



Stochastic parameterization of geophysical flows through modelling under location uncertainty

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Stochastic parametrization of geophysical flows through modeling under location uncertainty

Valentin Resseguier,

Pierre Dérian,

Etienne Mémin,

Bertrand Chapron

Motivations

- Rigorously identified subgrid dynamics effects
- Injecting likely small-scale dynamics
- Studying bifurcations and attractors
 - Climate projections
- Quantification of modeling errors
 - Ensemble forecasts and data assimilation

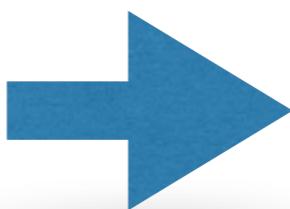
Contents

- Randomized dynamics
- SQG under Moderate Uncertainty
- Lorenz under location uncertainty

Randomized dynamics

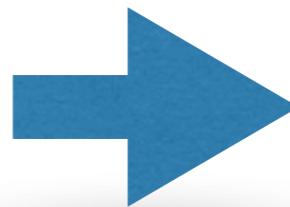
Random equations

- Random initial conditions



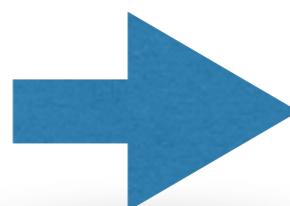
Underdispersive +
need large ensemble

- Arbitrary Gaussian forcing



Adding energy
+ wrong phase

- Averaging, homogenization



Assumptions and
energy issues

- Adding white
random velocity



$$v = w + \sigma \dot{B}$$

Advection of tracer Θ

$$\frac{D\Theta}{Dt} = 0$$

Advection of tracer Θ

Advection of tracer Θ

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection of tracer Θ

Advection

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection of tracer Θ

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection

Diffusion

Advection of tracer Θ

$$\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{\mathbf{B}} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

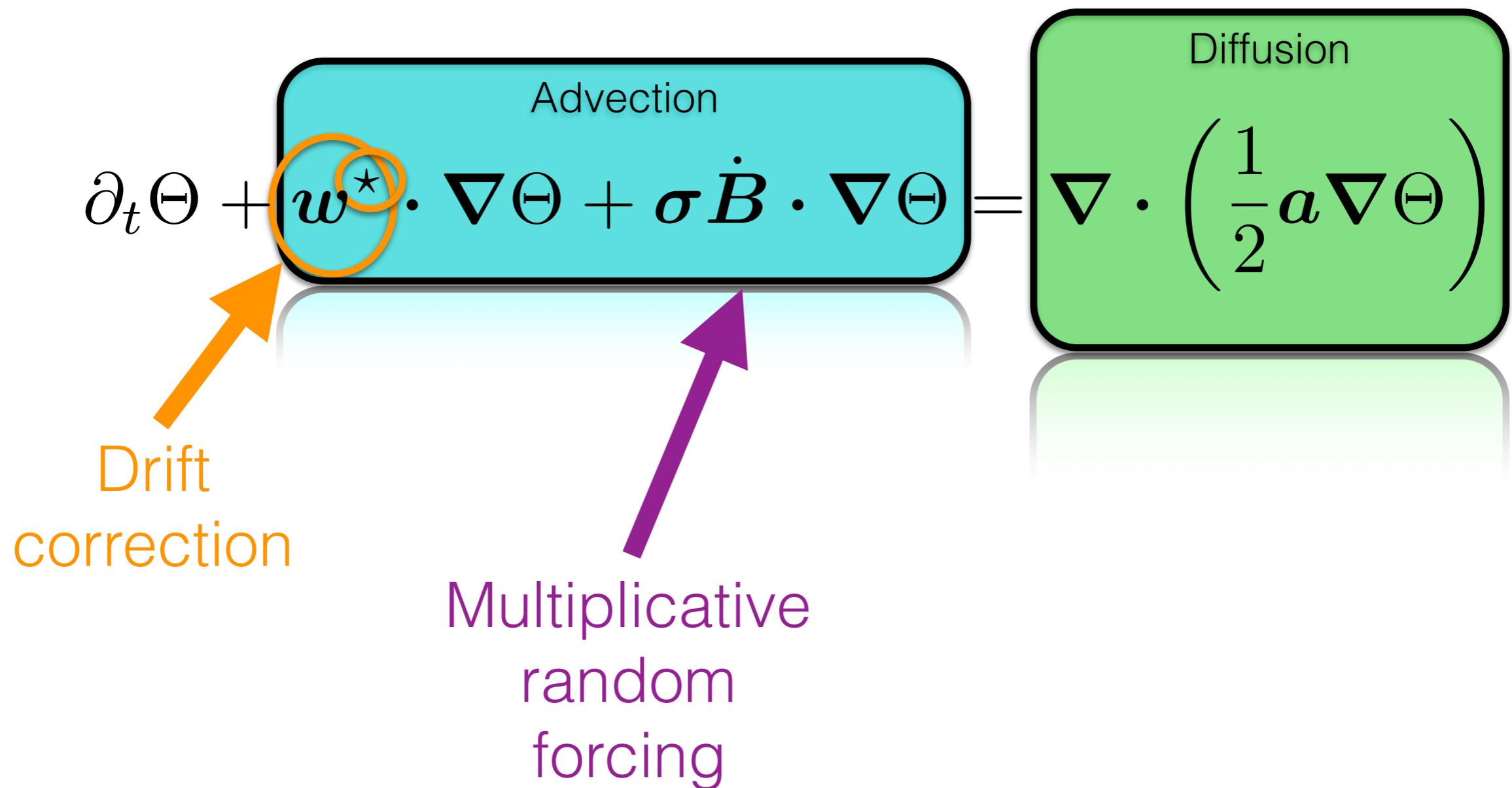
Advection

Diffusion

Drift correction

The diagram illustrates the components of the tracer advection equation. The left side of the equation, $\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{\mathbf{B}} \cdot \nabla \Theta$, is enclosed in a light blue rounded rectangle labeled 'Advection'. The right side, $\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$, is enclosed in a light green rounded rectangle labeled 'Diffusion'. An orange magnifying glass highlights the term $\mathbf{w}^* \cdot \nabla \Theta$. An orange arrow points from the text 'Drift correction' at the bottom left towards this highlighted term.

Advection of tracer Θ



Advection of tracer Θ

$$\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{\mathcal{B}} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection

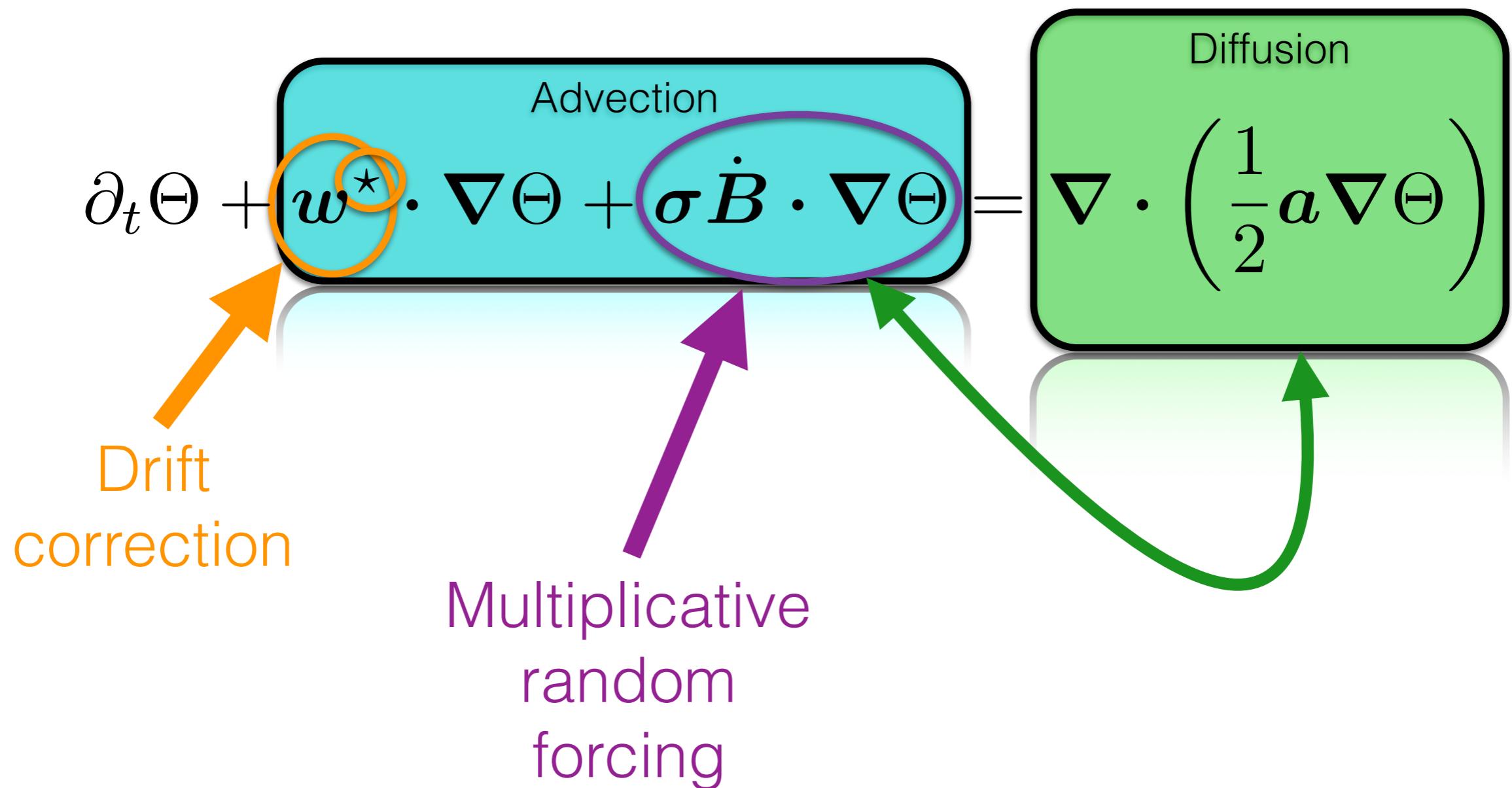
Diffusion

Drift correction

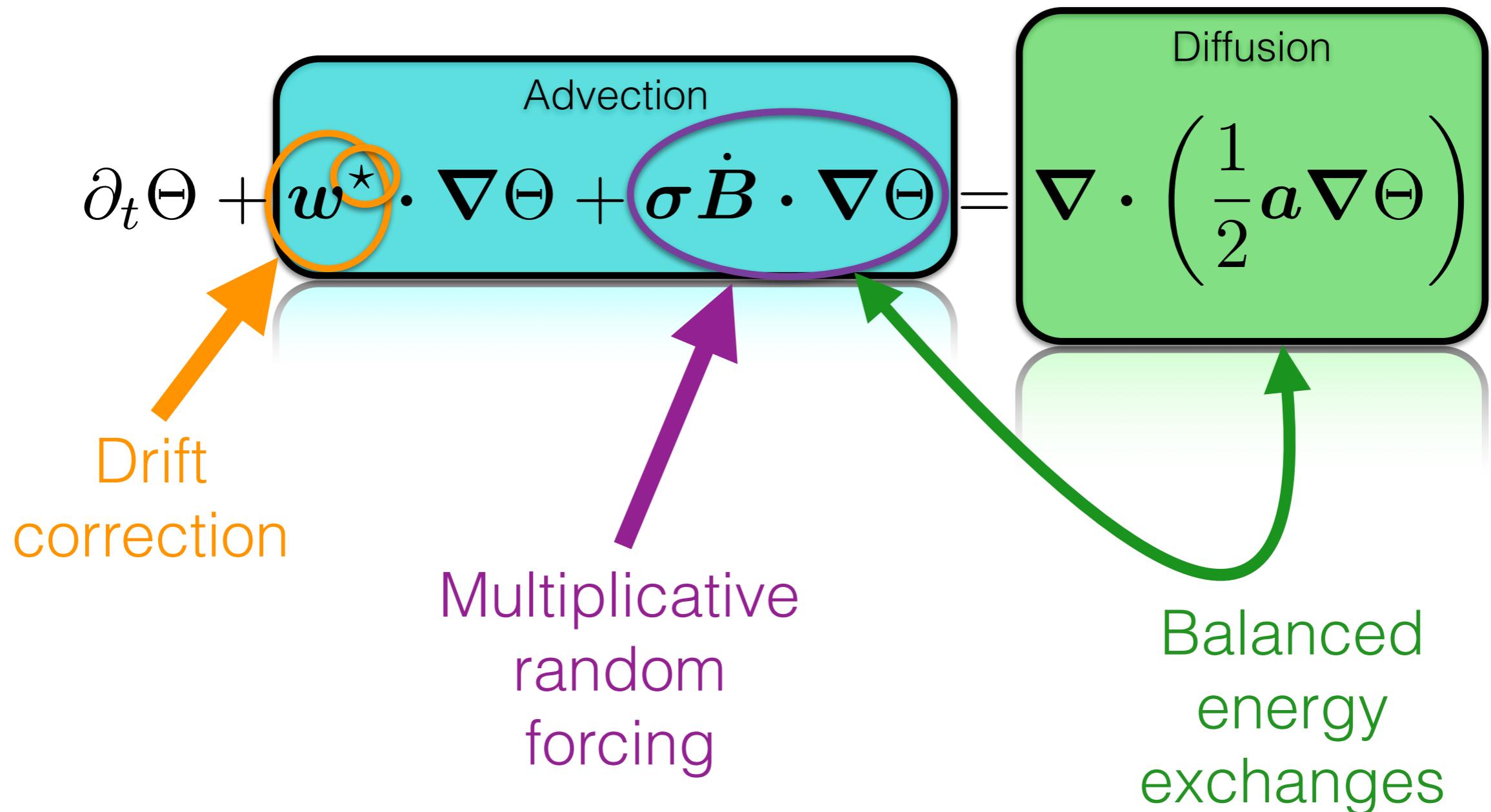
Multiplicative random forcing

The diagram illustrates the decomposition of advection into two components. On the left, the full advection term $\mathbf{w}^* \cdot \nabla \Theta$ is shown, with the velocity vector \mathbf{w}^* highlighted by an orange circle and arrow, labeled 'Drift correction'. On the right, the multiplicative random forcing term $\sigma \dot{\mathcal{B}} \cdot \nabla \Theta$ is highlighted by a purple circle and arrow, labeled 'Multiplicative random forcing'. The two terms are separated by a plus sign, indicating they are added together. The resulting equation is then equated to the diffusion term $\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$, which is enclosed in a green box labeled 'Diffusion'.

Advection of tracer Θ



Advection of tracer Θ



Derived random models

Conservations
(mass, linear
momentum, ...)

$$\frac{D}{Dt}$$

Navier-Stokes

Boussinesq

Derived random models

Conservations
(mass, linear
momentum, ...)

