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Stochastic parametrization of geophysical flows through modeling under location uncertainty

Valentin Resseguier,

Pierre Dérian,

Etienne Mémin,

Bertrand Chapron

Motivations

- Rigorously identified subgrid dynamics effects
- Injecting likely small-scale dynamics
- Studying bifurcations and attractors



Climate projections

- Quantification of modeling errors



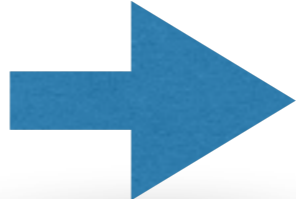
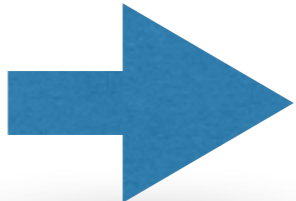
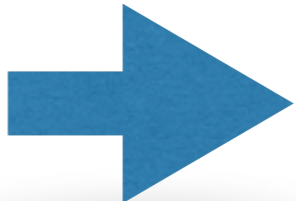

Ensemble forecasts and data assimilation

Contents

- Randomized dynamics
- SQG under Moderate Uncertainty
- Lorenz under location uncertainty

Randomized dynamics

Random equations

- Random initial conditions  Underdispersive + need large ensemble
- Arbitrary Gaussian forcing  Adding energy + wrong phase
- Averaging, homogenization  Assumptions and energy issues
- Adding white random velocity  $\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$

Advection of tracer Θ

$$\frac{D\Theta}{Dt} = 0$$

Advection of tracer Θ

Advection of tracer Θ

$$\partial_t \Theta + \boldsymbol{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} \boldsymbol{a} \nabla \Theta \right)$$

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Advection of tracer Θ

Drift correction

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection

Diffusion

Advection of tracer Θ

The diagram illustrates the advection of a tracer Θ . The equation is presented as follows:

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta}_{\text{Drift correction}} + \underbrace{\sigma \dot{B} \cdot \nabla \Theta}_{\text{Multiplicative random forcing}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

The left side of the equation is labeled "Advection" and is enclosed in a blue rounded rectangle. The right side is labeled "Diffusion" and is enclosed in a green rounded rectangle. An orange arrow points from the label "Drift correction" to the w^* term. A purple arrow points from the label "Multiplicative random forcing" to the $\sigma \dot{B}$ term.

Advection of tracer Θ

The diagram illustrates the advection of a tracer Θ . The equation is presented as follows:

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta}_{\text{Drift correction}} + \underbrace{\sigma \dot{B} \cdot \nabla \Theta}_{\text{Multiplicative random forcing}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

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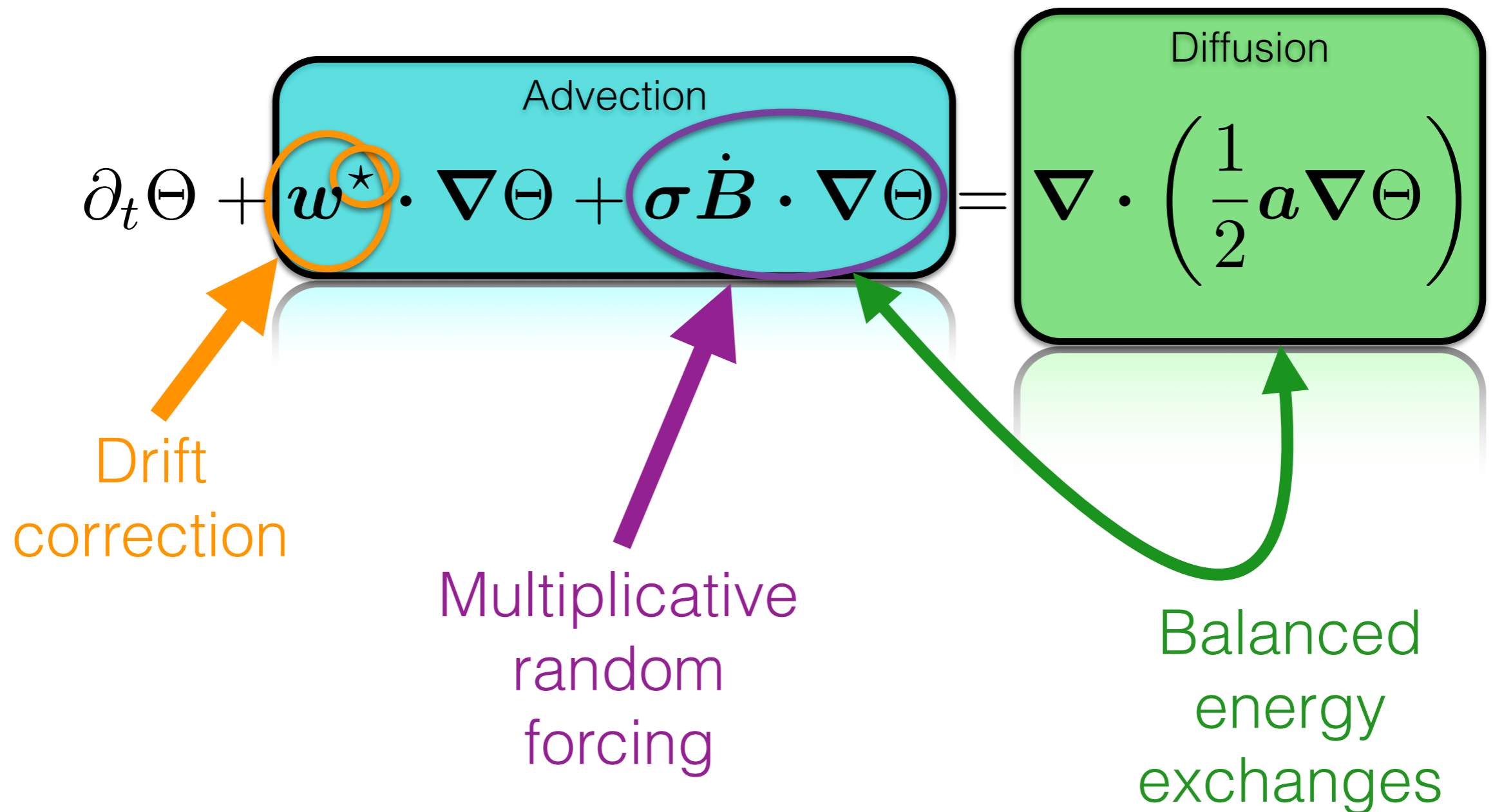
Advection of tracer Θ

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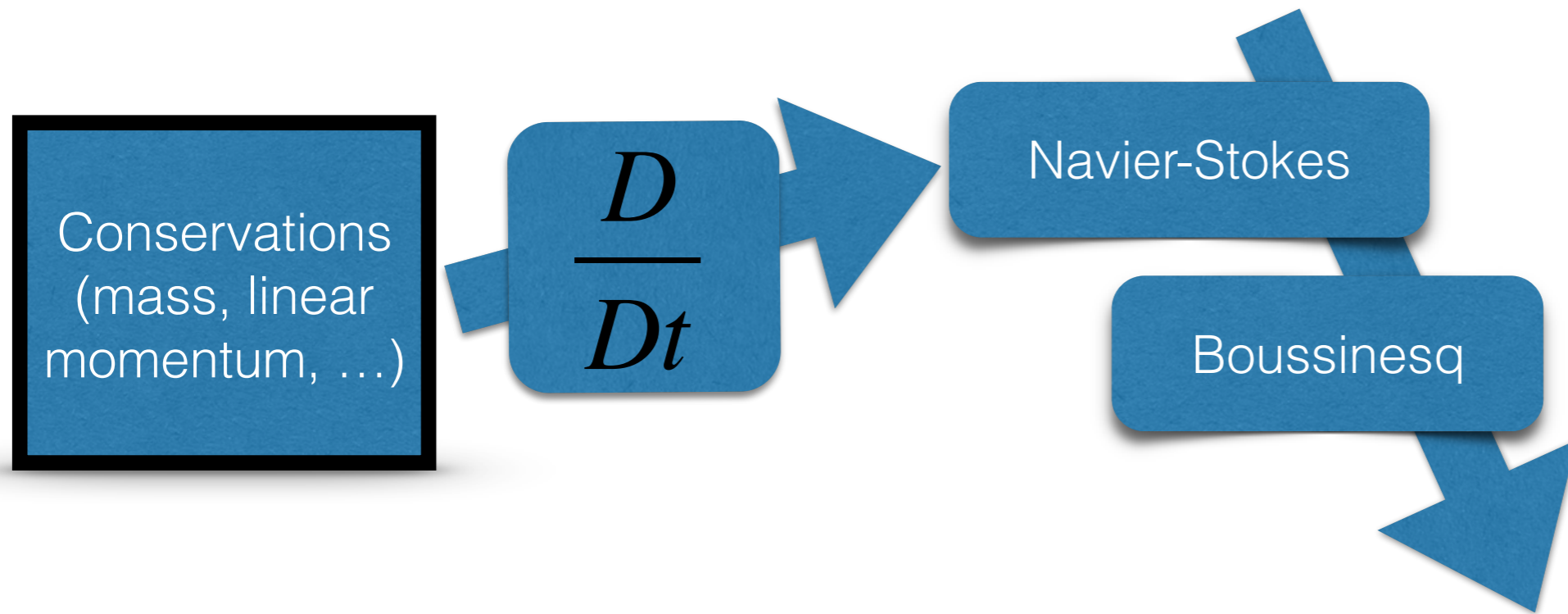
$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta}_{\text{Drift correction}} + \underbrace{\sigma \dot{B} \cdot \nabla \Theta}_{\text{Multiplicative random forcing}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

The left side of the equation is labeled "Advection" and is enclosed in a blue rounded rectangle. The right side is labeled "Diffusion" and is enclosed in a green rounded rectangle. An orange arrow points from the text "Drift correction" to the w^* term. A purple arrow points from the text "Multiplicative random forcing" to the $\sigma \dot{B}$ term. A green arrow points from the diffusion term back to the advection terms.

