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A type of Arithmetic Labels about Circulating ring

Ergen Liu Dan Wu Kewen Cai

(School of Basic Sciences, East China Jiaotong University, Nanchang,P.R.China,330013)

E-mail:leg_eg@sina.com

Absteact

The graph composed with several rings is a kind of important and interesting graph, many scholars studied on the gracefulness of this kind of the graph, The reference [1] is given the gracefulness of m kinds C_4 with one common point. In this paper, we researched the arithmetic labels of four kinds graph: $C_{8,1,n}$, $C_{8,2,n}$, $C_{8,3,n}$, $C_{8,4,n}$, and we proved they are all $(d,2d)$ -arithmetic graph.

Key Word: Arithmetic graph; labeling of graph; $C_{8,i,n}$

1. Introduction

The graph in this paper discussed are undirected、no multiple edges and simple graph, the unorganized state of definitions and terminology and the symbols in this graph referred to reference[2][3].

There are two kinds of labels of the graph: one is the reduced label, is to say that in order to get the label of one edge you should reduce the endpoints of the edge; the other is additive label, for the same you should active the endpoints of one edge to get the label of the edge. For example, the well-known of “Gracefulness” is reduced. the “Compatible labels” is additive. In 1990, B.D.Achaya and S.M.Hegde import the concept of “Arithmetic lables”(referred to reference[2]), which is a more extensive additive label, it have applied value on solution to question of the joint ventures in rights and obligations.

Definition 1.1 For $G=(V,E)$, if there is a mapping f (called the vertices v of the label) from $V(G)$ to the set of nonnegative integer N_0 , meet:

- (1) $f(u) \neq f(v)$, which $u \neq v$, and $u, v \in V(G)$;
 - (2) $\{f(u) + f(v) | uv \in E(G)\} = \{k, k + d, \dots, k + (q - 1)d\}$.
- Then we call graph G is a (k, d) -arithmetic graph.

2. Main results and certification

Theorem 2.1 $C_{8,1,n}$ is a $(d,2d)$ -arithmetic graph.

Proof As the graph shown on fig.1.

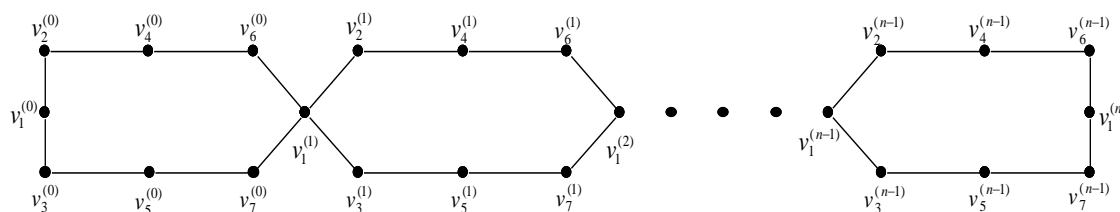


Fig.1.The Graf $C_{8,1,n}$

Label all vertices as follows:

$$\begin{aligned} f(v_1^{(i)}) &= 8id \ (i=0,1,2,\dots,n), & f(v_2^{(i)}) &= 8id + d \ (i=0,1,2,\dots,n-1), \\ f(v_3^{(i)}) &= 8id + 3d \ (i=0,1,2,\dots,n-1), & f(v_4^{(i)}) &= 8id + 6d \ (i=0,1,2,\dots,n-1), \\ f(v_5^{(i)}) &= 8id + 2d \ (i=0,1,2,\dots,n-1), & f(v_6^{(i)}) &= 8id + 5d \ (i=0,1,2,\dots,n-1), \\ f(v_7^{(i)}) &= 8id + 7d \ (i=0,1,2,\dots,n-1). \end{aligned}$$

Now we proof that the mapping f is arithmetic labeling of $C_{8,1,n}$.

We can see the mapping f meet $f(u) \neq f(v)$ which $u \neq v$ and $u, v \in V(C_{8,1,n})$.

Next we prove $\{f(u) + f(v) | uv \in E(C_{8,1,n})\}$ is an arithmetic progression in the way of mathematical induction.