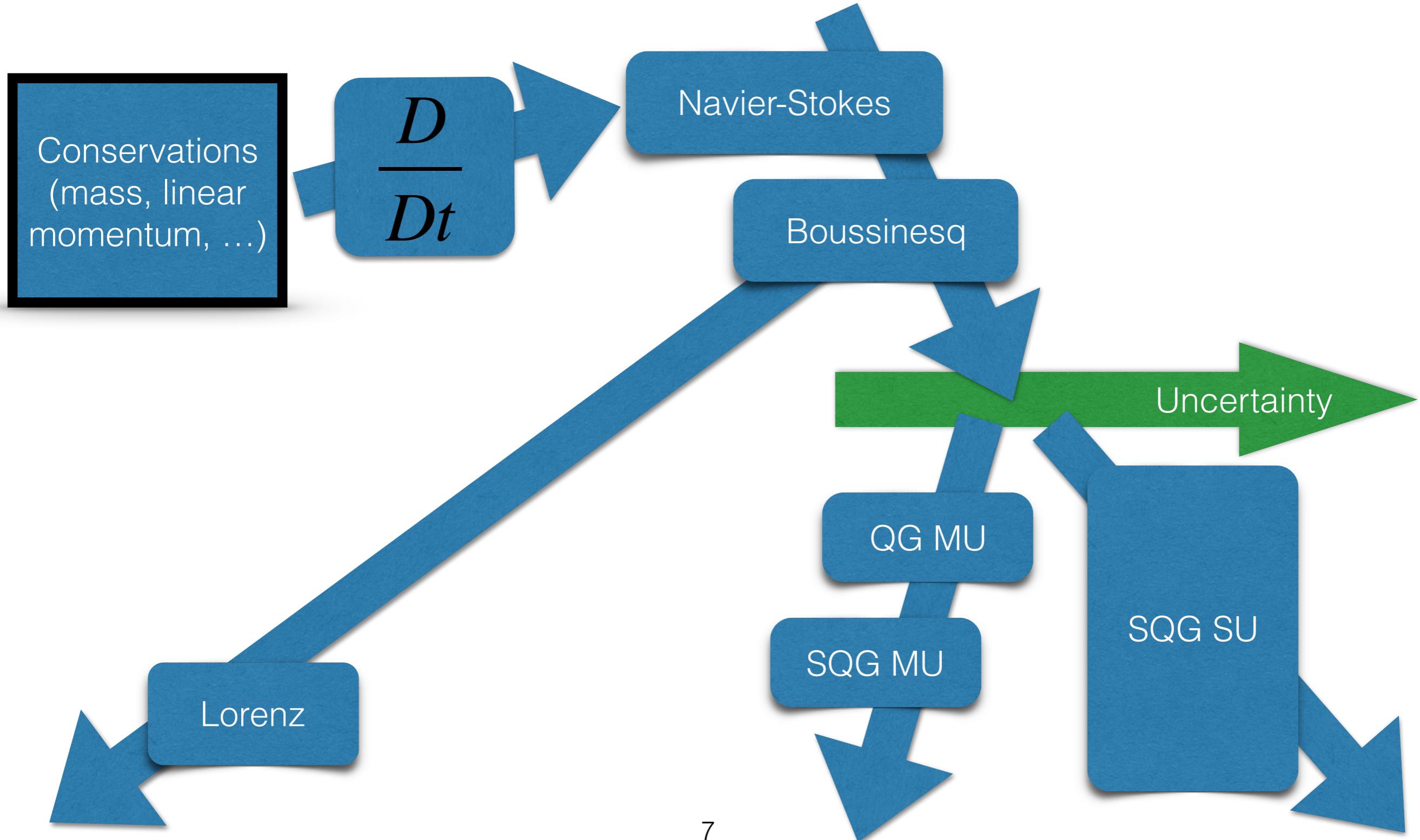
$$\frac{D}{Dt}$$

Navier-Stokes

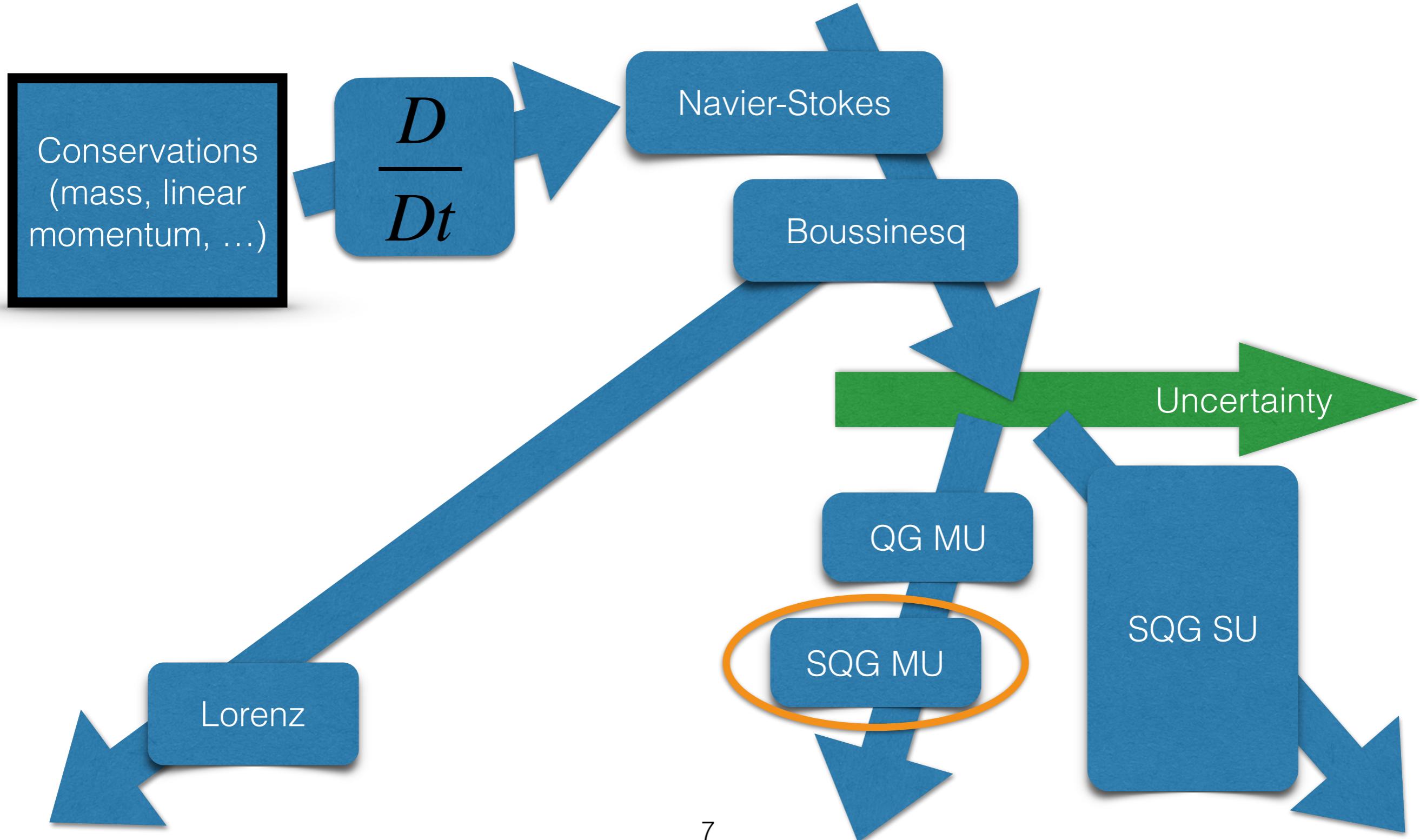
Boussinesq

Lorenz

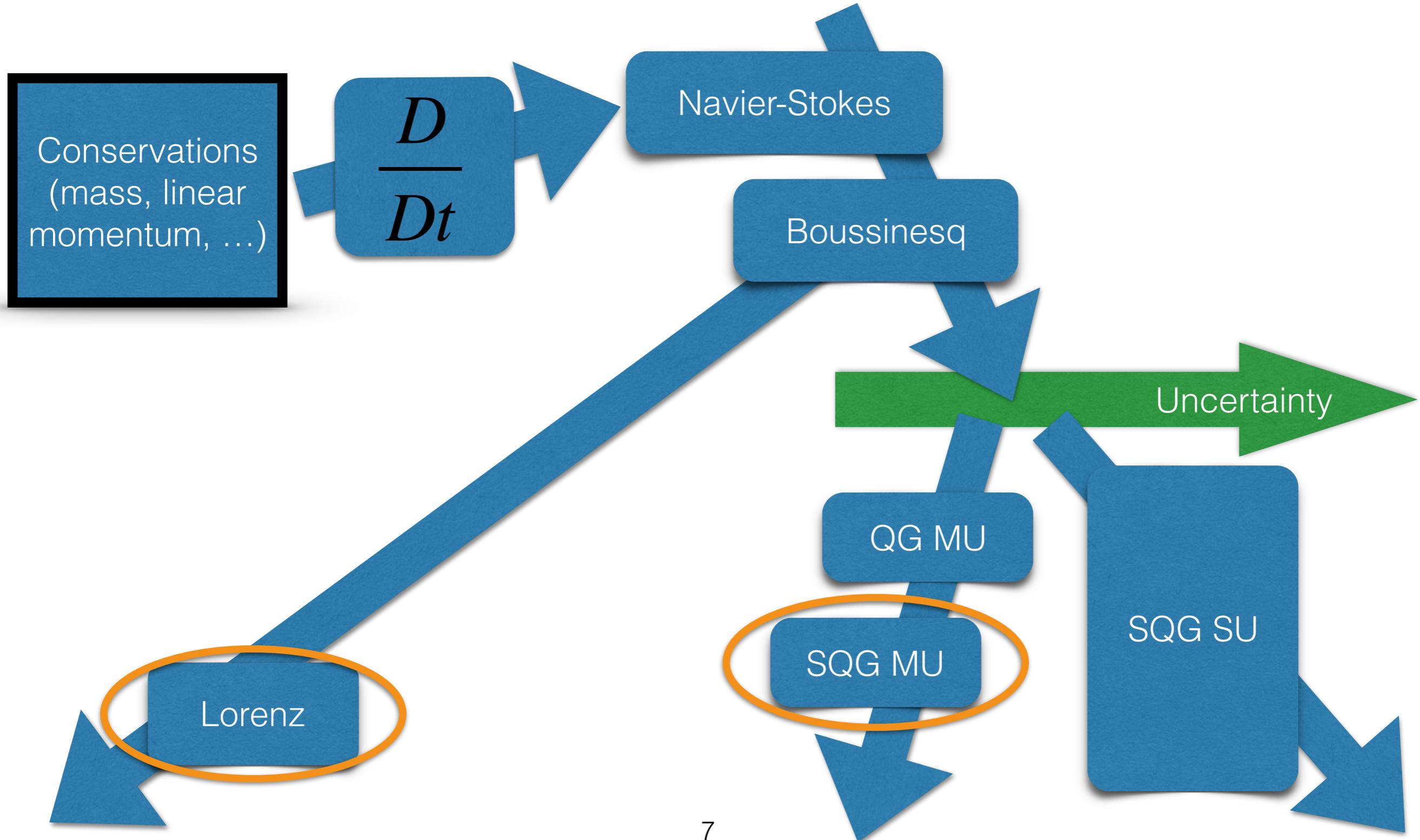
Derived random models



Derived random models



Derived random models



SQG under Moderate Uncertainty

SQG MU

Code available online

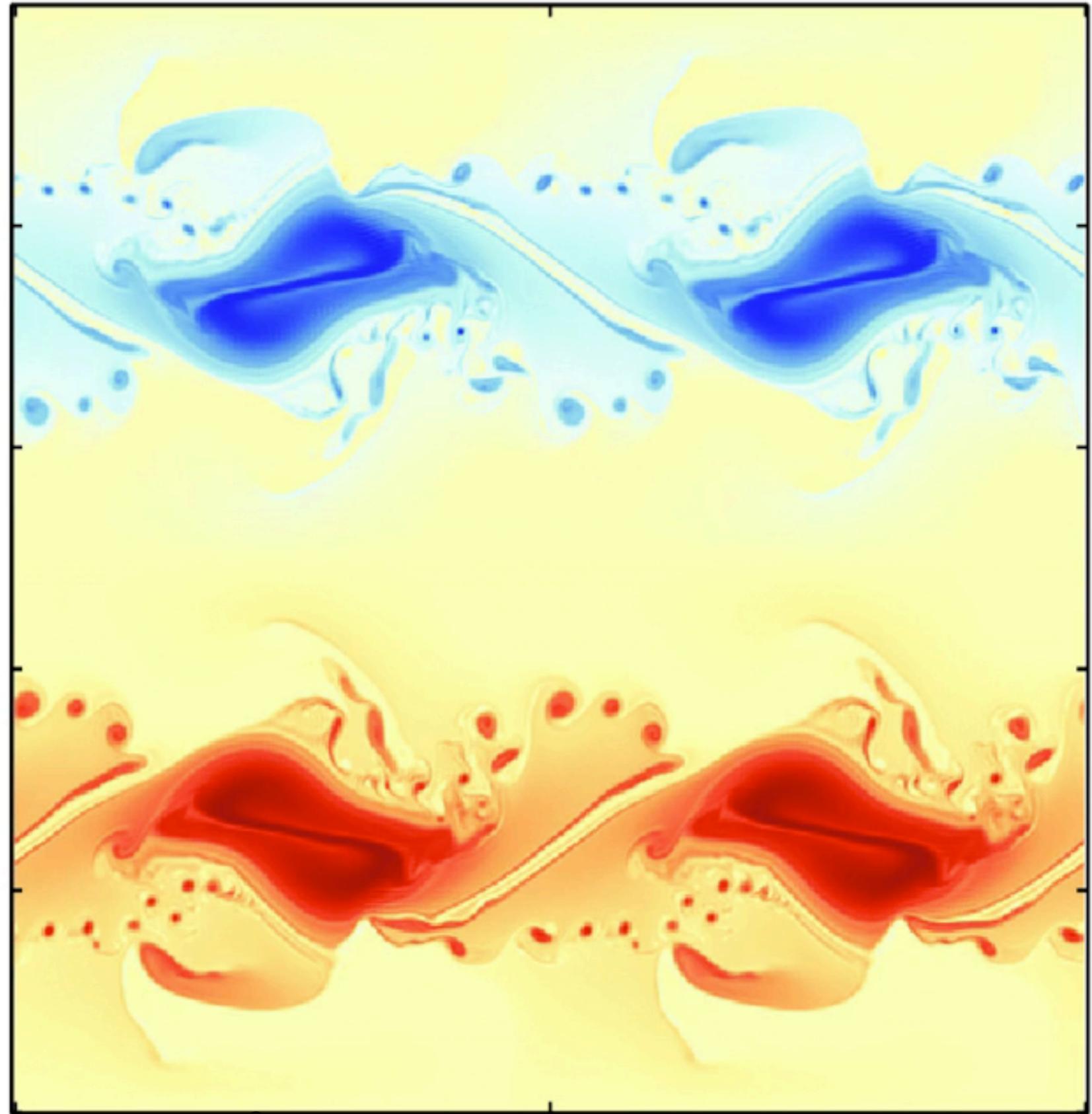
$t = 17$ days

Reference flow:

deterministic

SQG

512 x 512



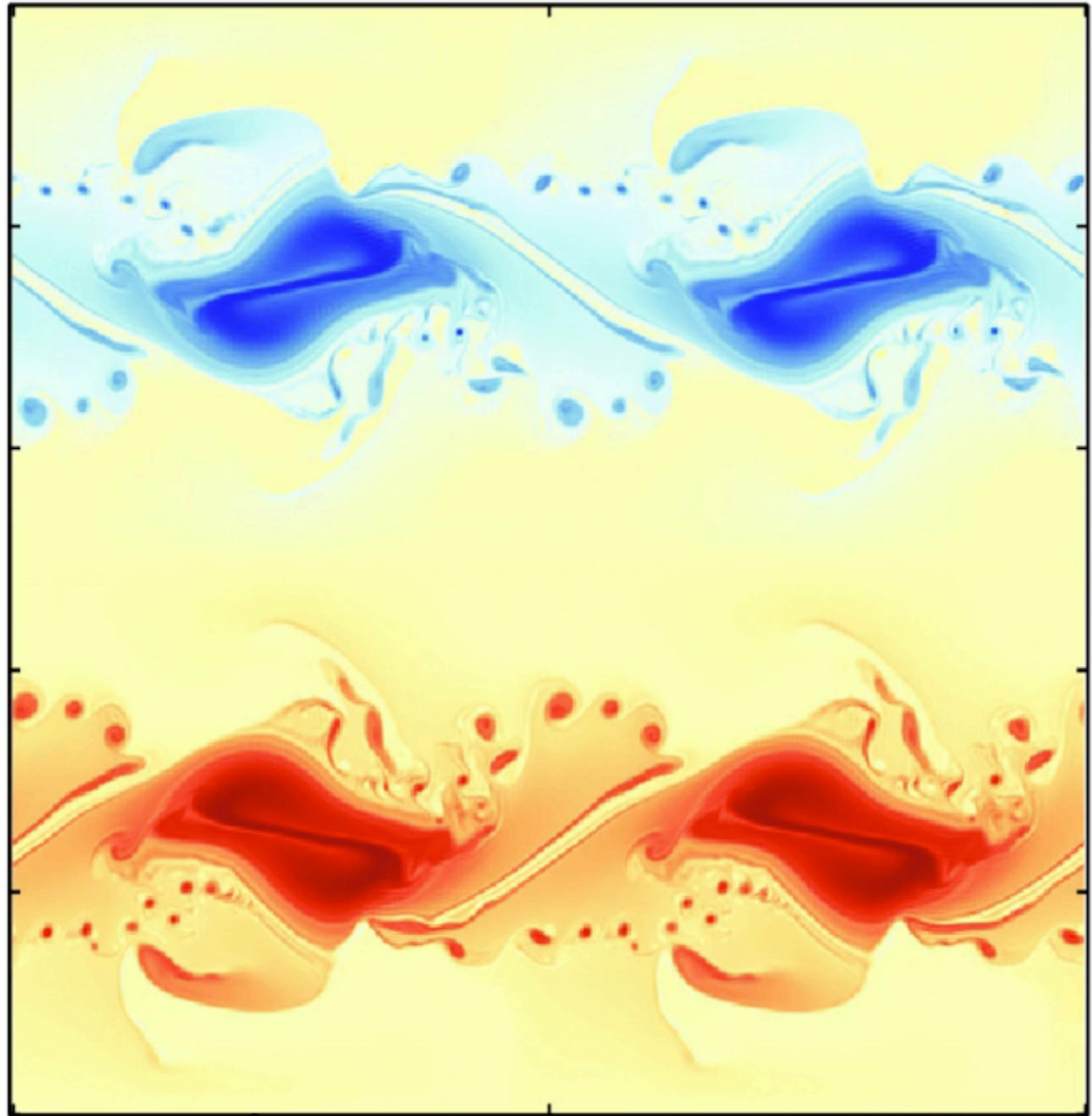
$t = 17$ days

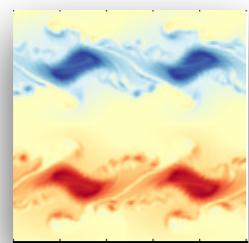
Reference flow:

deterministic

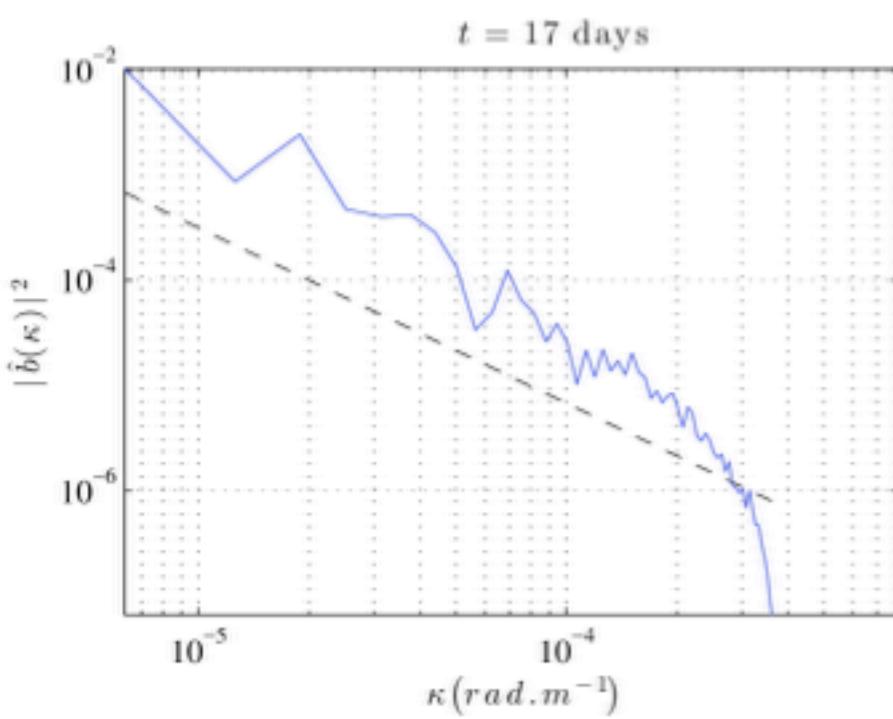
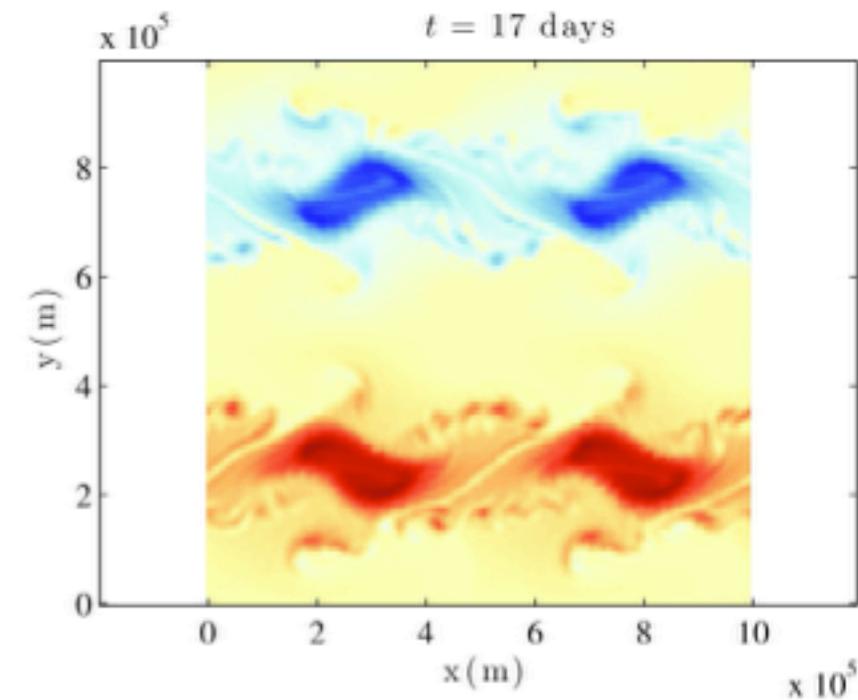
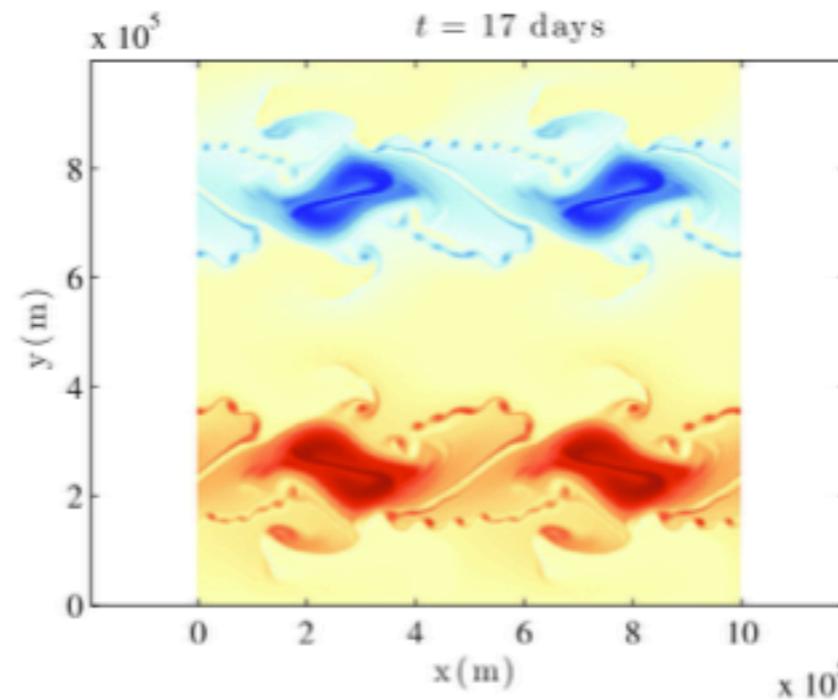
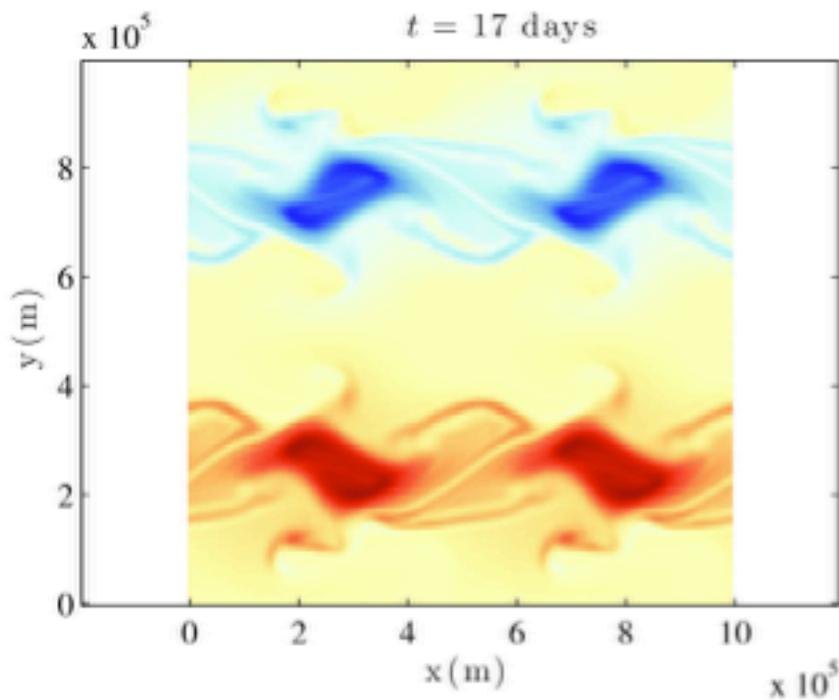
SQG