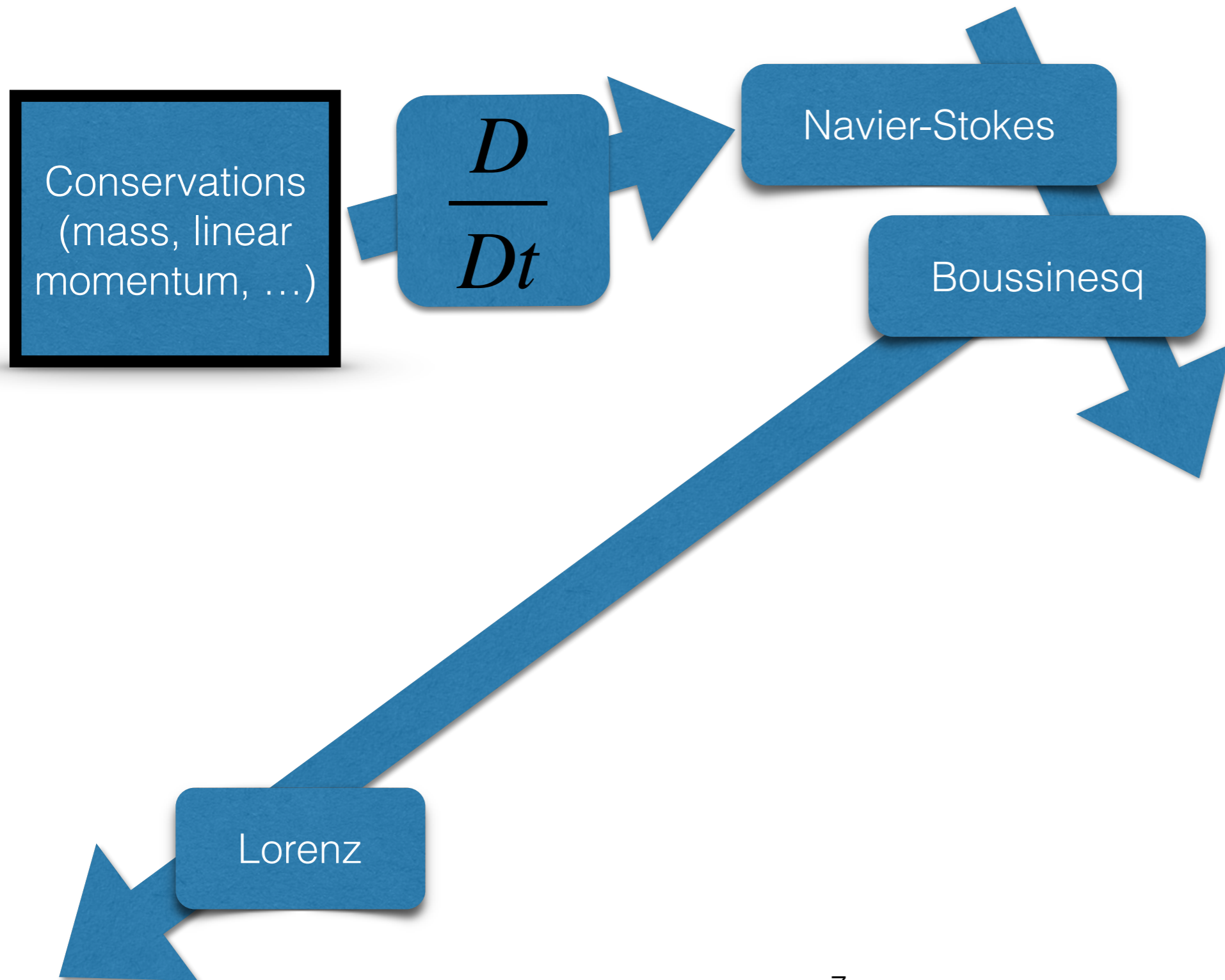
Advection of tracer Θ



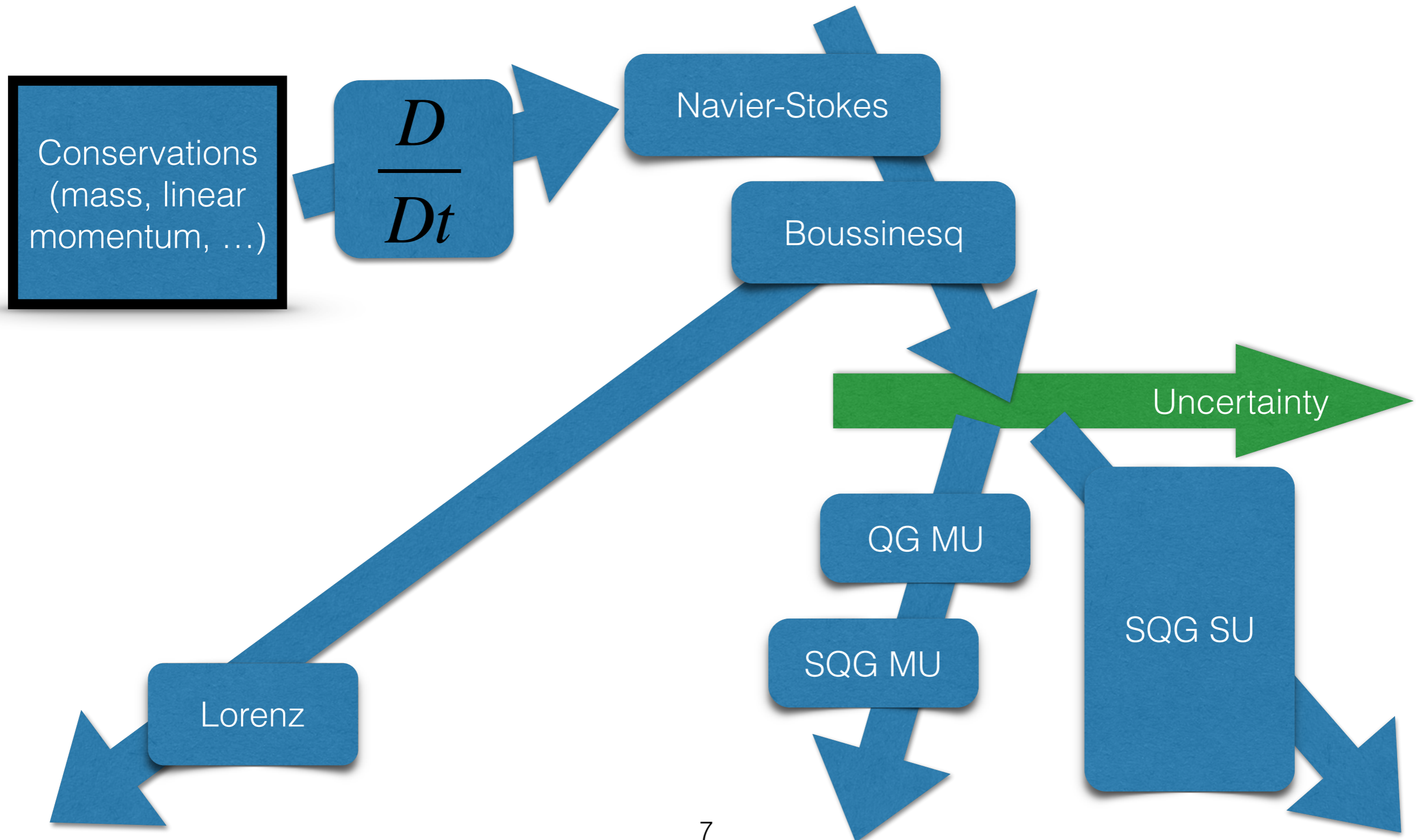
Derived random models



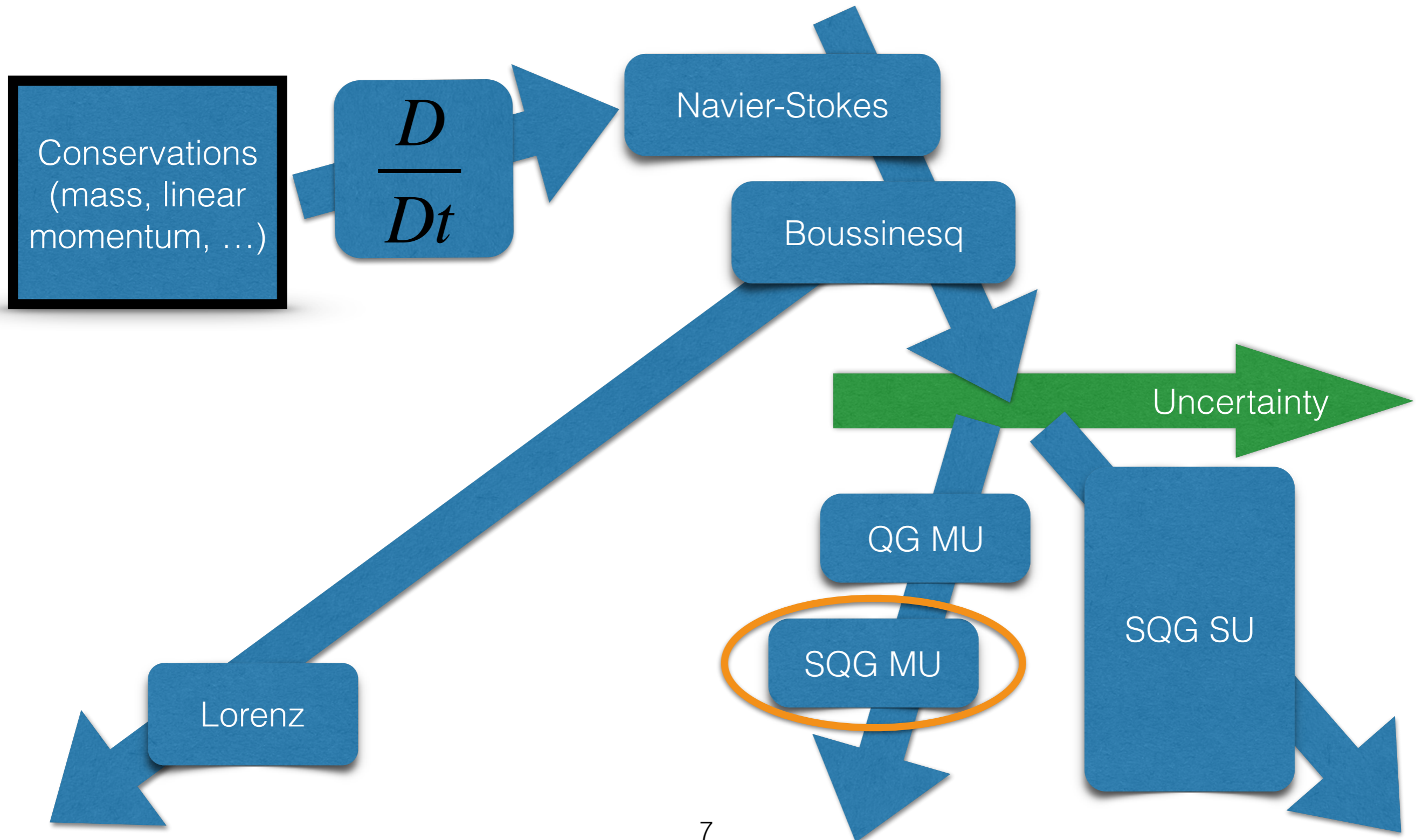
Derived random models



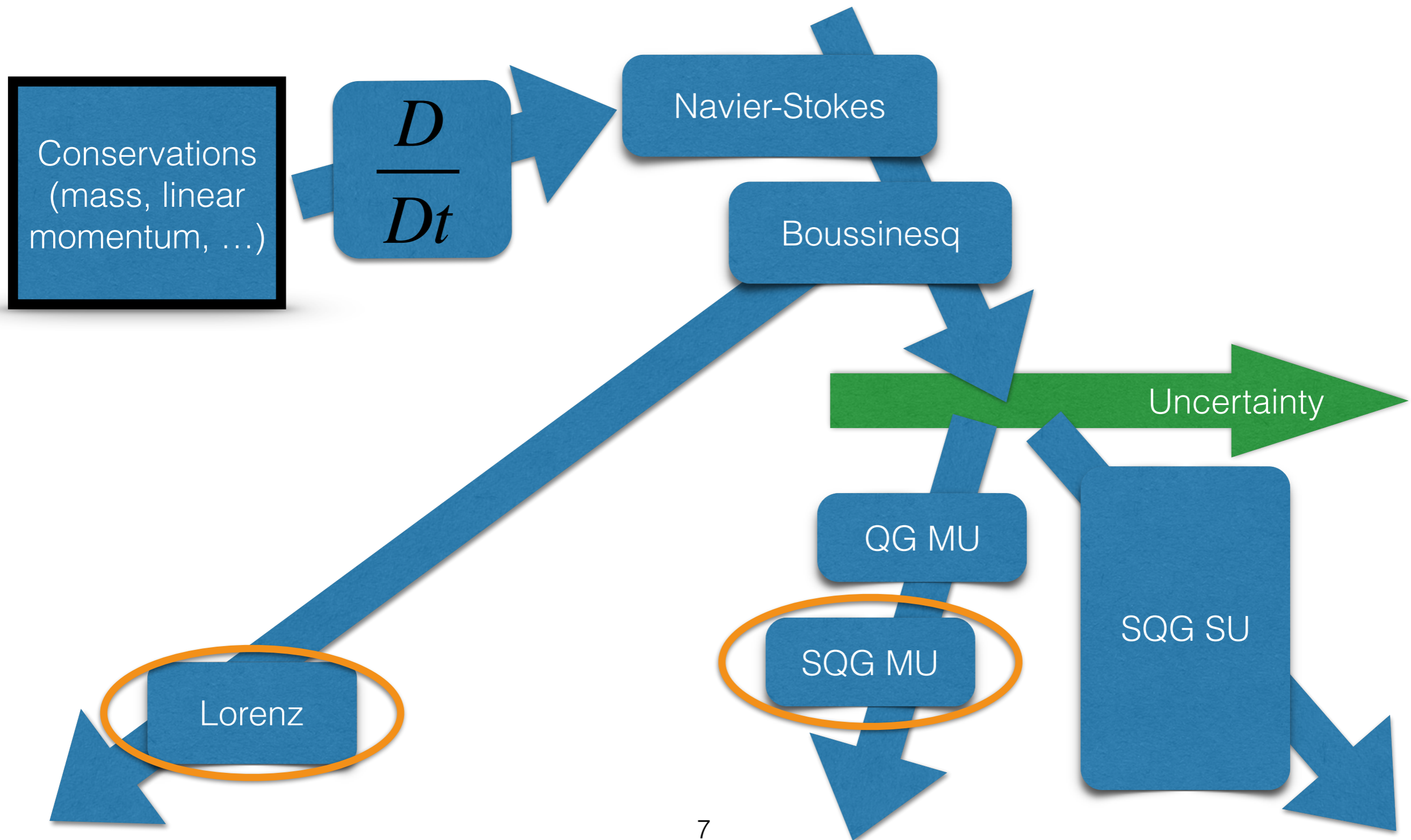
Derived random models



Derived random models



Derived random models



SQG under Moderate Uncertainty

SQG MU

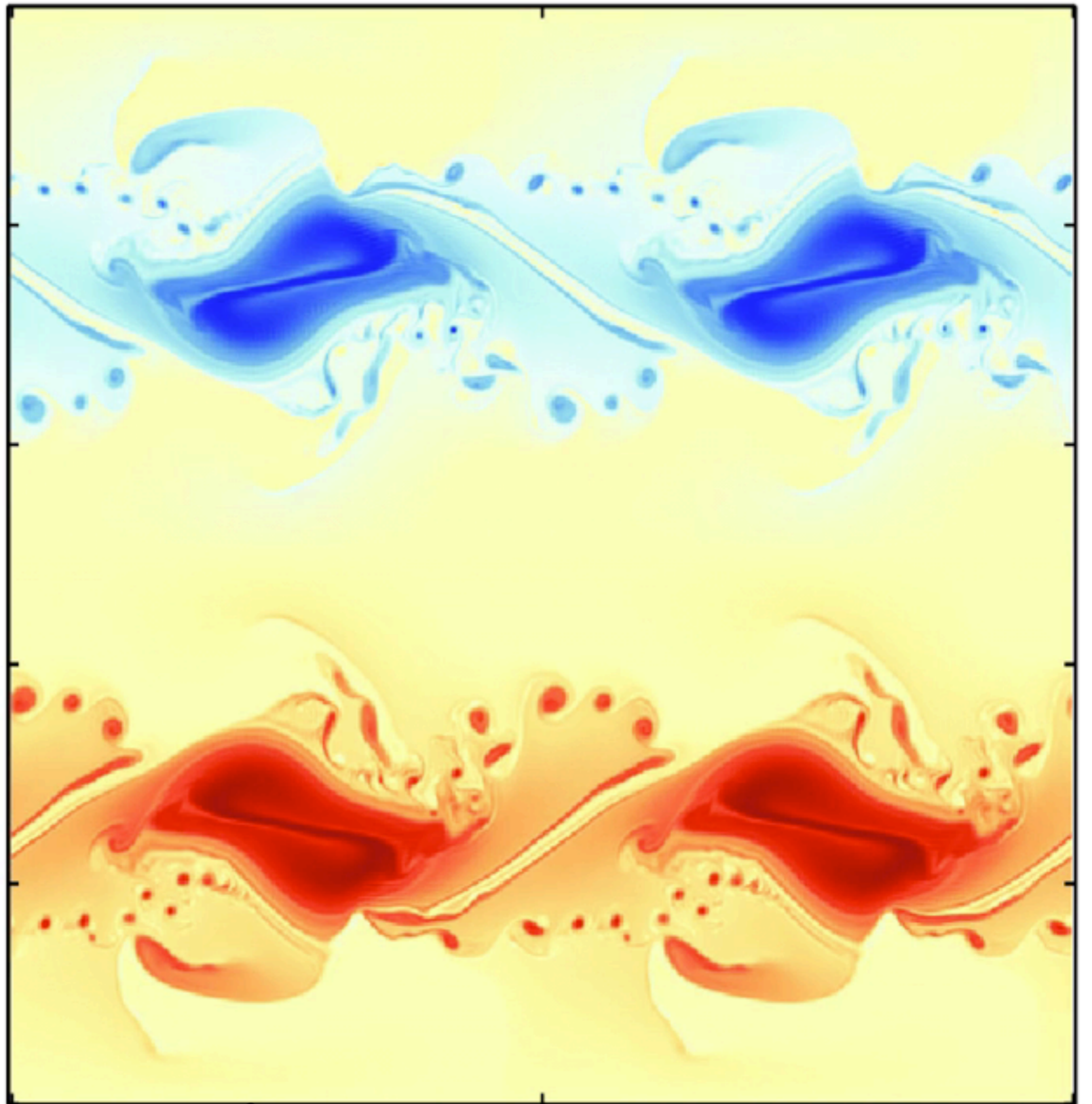
Code available online

$t = 17$ days

Reference flow:
deterministic

SQG

512 x 512

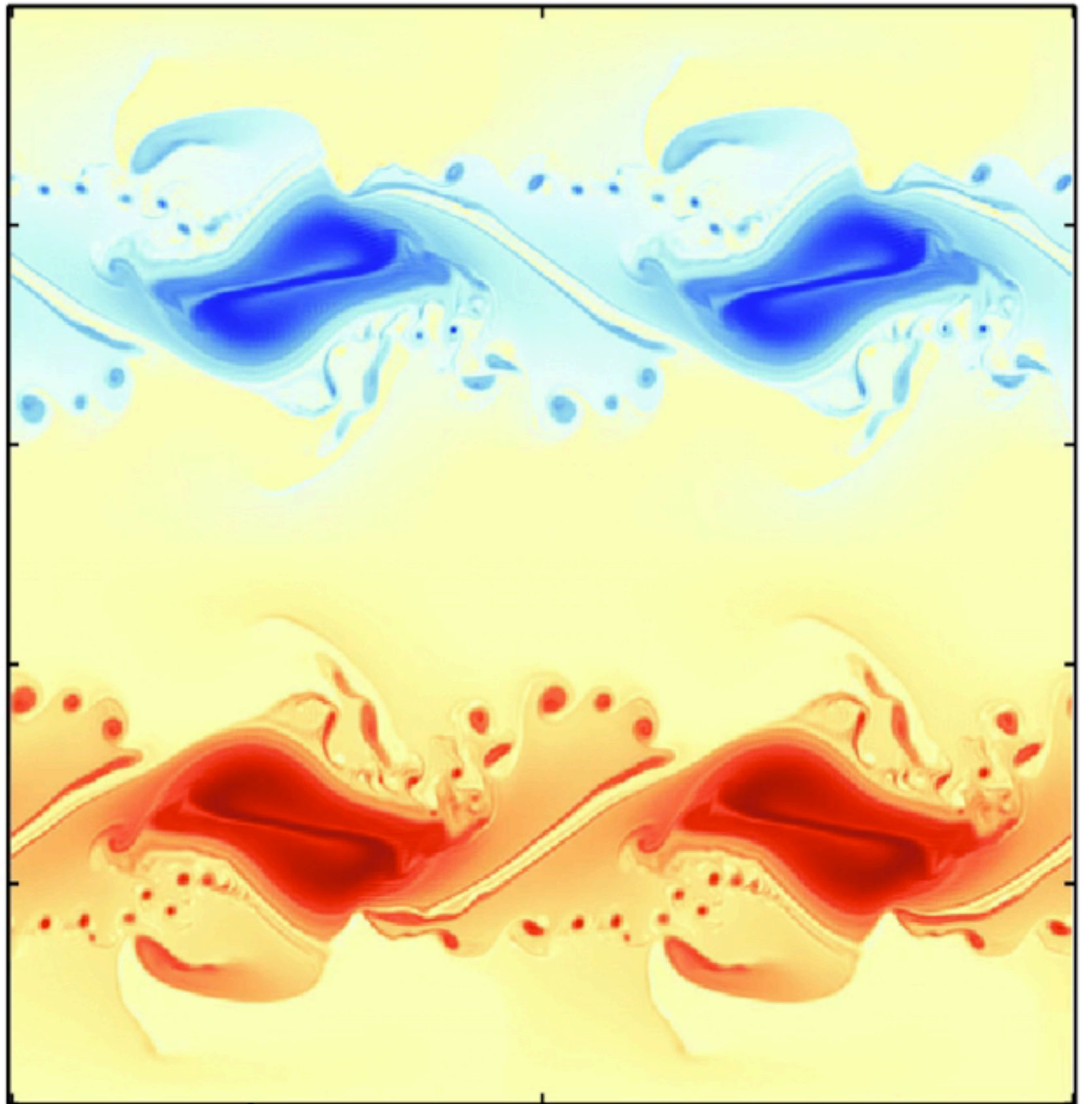


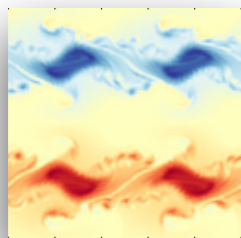
$t = 17$ days

Reference flow:
deterministic

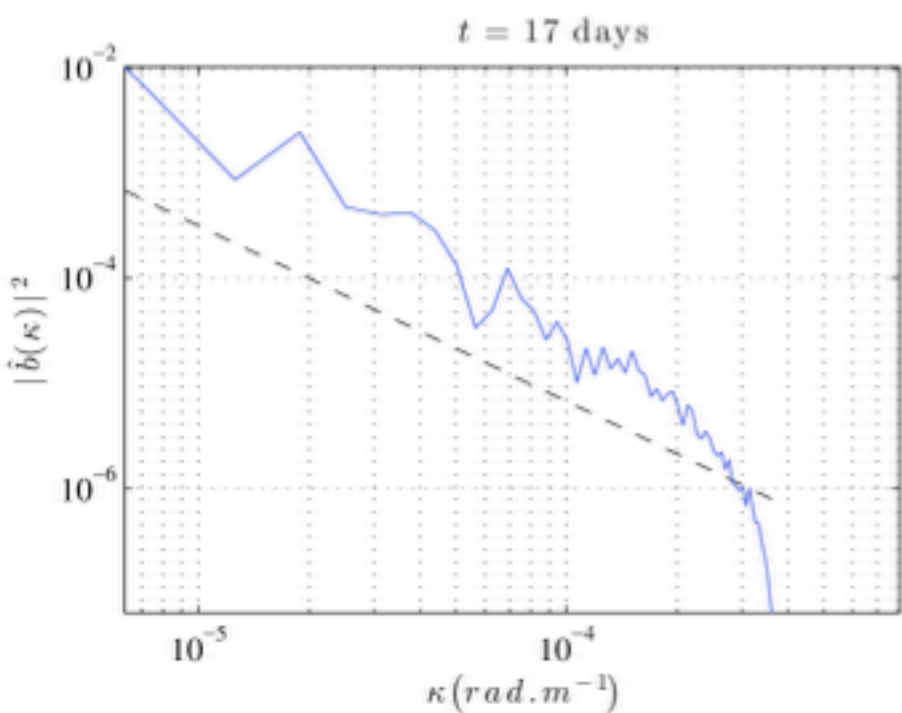
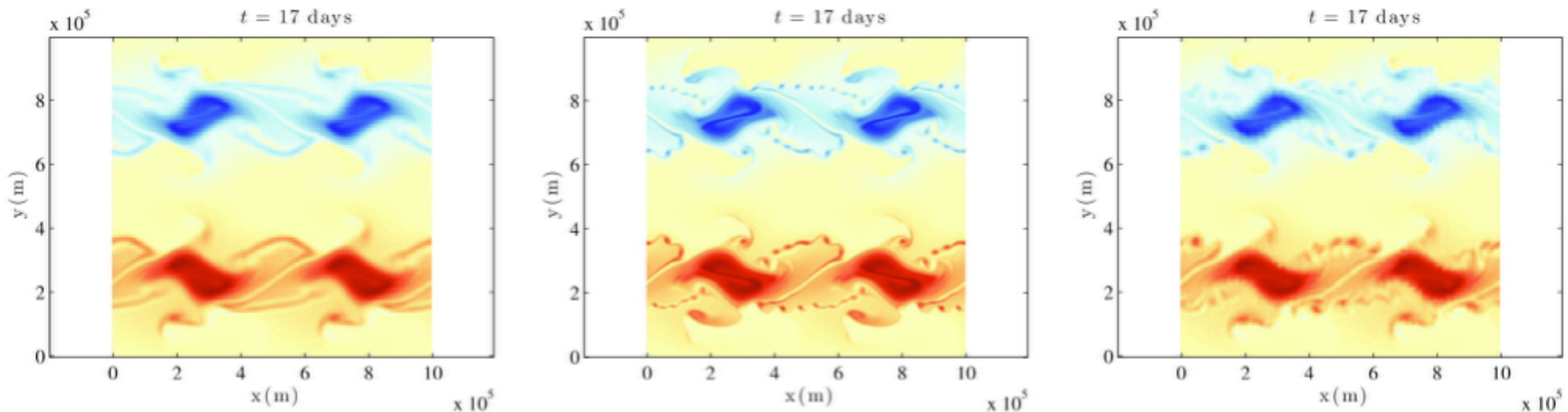
SQG

