industrial packages should be used [1, 21]. In our research, we use program tools of Xilinx to implement the FSM logic circuits.

## 5 Replacement LUTs by EMBs

It is possible to improve the characteristics of FSM replacing LUTs by embedded memory blocks (EMB) [5, 7, 9, 15, 18–20]. The EMBs of modern FPGA chips [2, 13] have a property of configurability. It means that such parameters as the number of cells and their outputs can be changed. There are the following typical configurations of EMBs:  $32K \times 1$ ,  $16K \times 2$ ,  $8K \times 4$ ,  $4K \times 8$ ,  $2K \times 16$ ,  $1K \times 32$  and  $512 \times 64$  (bits) [2, 13]. So, modern EMBs are very flexible and can be tuned to meet a particular design.

It is possible to implement an FSM circuit using only a single EMB [20]. It can be done if the following condition takes place:

$$(N + R) \cdot 2^{L+R} \leq V_0. \quad (18)$$

In (18), the symbol  $V_0$  stands for the number of cells in the configuration of EMB having  $t_F = 1$  outputs. Of course, the condition (18) has place only for rather simple FSMs.

To improve the characteristics of  $U_1$ , we propose to replace the LUTer2 by EMBer. The symbol EMBer stands for the circuit implemented with EMBs. Such a replacement leads to the FSM  $U_3$  (Fig. 4).

In this FSM, the EMBer implements the systems (10) and (11). Our analysis of the library [11] shows that the following condition takes place for all standard benchmarks [11]:

$$2^{R_Q+R_K}(N + R_I) \leq V_0. \quad (19)$$

It means that only a single EMB is enough for implementing FSM circuits based on the model  $U_3$ . This model can be used till the following condition takes place:

$$S_A \geq R_Q + R_K. \quad (20)$$



**Table 2.** Table of EMBer of Moore FSM  $S_1$ 

$K(Y_q)$	$K(I_K)$	Y	$\mathcal{T}$	$h$	$a_m$
$z_1z_2$	$v_1$	$y_1y_2y_3y_4$	$\tau_1\tau_2$		
00	0	0000	00	1	$a_1$
00	1	****	**	2	*
01	0	1100	01	3	$a_2$
01	1	****	**	4	*
10	0	001*	01	5	$a_3$
10	1	****	**	6	*
11	0	0001	01	7	$a_4$
11	1	0001	10	8	$a_5$

In (20), the symbol  $S_A$  stands for the number of address bits for some configuration of EMB.

There are the same steps 1–5 in design methods for  $U_2$  and  $U_3$ . But there is a difference in the executing the step 6. In the case of  $U_2$  it is necessary to construct the table of EMBer. It includes the following columns:  $K(Y_q)$ ,  $K(I_K)$ ,  $Y$ ,  $\mathcal{T}$ ,  $h$ . The first two columns determine an address of a memory cell. The table includes  $H_E$  lines where

$$H_E = 2^{R_Q+R_K}. \quad (21)$$

In the discussed case, there is  $R_Q = 2$  and  $R_K = 1$ . So, there is  $H_E = 8$ . Table 2 represents the table of EMBer for Moore FSM  $S_1$ .

We added the column  $a_m$  in Table 2 to show the correspondence among the pairs  $\langle Y_q, I_K \rangle$  and states  $a_m \in A$ . If a pair  $\langle K(Y_q), K(I_K) \rangle$  does not correspond to any state, then there are asterisks in the corresponding lines of Table 2. If a state  $a_m \in B_i$ , then the corresponding line includes the code of this class. For example, there is the relation  $a_2, a_3, a_4 \in B_2$ , so there is the code 01 in the lines 3,5 and 7 of Table 2.

Let us point out that the structure table of Moore FSM  $S_1$  in the case of  $U_3$  is the same as for  $U_2$  (Table 1). It means that systems (8)–(9) are represented as the system (16) for both models  $U_2$  and  $U_3$  of FSM  $S_1$ .

## 6 Results of investigations

We use the standard benchmarks [11] to investigate efficiency of proposed method. The library LGSynth93 includes 48 finite-state machines represented in the KISS2 format.

To use the benchmarks, we worked out the CAD tool named K2F. It translates the KISS2 file into VHDL model of the Moore FSM. We use Active-HDL environment to synthesize and simulate the FSM. To implement the FSM circuits, we use Xilinx ISE 14.1 package. We choose the FPGA device XC5vex30ff324 by

**Table 3.** Results of investigation for FSM  $U_2$ 

BM	$U_1$ auto	$U_1$ compact	JEDI	DEMAIN	$U_2$
cse	74	72	57	63	42
dk16	72	59	49	67	36
ex1	96	111	90	86	56
keyb	84	98	70	74	52
kirkman	77	85	68	70	50
planet	152	208	145	154	105
S1488	210	212	183	185	134
S298	542	495	443	465	315
scf	269	303	232	260	174
styr	178	191	156	158	114
Total	1754	1834	1482	1582	1078
Percentage	163%	170%	137%	147%	100%

Virtex-5 of Xilinx as a target platform. Some results of investigations are shown in Table 3.