When $n=1$

$$\begin{aligned} \text{Then } f(v_1^{(0)}) &= 0, \ f(v_2^{(0)}) = d, \ f(v_3^{(0)}) = 3d, \ f(v_4^{(0)}) = 6d, \ f(v_5^{(0)}) = 2d, \ f(v_6^{(0)}) = 5d, \\ f(v_7^{(0)}) &= 7d, \ f(v_1^{(1)}) = 8d. \end{aligned}$$

$$\begin{aligned} \text{Therefore } \{f(u) + f(v) | uv \in E(C_{8,1,1})\} \\ &= \{f(v_1^{(0)}) + f(v_2^{(0)}), f(v_1^{(0)}) + f(v_3^{(0)}), f(v_3^{(0)}) + f(v_5^{(0)}), f(v_2^{(0)}) + f(v_4^{(0)}), \\ &f(v_5^{(0)}) + f(v_7^{(0)}), f(v_4^{(0)}) + f(v_6^{(0)}), f(v_6^{(0)}) + f(v_1^{(1)}), f(v_7^{(0)}) + f(v_1^{(1)})\} \\ &= \{d, 3d, 5d, 7d, 9d, 11d, 13d, 15d\} \end{aligned}$$

is an arithmetic progression, and the common difference is $2d$.

Suppose when $n=k-1$, we know

$$\{f(u) + f(v) | uv \in E(C_{8,1,k-1})\} = \{d, d+1 \times 2d, d+2 \times 2d, \dots, d+(8k-9) \times 2d\}$$

is an arithmetic progression, and the common difference is $2d$.

Then when $n=k$

$$\begin{aligned} \{f(u) + f(v) | uv \in E(C_{8,1,k})\} \\ &= \{f(u) + f(v) | uv \in E(C_{8,1,k-1})\} \cup \{f(v_1^{(k-1)}) + f(v_2^{(k-1)}), f(v_1^{(k-1)}) + f(v_3^{(k-1)}), f(v_3^{(k-1)}) + f(v_5^{(k-1)}), \\ &f(v_2^{(k-1)}) + f(v_4^{(k-1)}), f(v_5^{(k-1)}) + f(v_7^{(k-1)}), f(v_4^{(k-1)}) + f(v_6^{(k-1)}), f(v_6^{(k-1)}) + f(v_1^{(k)}), f(v_7^{(k-1)}) + f(v_1^{(k)})\} \\ &= \{d, d+1 \times 2d, d+2 \times 2d, \dots, d+(8k-9) \times 2d\} \cup \{d+(8k-8) \times 2d, d+(8k-7) \times 2d, \\ &d+(8k-6) \times 2d, d+(8k-5) \times 2d, d+(8k-4) \times 2d, d+(8k-3) \times 2d, d+(8k-2) \times 2d, d+(8k-1) \times 2d\} \\ &= \{d, d+1 \times 2d, d+2 \times 2d, \dots, d+(8k-1) \times 2d\} \end{aligned}$$

is an arithmetic progression, and the common difference is $2d$.

In sum for the arbitrary $n \in N_0$, the mapping $f : V(C_{8,1,n}) \rightarrow N_0$ meet:

(1) $f(u) \neq f(v)$ when $u \neq v$ and $u, v \in V(C_{8,1,n})$;

(2) $\{f(u) + f(v) | uv \in E(C_{8,1,n})\} = \{d, d+1 \times 2d, d+2 \times 2d, \dots, d+(8n-1) \times 2d\}$.

Therefore $C_{8,1,n}$ is a $(d, 2d)$ --arithmetic graph.

Theorem 2.2 $C_{8,2,n}$ is a $(d, 2d)$ --arithmetic graph.

Proof As the graph shown on fig.2.

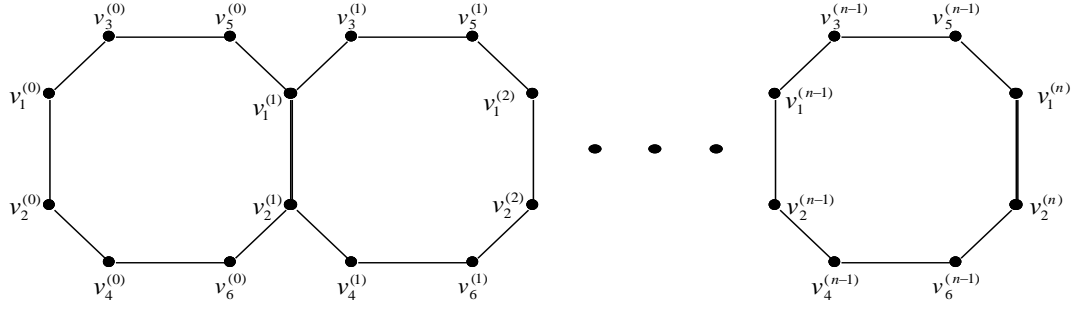


Fig.2.The Graf $C_{8,2,n}$

Label all vertices as follows:

$$\begin{aligned} f(v_1^{(i)}) &= 7id \quad (i=0,1,2,\dots,n), & f(v_4^{(i)}) &= 7id + 6d \quad (i=0,1,2,\dots,n-1), \\ f(v_2^{(i)}) &= 7id + d \quad (i=0,1,2,\dots,n), & f(v_5^{(i)}) &= 7id + 2d \quad (i=0,1,2,\dots,n-1), \\ f(v_3^{(i)}) &= 7id + 3d \quad (i=0,1,2,\dots,n-1), & f(v_6^{(i)}) &= 7id + 5d \quad (i=0,1,2,\dots,n-1). \end{aligned}$$

Now we proof that the mapping f is arithmetic labeling of $C_{8,2,n}$

We can see the mapping f meet $f(u) \neq f(v)$ which $u \neq v$ and $u, v \in V(C_{8,2,n})$.

Next we prove $\{f(u) + f(v) | uv \in E(C_{8,2,n})\}$ is an arithmetic progression in the way of mathematical induction.

When $n = 1$

Then $f(v_1^{(0)}) = 0$, $f(v_2^{(0)}) = d$, $f(v_3^{(0)}) = 3d$, $f(v_4^{(0)}) = 6d$, $f(v_5^{(0)}) = 2d$, $f(v_6^{(0)}) = 5d$,
 $f(v_1^{(1)}) = 7d$, $f(v_2^{(1)}) = 8d$.

$$\begin{aligned} \text{Therefore } \{f(u) + f(v) | uv \in E(C_{8,2,1})\} &= \{f(v_1^{(0)}) + f(v_2^{(0)}), f(v_1^{(0)}) + f(v_3^{(0)}), f(v_3^{(0)}) + f(v_5^{(0)}), \\ &f(v_2^{(0)}) + f(v_4^{(0)}), f(v_5^{(0)}) + f(v_1^{(1)}), f(v_4^{(0)}) + f(v_6^{(0)}), f(v_6^{(0)}) + f(v_2^{(1)}), f(v_1^{(1)}) + f(v_2^{(1)})\} \\ &= \{d, 3d, 5d, 7d, 9d, 11d, 13d, 15d\} \end{aligned}$$

is an arithmetic progression, and the common difference is $2d$.

Suppose when $n = k - 1$, we know

$$\{f(u) + f(v) | uv \in E(C_{8,2,k-1})\} = \{d, d + 1 \times 2d, d + 2 \times 2d, \dots, d + (7k - 7) \times 2d\}$$

is an arithmetic progression, and the common difference is $2d$.

Then when $n = k$

$$\begin{aligned} \{f(u) + f(v) | uv \in E(C_{8,2,k})\} &= \{f(u) + f(v) | uv \in E(C_{8,2,k-1})\} \cup \{f(v_1^{(k-1)}) + f(v_3^{(k-1)}), f(v_3^{(k-1)}) + f(v_5^{(k-1)}), \\ &f(v_2^{(k-1)}) + f(v_4^{(k-1)}), f(v_5^{(k-1)}) + f(v_1^{(k)}), f(v_4^{(k-1)}) + f(v_6^{(k-1)}), f(v_6^{(k-1)}) + f(v_2^{(k)}), f(v_1^{(k)}) + f(v_2^{(k)})\} \\ &= \{d, d + 1 \times 2d, d + 2 \times 2d, \dots, d + (7k - 7) \times 2d\} \cup \{d + (7k - 6) \times 2d, d + (7k - 5) \times 2d, \\ &d + (7k - 4) \times 2d, d + (7k - 3) \times 2d, d + (7k - 2) \times 2d, d + (7k - 1) \times 2d, d + 7k \times 2d\} \\ &= \{d, d + 1 \times 2d, d + 2 \times 2d, \dots, d + 7k \times 2d\} \end{aligned}$$

is an arithmetic progression, and the common difference is $2d$.