512 x 512

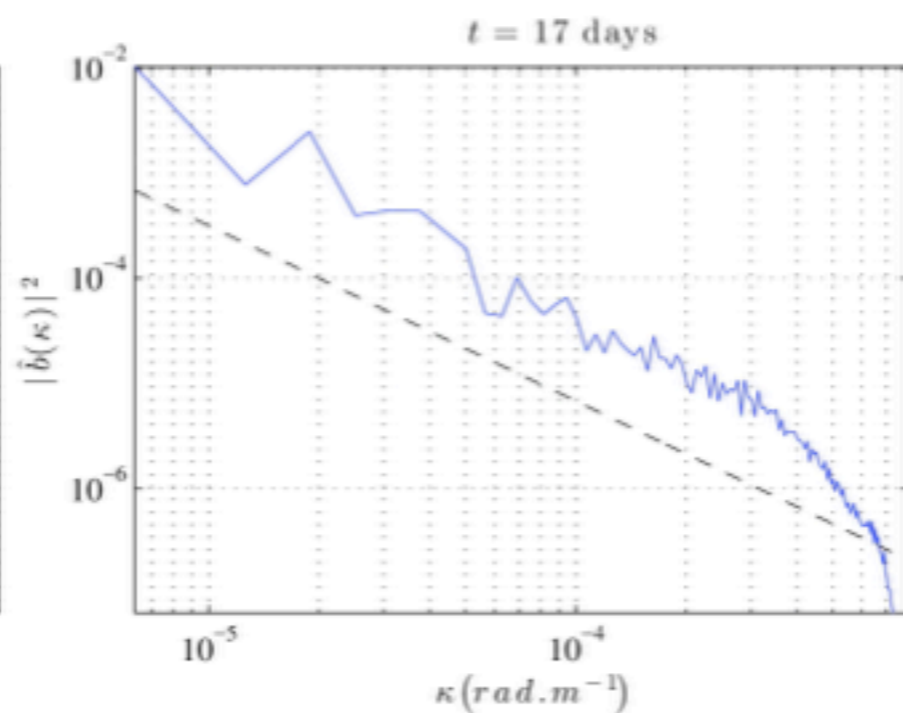




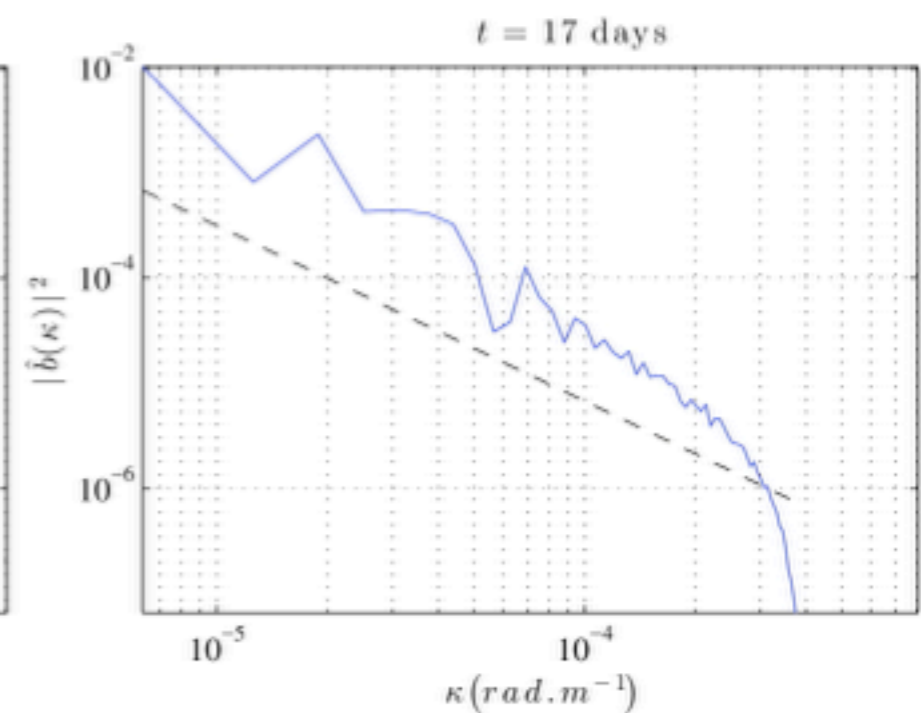
One realization



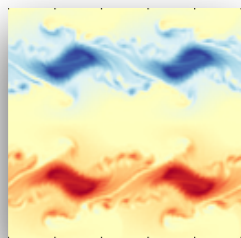
Deterministic 128x128



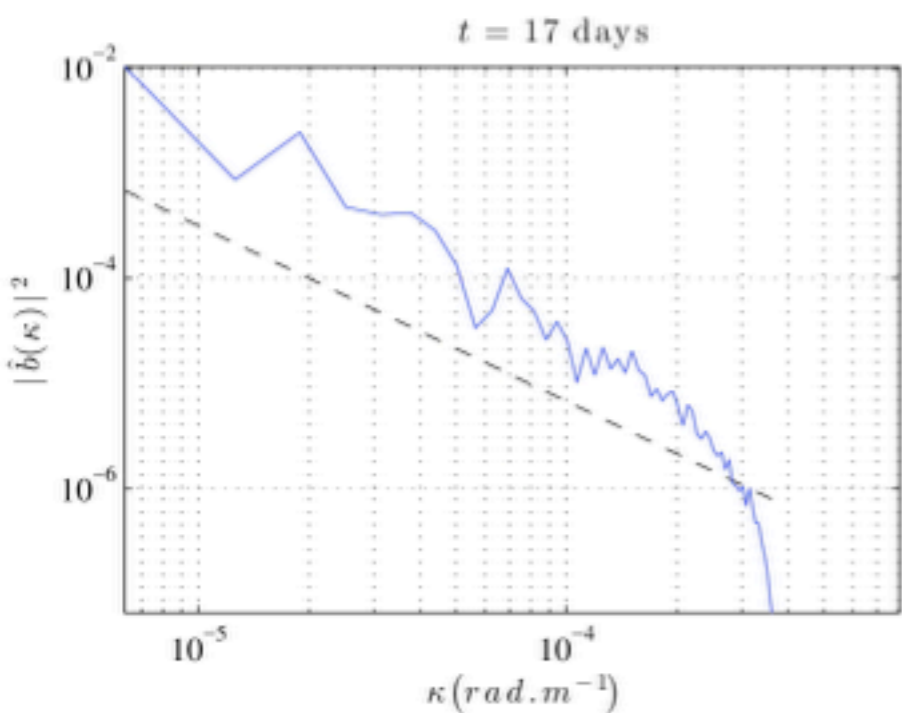
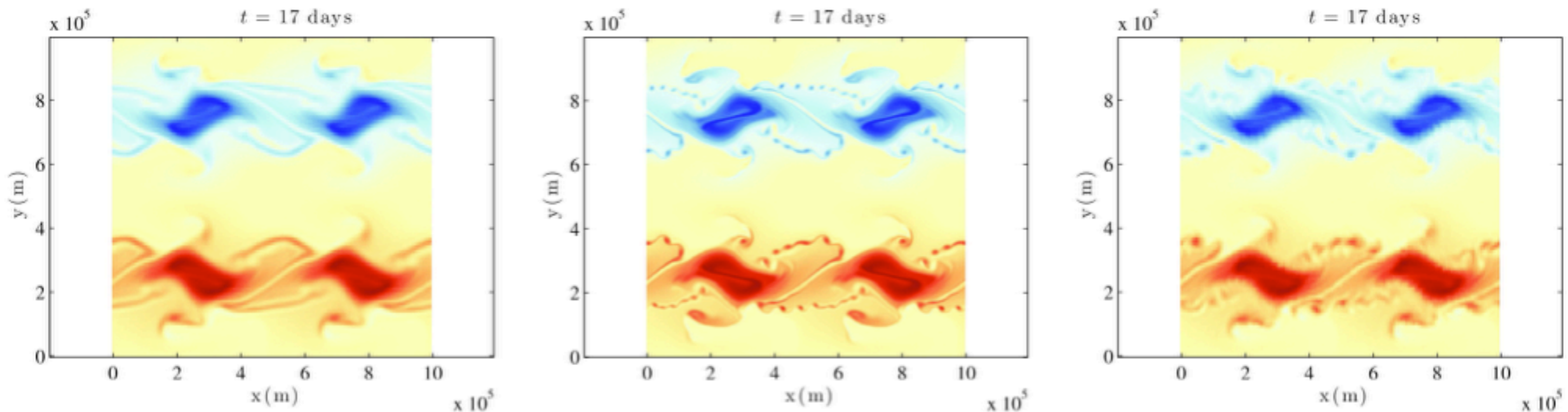
Deterministic 512x512