512 x 512

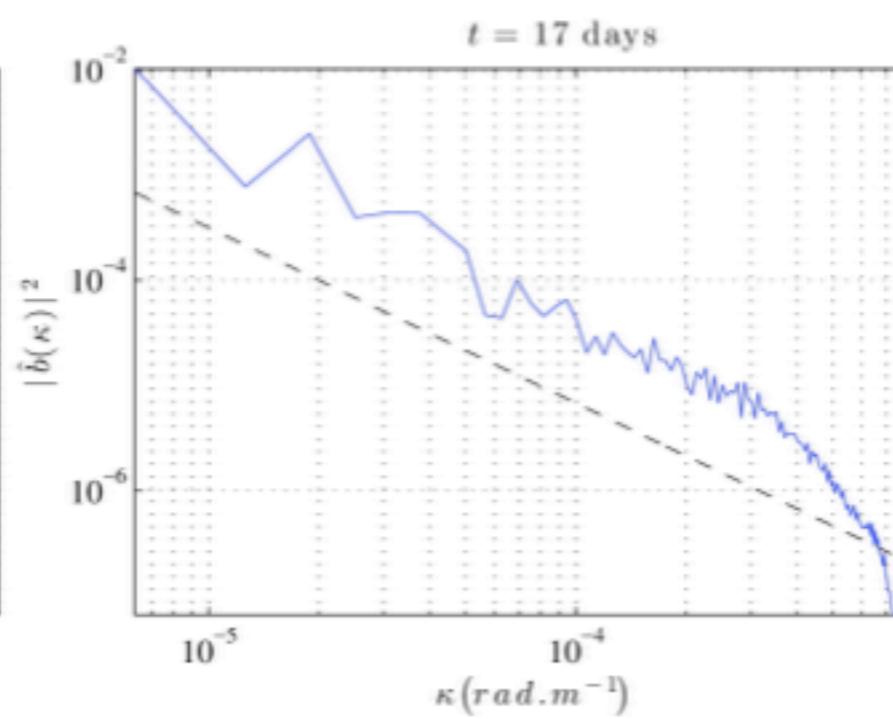




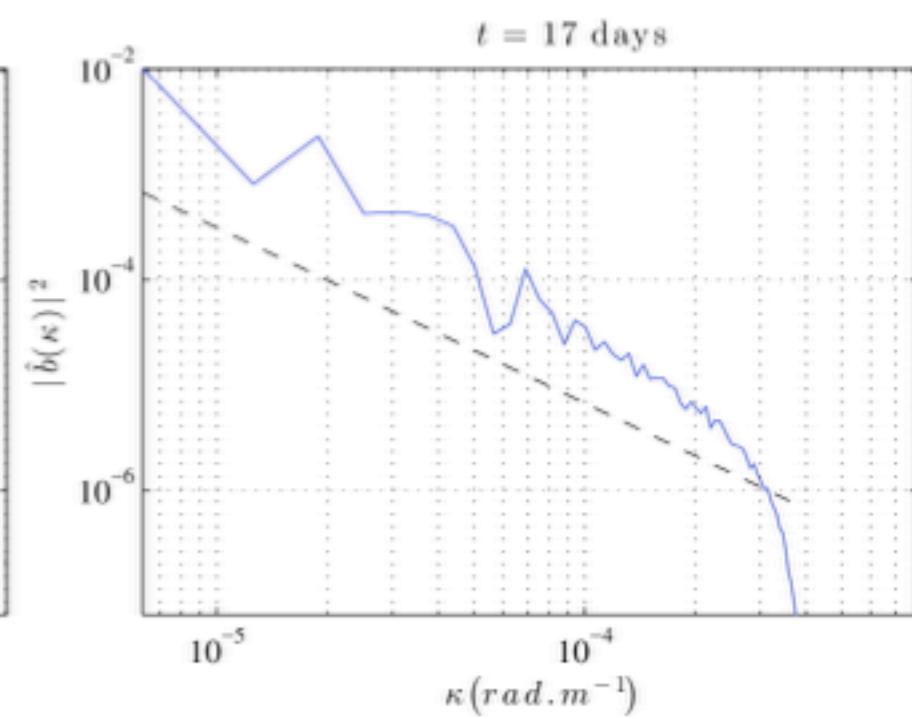
One realization



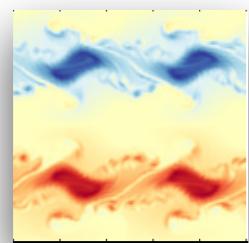
Deterministic 128x128



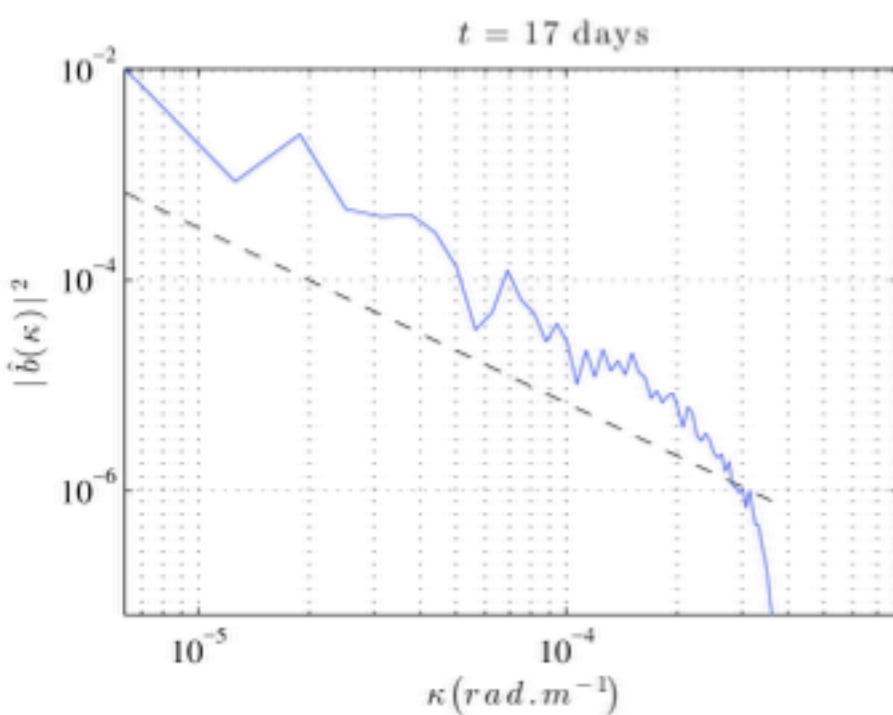
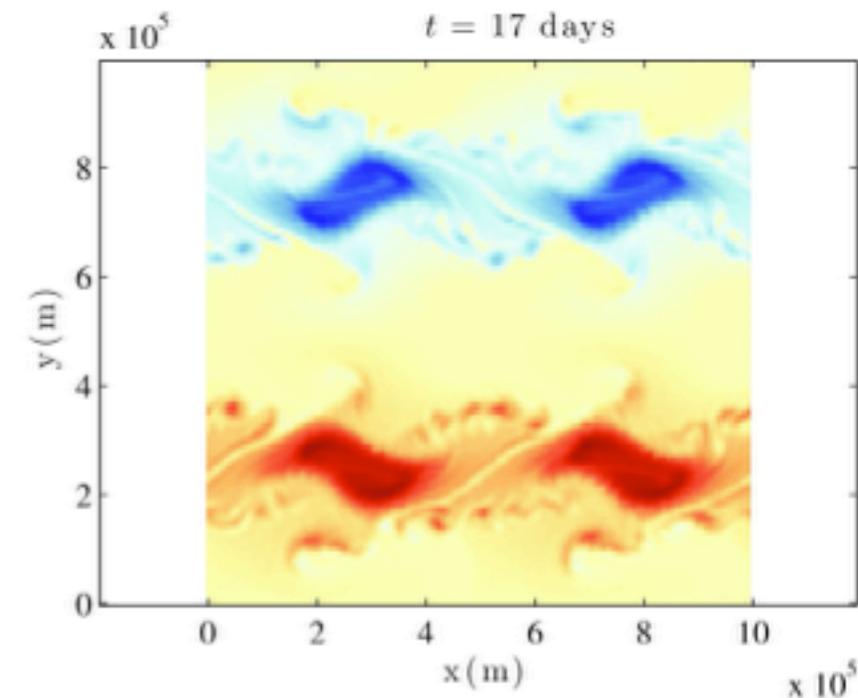
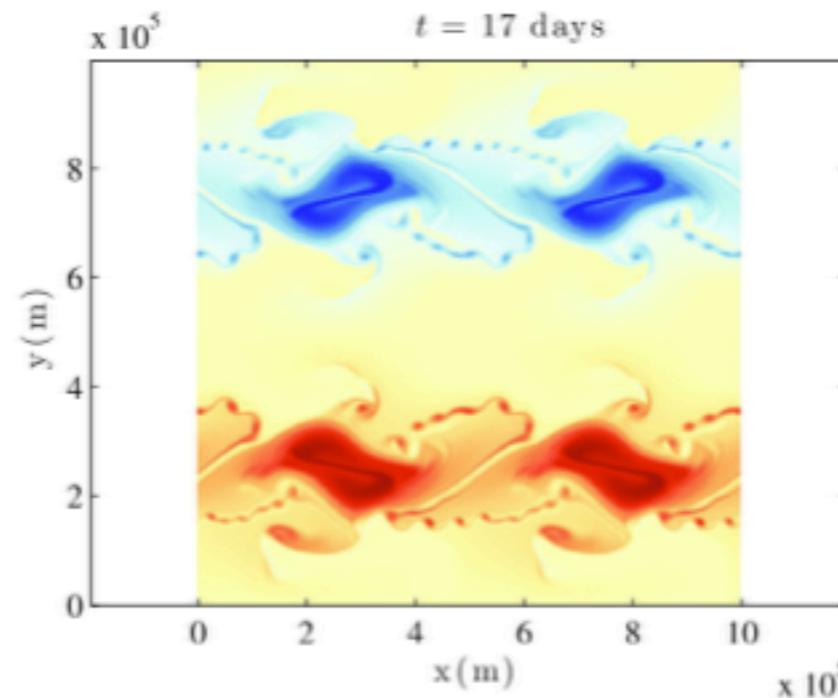
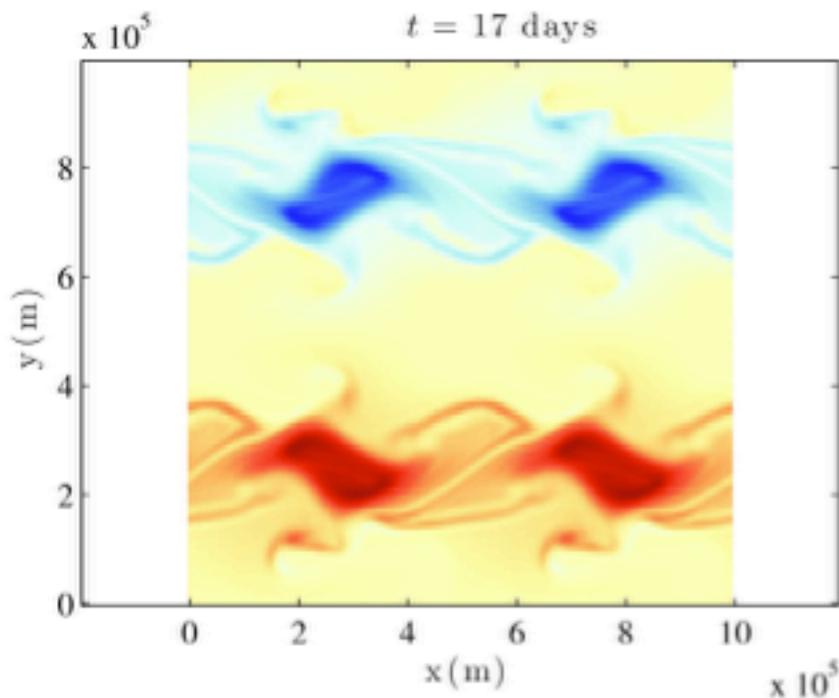
Deterministic 512x512
10



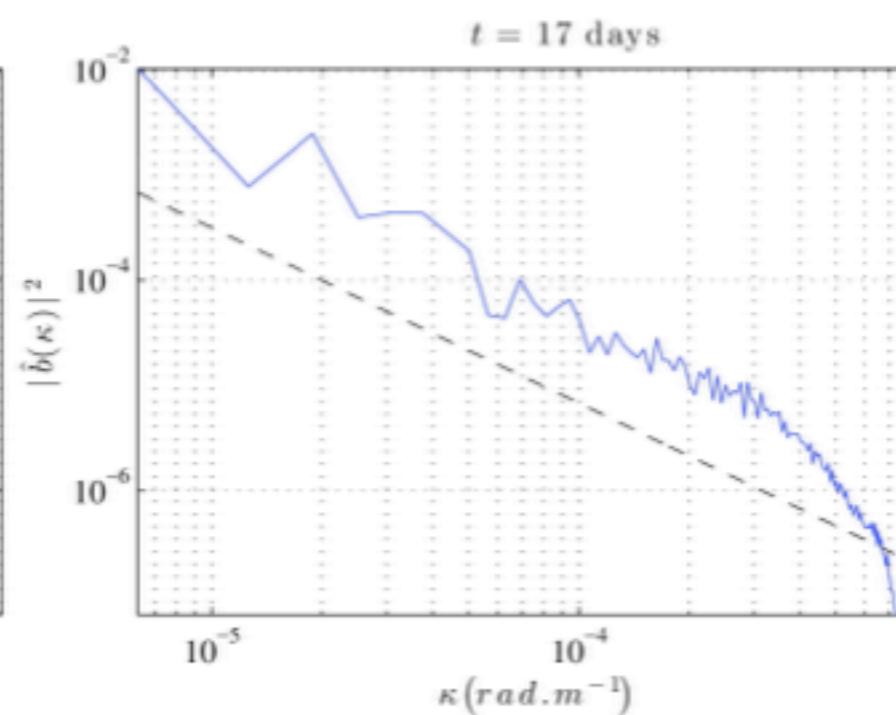
Stochastic 128x128



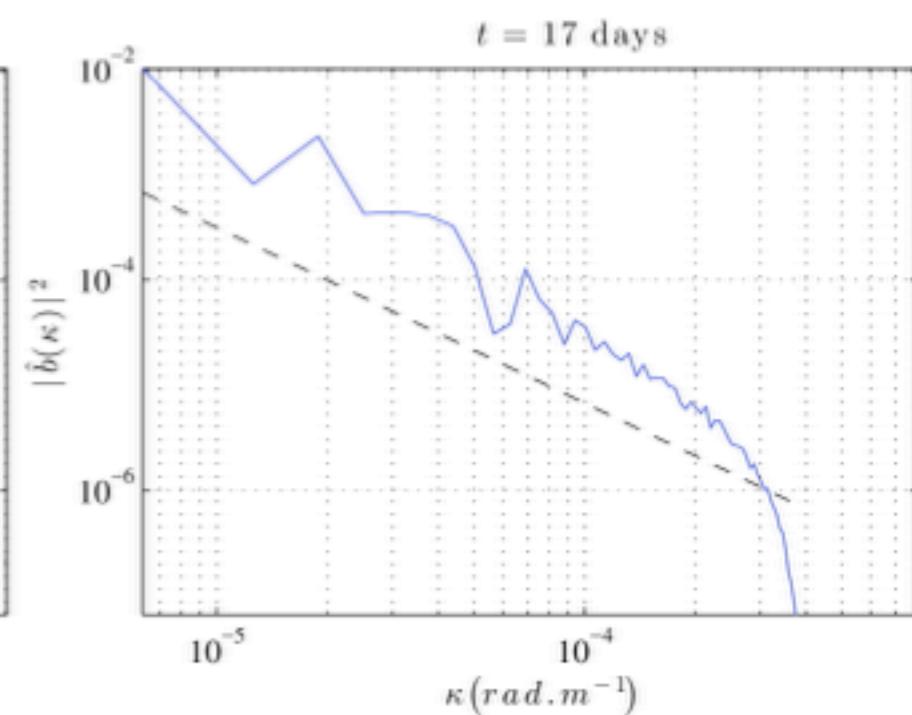
One realization



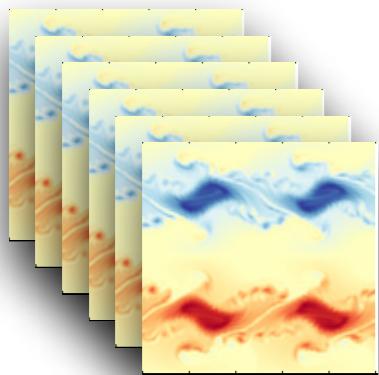
Deterministic 128x128



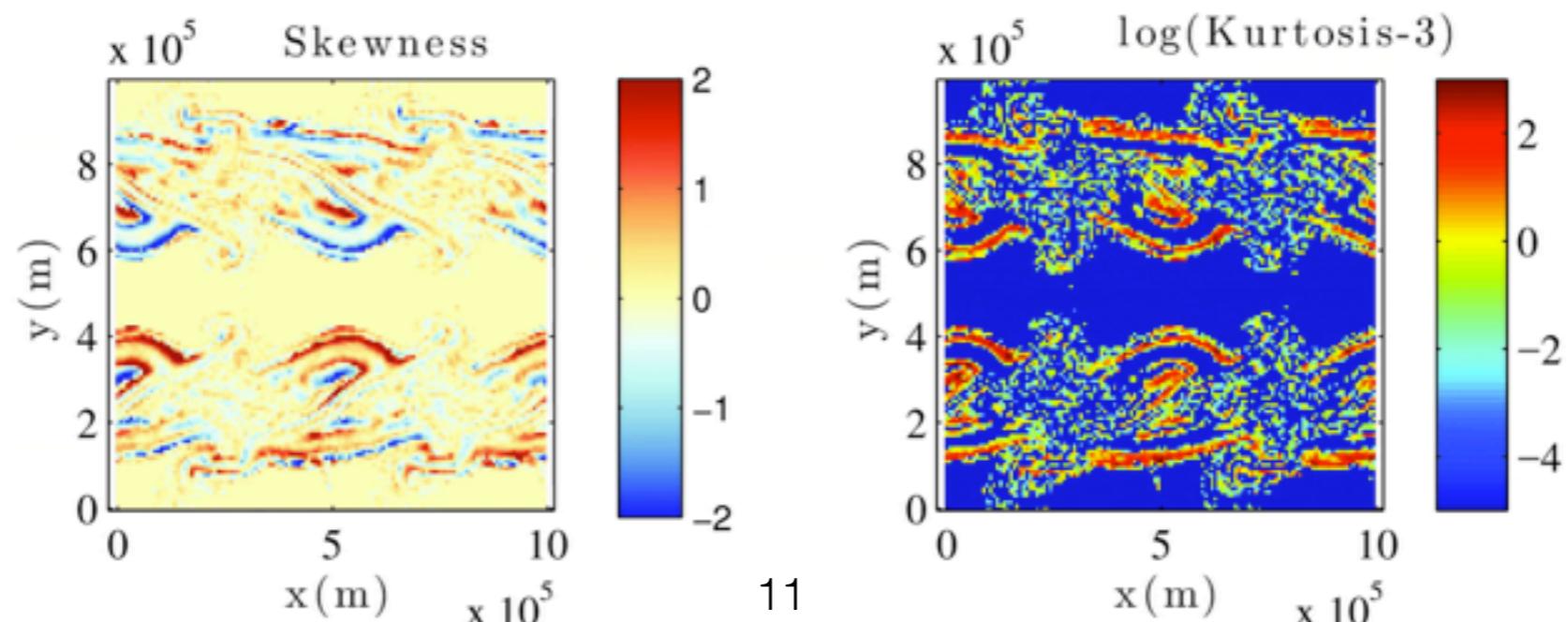
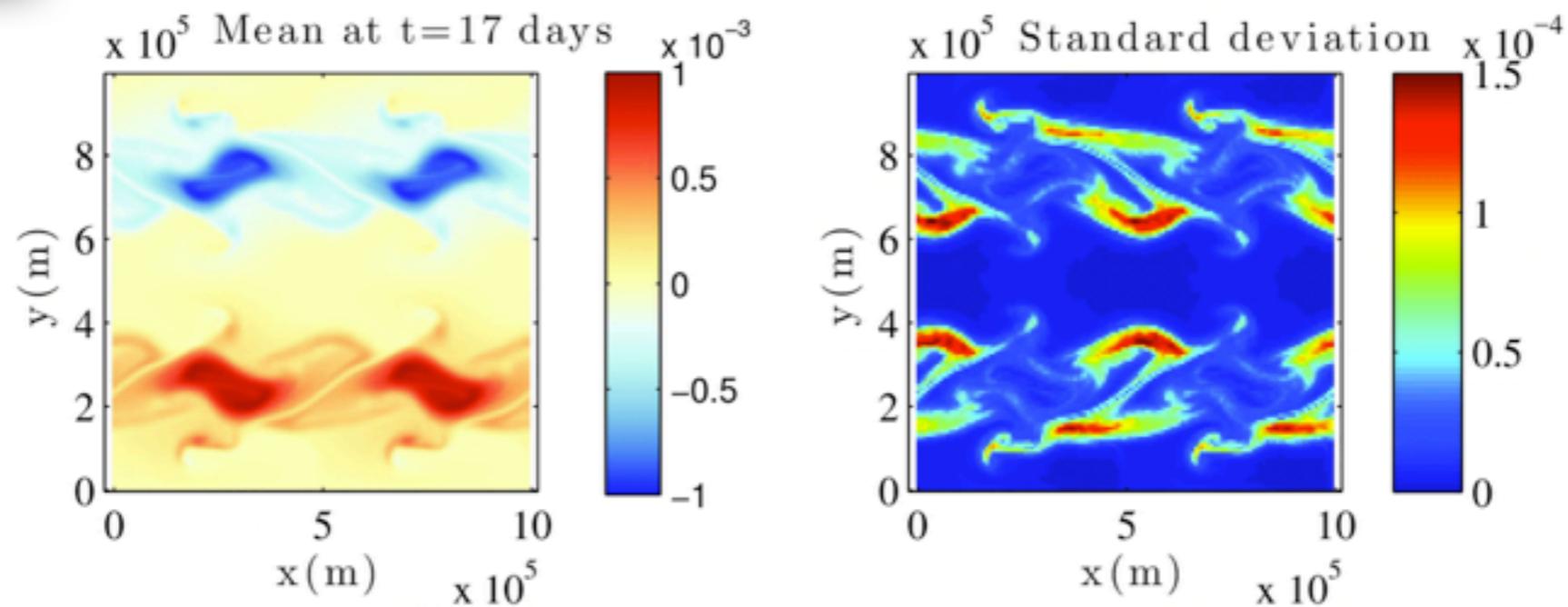
Deterministic 512x512
10

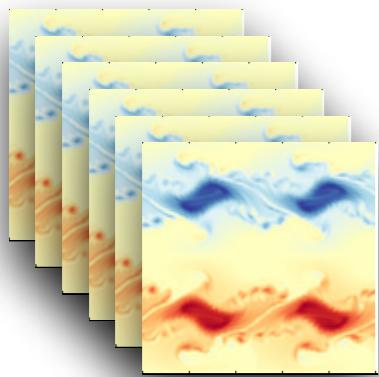


Stochastic 128x128

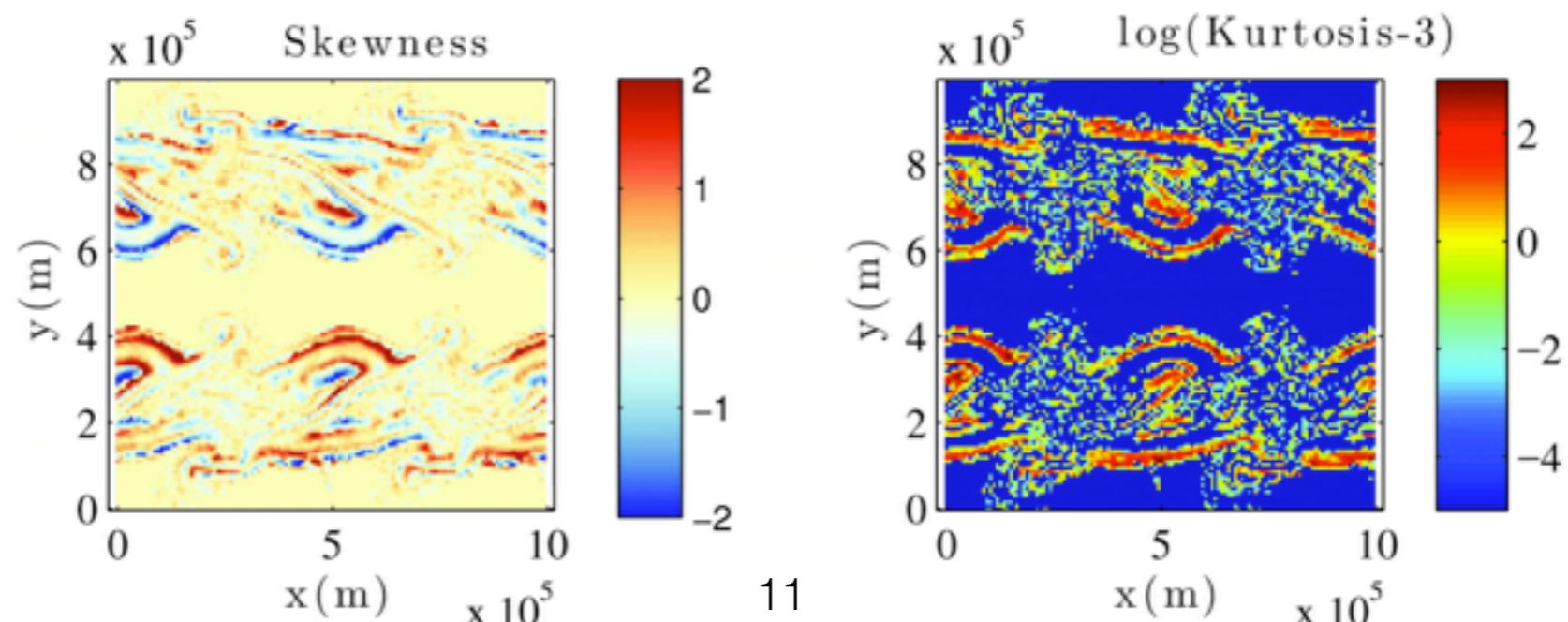
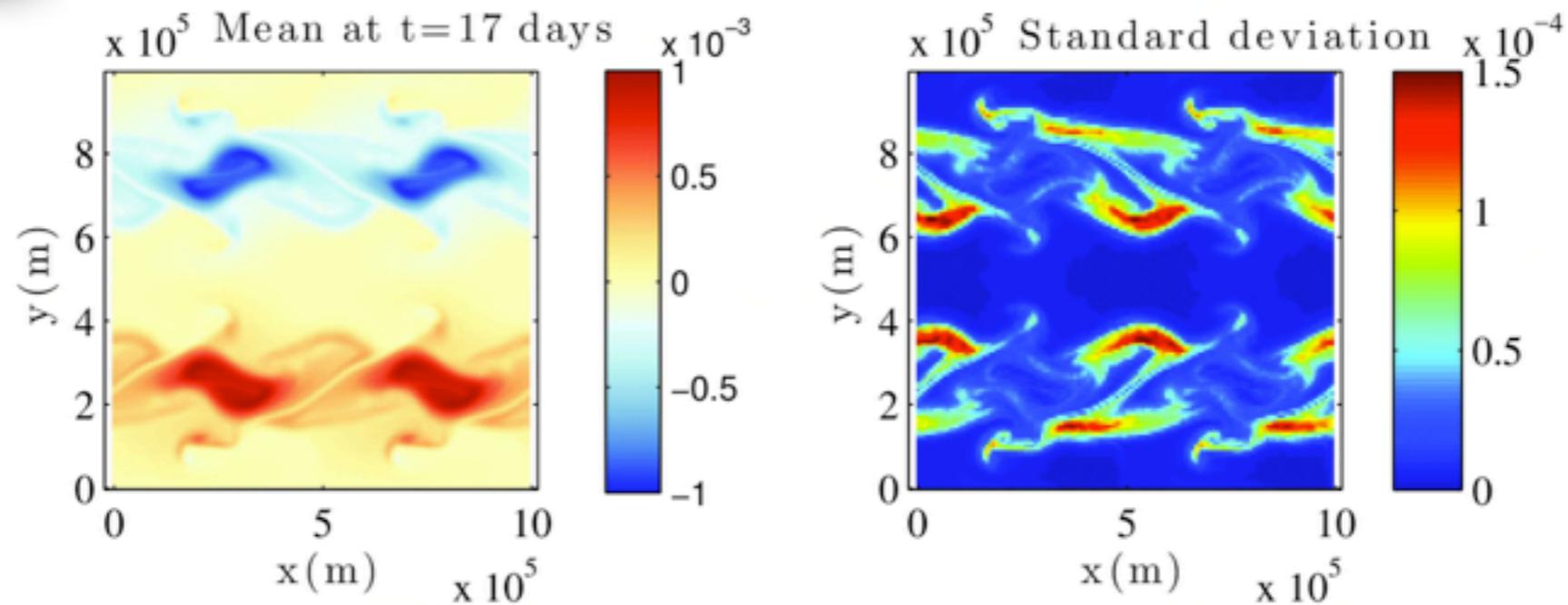