In sum for the arbitrary $n \in N_0$, the mapping $f : V(C_{8,2,n}) \rightarrow N_0$ meet:

- (1) $f(u) \neq f(v)$ when $u \neq v$ and $u, v \in V(C_{8,2,n})$;
(2) $\{f(u) + f(v) | uv \in E(C_{8,2,n})\} = \{d, d + 1 \times 2d, d + 2 \times 2d, \dots, d + 7n \times 2d\}$.

Therefore $C_{8,2,n}$ is a $(d, 2d)$ --arithmetic graph.

Theorem 2.3 $C_{8,3,n}$ is a $(d, 2d)$ --arithmetic graph .

Proof As the graph shown on fig.3.

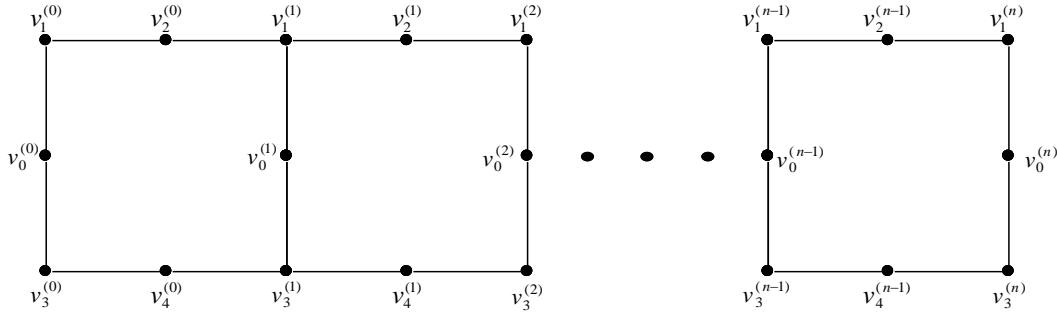


Fig.3.The Graf $C_{8,3,n}$

Label all vertices as follows:

$$\begin{aligned} f(v_0^{(i)}) &= 8id \quad (i = 0, 1, 2, \dots, n), & f(v_1^{(i)}) &= 4id + d \quad (i = 0, 1, 2, \dots, n), \\ f(v_3^{(i)}) &= 4id + 3d \quad (i = 0, 1, 2, \dots, n), & f(v_2^{(i)}) &= 8id + 6d \quad (i = 0, 1, 2, \dots, n-1), \\ f(v_4^{(i)}) &= 8id + 2d \quad (i = 0, 1, 2, \dots, n-1). \end{aligned}$$

Proof in imitation of Theorem 2.1.

For the arbitrary $n \in N_0$, the mapping $f : V(C_{8,3,n}) \rightarrow N_0$ meet:

- (1) $f(u) \neq f(v)$ when $u \neq v$ and $u, v \in V(C_{8,3,n})$;
(2) $\{f(u) + f(v) | uv \in E(C_{8,3,n})\} = \{d, d + 1 \times 2d, d + 2 \times 2d, \dots, d + (6n + 1) \times 2d\}$.

By (1), (2) we know the mapping f is arithmetic labeling of $C_{8,3,n}$. Therefore $C_{8,3,n}$ is a $(d, 2d)$ --arithmetic graph.

Theorem 2.4 $C_{8,4,n}$ is a $(d, 2d)$ --arithmetic graph.

Proof As the graph shown on fig.4.