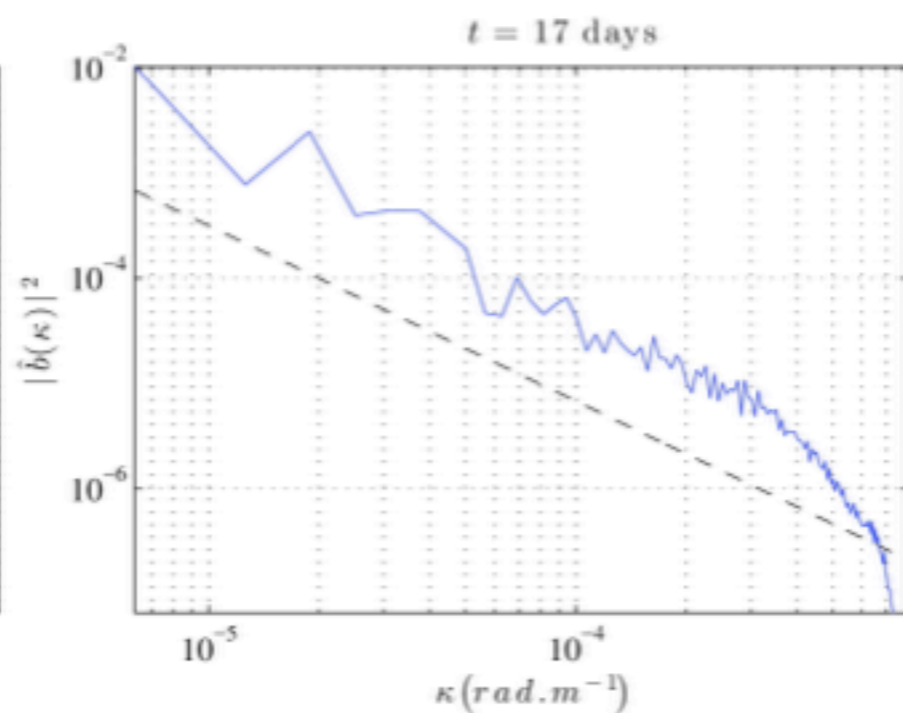
Stochastic 128x128



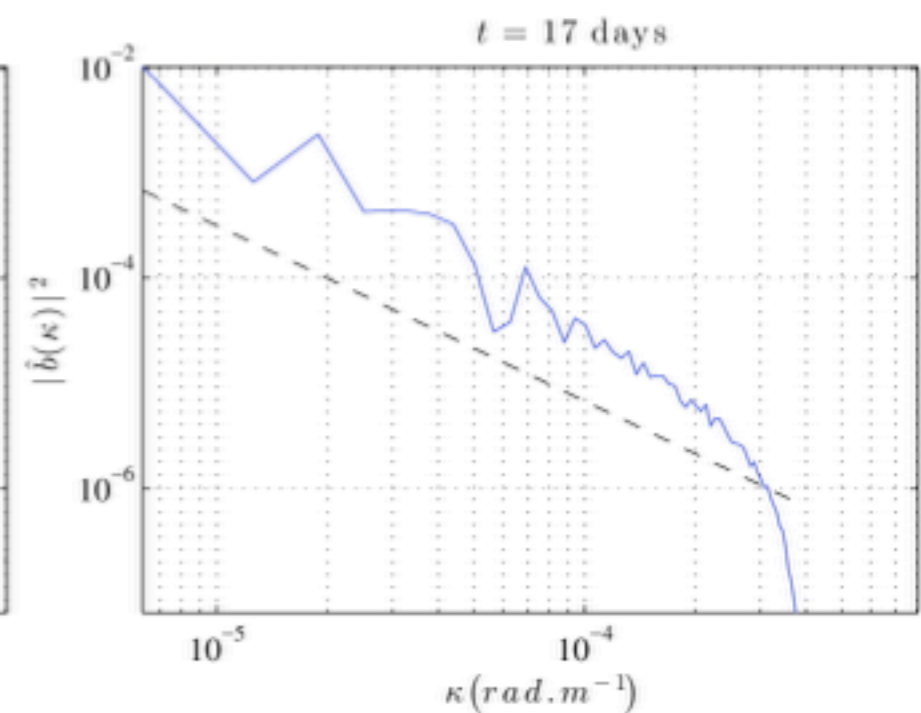
One realization



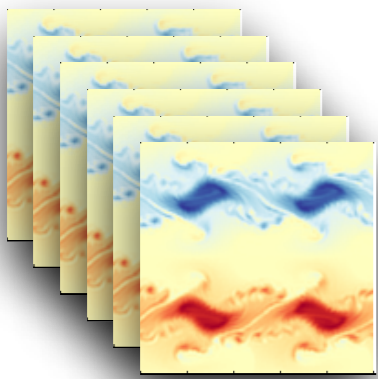
Deterministic 128x128



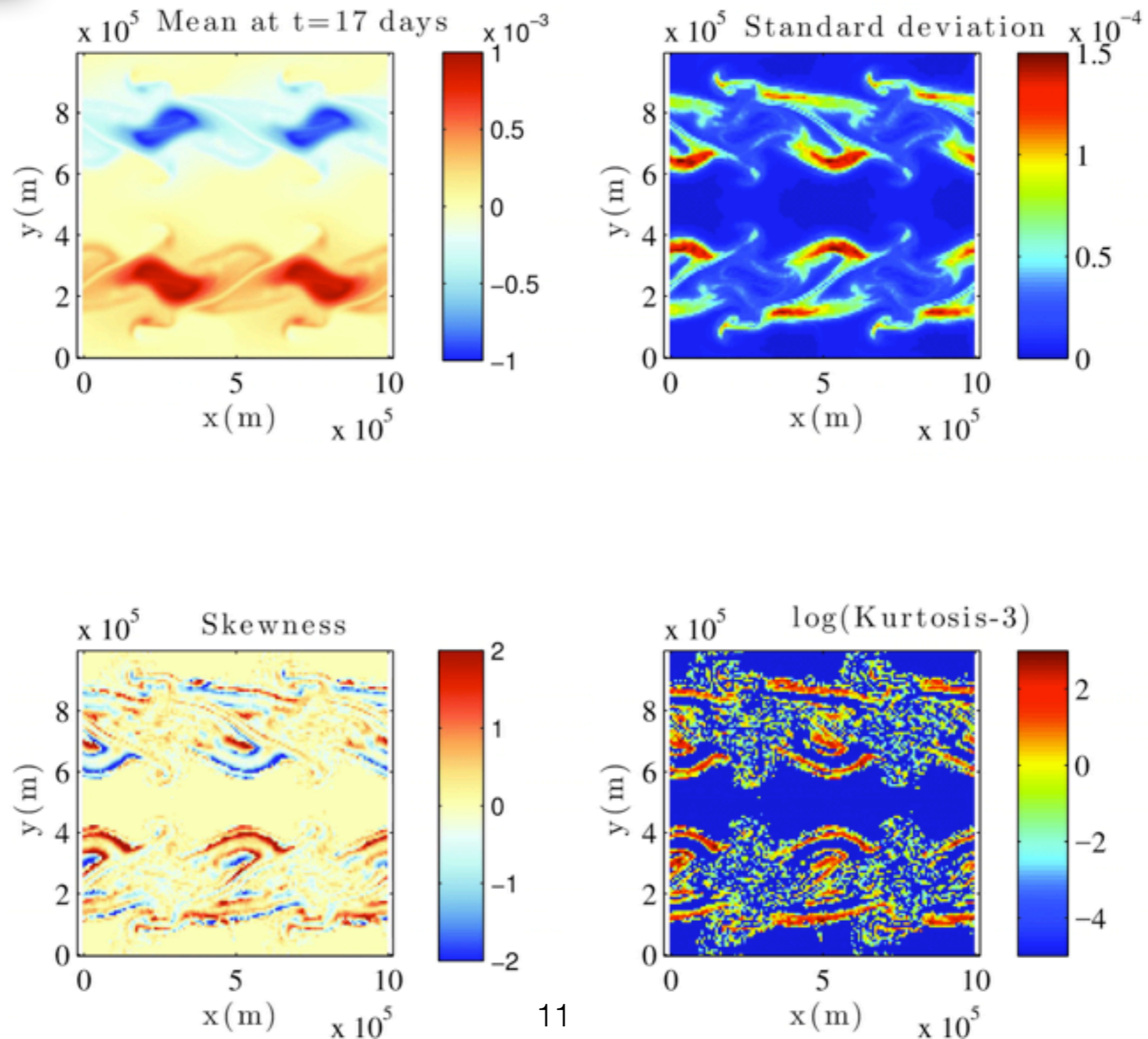
Deterministic 512x512

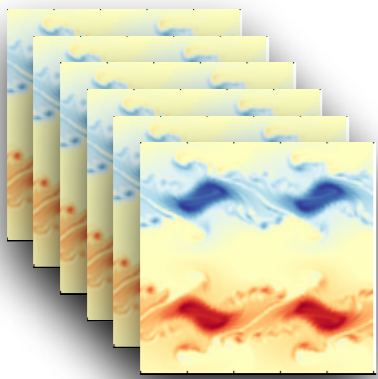


Stochastic 128x128

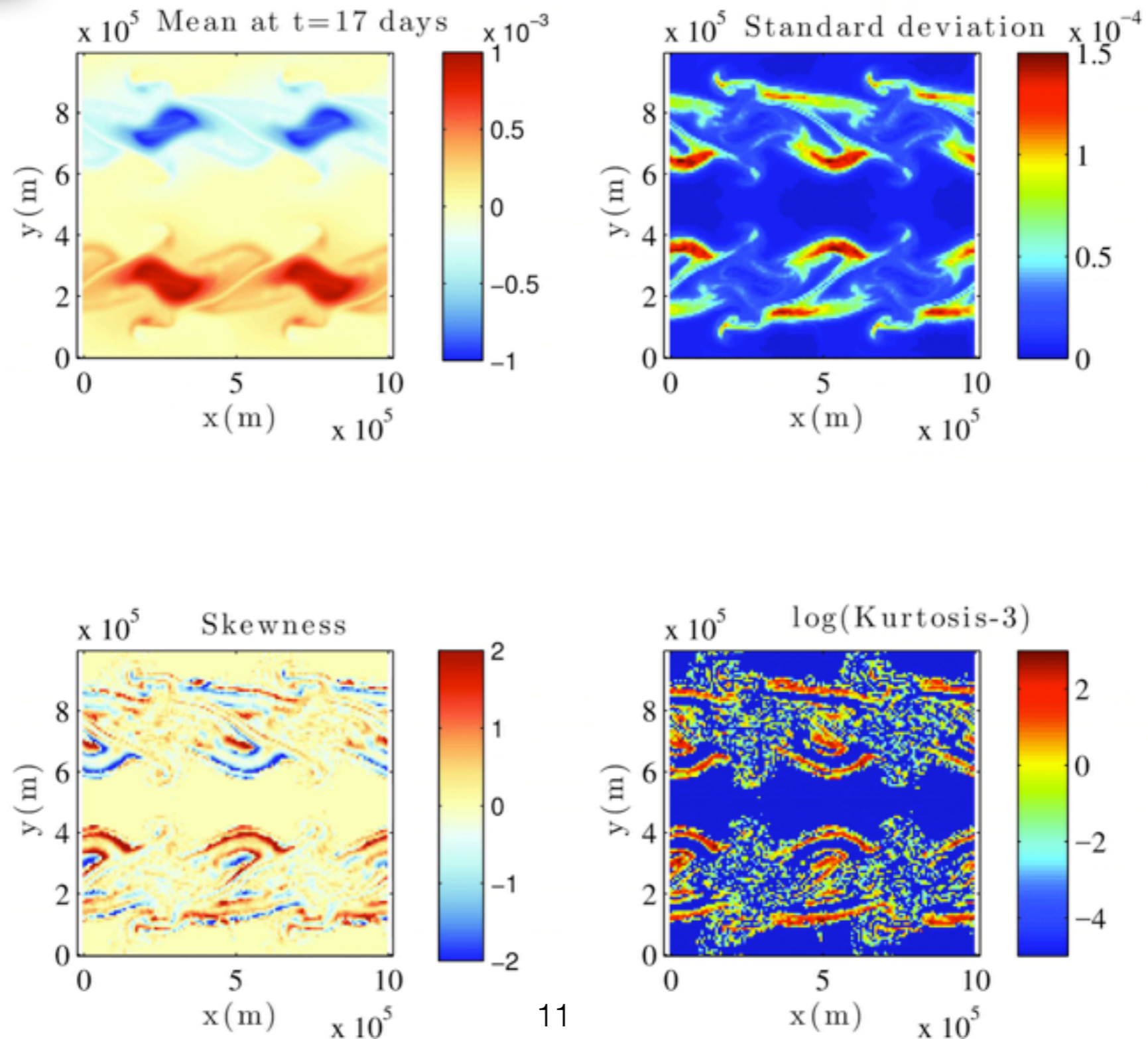


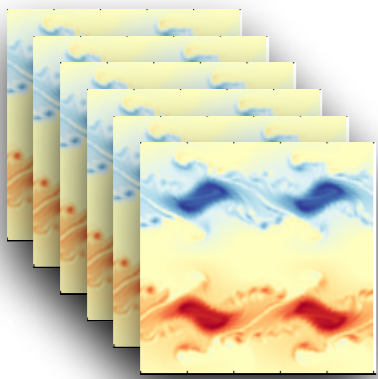
Ensemble



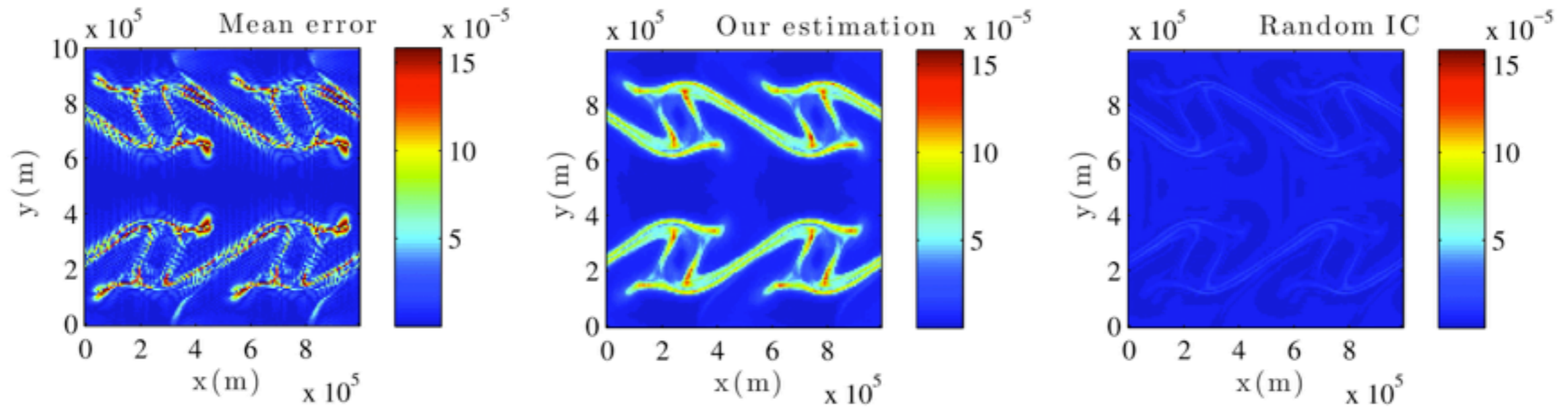


Ensemble

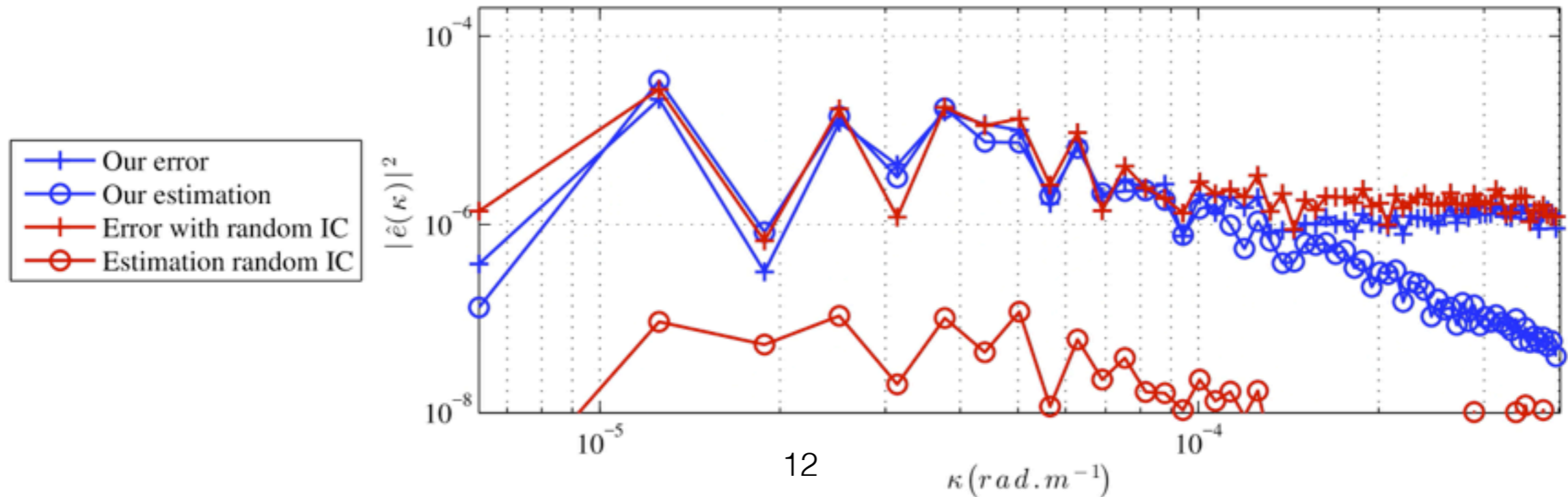


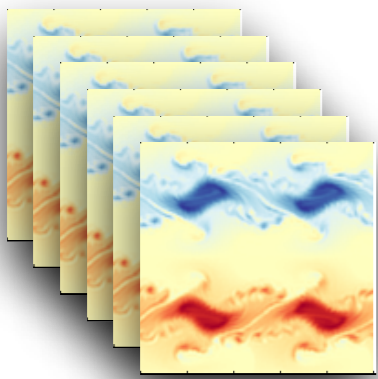


Ensemble

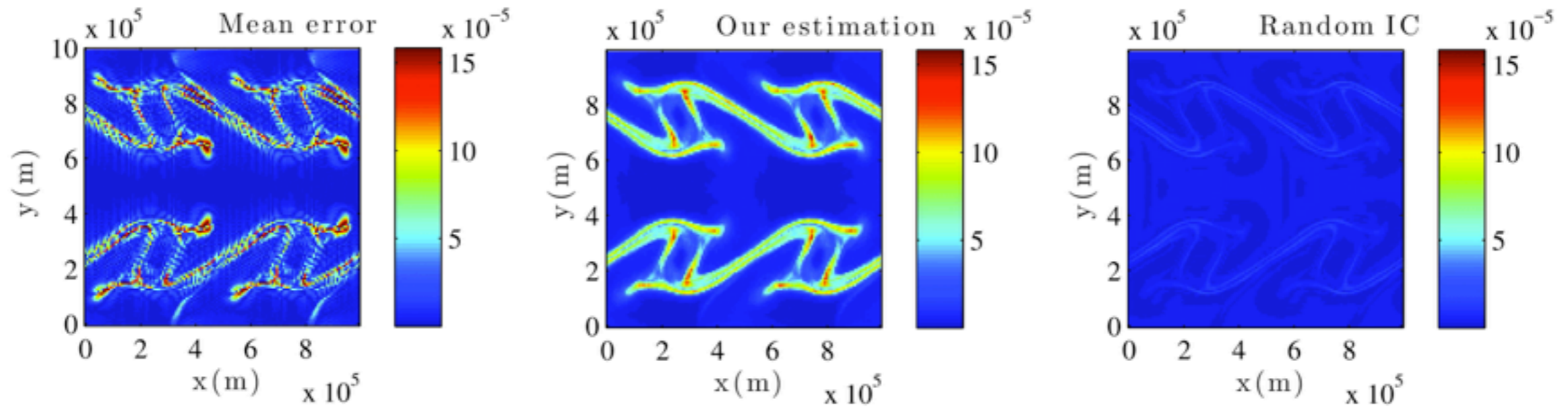


Spectrum of the errors and its estimation at $t=12$ days

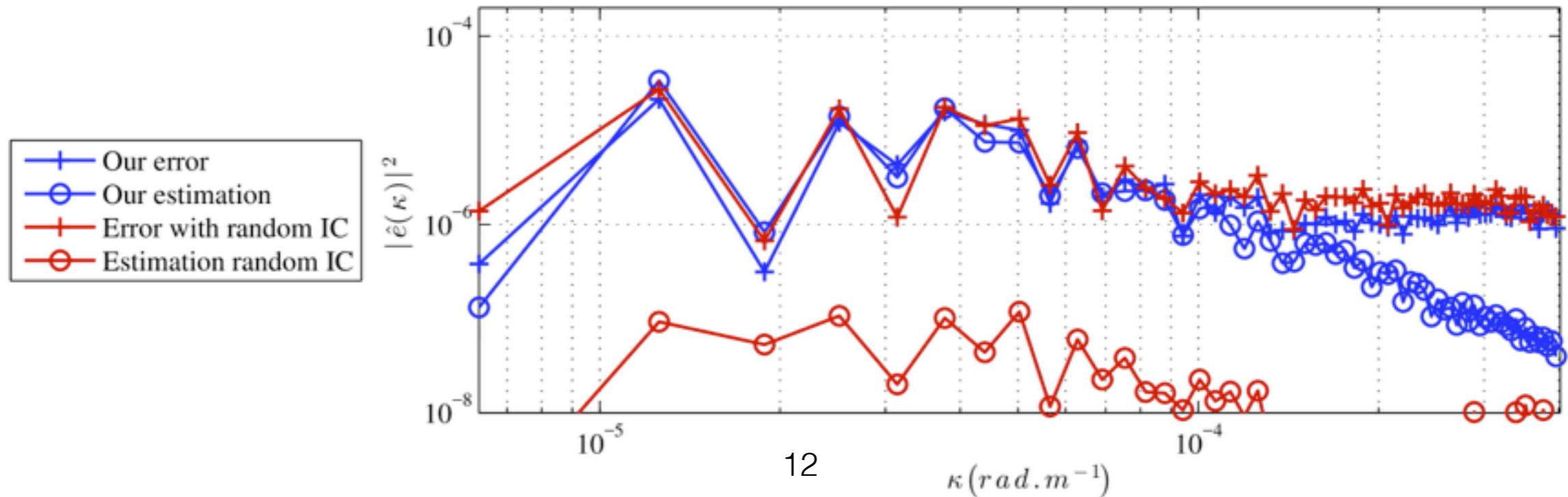




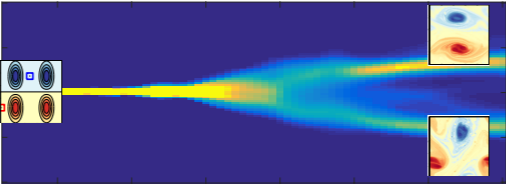
Ensemble



Spectrum of the errors and its estimation at $t=12$ days



Summary of QG models

- Better small scales
- Estimate position and amplitude of errors
- Extreme events
- Bifurcations 
- under Strong Uncertainty:
Simple 2D description of frontolysis/frontogenesis

Code SQG MU:
link from Fluminance website - V. Resseguier

Lorenz model under location uncertainty

Do large-scale (diffusive) models lead to
over-representing "stable"-states
in ensemble simulations?

Lorenz model(s)

$$\frac{dX}{dt} = \text{Pr} (Y - X) - \frac{4}{2\Upsilon} X$$

$$dY = \left[X(\rho - Z) - Y - \frac{4}{2\Upsilon} Y \right] dt + \frac{\rho - Z}{\Upsilon^{1/2}} dB_t$$

$$dZ = \left[XY - bZ - \frac{8}{2\Upsilon} bZ \right] dt + \frac{Y}{\Upsilon^{1/2}} dB_t$$

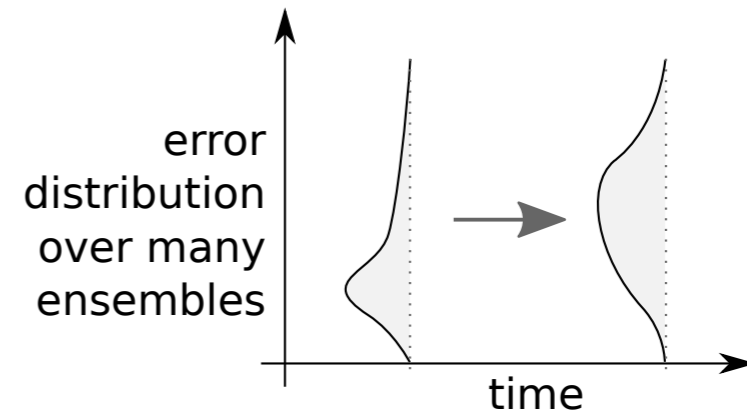
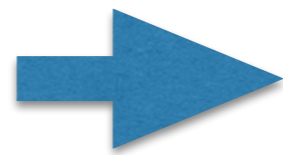
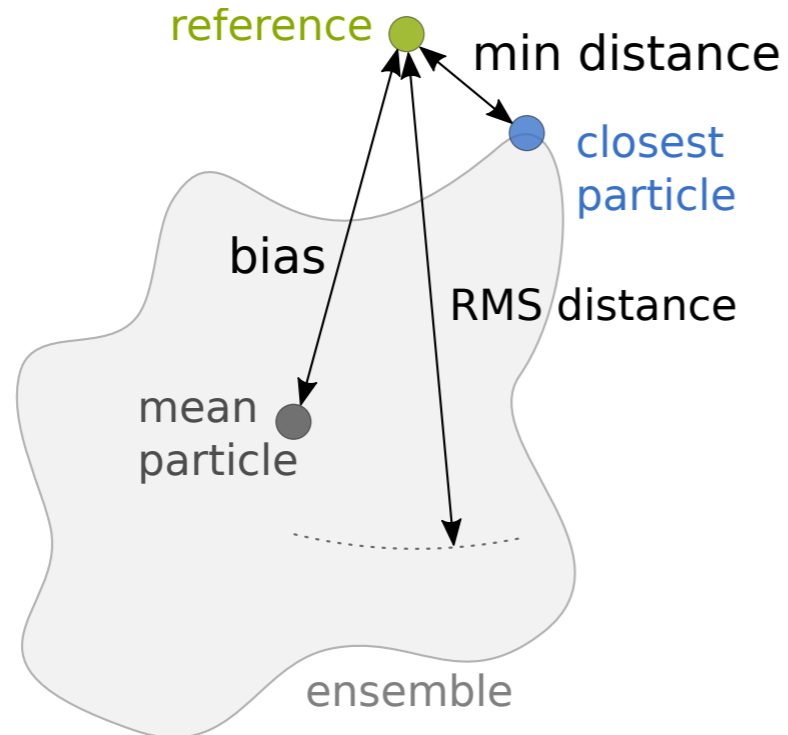
- (usual) **deterministic** model ~ DNS, accurate but impossible to compute
- (deterministic) **diffusive** model ~ LES
- **stochastic** model under location uncertainty

➔ behaviors of ensembles?



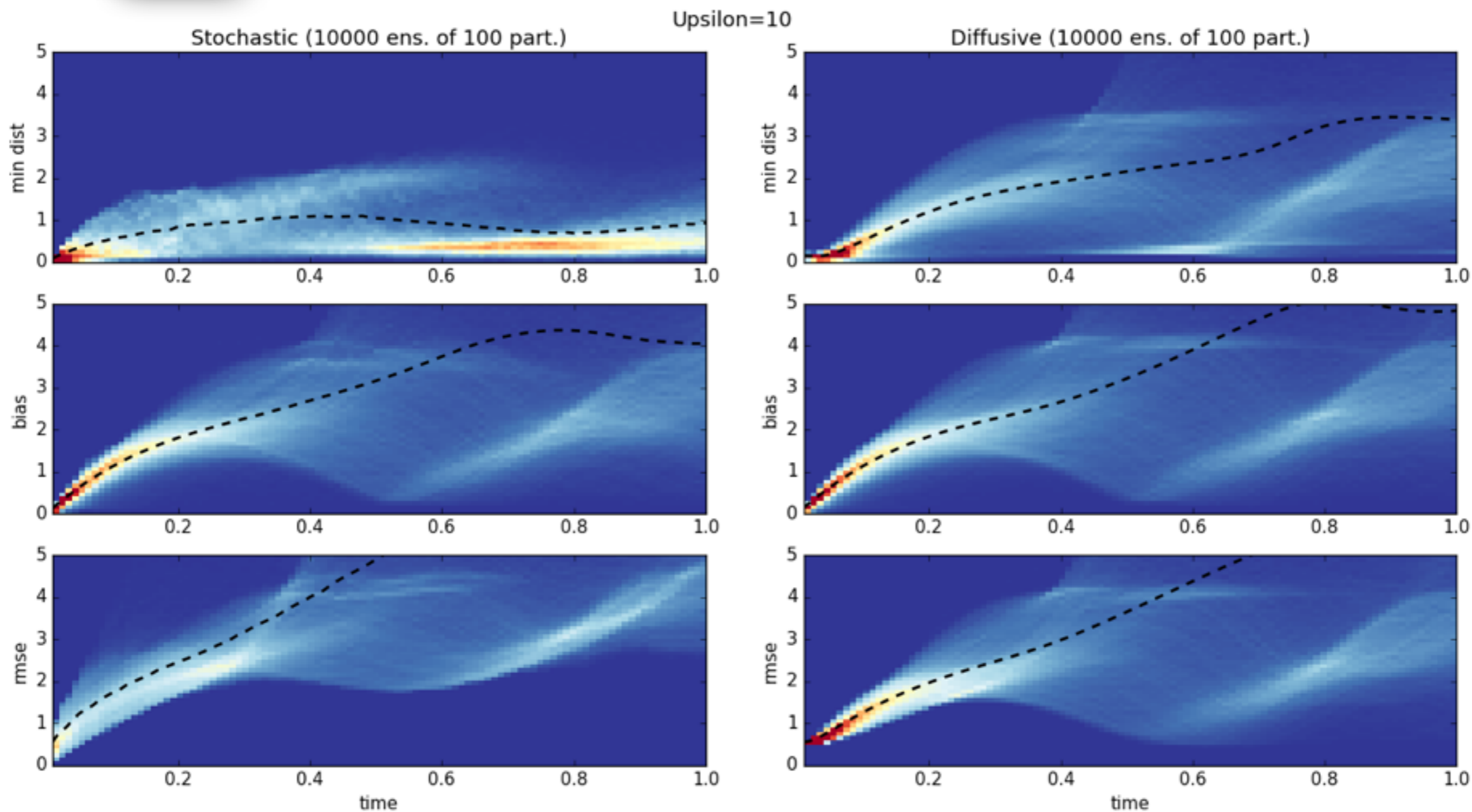
Short time behavior

Comparison ensemble \leftrightarrow reference
3 metrics: minimum distance, bias, RMS distance