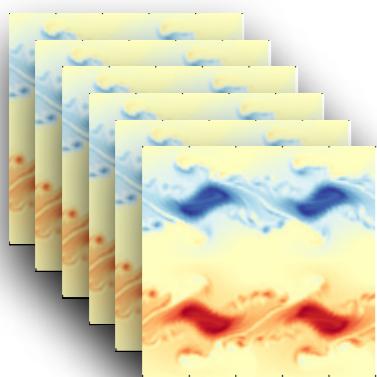
Ensemble



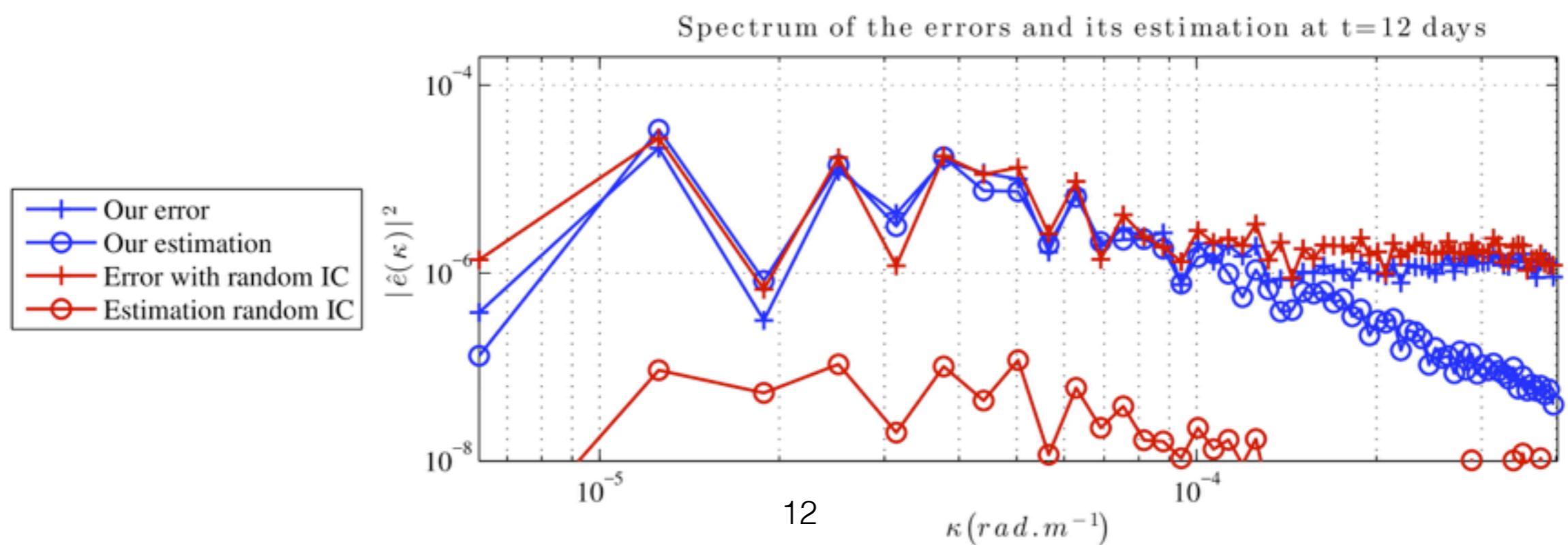
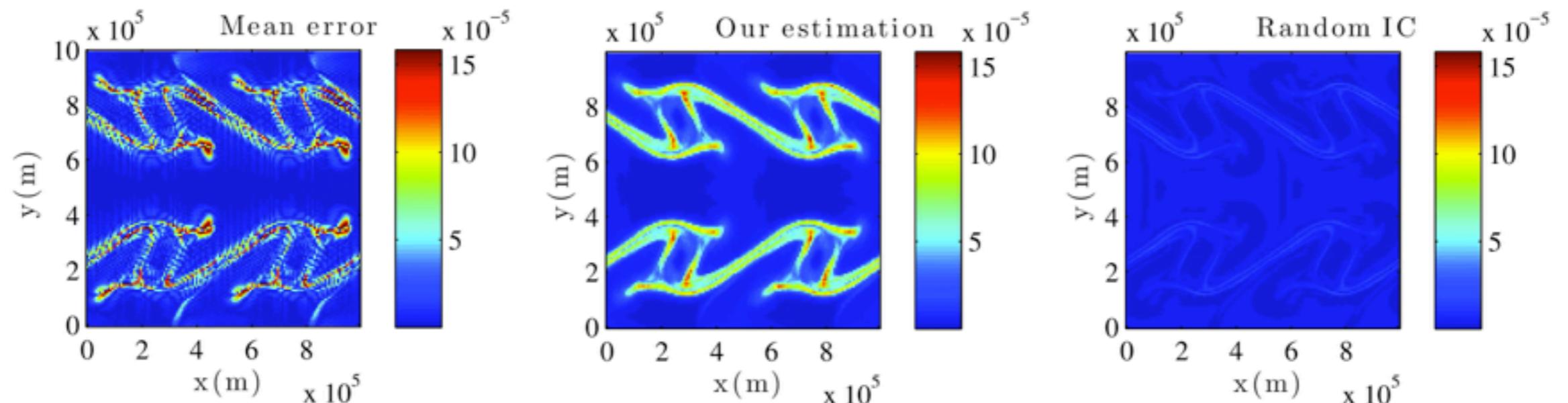


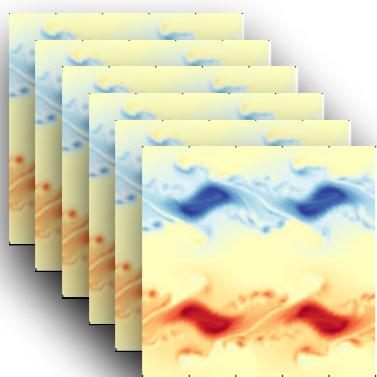
Ensemble



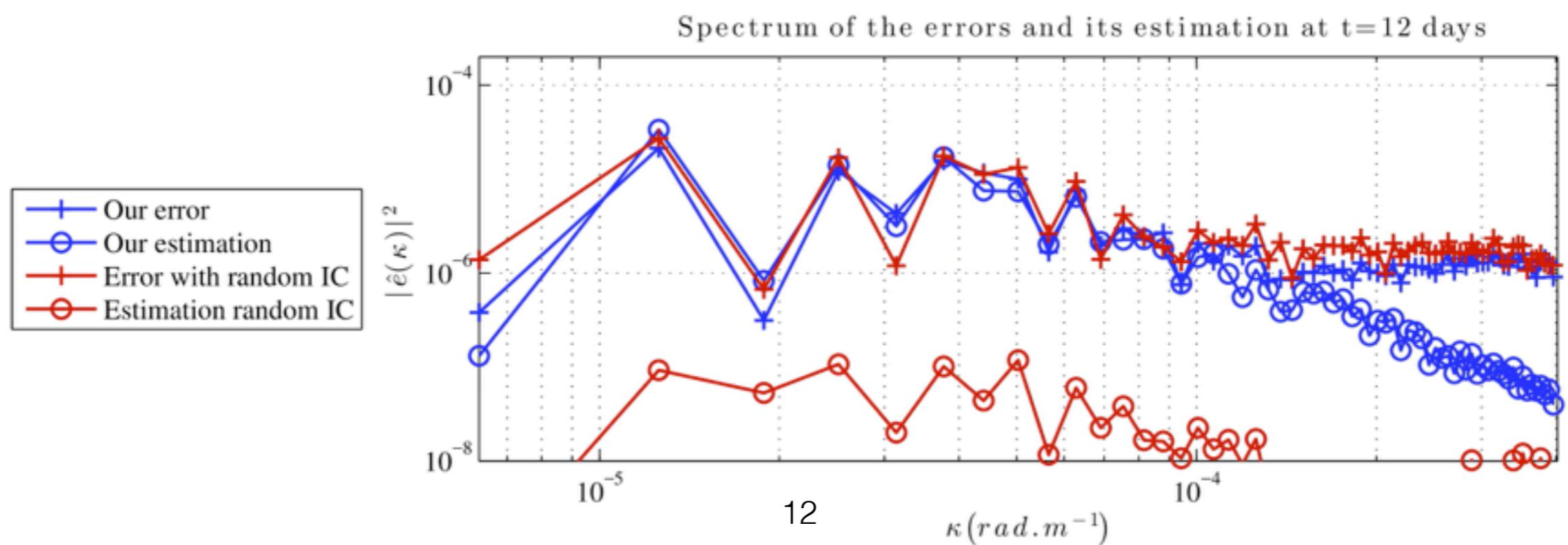
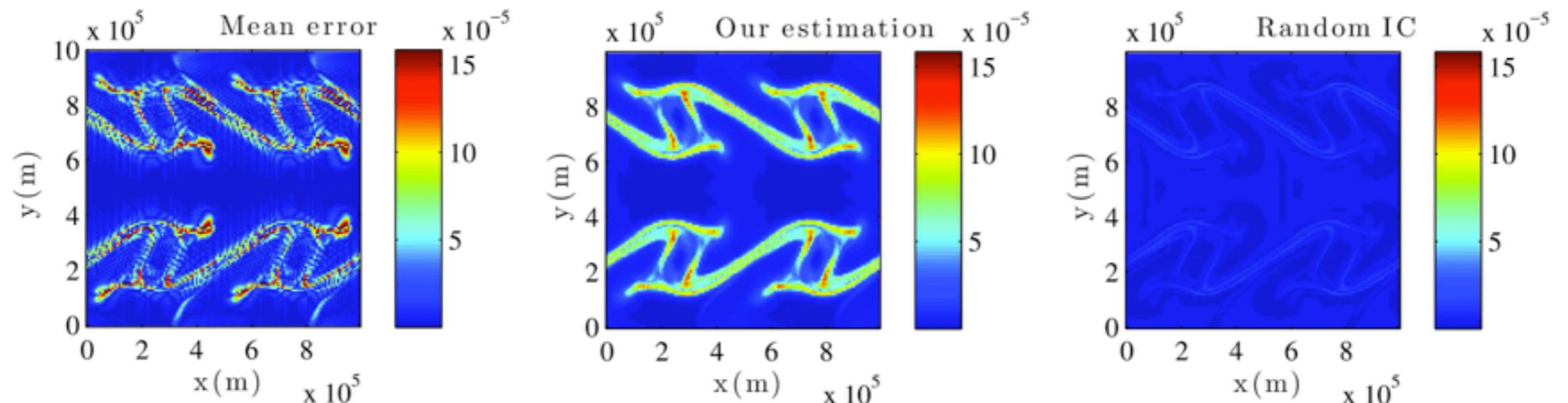


Ensemble

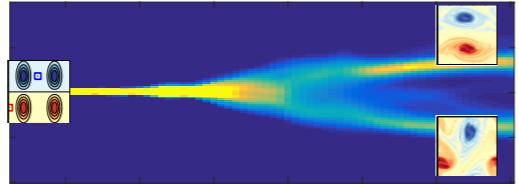




Ensemble



Summary of QG models

- Better small scales
- Estimate position and amplitude of errors
- Extreme events
- Bifurcations 
- under Strong Uncertainty:
Simple 2D description of frontolysis/frontogenesis

Code SQG MU:
link from Fluminance website - V. Resseguier

Lorenz model under location uncertainty

Do large-scale (diffusive) models lead to
over-representing "stable"-states
in ensemble simulations?

Lorenz model(s)

$$\frac{dX}{dt} = \Pr(Y - X) - \frac{4}{2\Upsilon} X$$

$$dY = \left[X(\rho - Z) - Y - \frac{4}{2\Upsilon} Y \right] dt + \frac{\rho - Z}{\Upsilon^{1/2}} dB_t$$

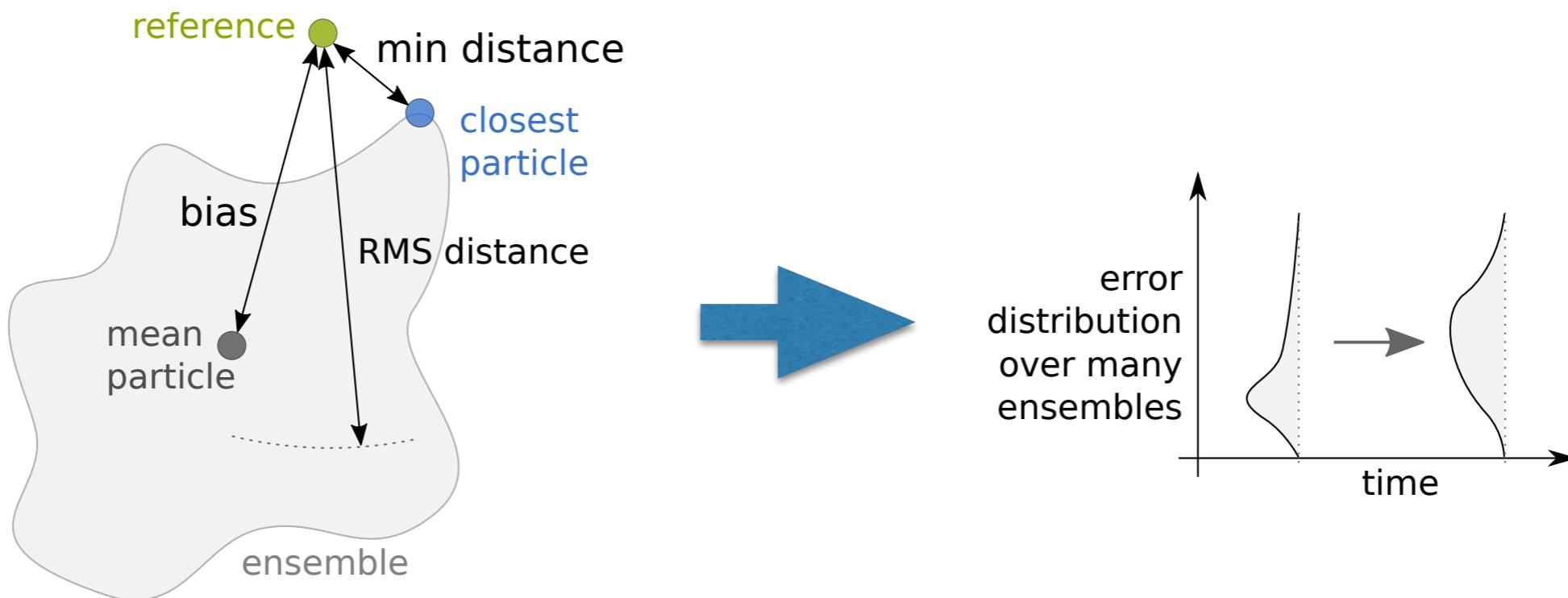
$$dZ = \left[XY - bZ - \frac{8}{2\Upsilon} bZ \right] dt + \frac{Y}{\Upsilon^{1/2}} dB_t$$

- (usual) **deterministic** model ~ DNS, accurate but impossible to compute
- (deterministic) **diffusive** model ~ LES
- **stochastic** model under location uncertainty
 - behaviors of ensembles?



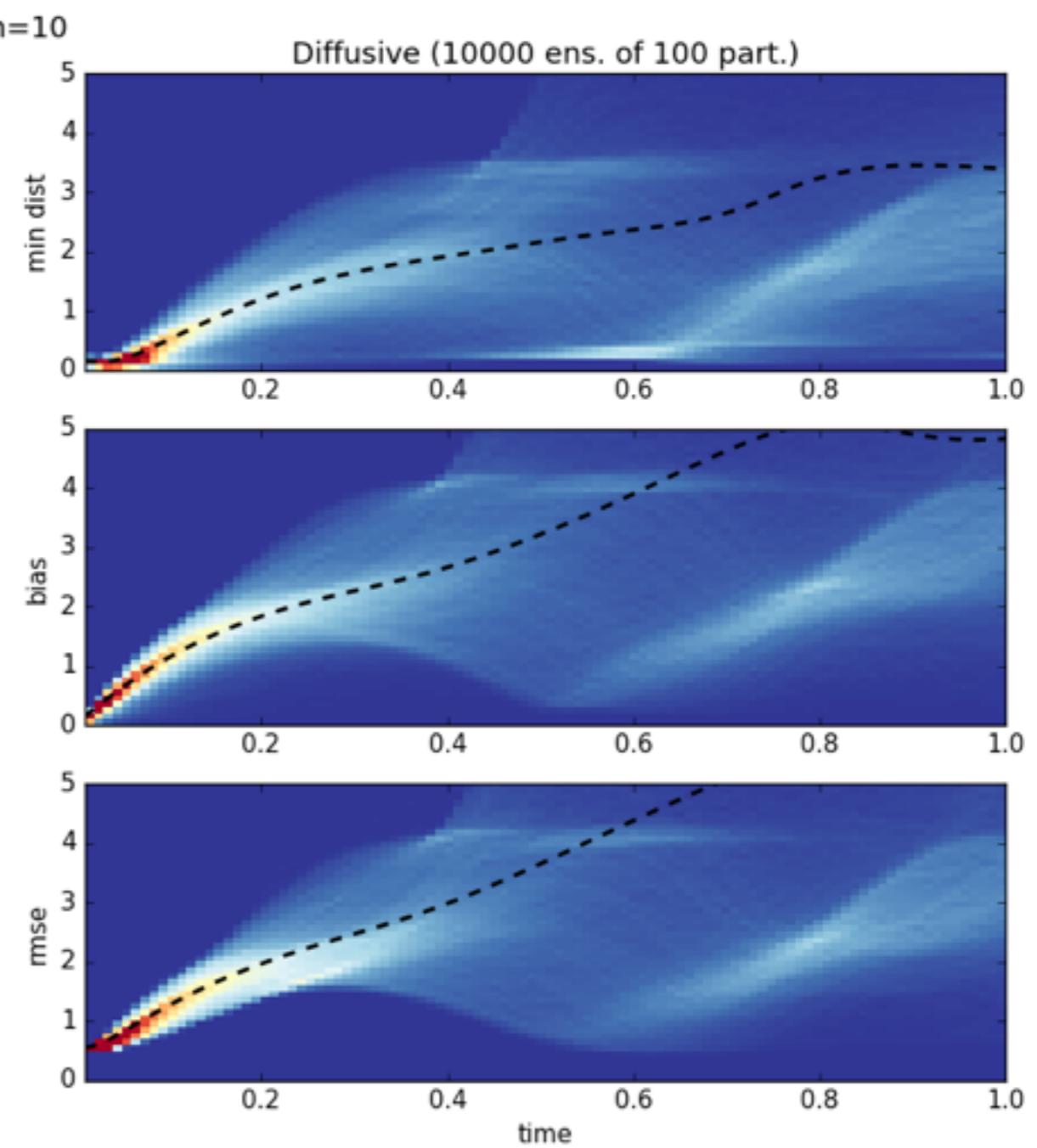
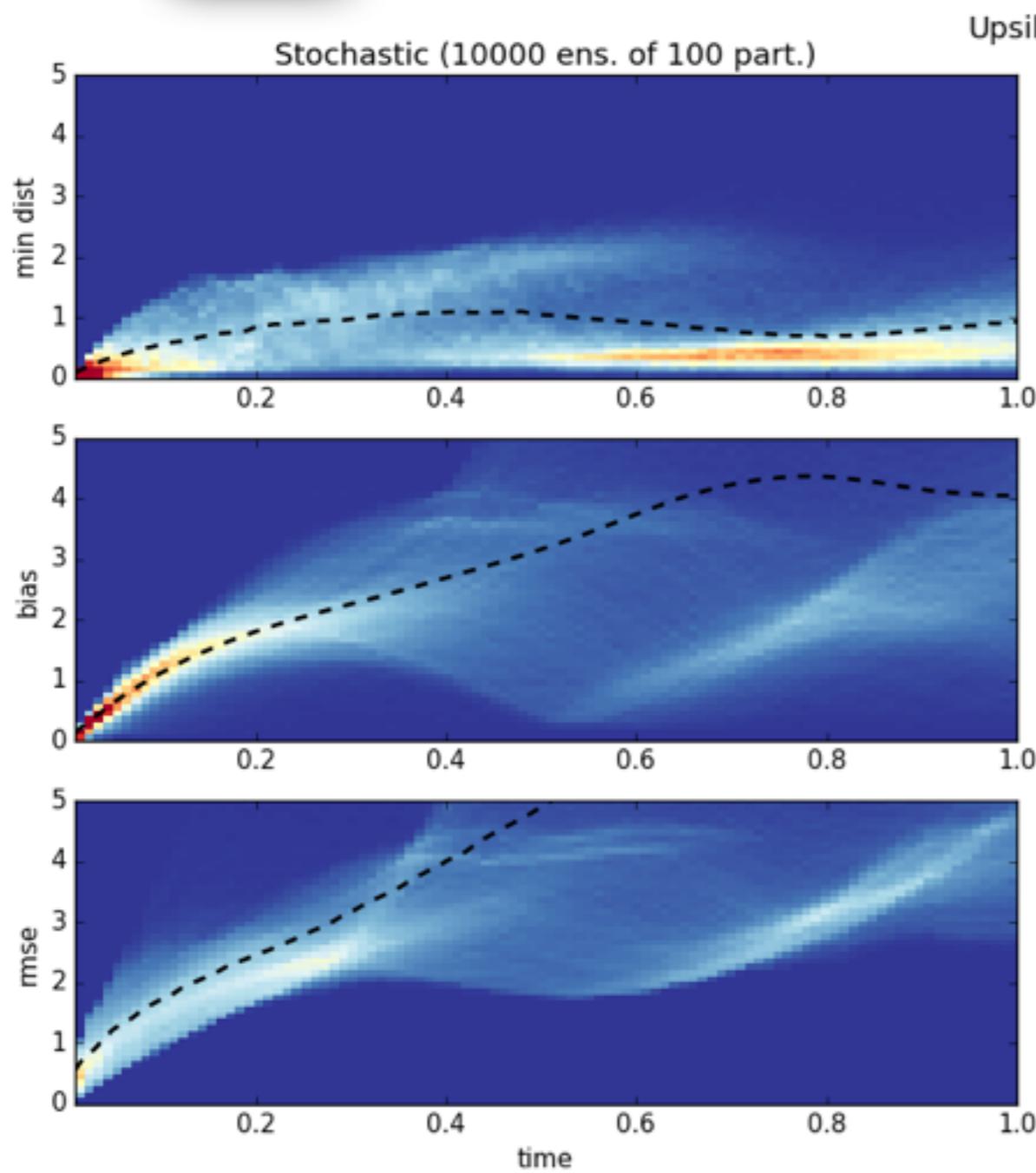
Short time behavior

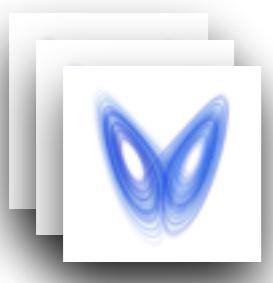
Comparison ensemble \leftrightarrow reference
3 metrics: minimum distance, bias, RMS distance





