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# Comparison of damage localization in mechanical systems based on Stochastic Subspace Identification method

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## Context

- Vibration-based damage localization on structures relates to the monitoring of the changes in the modal parameters
- Based on finite Element model of the structure and output-only measurement data in the reference and damaged states
- Comparison of Stochastic Dynamic Damage Locating Vector (SDDLTV) approach and Subspace Fitting (SF) method, for damage localization

## Covariance computation

- System matrices are subject to uncertainties due to unknown excitation, noise and finite data length
- Estimation from subspace identification, based on covariance-driven Hankel matrix  $\mathcal{H}$
- Let  $f$  be a function of  $\mathcal{H}$ , then  $\text{cov}(f(\mathcal{H})) \approx \mathcal{J}_{f,\mathcal{H}} \hat{\Sigma}_{\mathcal{H}} \mathcal{J}_{f,\mathcal{H}}^T$  with sensitivity  $\mathcal{J}_{f,\mathcal{H}} = \partial f(\mathcal{H}) / \partial \text{vec}(\mathcal{H})$

## SDDLTV

$$\text{Transfer matrix} \begin{cases} G(s) = R(s)D_c \\ R(s) = C_c(sI - A_c)^{-1} \begin{bmatrix} C_c A_c \\ C_c \end{bmatrix}^+ \begin{bmatrix} I \\ 0 \end{bmatrix} \end{cases}$$

### Damage localization procedure

- From data: changes in the transfer matrix between both reference and damaged state:  $\partial R(s)^T = \hat{R}(s)^T - R(s)^T$  load vector  $v(s)$  in the null space of  $\partial R(s)^T$
- From FE model: apply load vector  $v(s)$  to model  
Stress computation:  $\hat{S}(s) = \mathcal{L}_{\text{model}}(s)v(s)$
- Damaged element indicated by stress = 0 (or close to 0)

### Uncertainty propagation and covariance computation

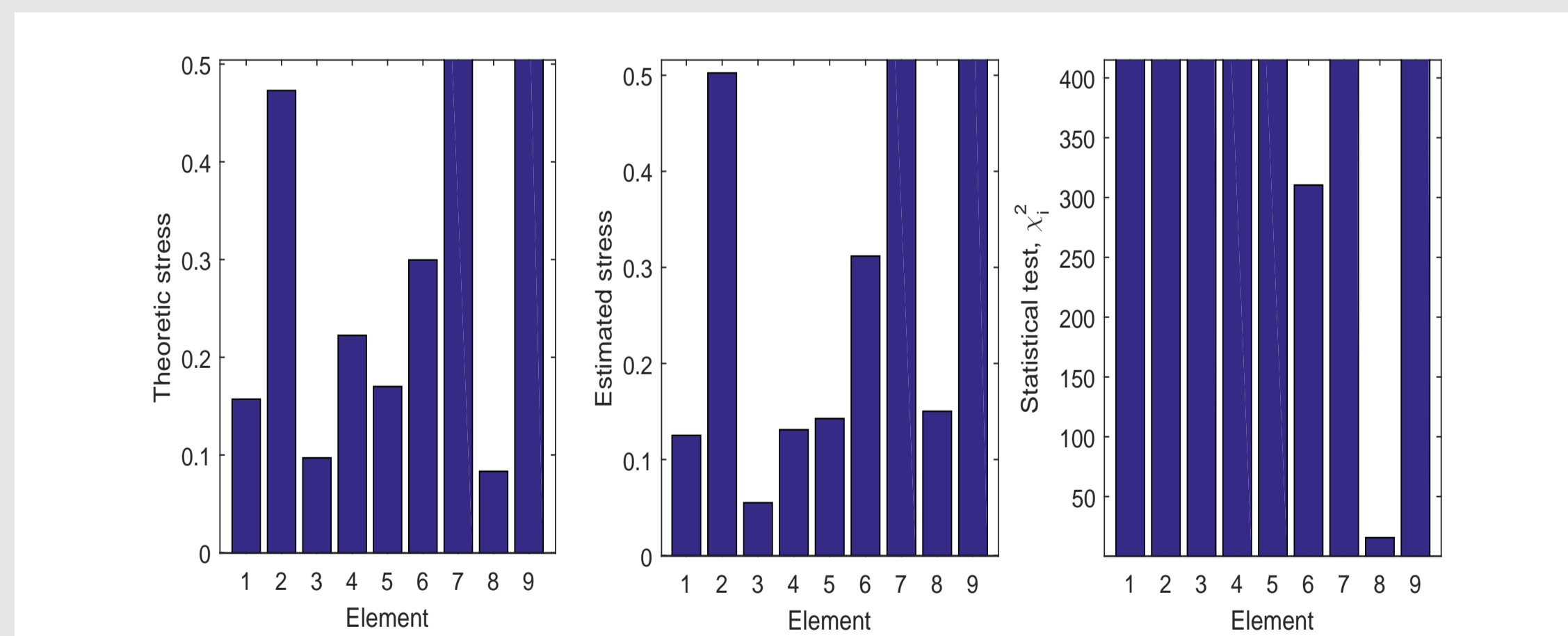
- Computed stress for damage localization is afflicted with uncertainties  
– Sensitivity-based uncertainty propagation from measurement data ( $\mathcal{H}$ ) to computed stress