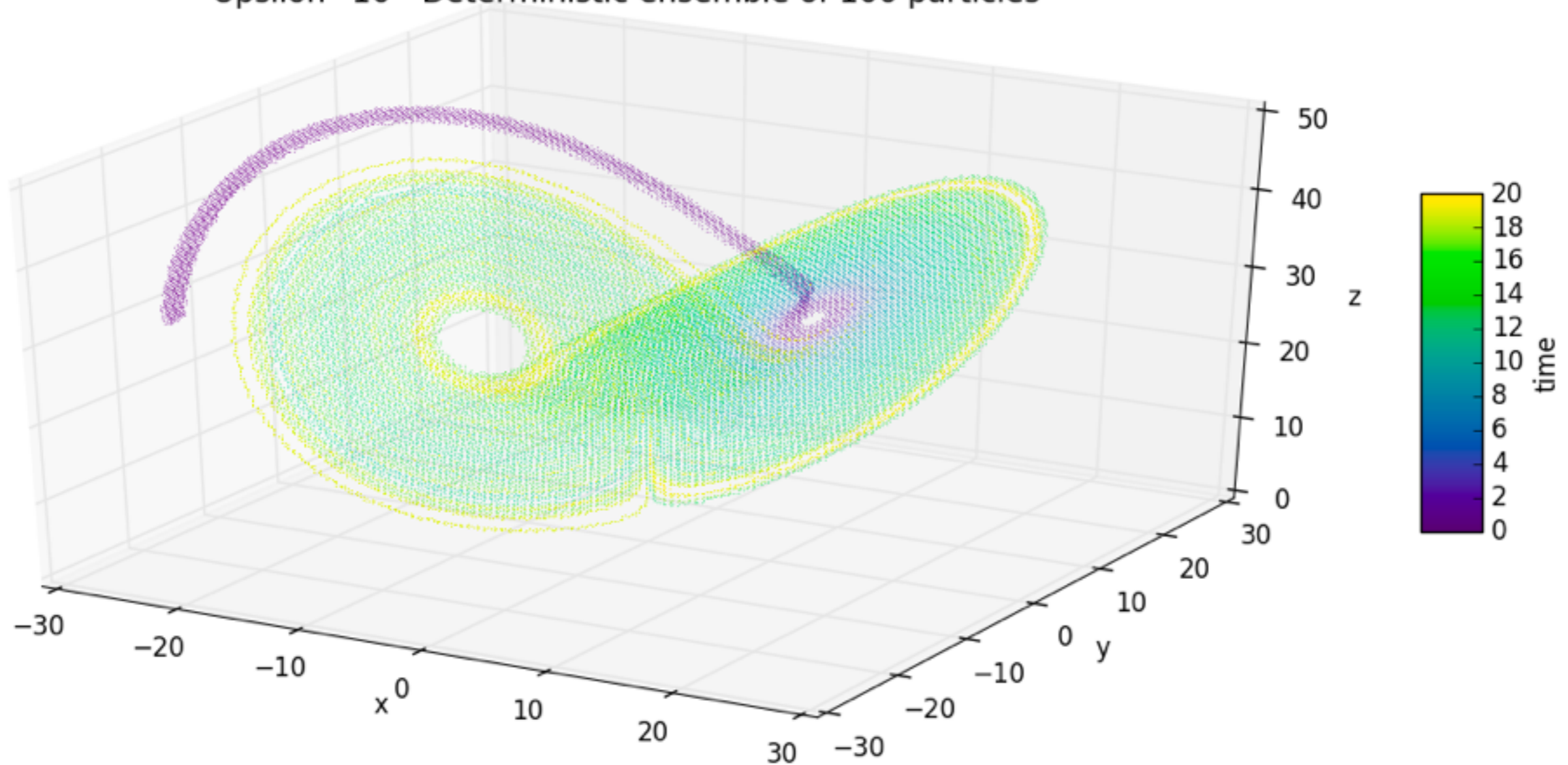
Short time behavior





Long time

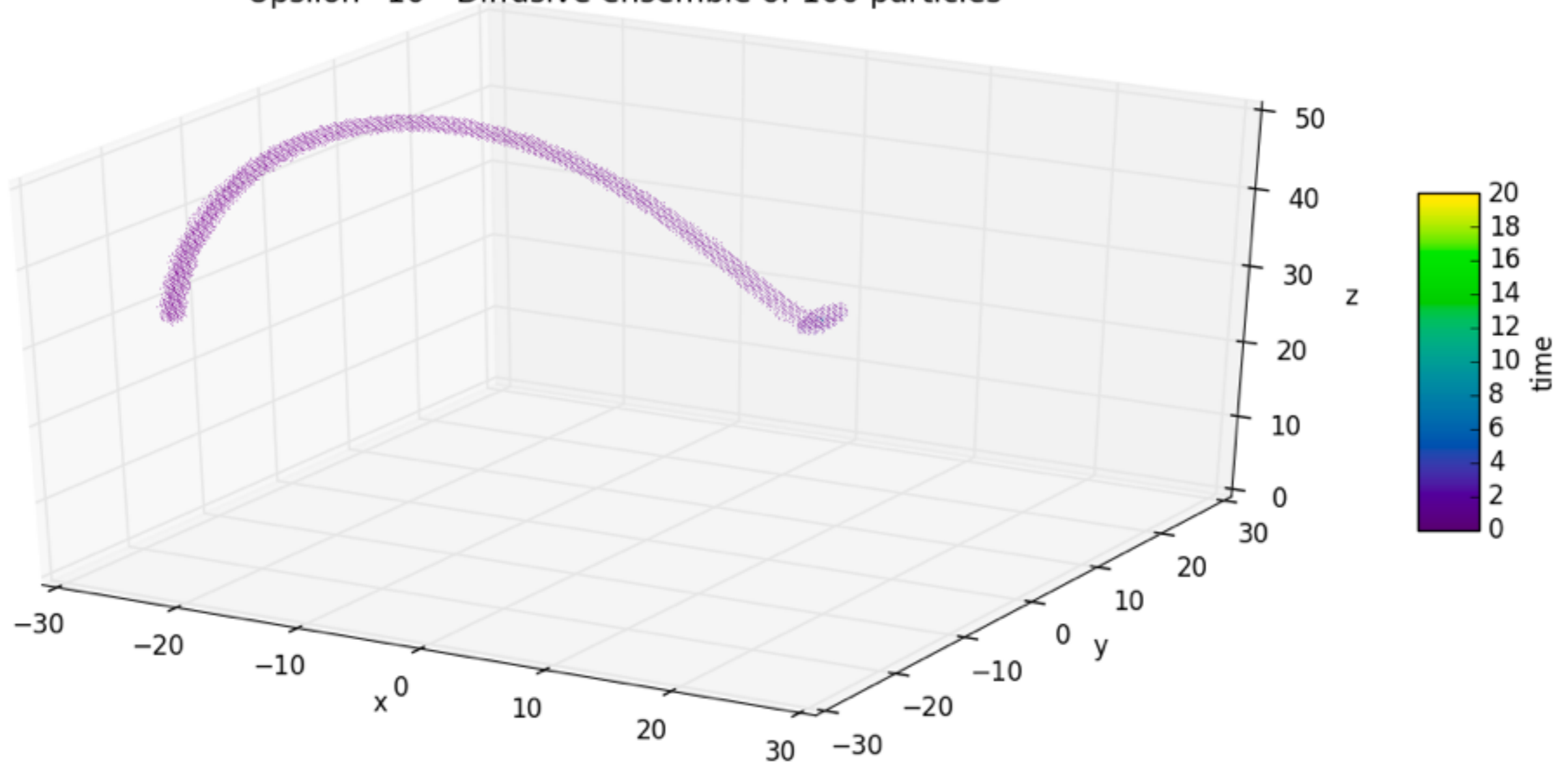
Upsilon=10 - Deterministic ensemble of 100 particles





Long time

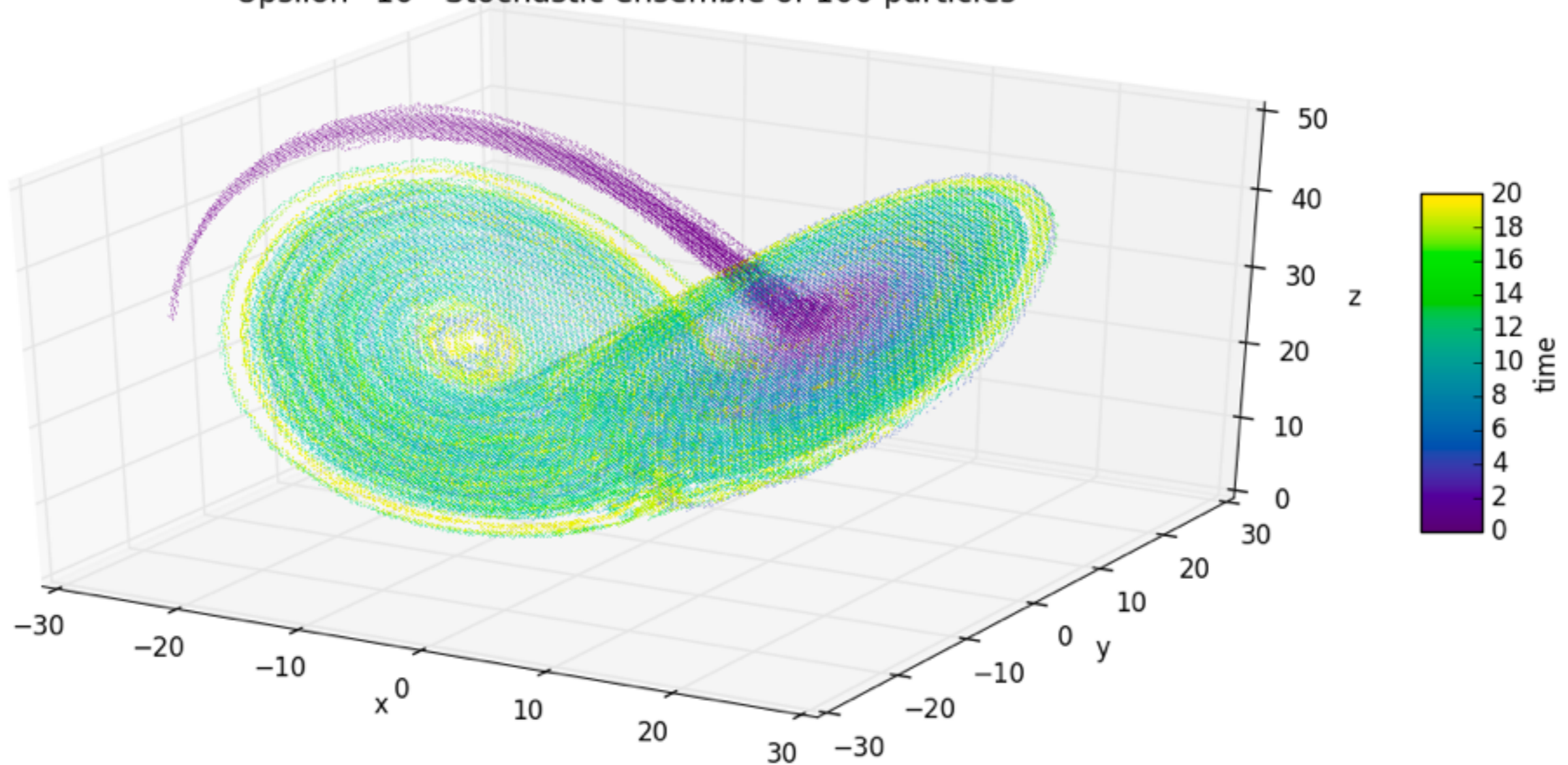
Upsilon=10 - Diffusive ensemble of 100 particles





Long time

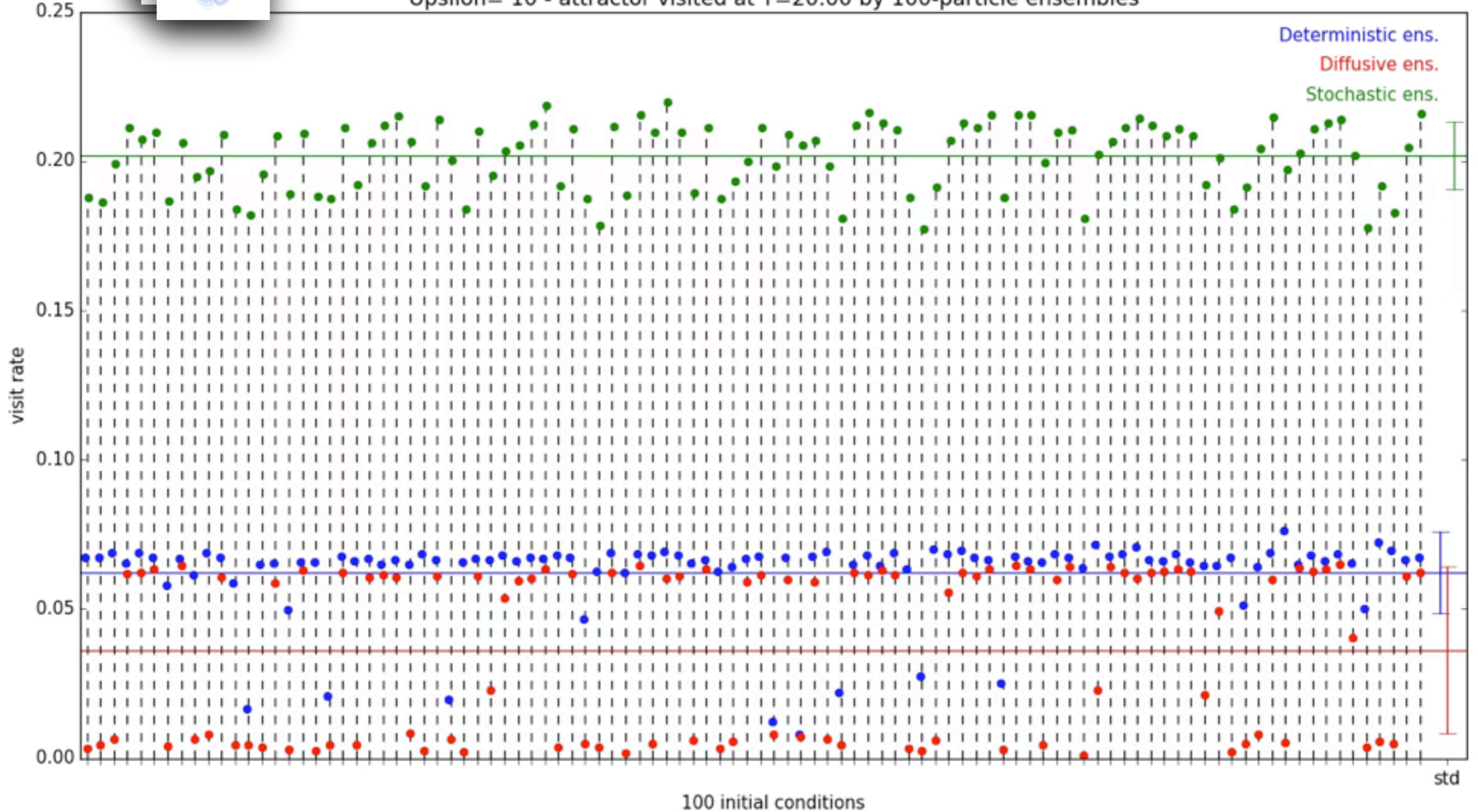
Upsilon=10 - Stochastic ensemble of 100 particles



Attractor visit rates



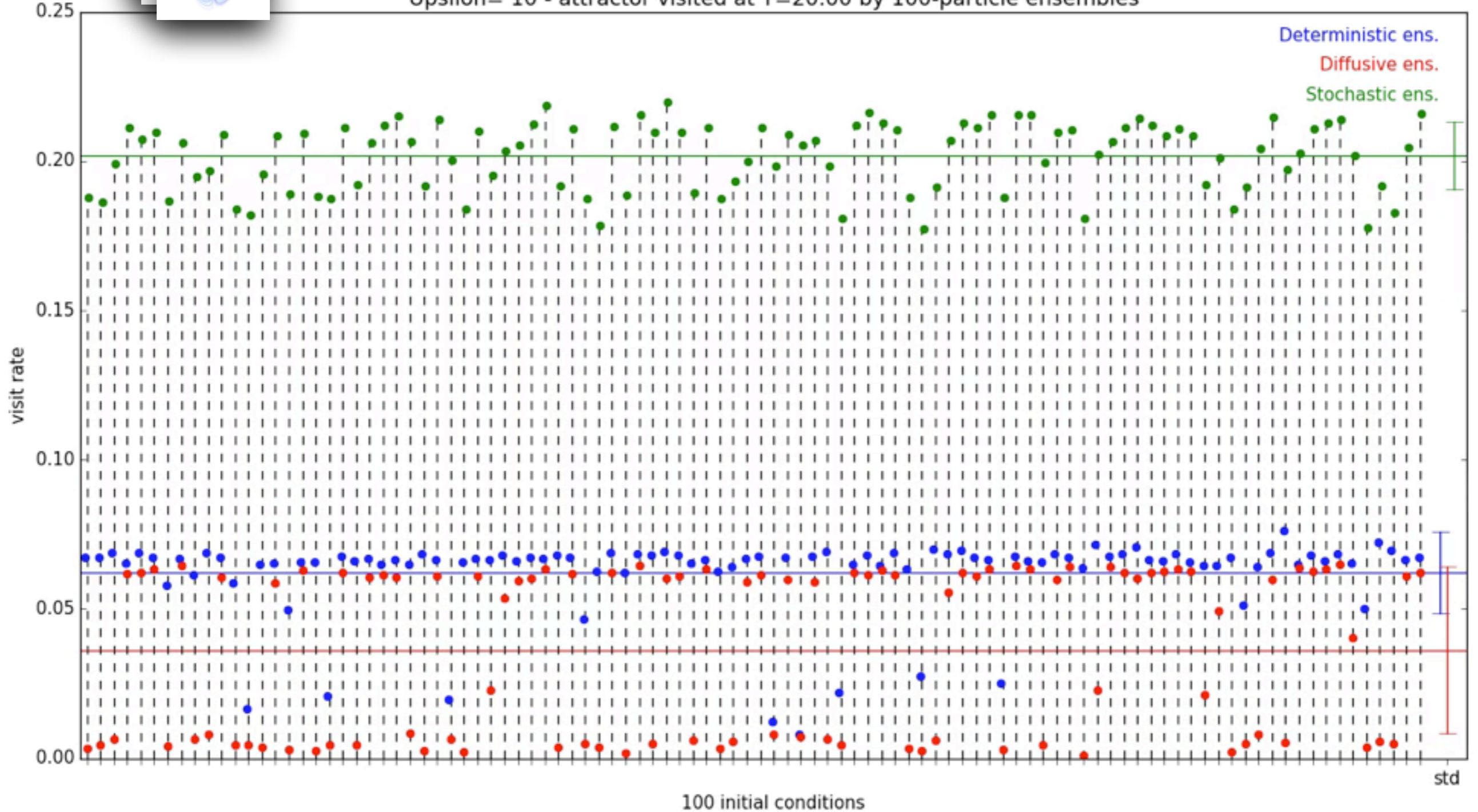
Upsilon= 10 - attractor visited at T=20.00 by 100-particle ensembles



Attractor visit rates



Upsilon= 10 - attractor visited at T=20.00 by 100-particle ensembles



Conclusion

Conclusion

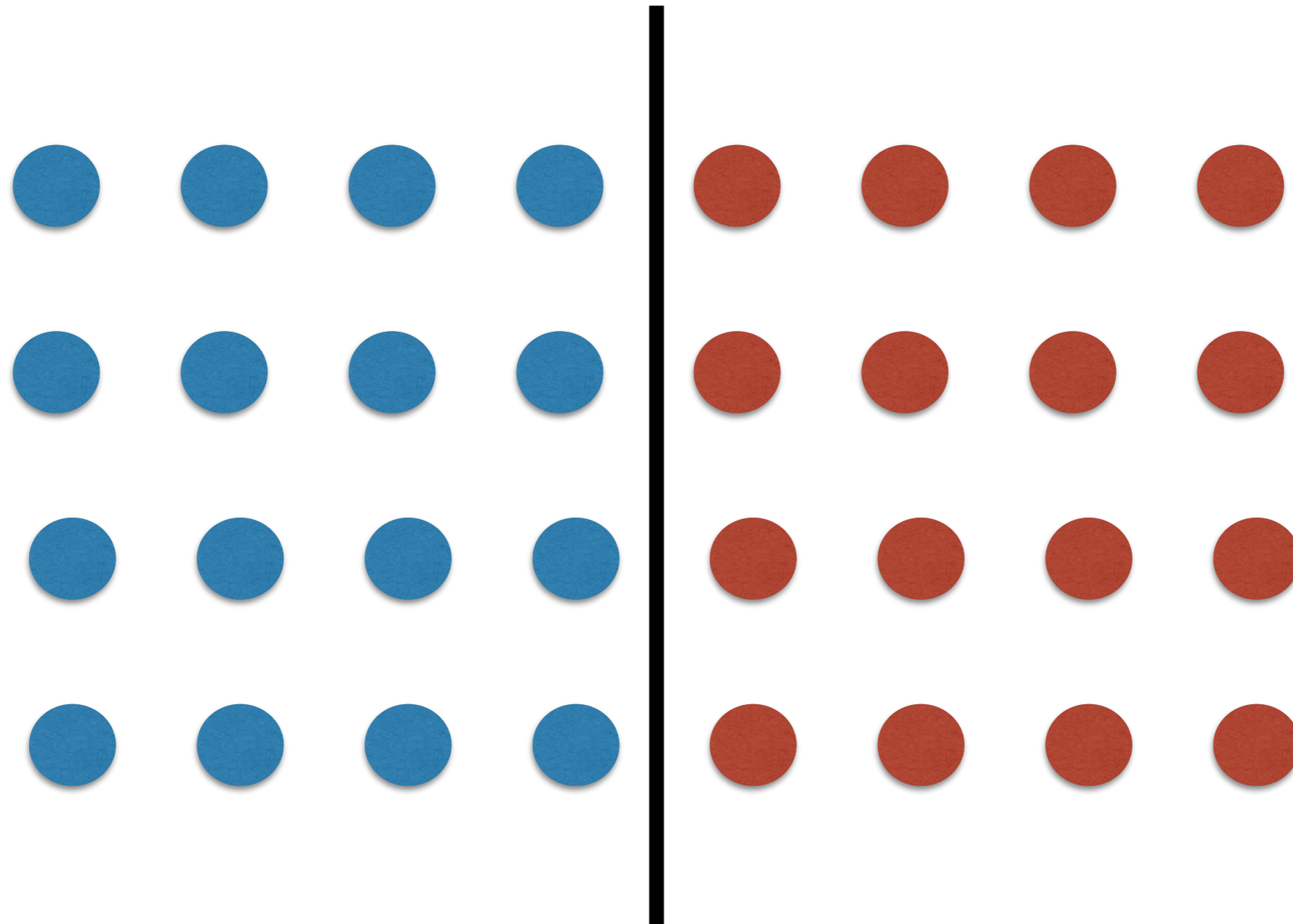
- Random transport applicable to any dynamics
- Better small scales
- Efficient spreading of the ensemble
- Likely scenarios
- Exploration of the attractor