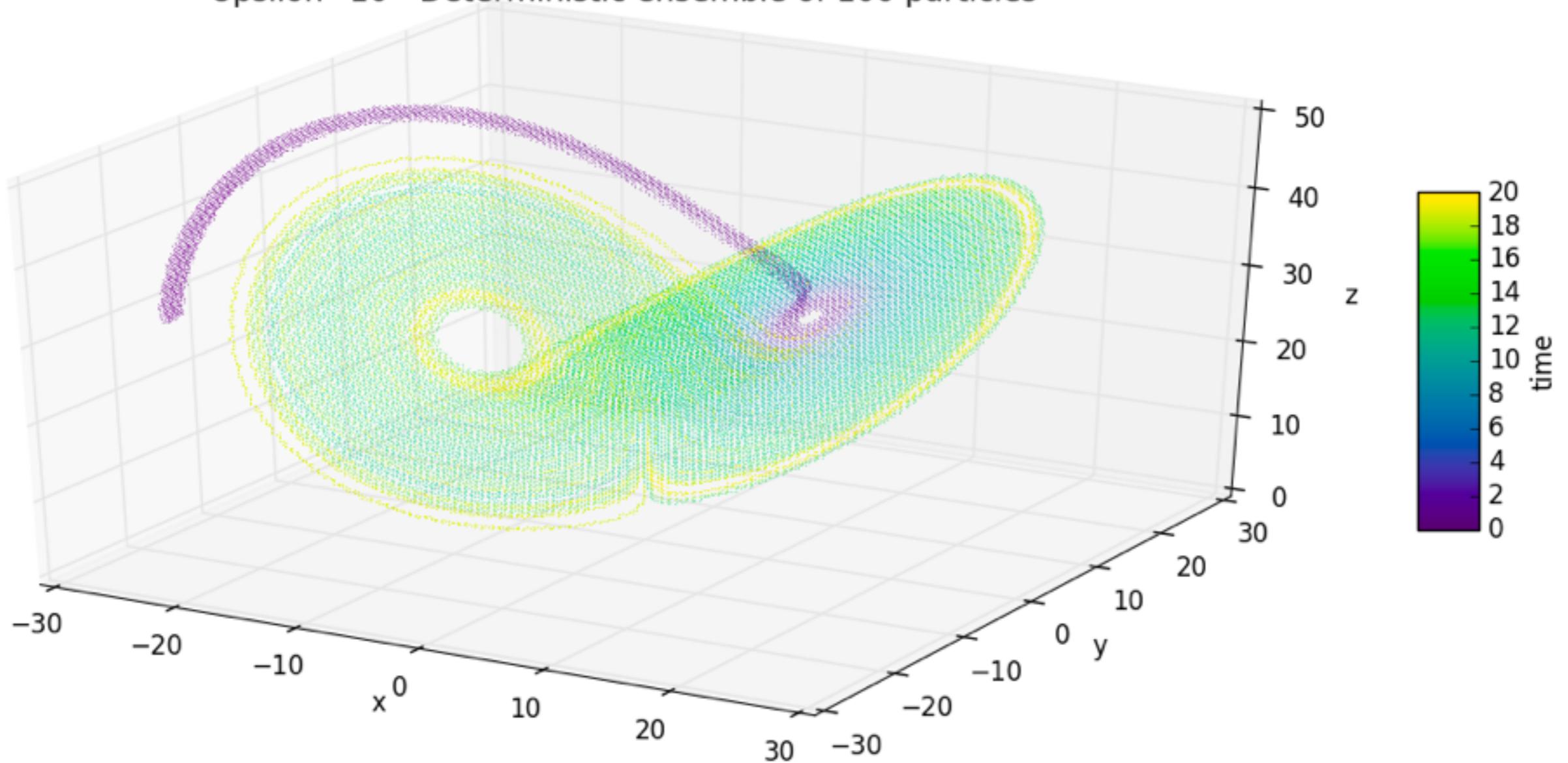
Short time behavior

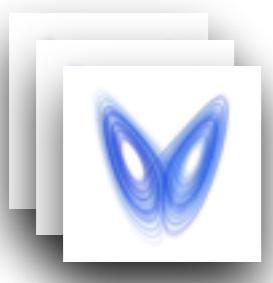




Long time

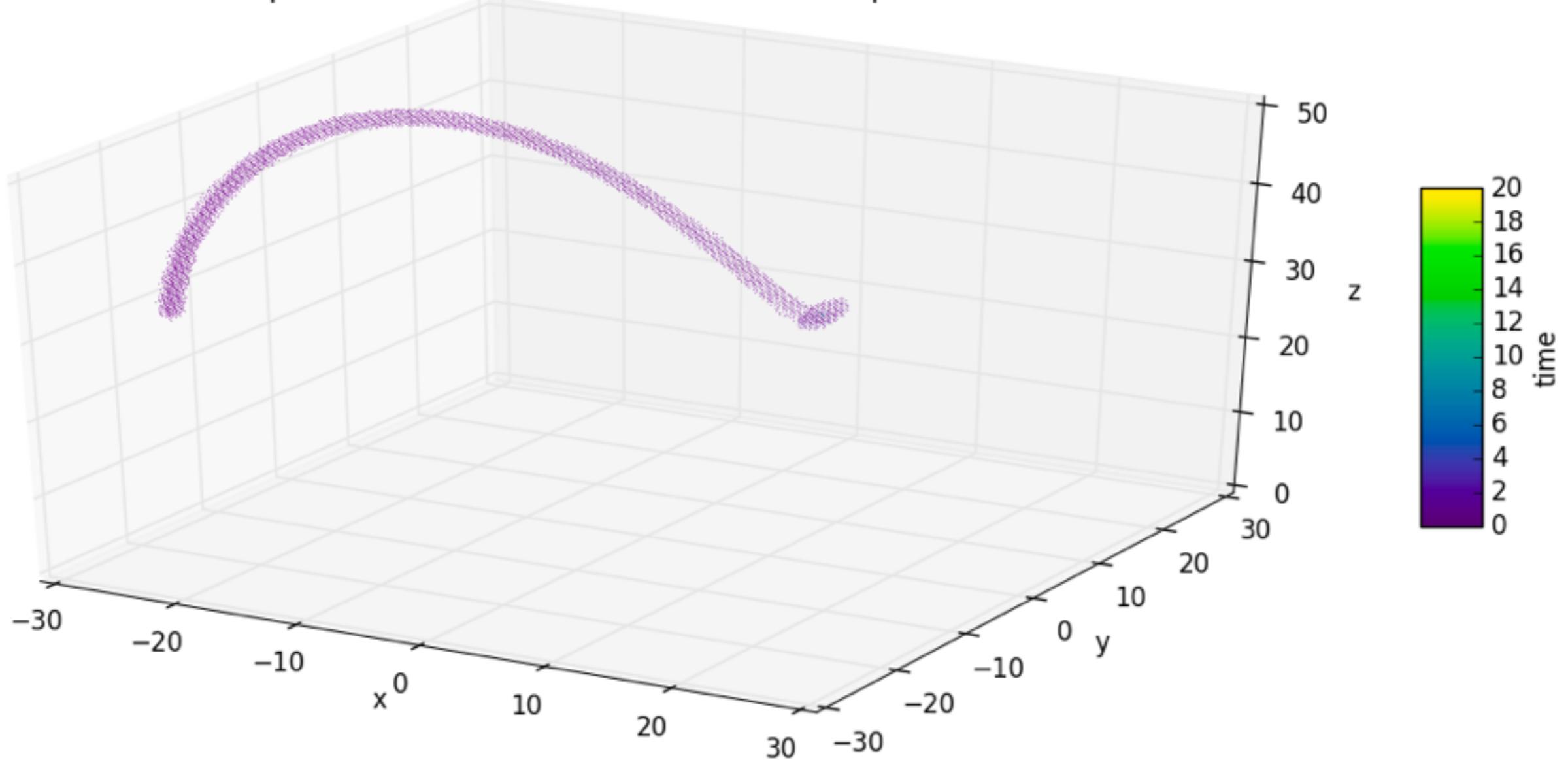
Upsilon=10 - Deterministic ensemble of 100 particles





Long time

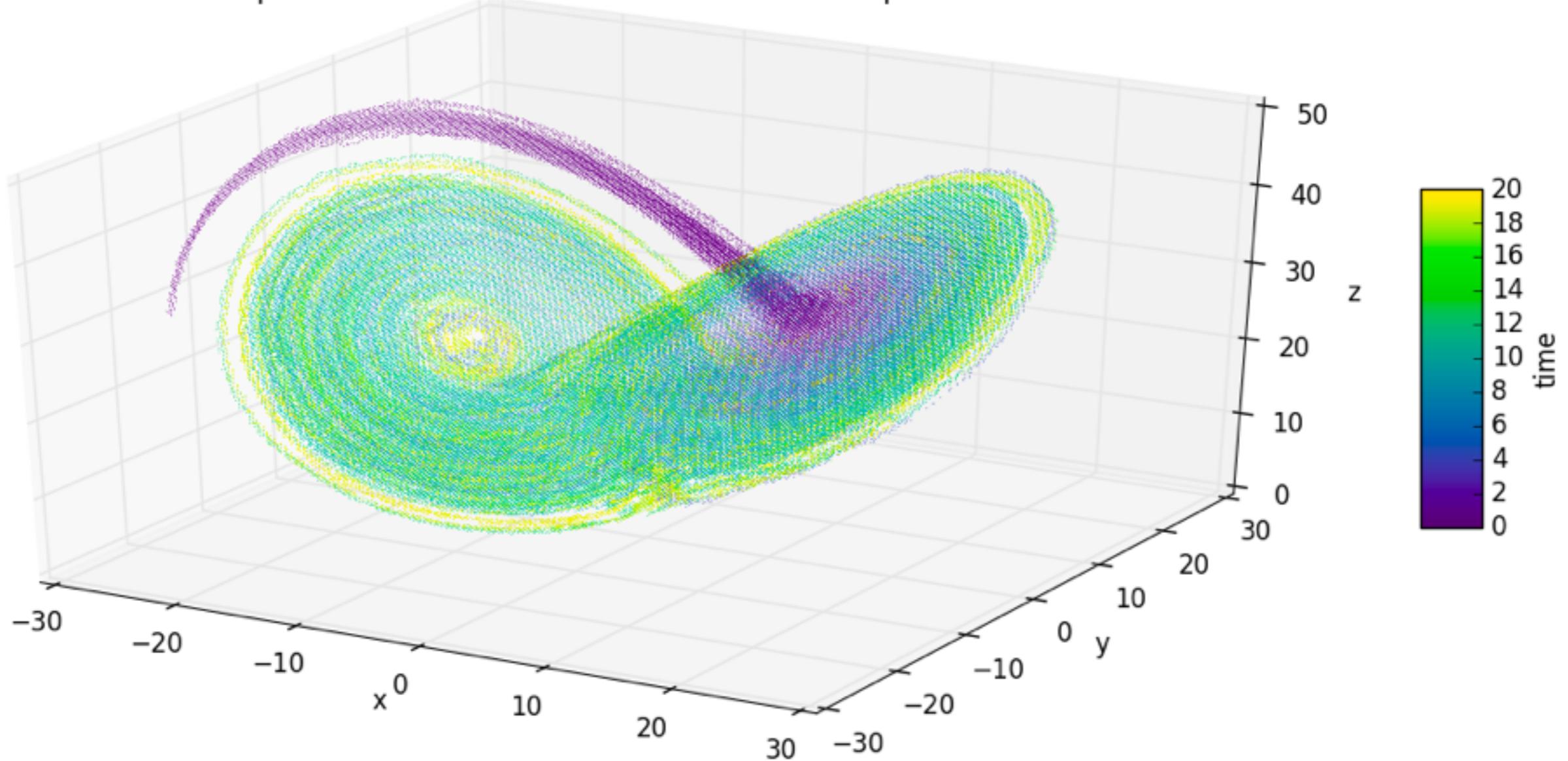
Upsilon=10 - Diffusive ensemble of 100 particles





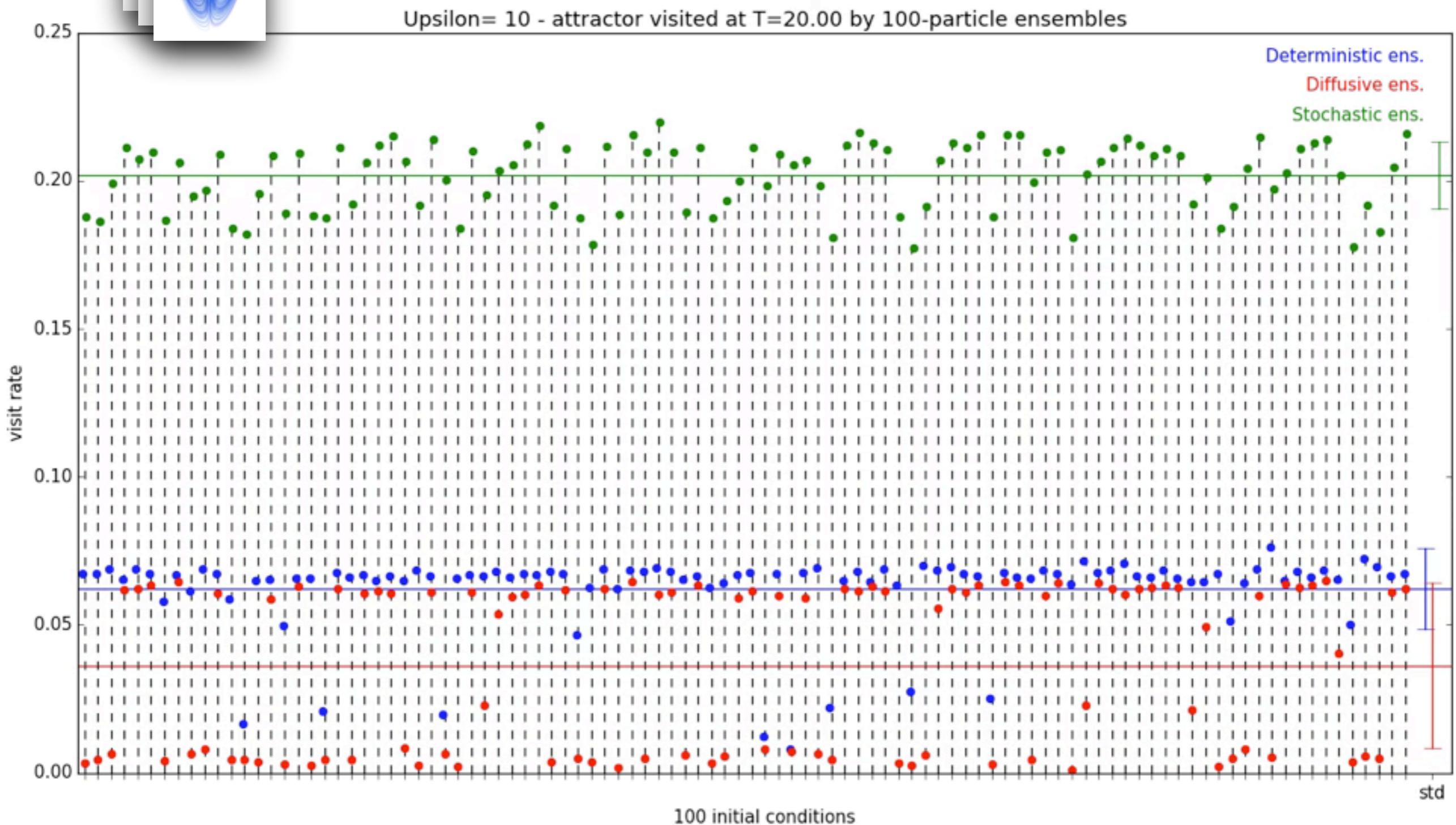
Long time

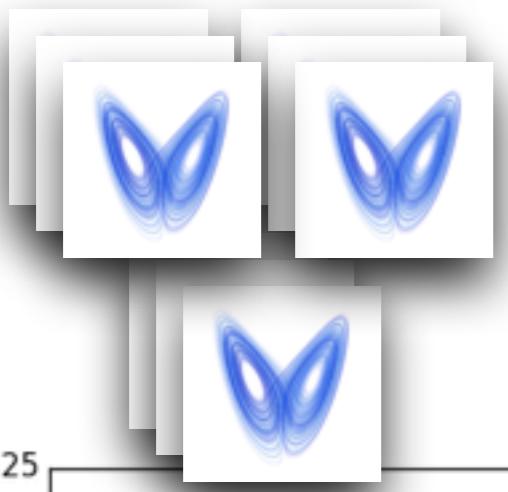
Upsilon=10 - Stochastic ensemble of 100 particles



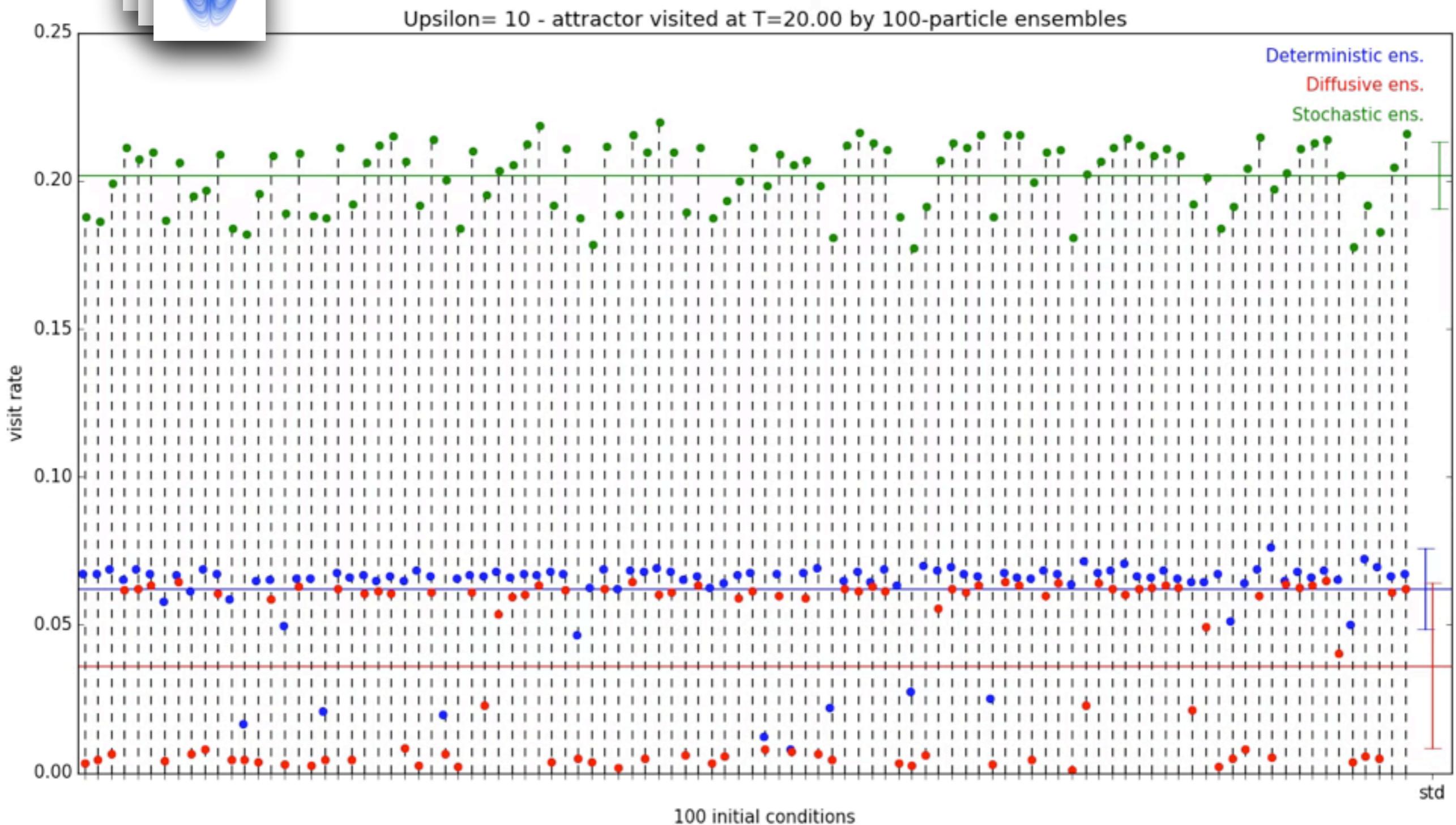


Attractor visit rates





Attractor visit rates



Conclusion

Conclusion

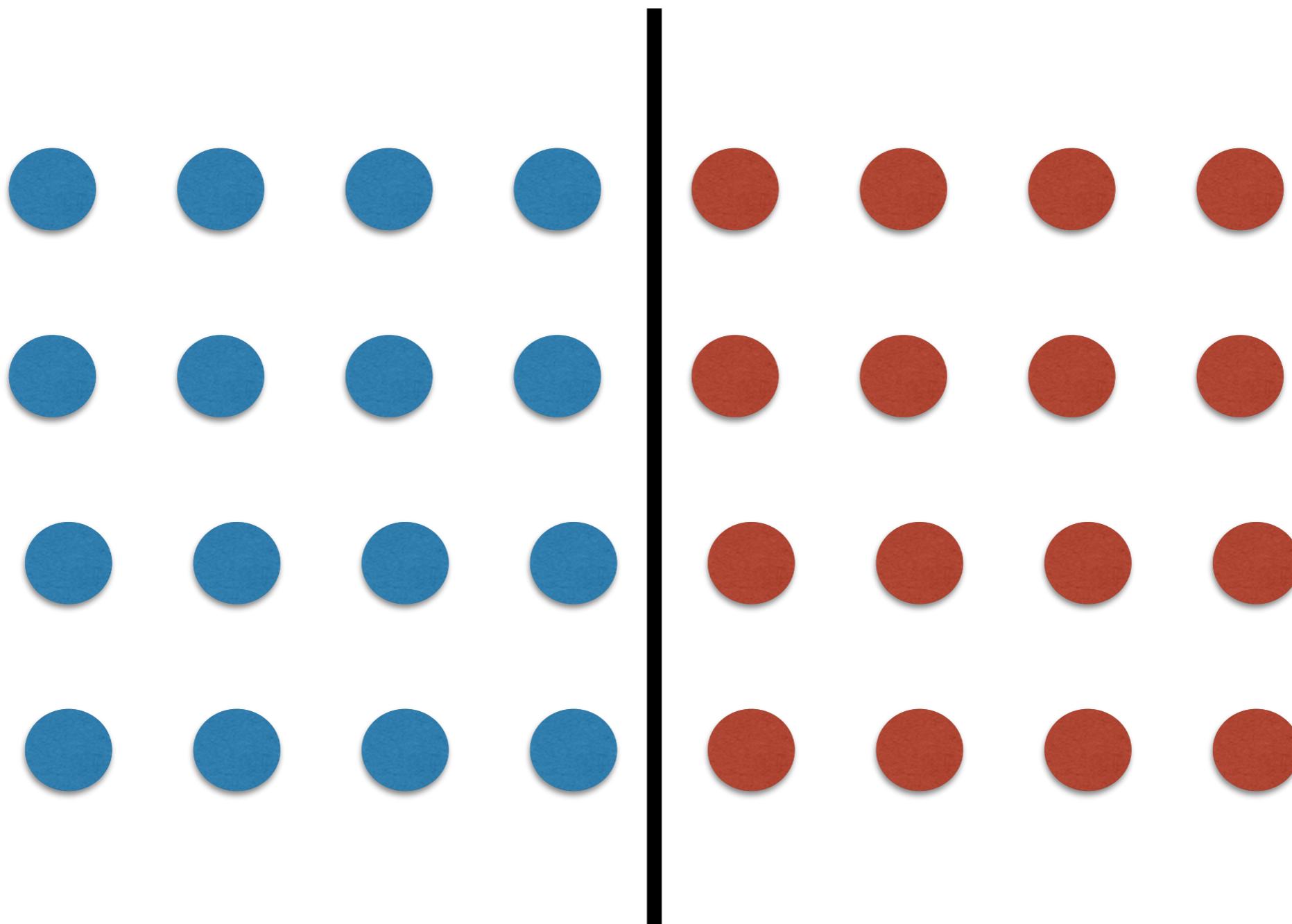
- Random transport applicable to any dynamics
- Better small scales
- Efficient spreading of the ensemble
- Likely scenarios
- Exploration of the attractor