$$\text{Reference state: } \mathcal{H}^{\text{ref}} \xrightarrow{\mathcal{J}_{A,\mathcal{H}}} A, C \xrightarrow{\mathcal{J}_{R,A}} R \quad \delta R \xrightarrow{\mathcal{J}_{v,R}} v(s) \xrightarrow{\mathcal{J}_{S(s)}} S(s)$$

$$\text{Damaged state: } \mathcal{H}^{\text{dam}} \xrightarrow{\mathcal{J}_{A,\hat{\mathcal{H}}}} \hat{A}, \hat{C} \xrightarrow{\mathcal{J}_{R,\hat{A}}} \hat{R}$$

$$\text{cov}(S(s)) = \mathcal{J}_{S(s)}(\text{cov}(\text{vec}(\hat{R}^T)_{\text{re}})) + (\text{cov}(\text{vec}(\hat{R}^T)_{\text{re}})) \mathcal{J}_{S(s)}^T$$

- $\chi^2$  test  $S_i^T \text{cov}(S_i)^{-1} S_i$  for each element  $i$



## Mathematical model

$$\text{Mechanical model} \quad M\ddot{v}(t) + C\dot{v}(t) + K v(t) = u(t)$$

$$\text{State space model} \quad \begin{cases} \dot{x}(t) = Ax(t) + B_c e(t) \\ y(t) = C_c x(t) + D_c e(t) \end{cases}$$

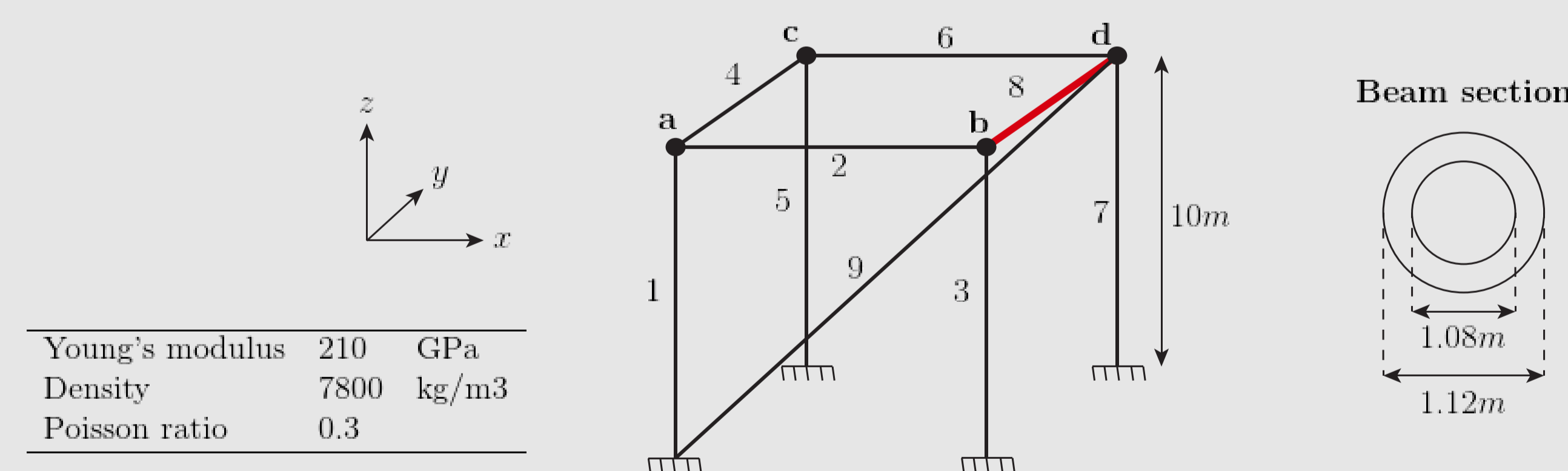
## Subspace Identification

$$\mathcal{Y}^- = \begin{bmatrix} y_q & y_{q+1} & \dots & y_{N+q-1} \\ y_{q-1} & y_q & \dots & y_{N+q-2} \\ \vdots & \vdots & \ddots & \vdots \\ y_1 & y_2 & \dots & y_N \end{bmatrix}, \quad \mathcal{Y}^+ = \begin{bmatrix} y_{q+1} & y_{q+2} & \dots & y_{N+q} \\ y_{q+2} & y_{q+3} & \dots & y_{N+q+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{q+p+1} & y_{q+p+2} & \dots & y_{N+p+q} \end{bmatrix}$$

$$\hat{\mathcal{H}} = \frac{1}{N} \mathcal{Y}^+ (\mathcal{Y}^-)^T = \mathbf{U} \Delta \mathbf{V}^T = [\mathbf{U}_1 \ \mathbf{U}_0] \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{bmatrix} \mathbf{V}^T \approx \mathbf{U}_1 \Delta_1 \mathbf{V}_1^T, \quad \hat{\mathcal{O}} = \mathbf{U}_1$$