Thank you for your attention

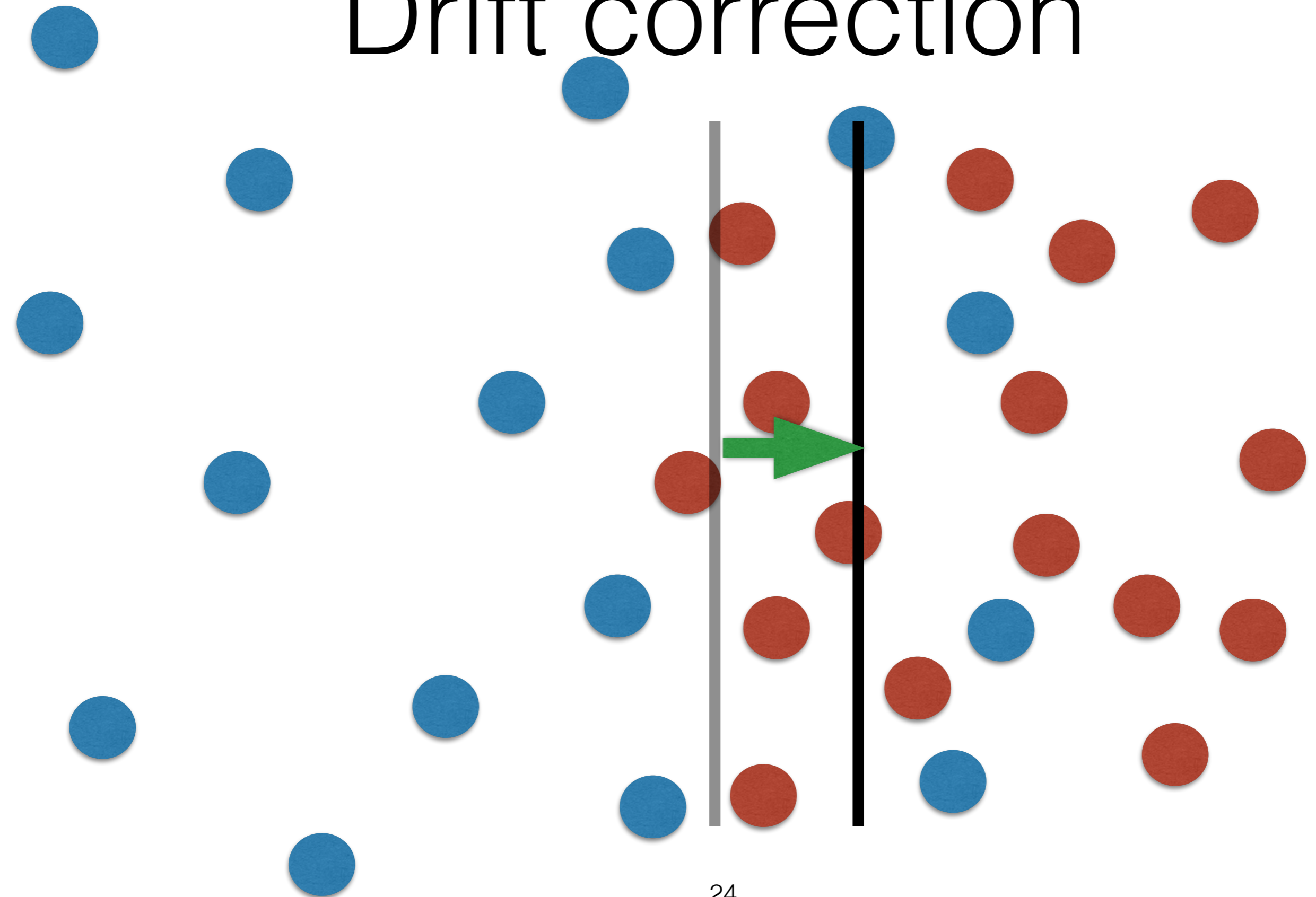
Code SQG MU:
link from Fluminance website - V. Resseguier

Drift correction

Drift correction

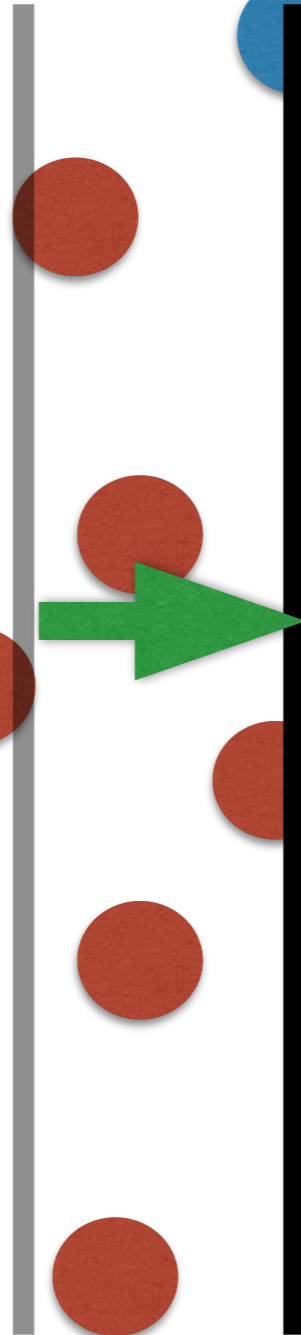


Drift correction



Drift correction

$$w^* = w - \frac{1}{2} (\nabla \cdot \mathbf{a})^T$$



Bifurcations in SQG

tracked by SQG MU

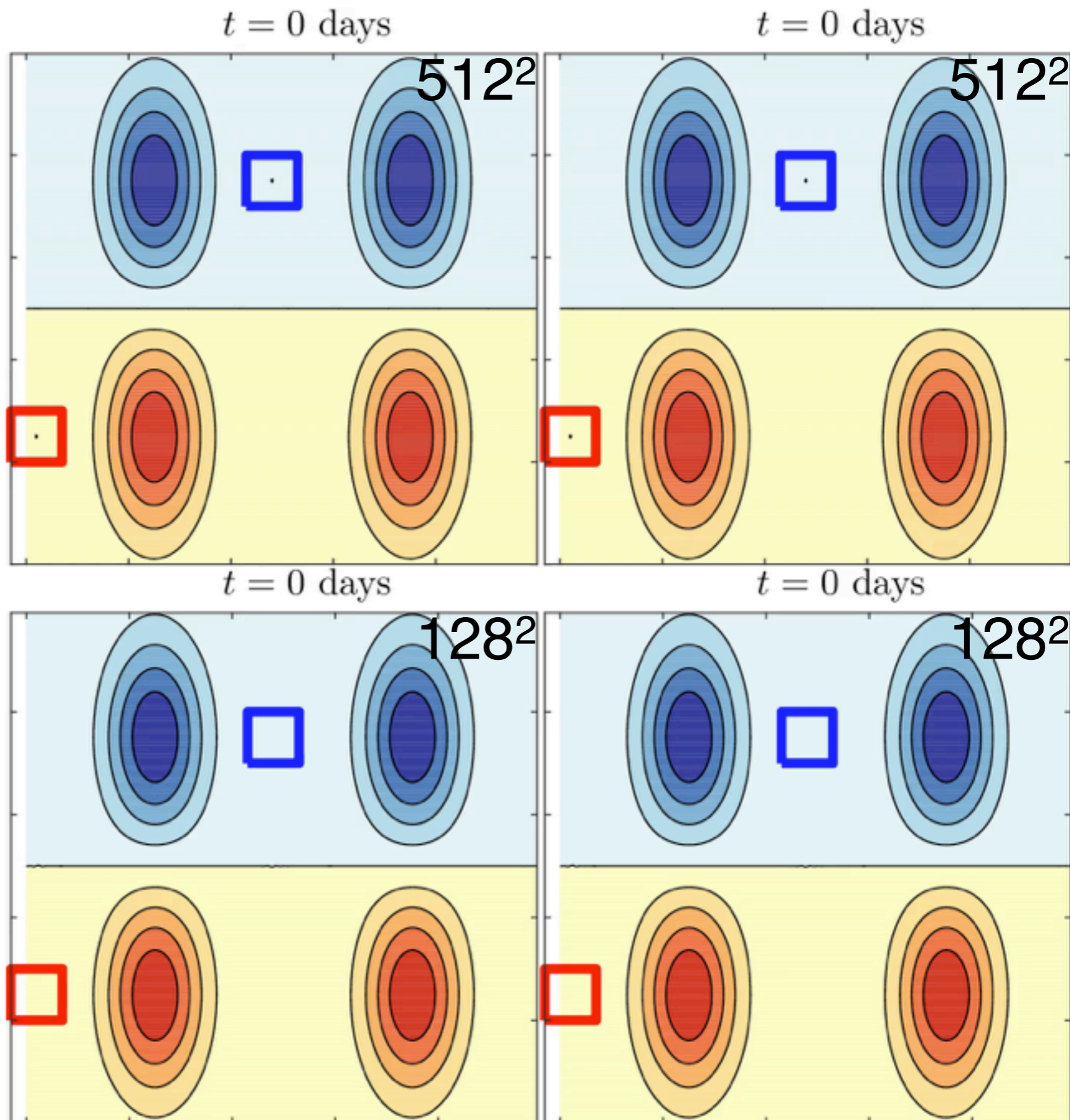
Reference flow:
deterministic
SQG

512² versus 128²

Initial condition 1



?



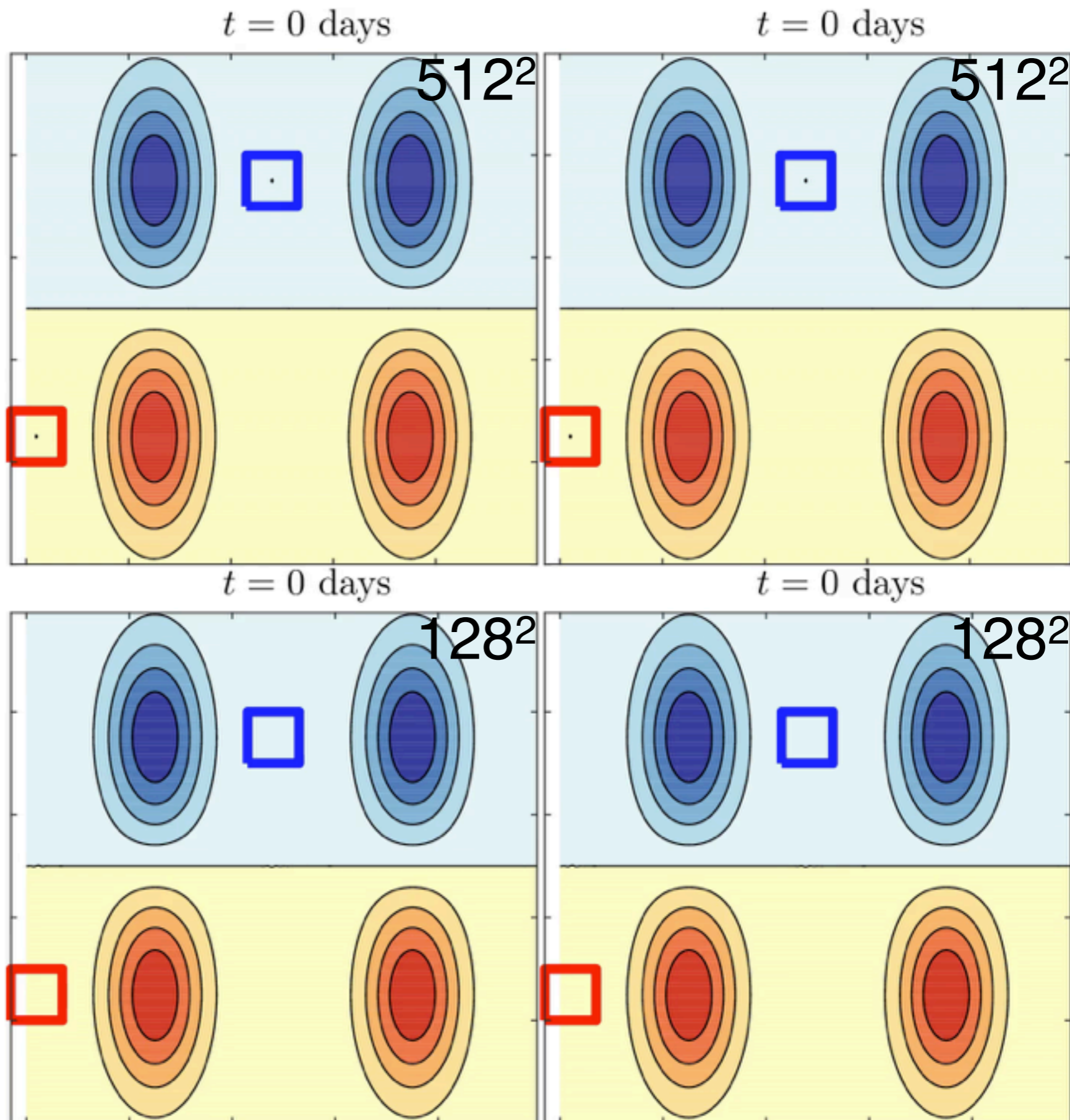
Reference flow:
deterministic
SQG

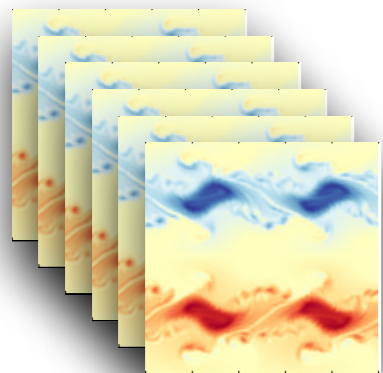
512² versus 128²

Initial condition 1

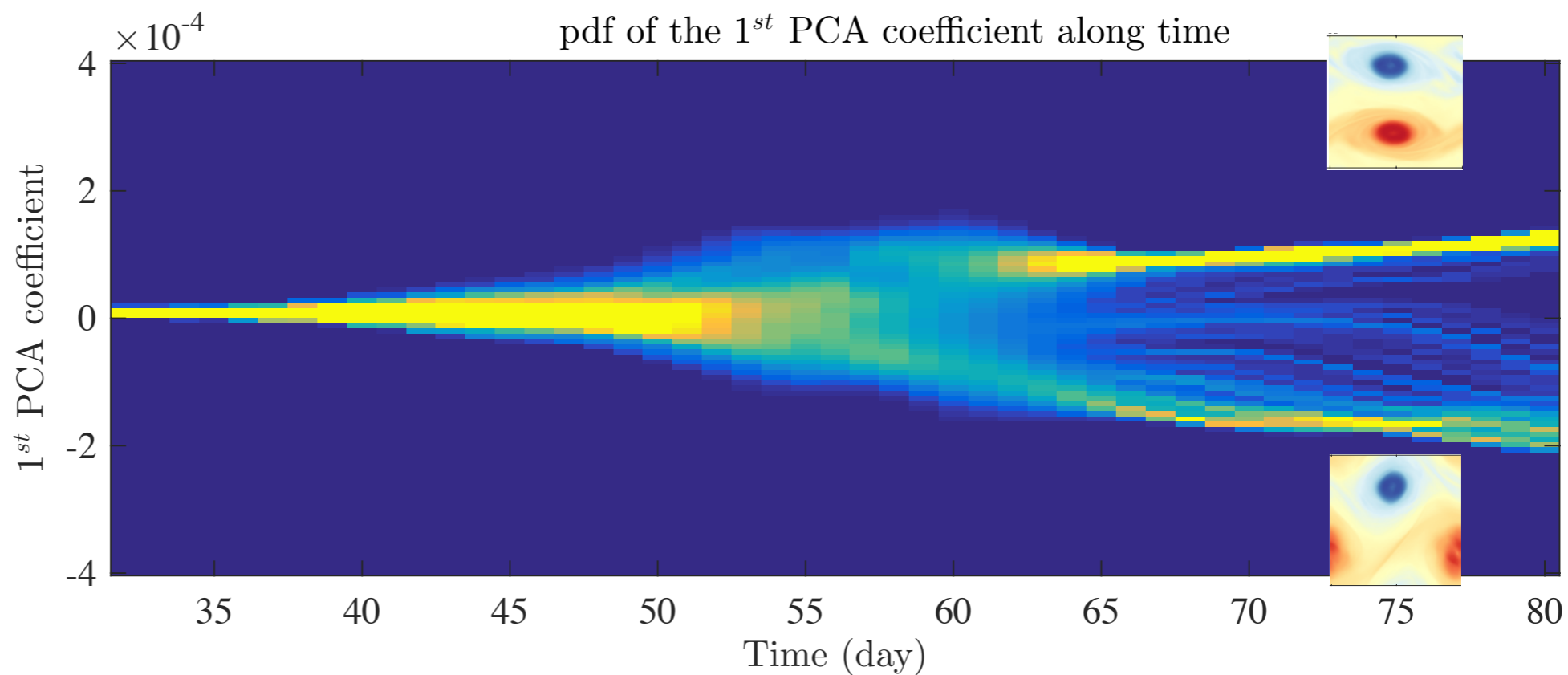


?