Thank you for your attention

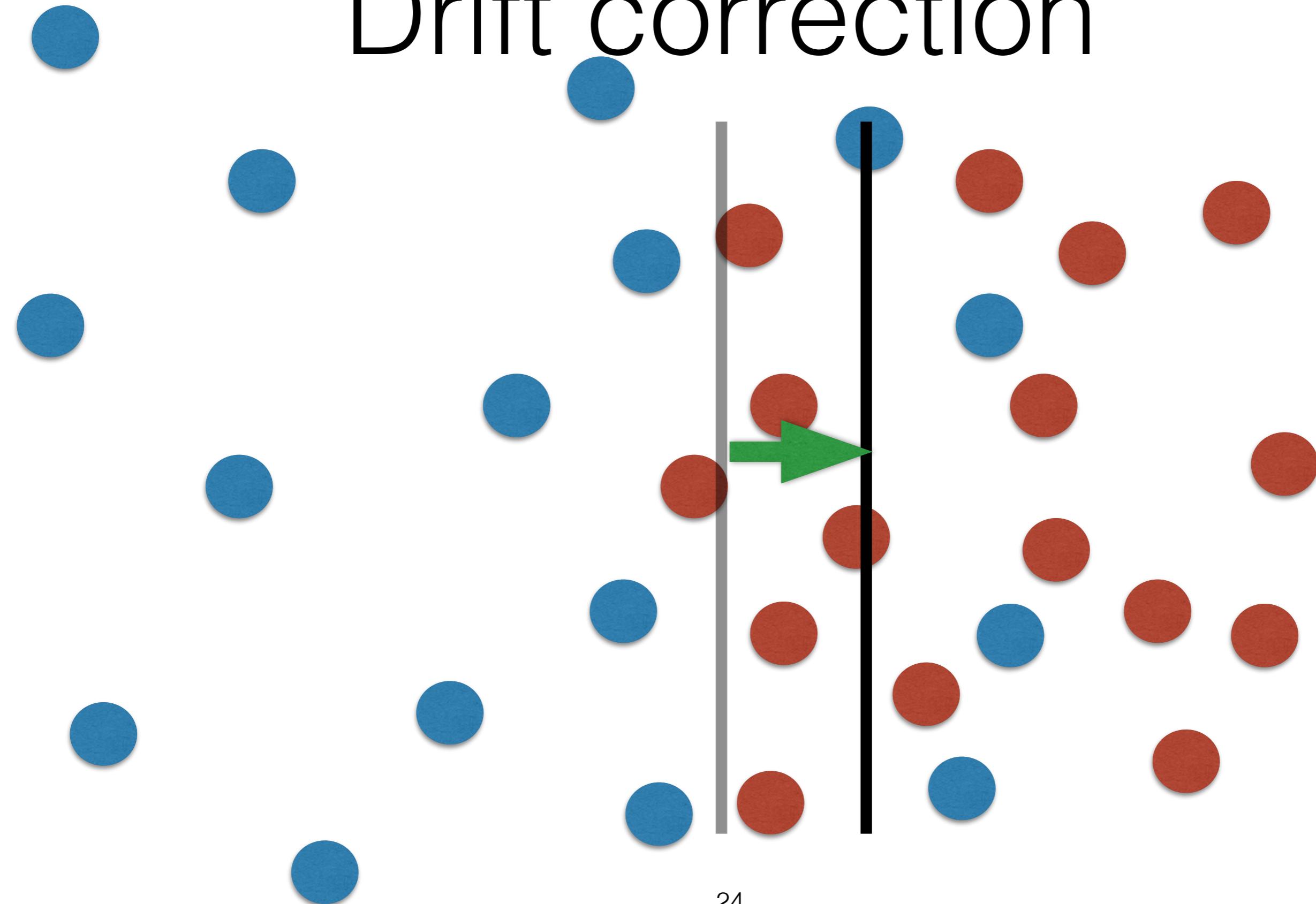
Code SQG MU:
link from Fluminance website - V. Resseguier

Drift correction

Drift correction

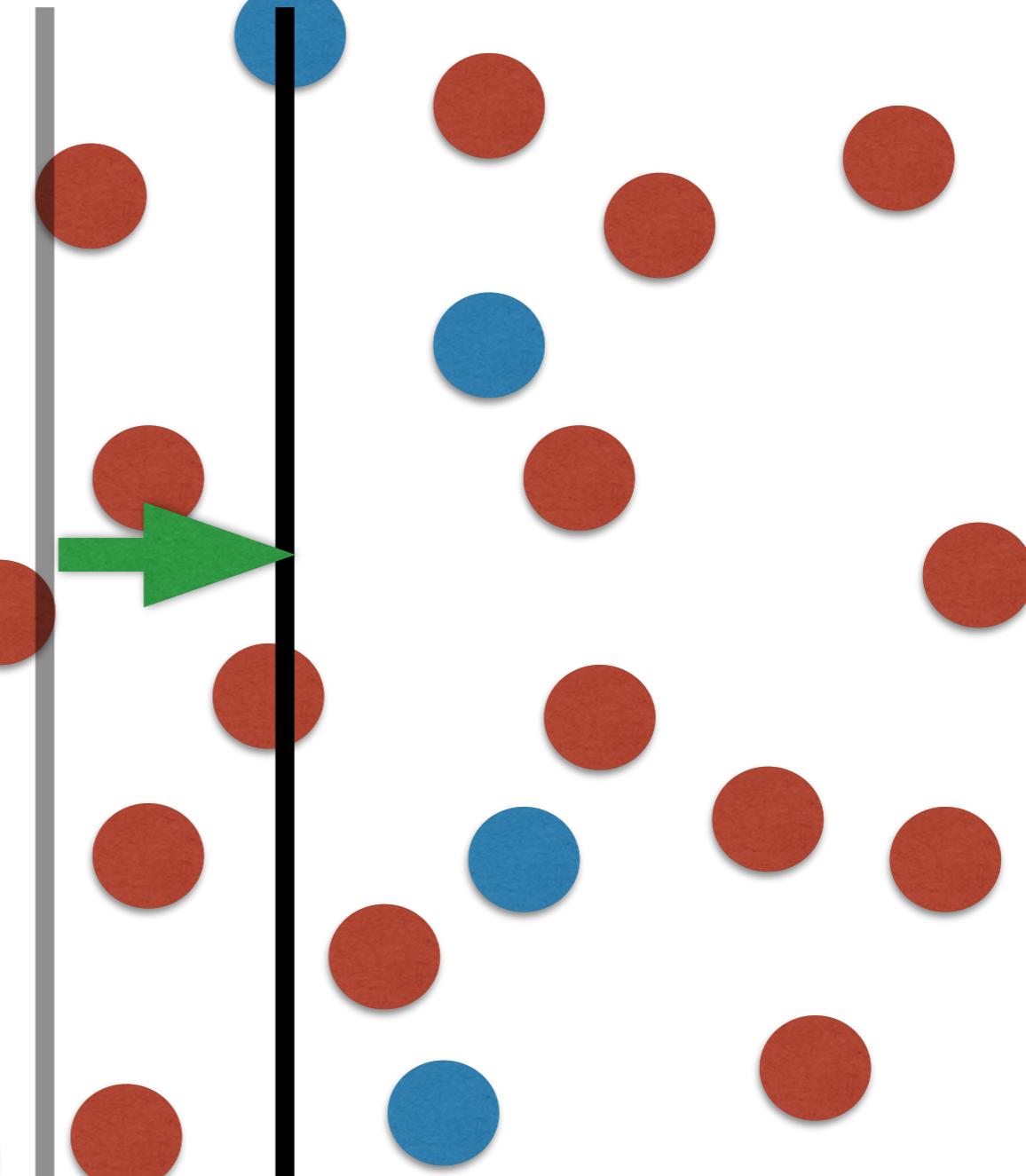


Drift correction



Drift correction

$$w^* = w - \frac{1}{2} (\nabla \cdot a)^T$$



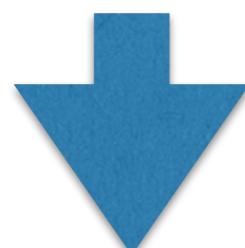
Bifurcations in SQG

tracked by SQG MU

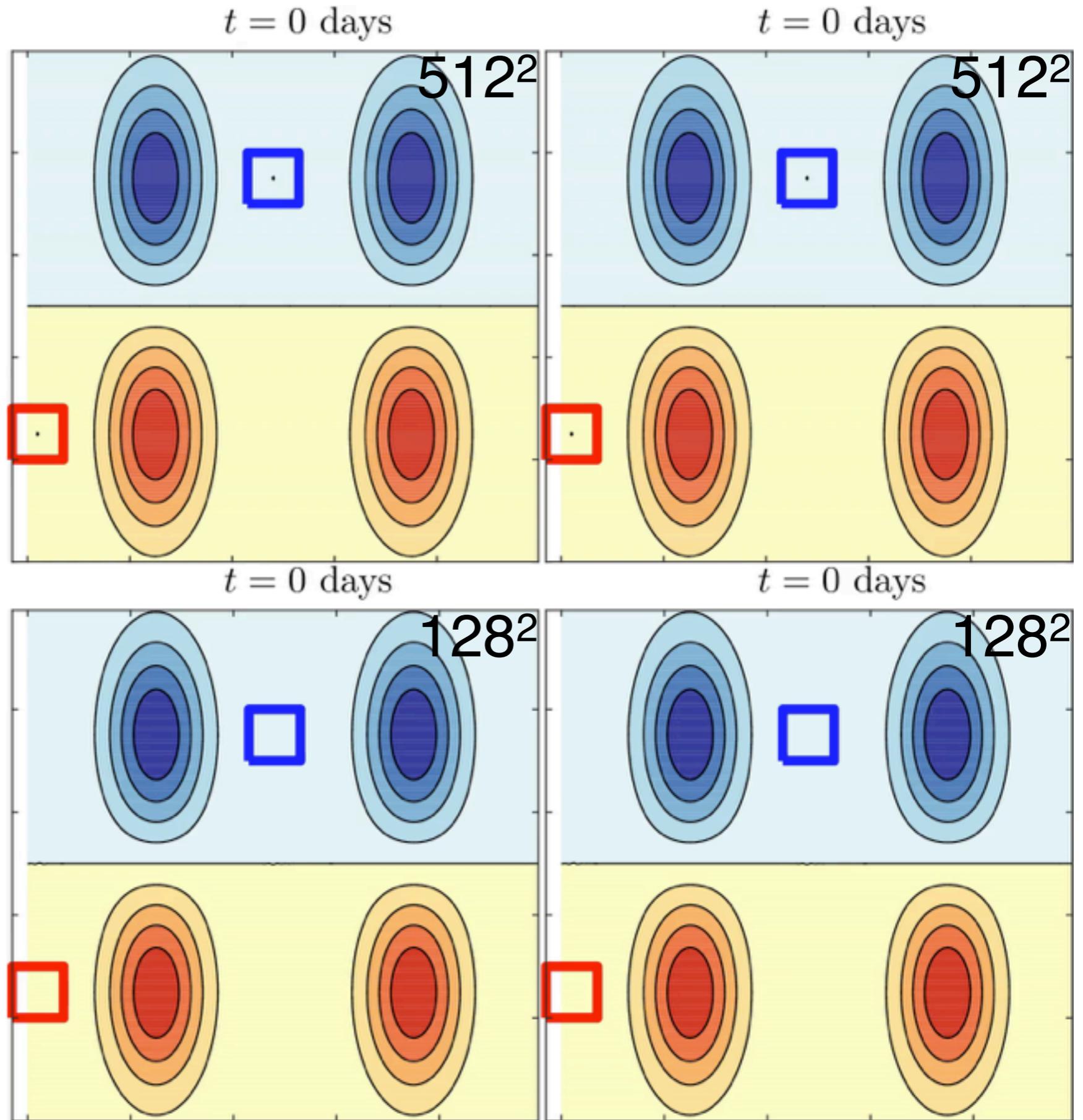
Reference flow:
deterministic
SQG

512² versus 128²

Initial condition 1



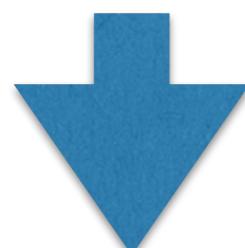
?



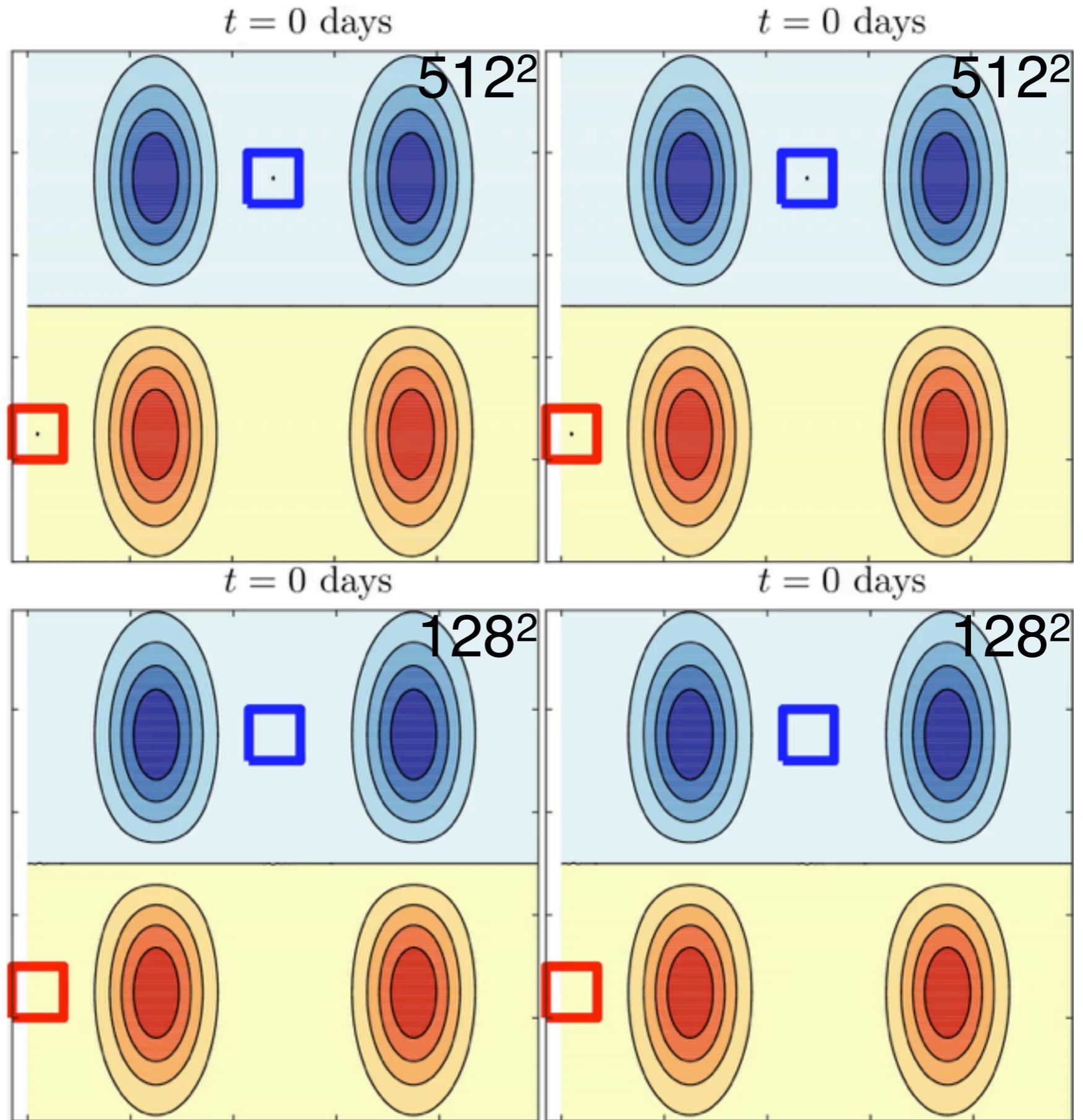
Reference flow:
deterministic
SQG

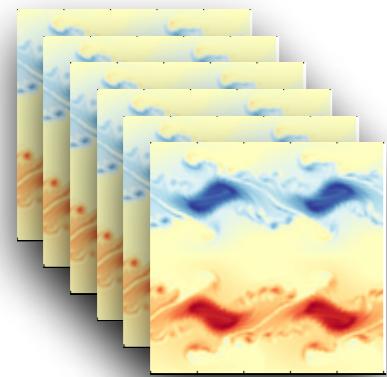
512² versus 128²

Initial condition 1

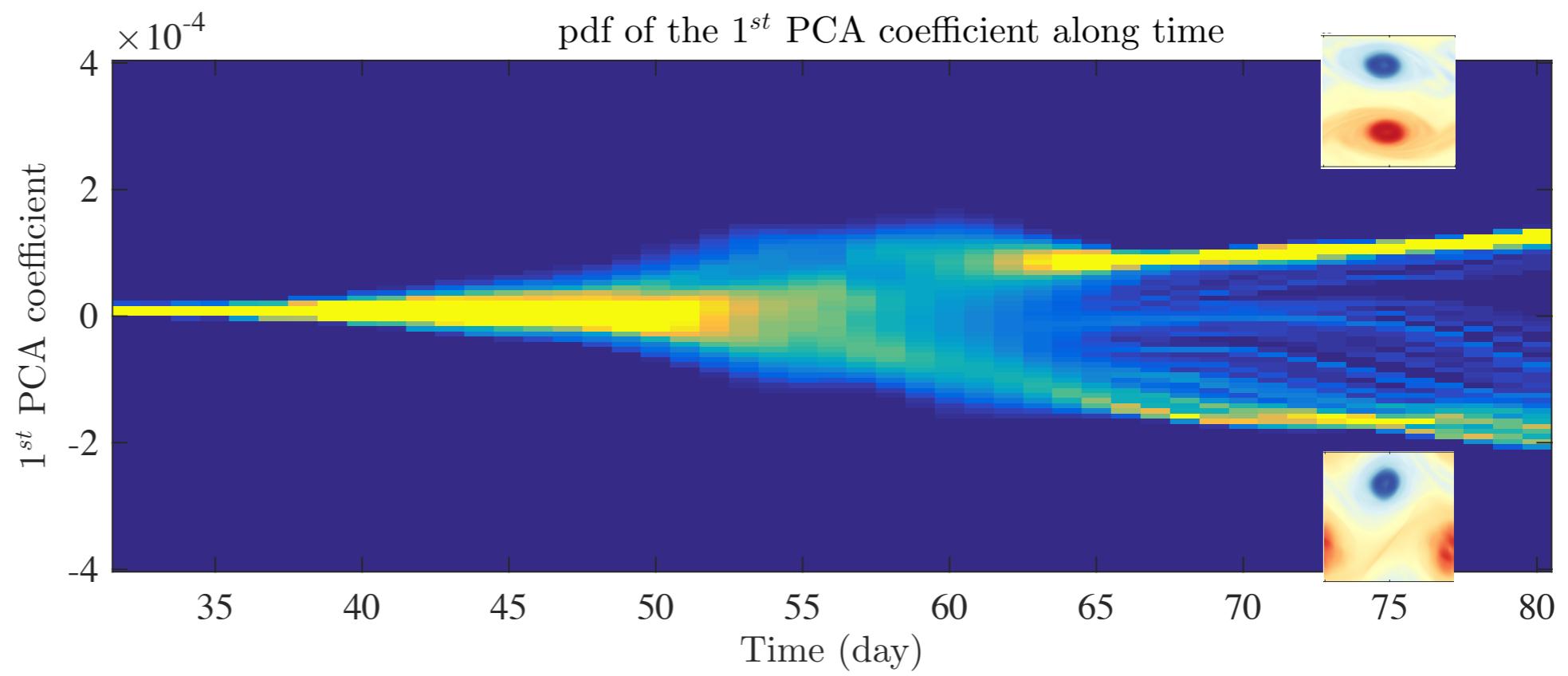


?

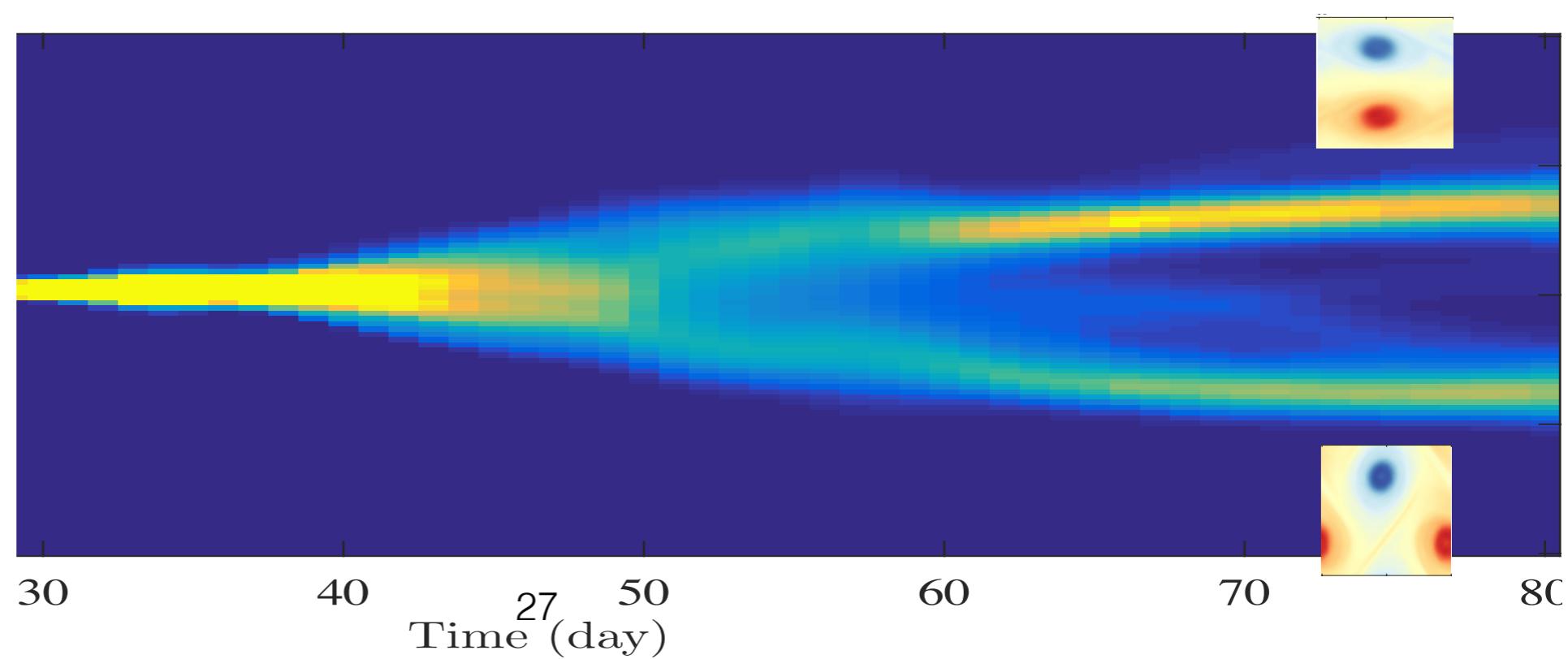




Random
initial
conditions



Under
location
uncertainty



SQG under Strong Uncertainty

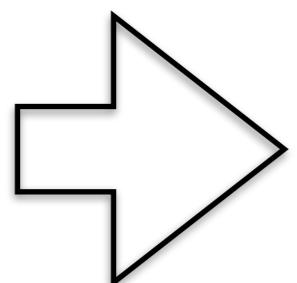
SQG SU

Mesoscale divergence

Geostrophic balance

$$\mathbf{f} \times \mathbf{u} = -\frac{1}{\rho_b} \nabla p' + \frac{a}{2} \Delta \mathbf{u}$$