## Application

- Number of degree of freedom: 24
- Damping ratio = 2%, noise = 5%
- Output sensors:  $x$  and  $y$  directions at node a and d
- Number of sample: 200000
- Damaged state: decreasing 25% Young modulus at element 8



## SF

$$\text{Observability matrix} \quad \mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^p \end{bmatrix}$$

### Damage localization procedure

- FE model updating procedure to identify structure parameters:  
 $\theta = \text{argmin} \|\mathbf{r}\|_2^2$   
 $\mathbf{r} = [\mathbf{I}_{2n} \otimes (\mathbf{I}_{(p+1)r} - \hat{\mathcal{O}}\hat{\mathcal{O}}^+)] \text{vec} \{ \mathcal{O}^h(\theta^h) \}$   
 $\theta_k = \theta_{k-1} - \mathcal{J}_{\mathbf{r}}^+ \mathbf{r}_k$
- $\theta$  related to damage (element stiffness reduction)

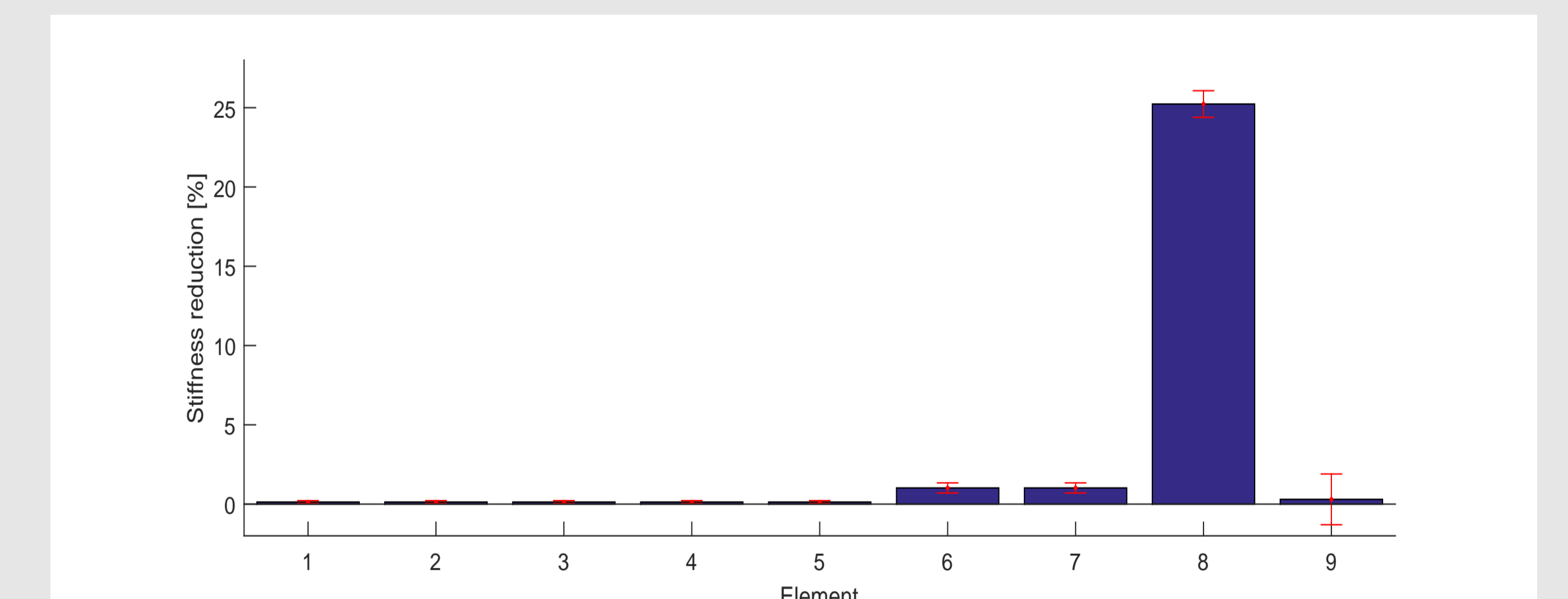
### Uncertainty propagation and covariance computation

- Updating parameters for damage localization is afflicted with uncertainties  
– Sensitivity-based uncertainty propagation from measurement data ( $\mathcal{H}$ ) to updated parameters, through each iteration step of the updating minimization problem

$$\text{Reference state: } \mathcal{H}^{\text{ref}} \xrightarrow{\mathcal{J}_{\theta^{\text{ref}},\mathcal{H}}} \theta^{\text{ref}}$$

$$\text{Damaged state: } \mathcal{H}^{\text{dam}} \xrightarrow{\mathcal{J}_{\theta^{\text{dam}},\mathcal{H}}} \theta^{\text{dam}}$$

- Damaged if  $\theta_i^{\text{dam}} \pm \sigma_{\theta_i^{\text{dam}}} - \theta_i^{\text{ref}} > \sigma_{\theta_i^{\text{ref}}}$  for each element  $i$



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