There are the numbers of LUTs necessary for implementing logic circuits of different FSMs for given benchmarks in Table 3. There is the name of a benchmark in the column BM. The following methods were taken for comparison with  $U_2$ : 1) FSM  $U_1$  designed with method “Auto” of ISE 14.1; 2) FSM  $U_1$  designed with method “Compact” of ISE 14.1; 3) Moore FSM used the state assignment by JEDI from SIS [17]; 4) Moore FSM designed by DEMAIN [13].

We show the most complex benchmarks from [11] into Table 3. As follows from Table 3, the FSM  $U_2$  always produces the best results. The row “Total” gives the overall numbers of LUTs for all benchmarks (for the given design method). The row “Percentage” shows the percentage of the total number of LUTs regarding the FSM  $U_2$ . We have taken the total number of LUTs for  $U_2$  as 100%.

Our approach gives better results for 60% of all benchmarks from [11]. If an FSM has less than 20 states, then better results belong to JEDI. Our methods produces better results because: 1) we take into account the existence of PES and 2) we encode the collections of output functions. The JEDI uses PES but there is no encoding of COF for this method. Also, the JEDI uses state variables  $T_r \in T$  for representing classes of PES. We use some additional variables  $\tau_r \in \mathcal{T}$ . Due to these two features, our method produces better circuits than JEDI (and all other methods from Table 3).

We also investigated the efficiency of the model  $U_3$ . The same FSMs were taken for comparison with  $U_3$  as it was for the case of  $U_2$ . The results of investigations for FSM  $U_3$  are shown in Table 4.

**Table 4.** Results of investigation for FSM  $U_3$ 

BM	$U_1$ auto	$U_1$ compact	JEDI	DEMAIN	$U_3$
cse	64	62	47	53	32
dk16	65	52	42	60	29
ex1	84	99	78	74	44
keyb	73	87	59	63	41
kirkman	68	76	59	61	41
planet	128	185	122	131	82
S1488	184	186	157	159	108
S298	529	482	430	452	302
scf	241	275	204	232	146
styr	164	177	142	144	100
Total	1601	1681	1329	1429	925
Percentage	173%	182%	144%	154%	100%

Our analysis of the library [16] shows that it is enough a single EMB to implement output functions in the cases of  $U_1$  auto,  $U_1$  compact, JEDI and DEMAIN. The same is true for the case of  $U_3$ . So, there are only numbers of LUTs shown in Table 4.

As follows from Table 4, our approach gives better results for all complex benchmarks from [16]. The reasons for it are stated during the analysis of Table 3. But if EMBs are used, then there is a gain exceeds the gain for the case of LUT-based FSMs. It follows from comparison the rows “Percentage” from Tables 3 and 4.

To explain this conclusion, let us construct a table with comparison of numbers of LUTs necessary in the case of LUT-based and EMB-based FSMs. It is Table 5. There is a ratio of numbers of LUTs for corresponding cells of Table 3 and Table 4.

In all cases, practically the same number of LUTs is replaced by a single EMB. It is equal to  $N$  for  $U_1$  auto,  $U_1$  compact, JEDI and DEMAIN. It is equal to  $N + R_I$  for  $U_2$  and  $U_3$ . So, the less number of LUTs in LUTer1 (or LUTer), the better results can be obtained due to replacement LUTer2 by EMBer. For example, in the case of  $U_1$  auto saving is equal to 12%. But transition from  $U_2$  to  $U_3$  saves in average 22% of LUTs (see the last column of Table 4).

## 7 Conclusion

The proposed method is based on representing state codes as concatenations of class codes and codes of collections of output functions. It targets LUT-based FSM circuits. Our investigation shows that the method produces FSM circuits having fewer LUTs than the methods used by SIS, DEMAIN and ISE 14.1. The

**Table 5.** Relations between Tables 3 and 4

BM	$U_1$ auto	$U_1$ compact	JEDI	DEMAIN	$U_2/U_3$
cse	1,16	1,16	1,21	1,19	1,31
dk16	1,11	1,13	1,16	1,12	1,24
ex1	1,14	1,12	1,15	1,16	1,27
keyb	1,15	1,13	1,19	1,17	1,27
kirkman	1,13	1,12	1,15	1,15	1,22
planet	1,18	1,12	1,19	1,17	1,28
S1488	1,14	1,15	1,17	1,16	1,24
S298	1,02	1,03	1,03	1,03	1,04
scf	1,11	1,10	1,14	1,12	1,21
styr	1,09	1,08	1,10	1,10	1,14
Total	11,23	11,14	11,49	11,37	12,22
Average	1,12	1,11	1,15	1,14	1,22

method could be applied for rather complex Moore FSMs having more than 20 states.

The future direction of our research is the development of a method for encoding of collections of output functions. It is necessary if the following condition is violated:

$$R_Q > S. \quad (22)$$

The method should diminish the number of arguments in Boolean function (8). It allows hardware reduction in the circuit of block LUTer2.

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