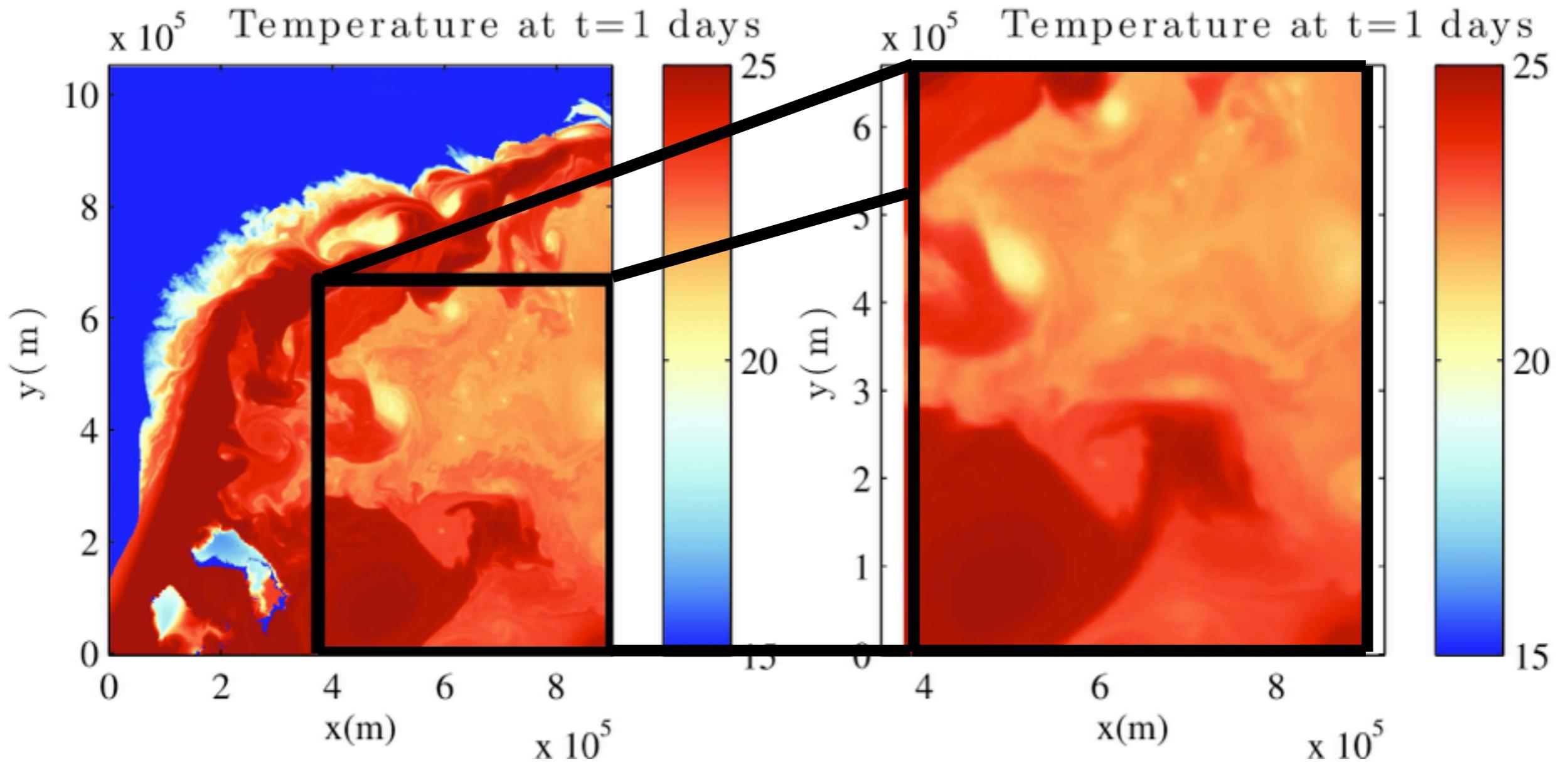
Horizontal
Diffusion



$$\nabla \cdot \mathbf{u} \propto \Delta \nabla^\perp \cdot \mathbf{u}$$

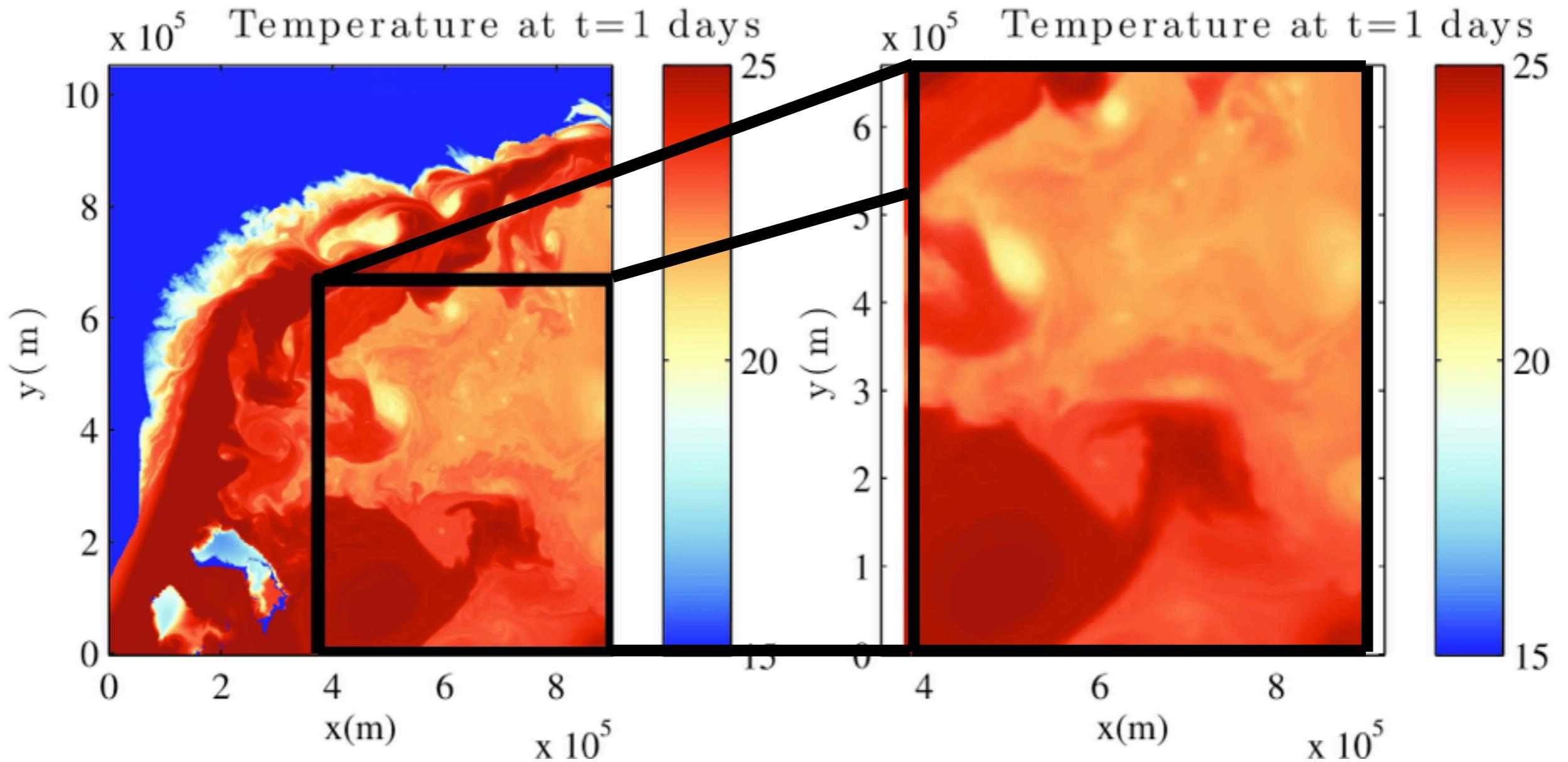
Filtering of model outputs:

Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
"Gulf Stream dynamics along the southeastern US seaboard."
Journal of Physical Oceanography 45.3 (2015): 690-715.

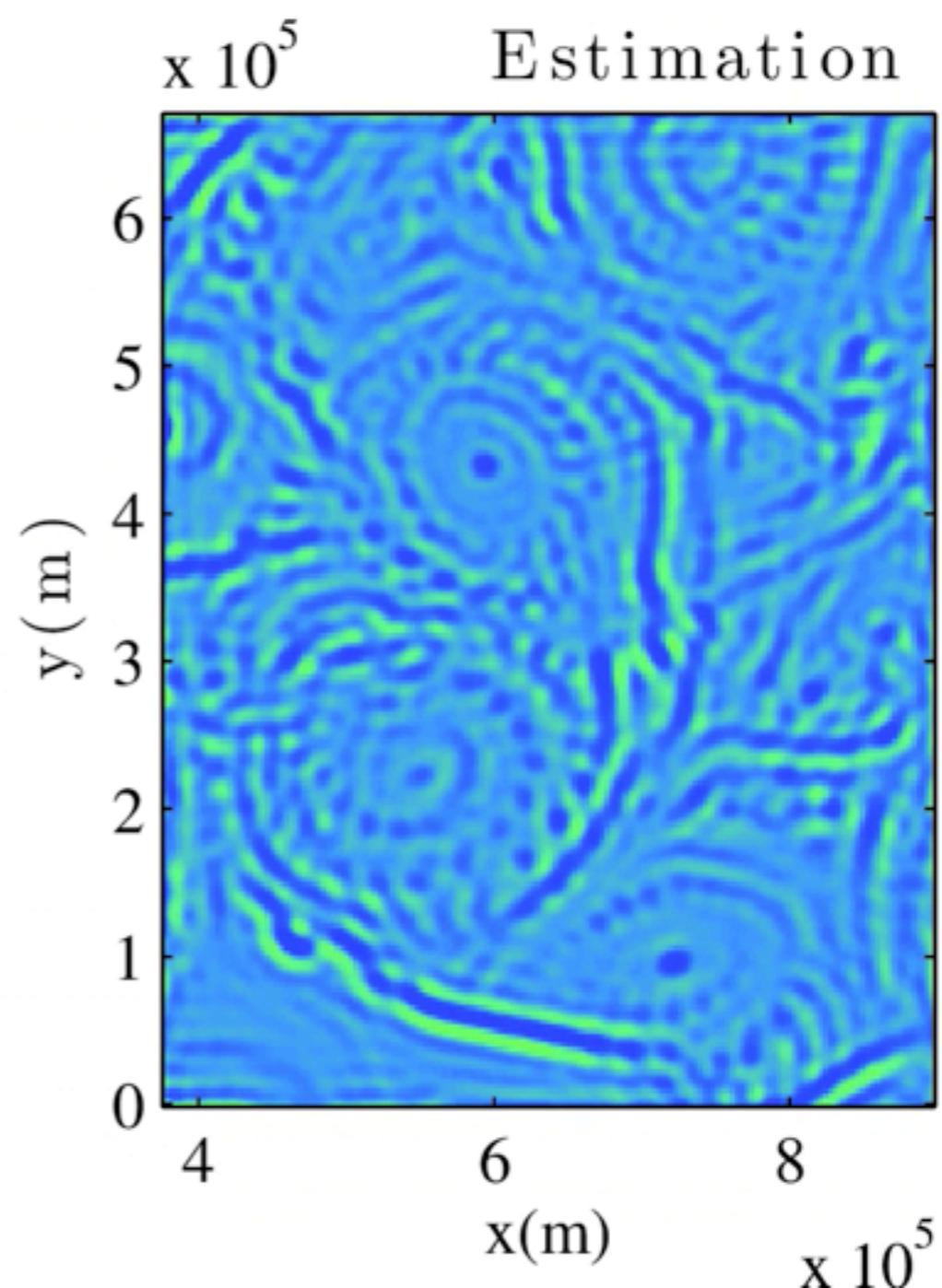
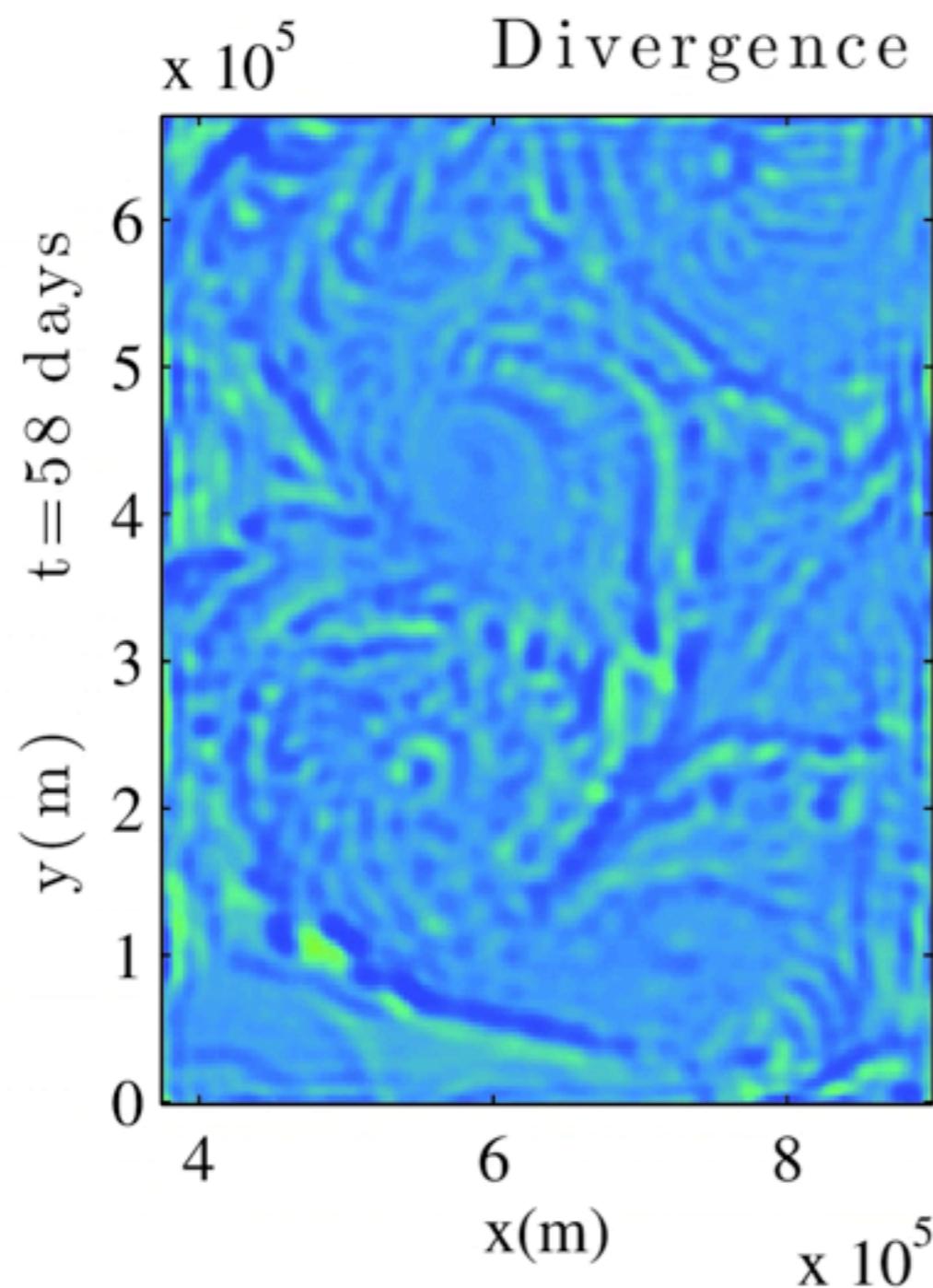


Filtering of model outputs:

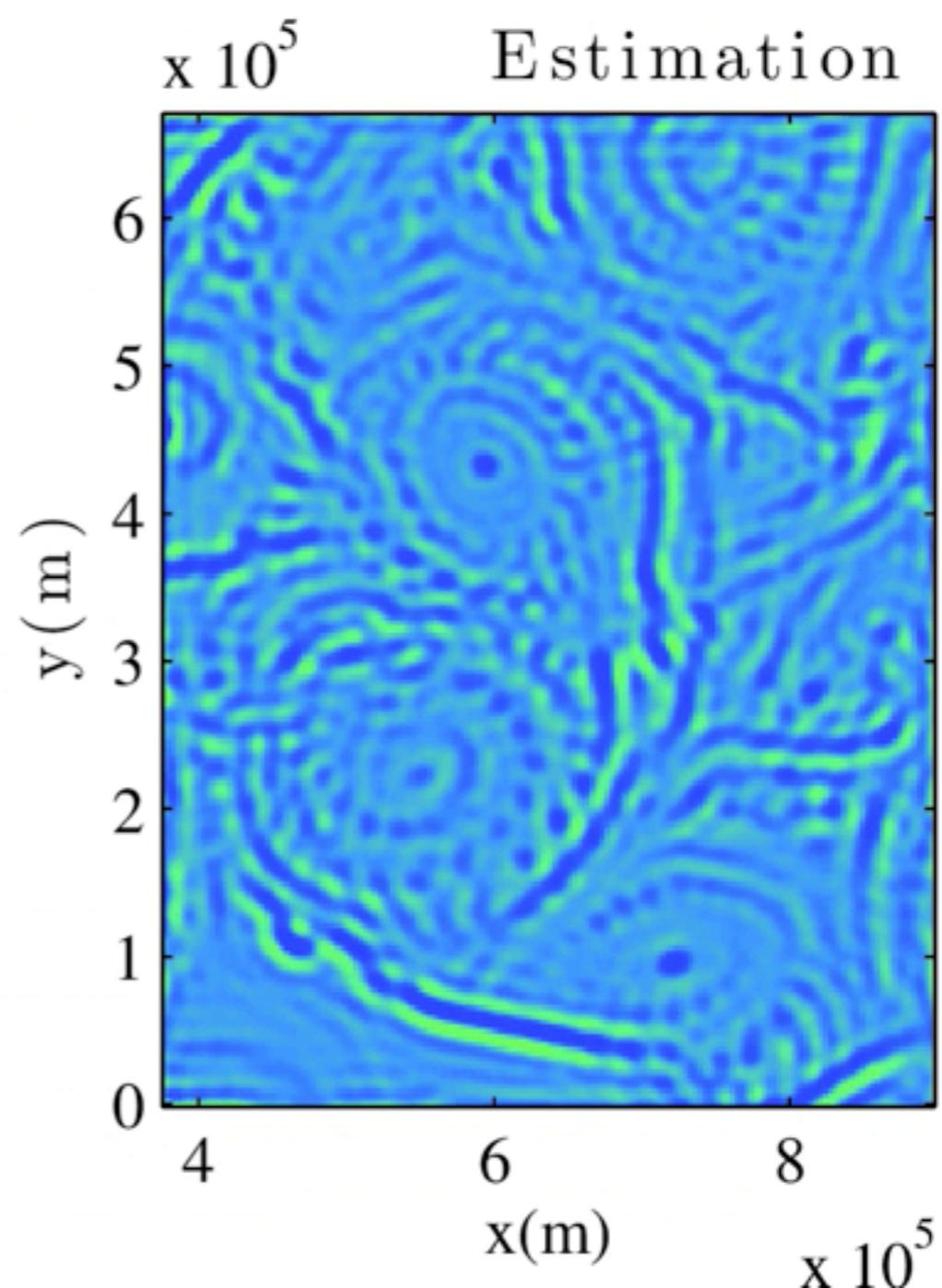
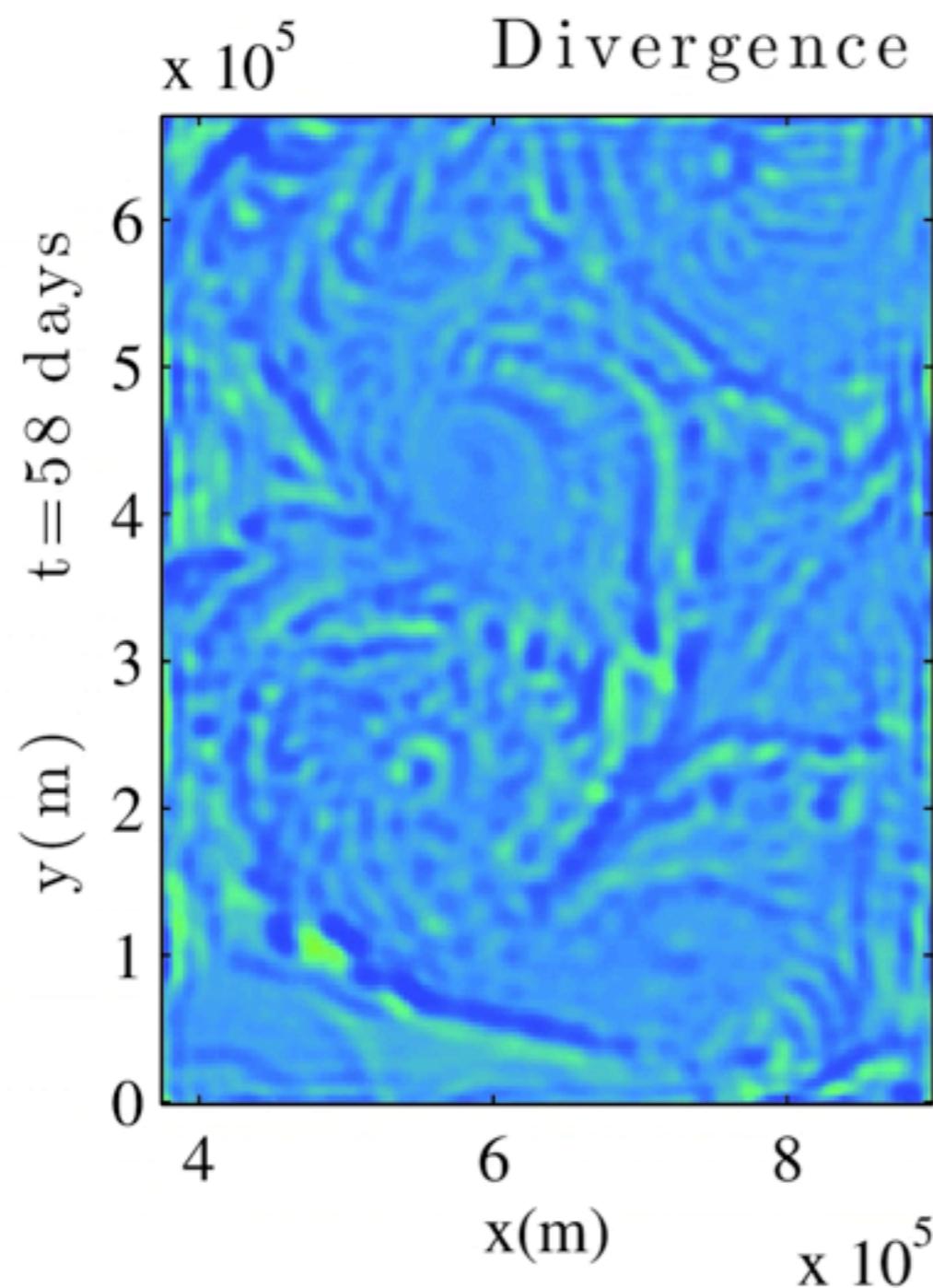
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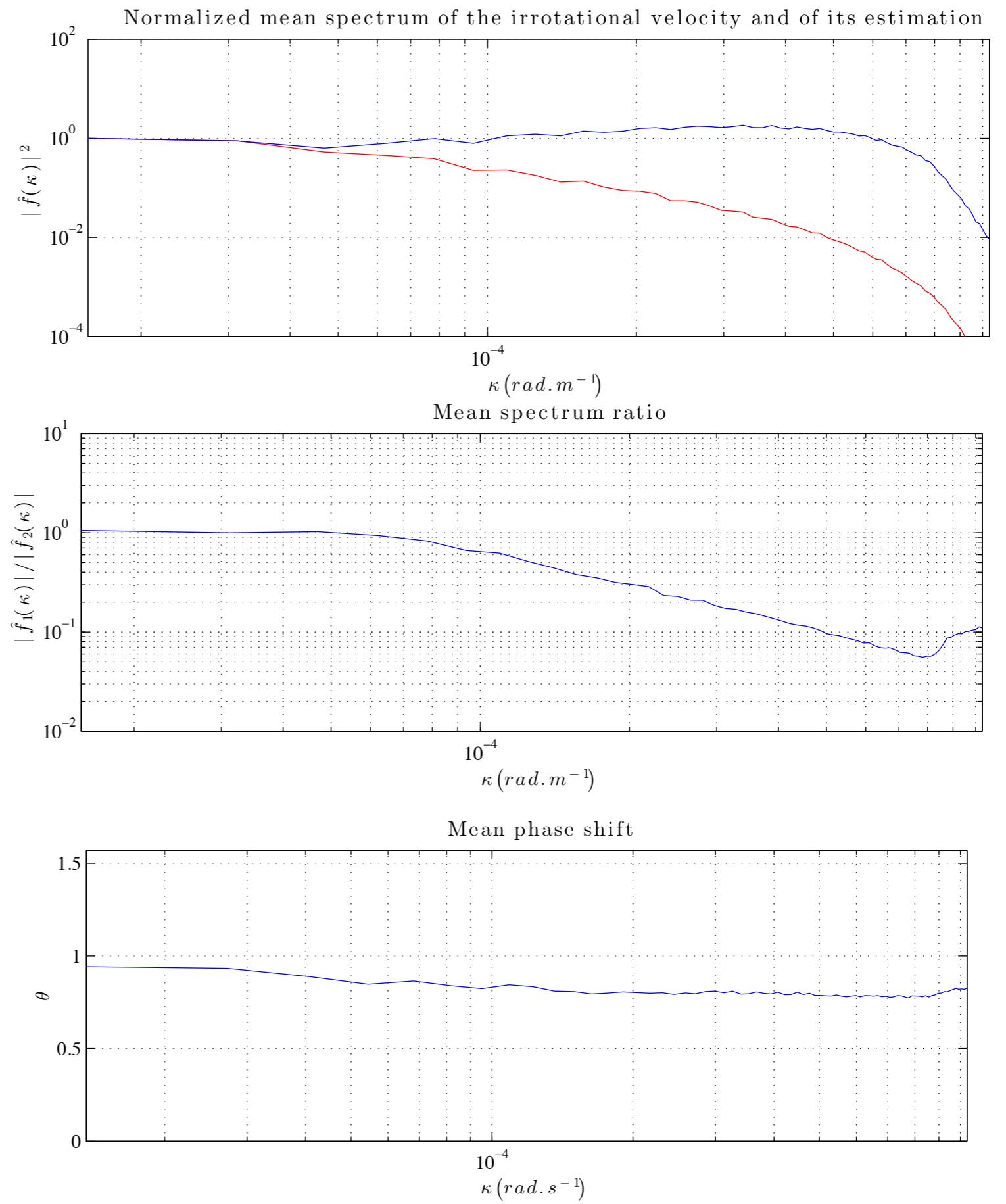
Spatial test