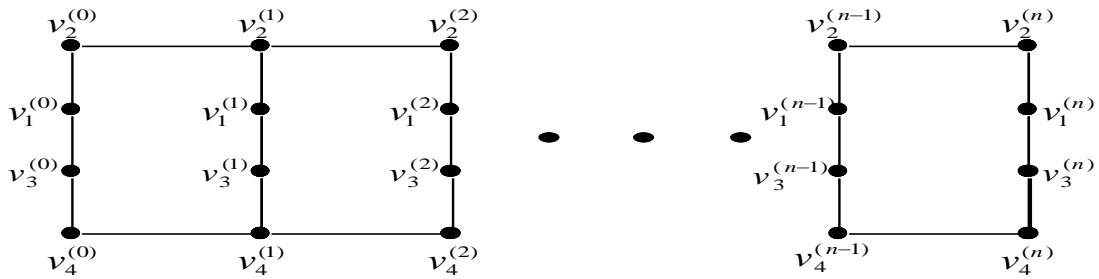


Fig.4.The Graf $C_{8,4,n}$

Label all vertices as follows:

$$\begin{aligned} f(v_1^{(i)}) &= 5id \quad (i = 0, 1, 2, \dots, n), & f(v_2^{(i)}) &= 5id + d \quad (i = 0, 1, 2, \dots, n), \end{aligned}$$

$$f(v_3^{(i)}) = 5id + 3d (i = 0, 1, 2, \dots, n), \quad f(v_4^{(i)}) = 5id + 2d (i = 0, 1, 2, \dots, n).$$

Proof in imitation of Theorem 2.2.

For the arbitrary $n \in N_0$, the mapping $f : V(C_{8,4,n}) \rightarrow N_0$ meet:

(1) $f(u) \neq f(v)$ when $u \neq v$ and $u, v \in V(C_{8,4,n})$;

(2) $\{f(u) + f(v) \mid uv \in E(C_{8,4,n})\} = \{d, d + 1 \times 2d, d + 2 \times 2d, \dots, d + (5n + 2) \times 2d\}$.

By (1), (2) we know the mapping f is arithmetic labeling of $C_{8,4,n}$. Therefore $C_{8,4,n}$ is a $(d, 2d)$ -arithmetic graph.

3. Labels of Some Special Graph

In order to explain the correctness of the aforementioned labels, we give the arithmetic labeling of $C_{8,1,3}$, $C_{8,2,3}$, $C_{8,3,3}$ and $C_{8,4,3}$.

(1) The arithmetic labeling of $C_{8,1,3}$ on fig.5.

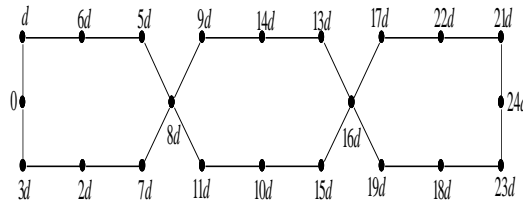


Fig.5. The Graf $C_{8,1,3}$

(2) The arithmetic labeling of $C_{8,2,3}$ on fig.6.

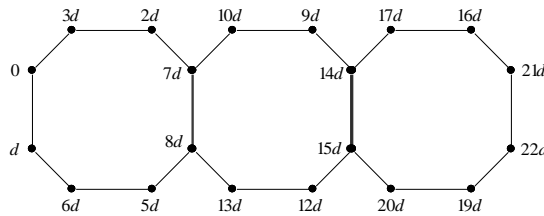


Fig.6. The Graf $C_{8,2,3}$

(3) The arithmetic labeling of $C_{8,3,3}$ on fig.7.

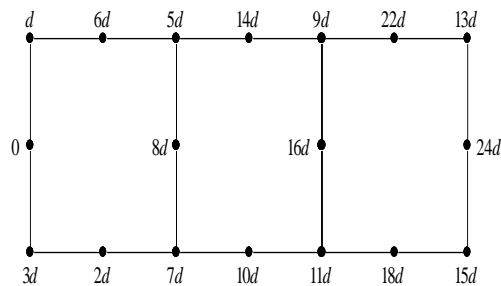


Fig.7. The Graf $C_{8,3,3}$

(4) The arithmetic labeling of $C_{8,4,3}$ on fig.8.

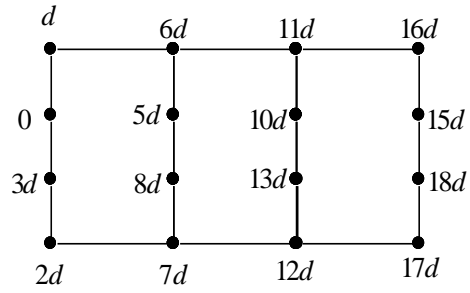


Fig.8. The Graf $C_{8,4,3}$

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