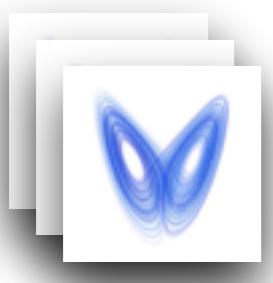


Spatial test

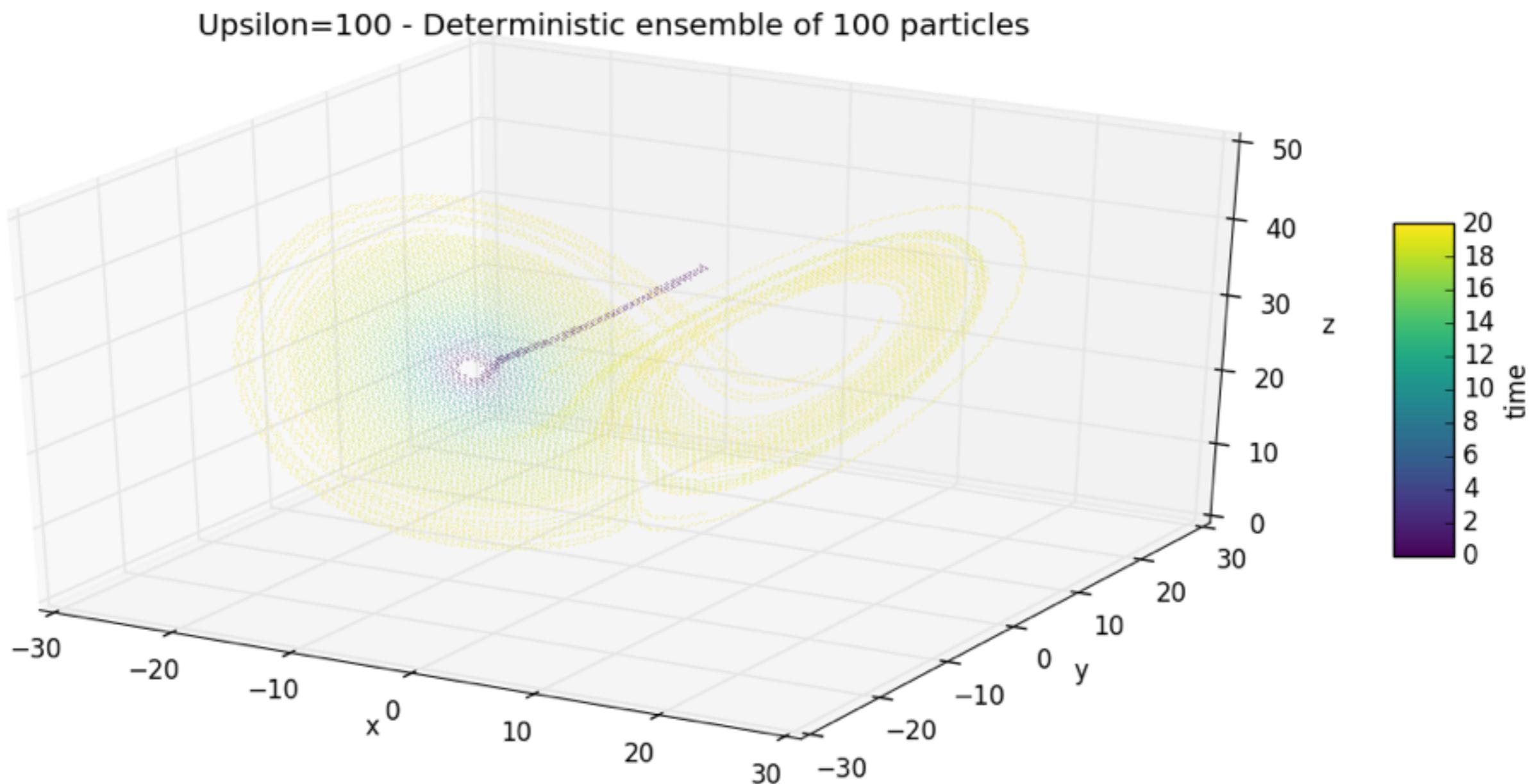


Spectral test



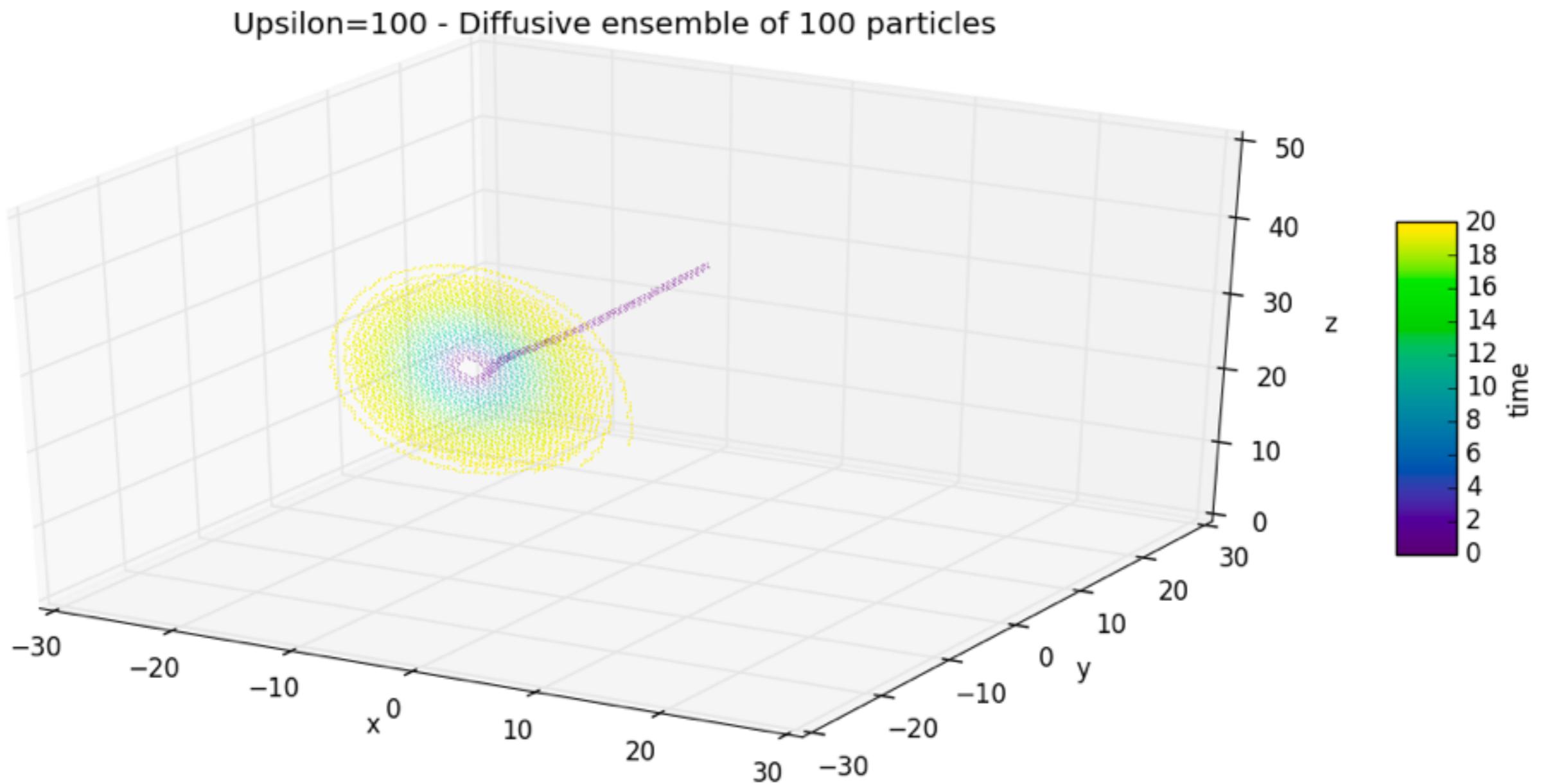


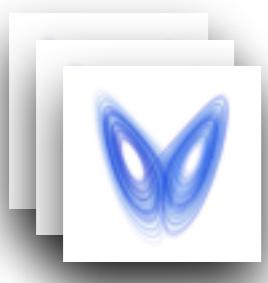
Long time: weak noise



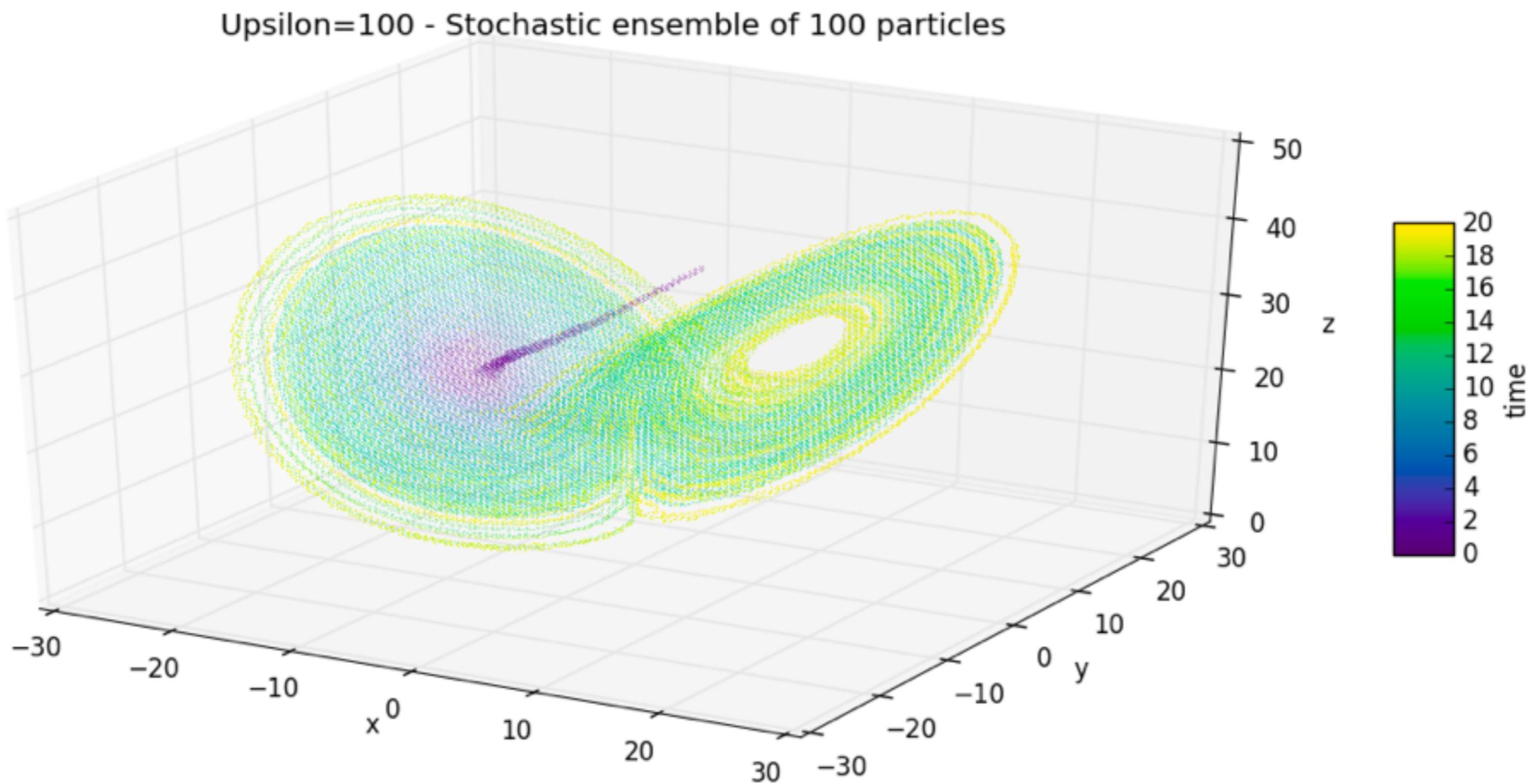


Long time: weak noise



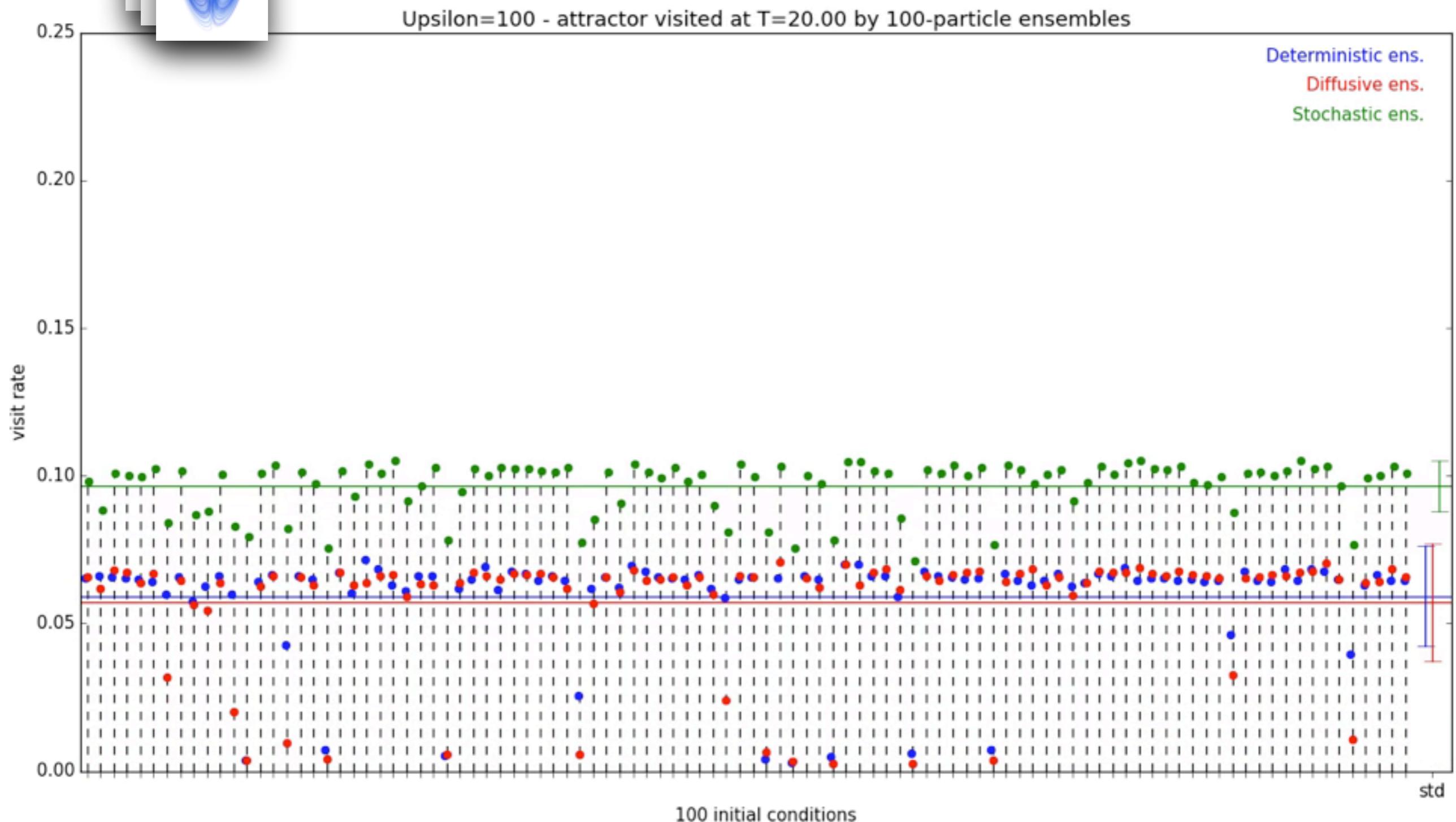


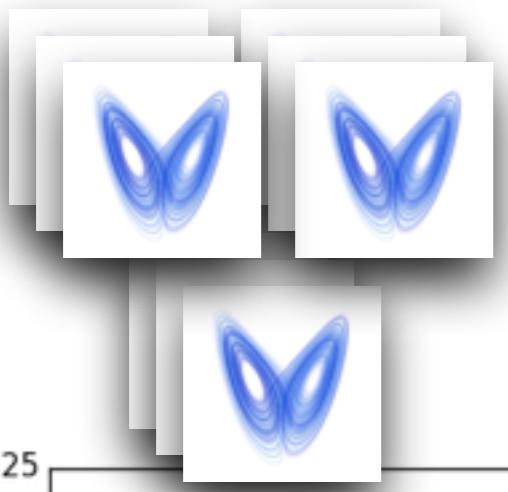
Long time: weak noise



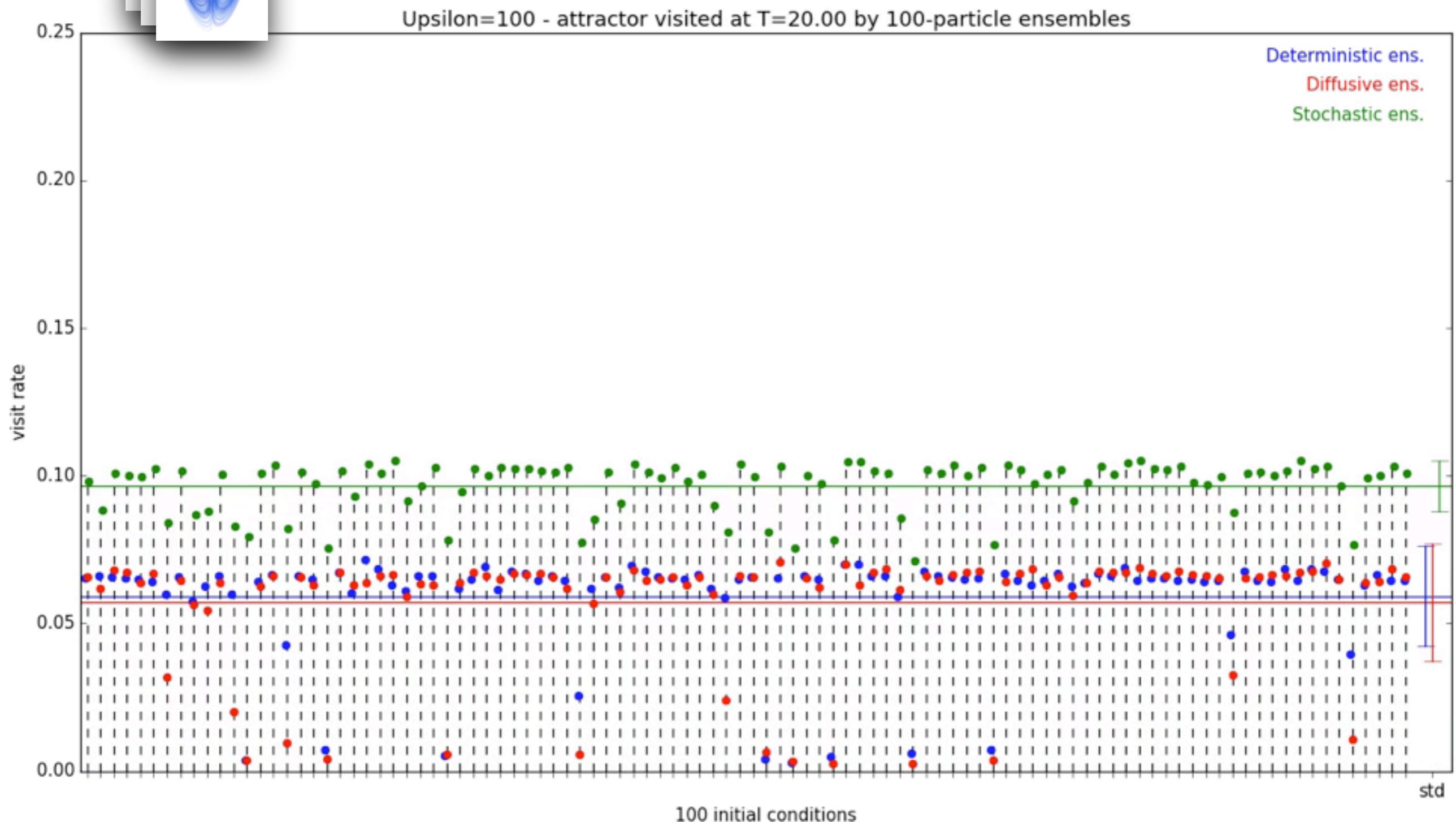


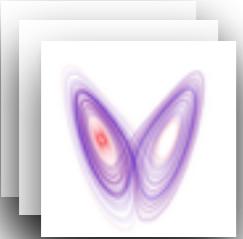
Attractor visit rates





Attractor visit rates





Computing the visit rate

- Discrete covering of the Lorenz attractor (GAIO)
→ 611,550 cubic boxes of radius=0.15625
- For each ensemble, the visit rate:

$$\tau(T) = \frac{\#\{ \text{unique boxes visited by ensemble over } [0; T] \}}{\text{total } \# \text{ of boxes}}$$