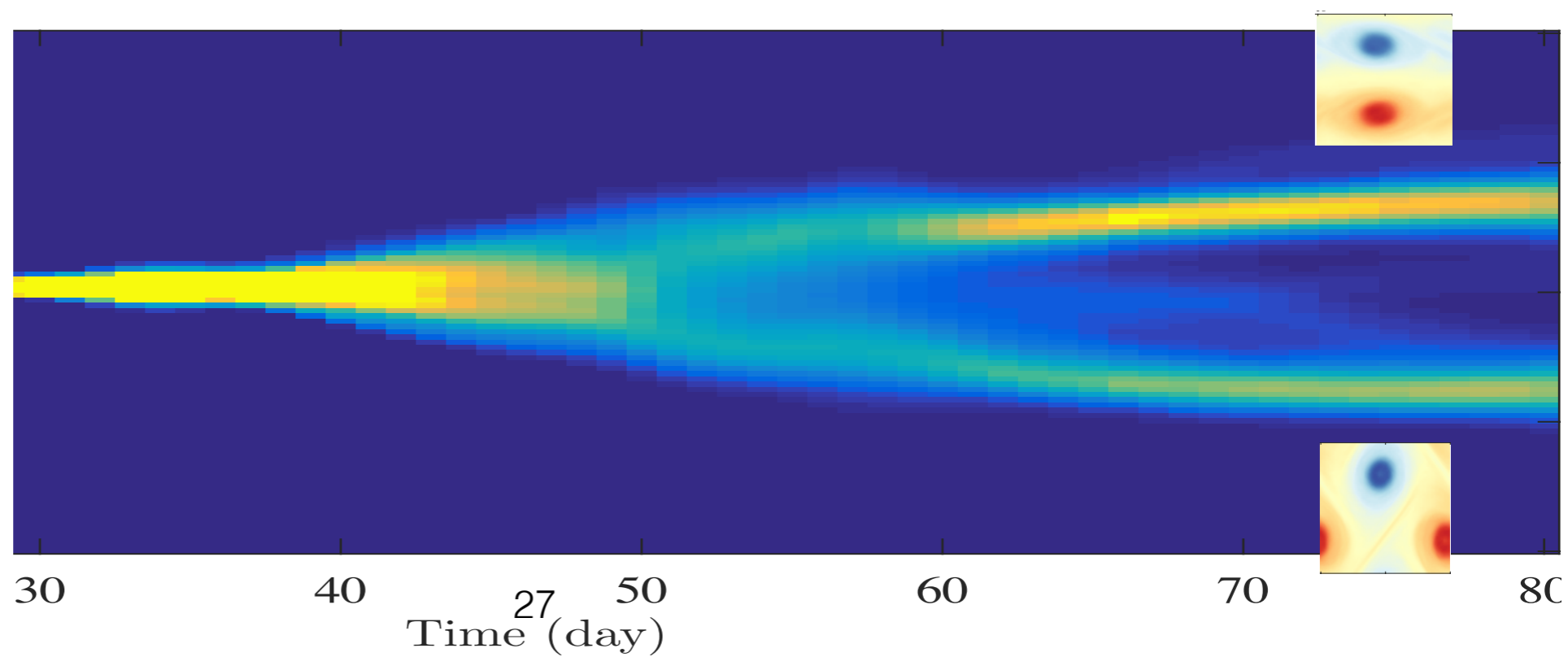




Random
initial
conditions



Under
location
uncertainty



SQG under Strong Uncertainty

SQG SU

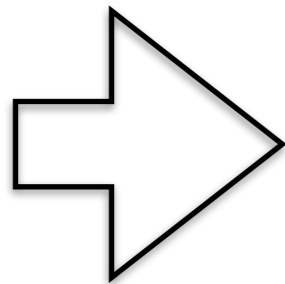
Mesoscale divergence

Geostrophic balance

$$\mathbf{f} \times \mathbf{u} = -\frac{1}{\rho_b} \nabla p'$$

Horizontal
Diffusion

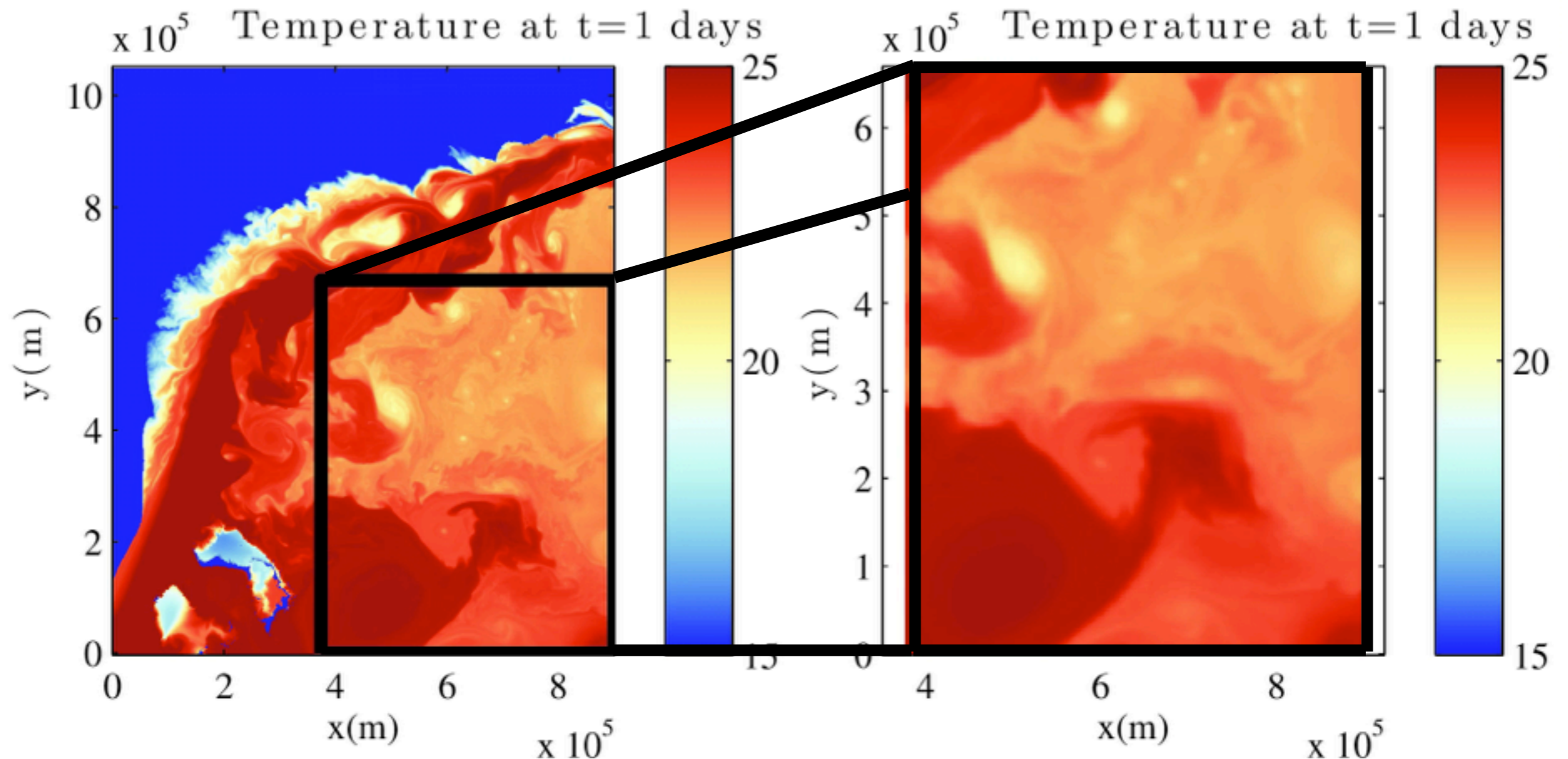
$$+ \frac{a}{2} \Delta \mathbf{u}$$



$$\nabla \cdot \mathbf{u} \propto \Delta \nabla^\perp \cdot \mathbf{u}$$

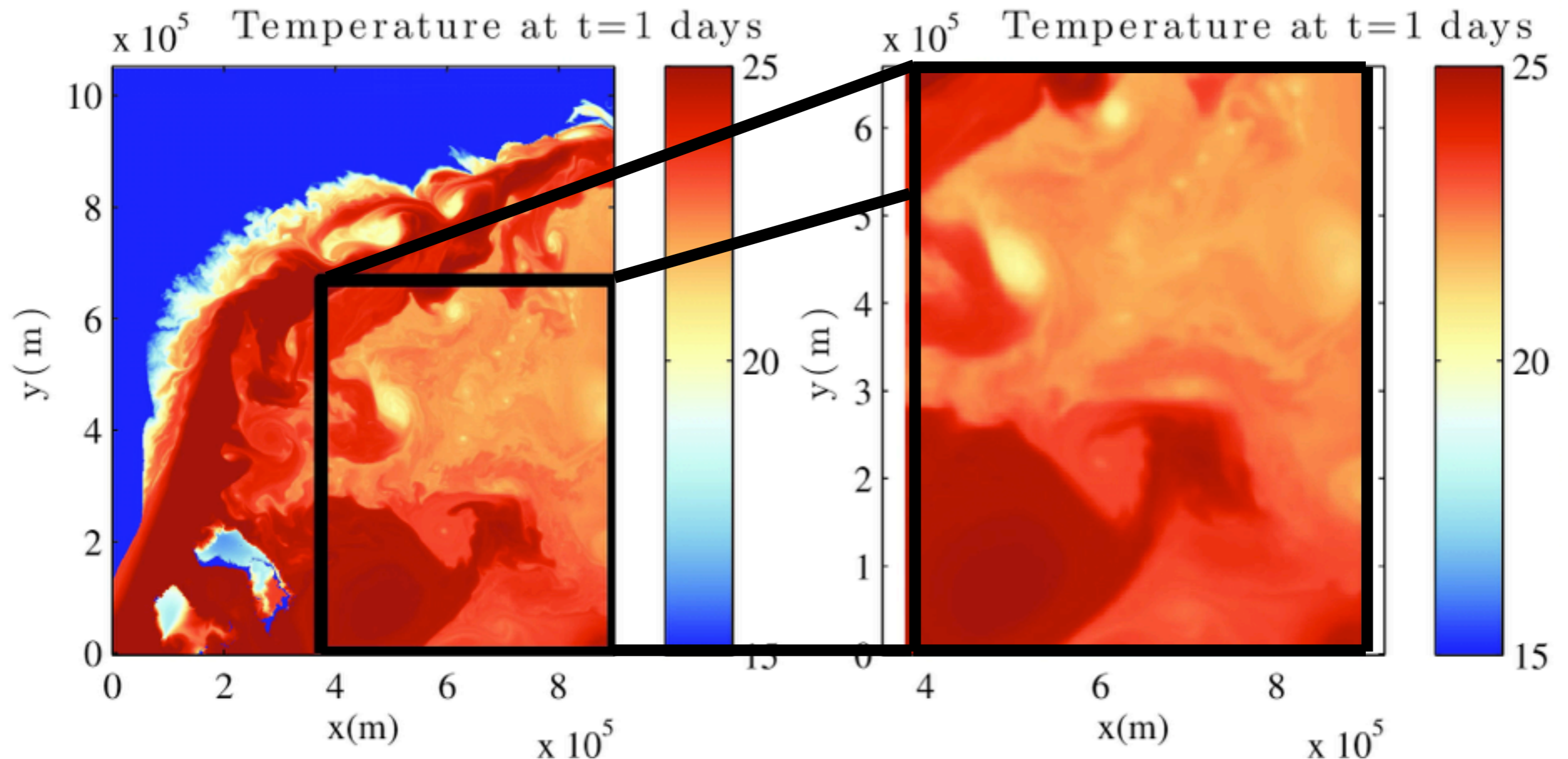
Filtering of model outputs:

Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
"Gulf Stream dynamics along the southeastern US seaboard."
Journal of Physical Oceanography 45.3 (2015): 690-715.

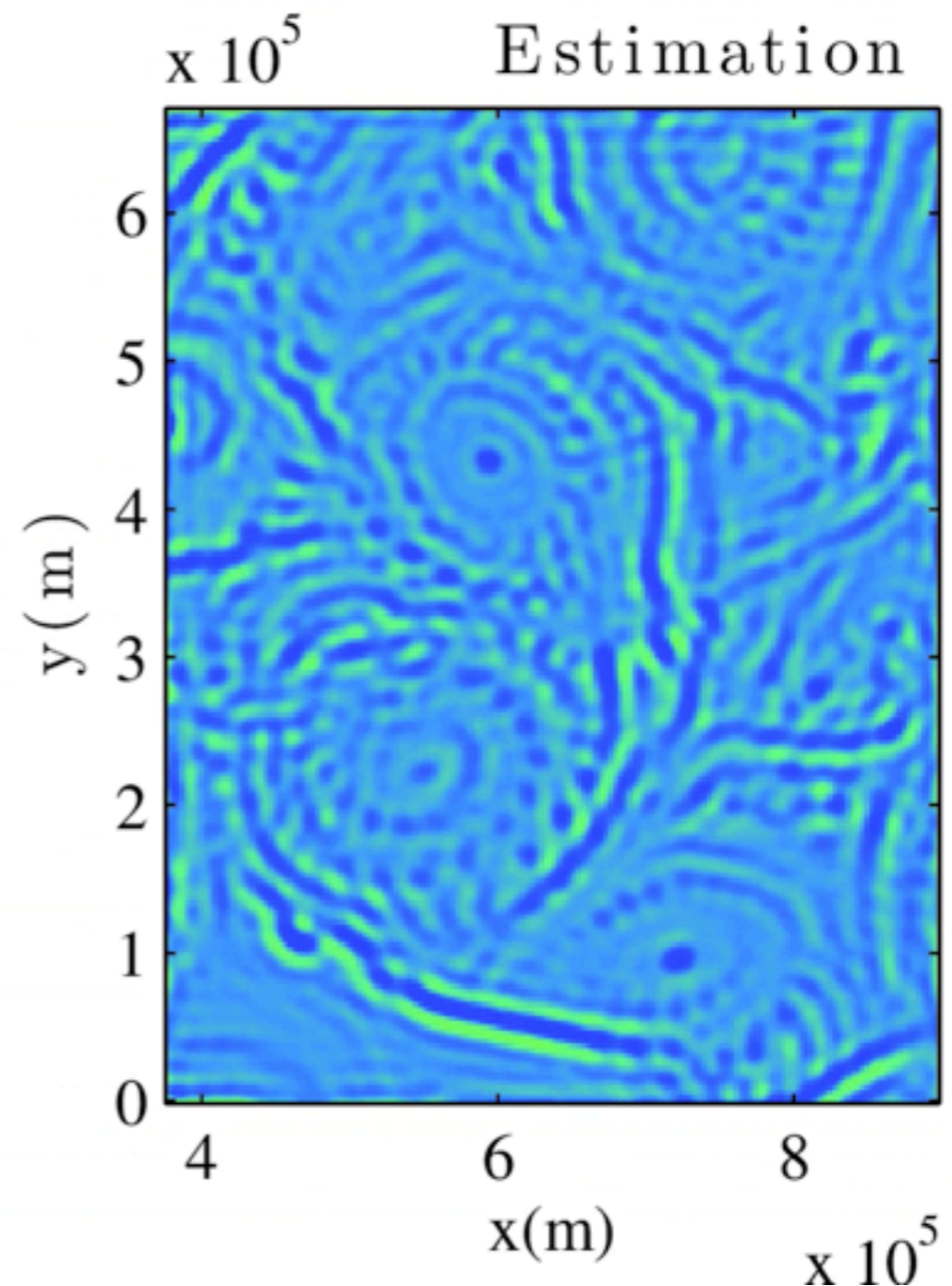
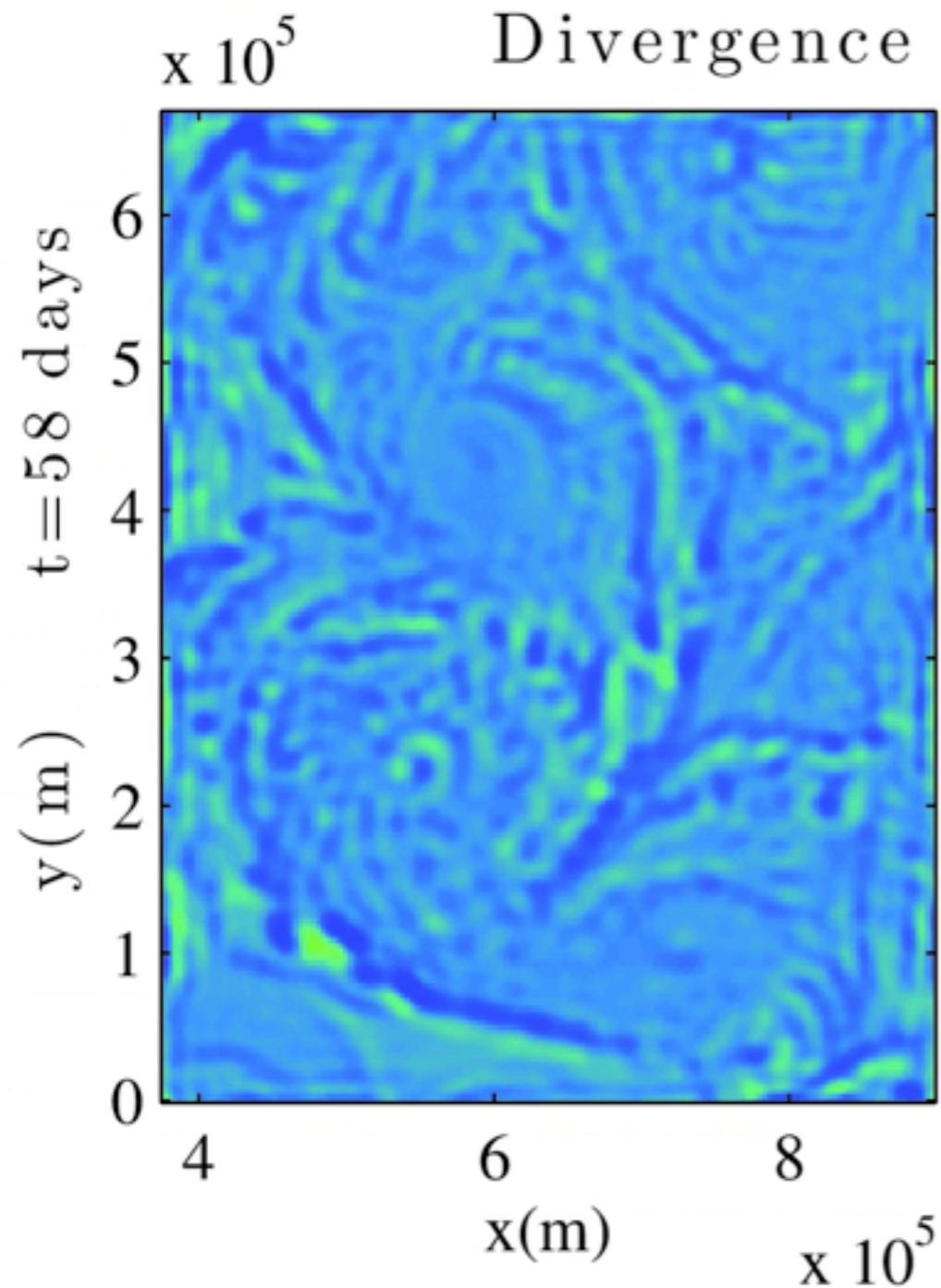


Filtering of model outputs:

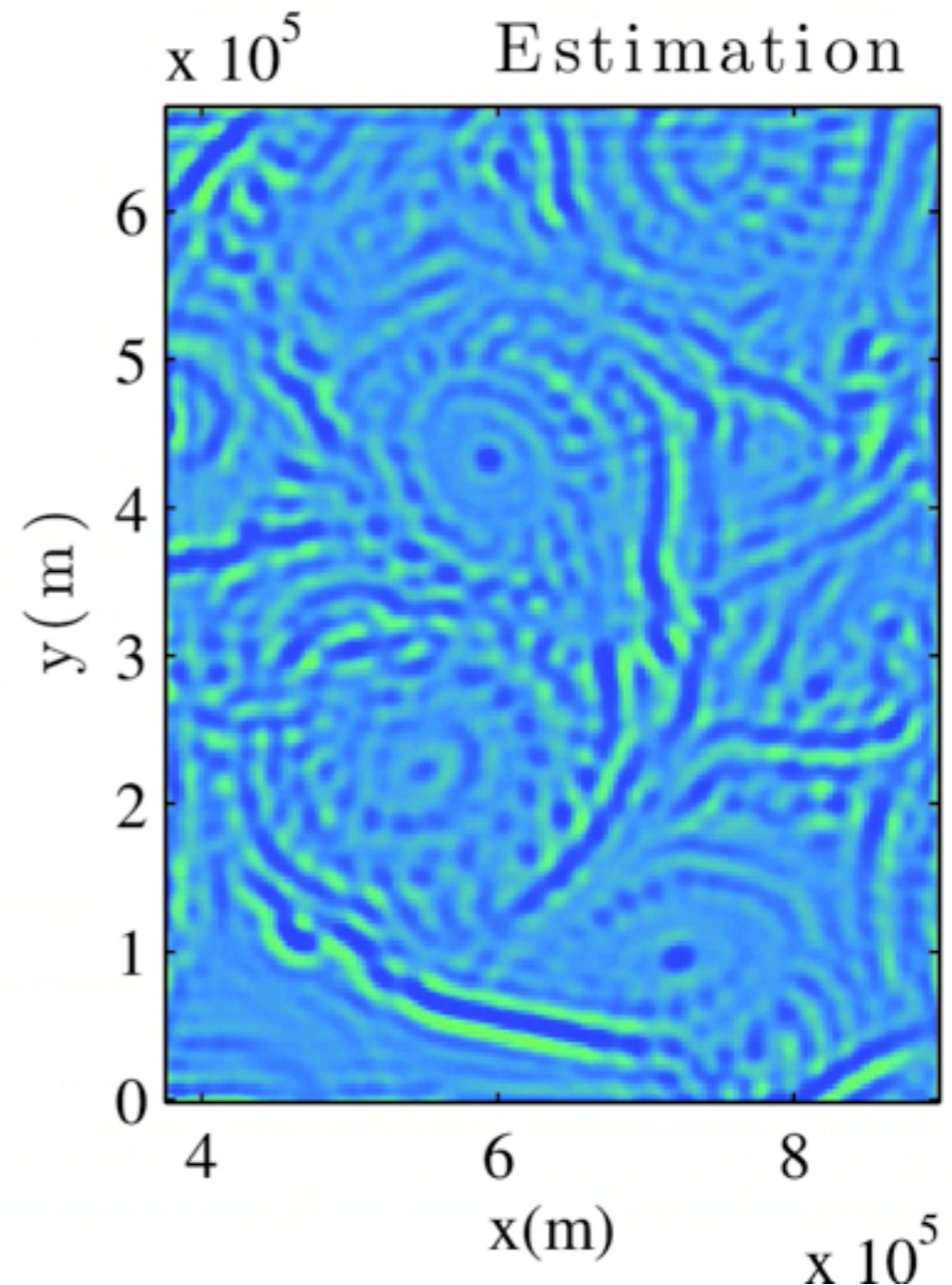
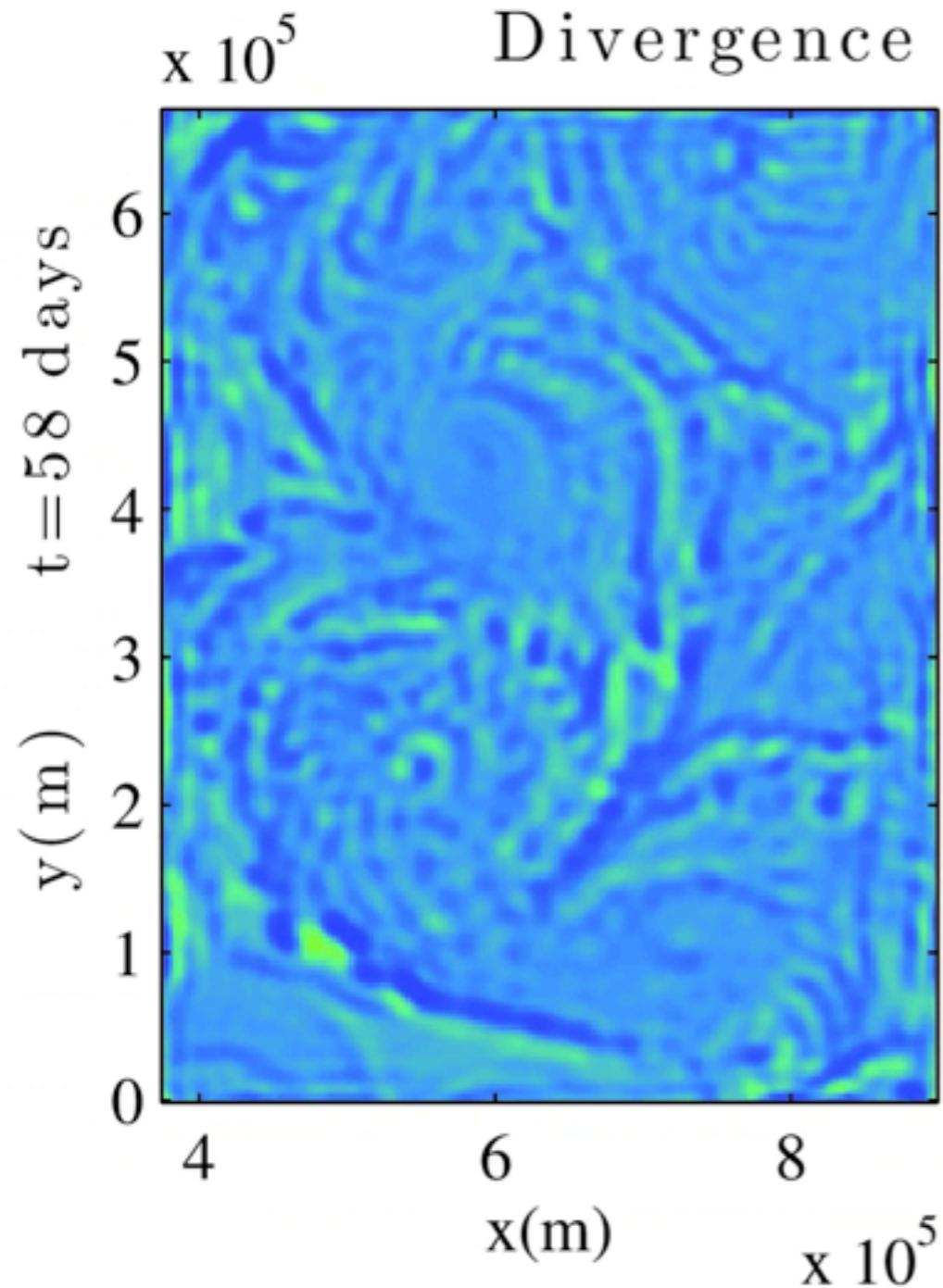
Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
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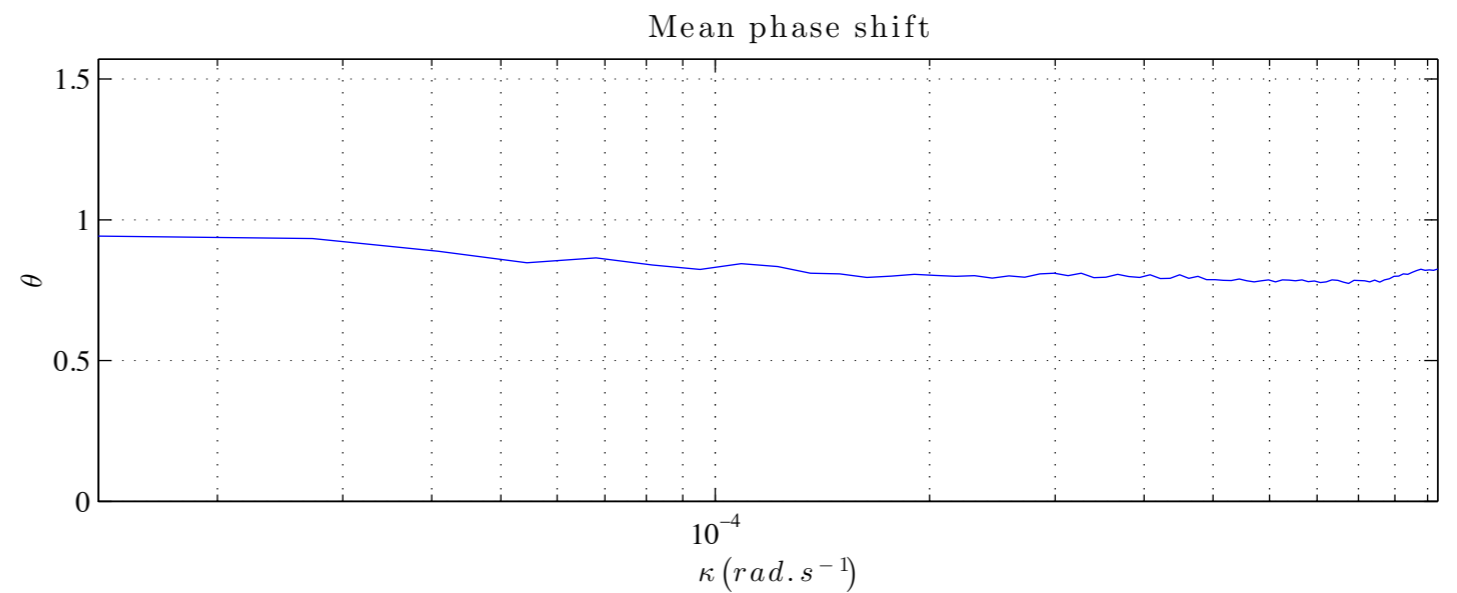
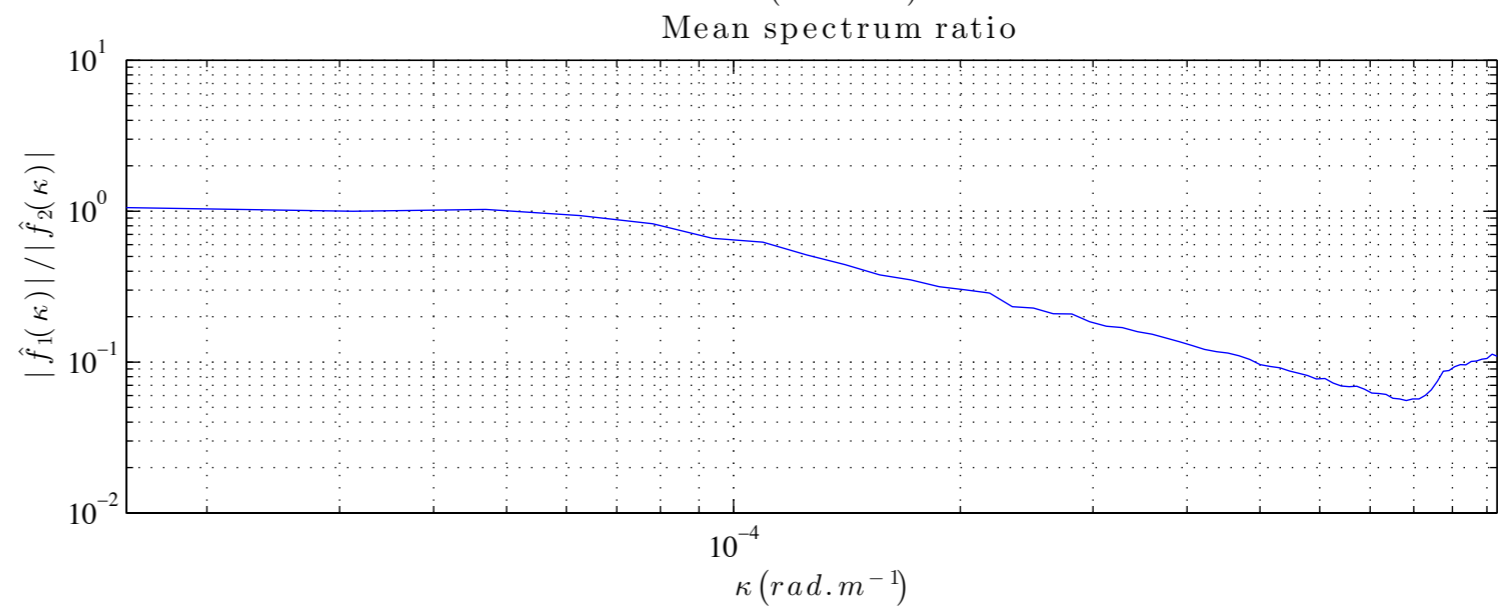
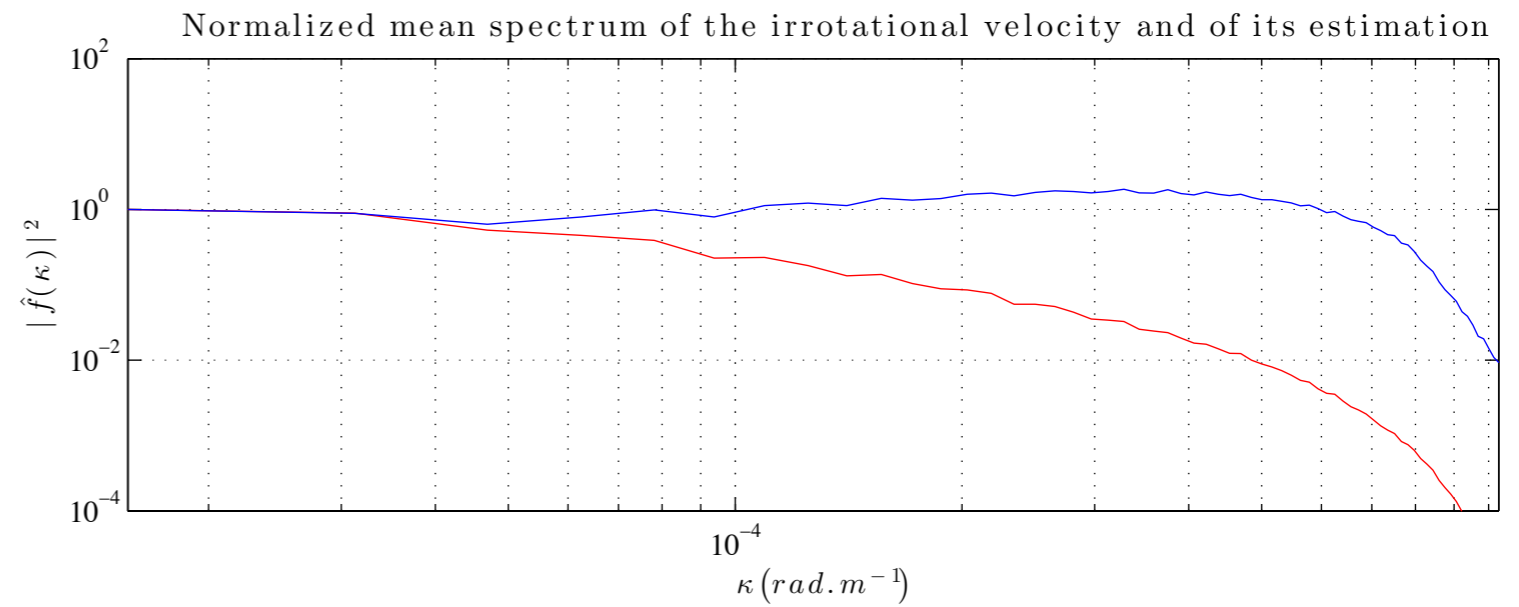
Spatial test



Spatial test



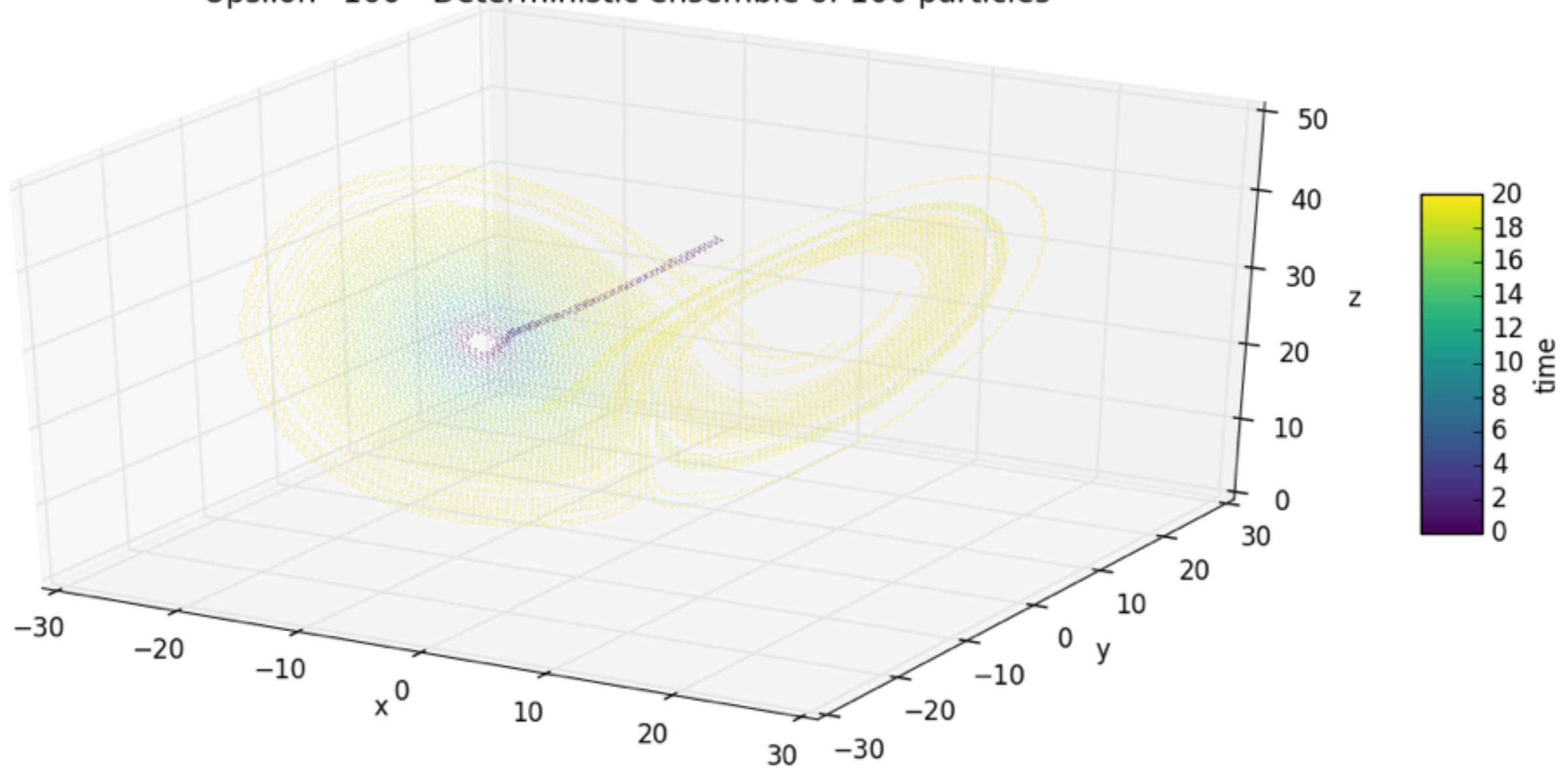
Spectral test





Long time: weak noise

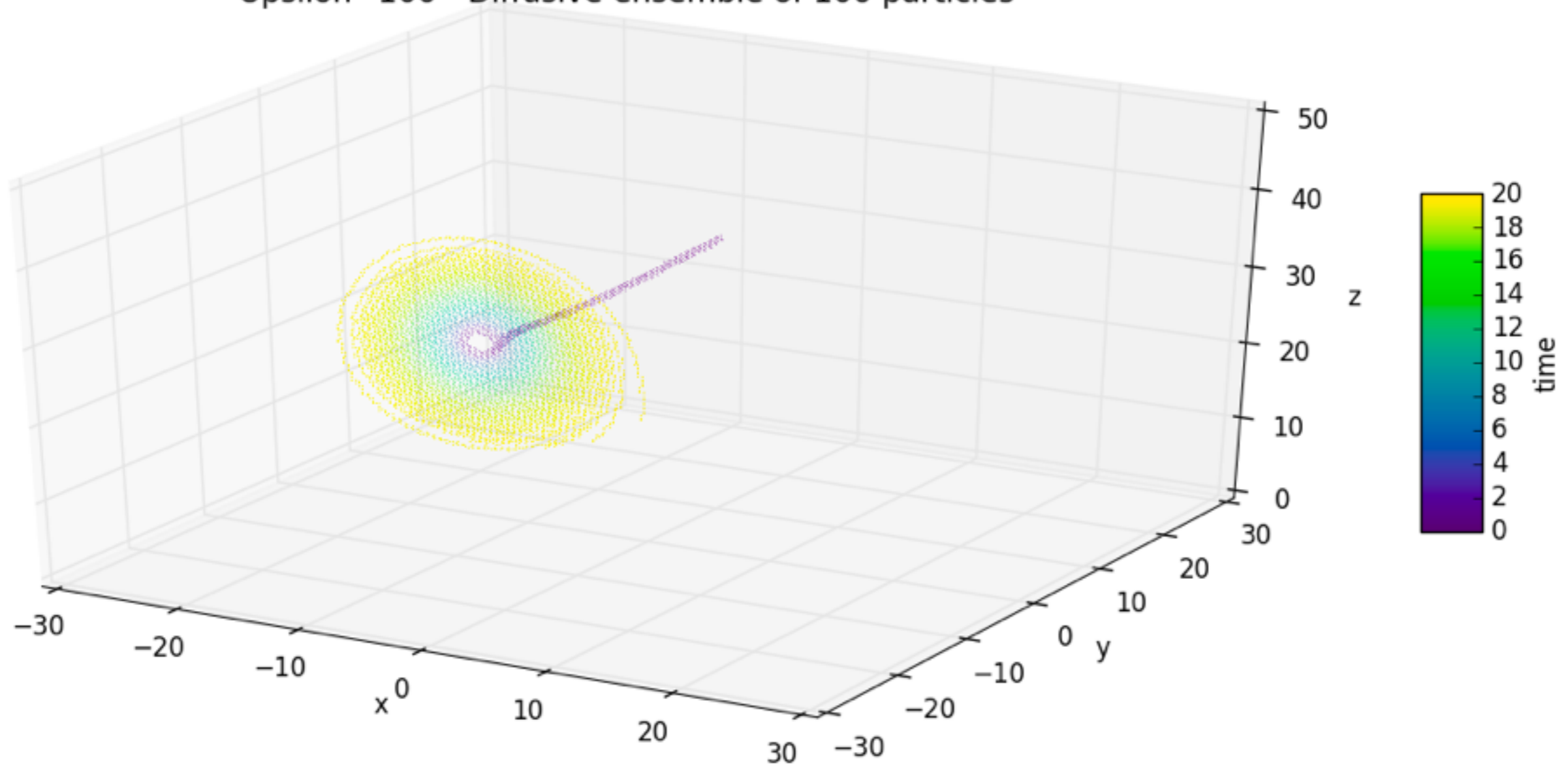
Upsilon=100 - Deterministic ensemble of 100 particles





Long time: weak noise

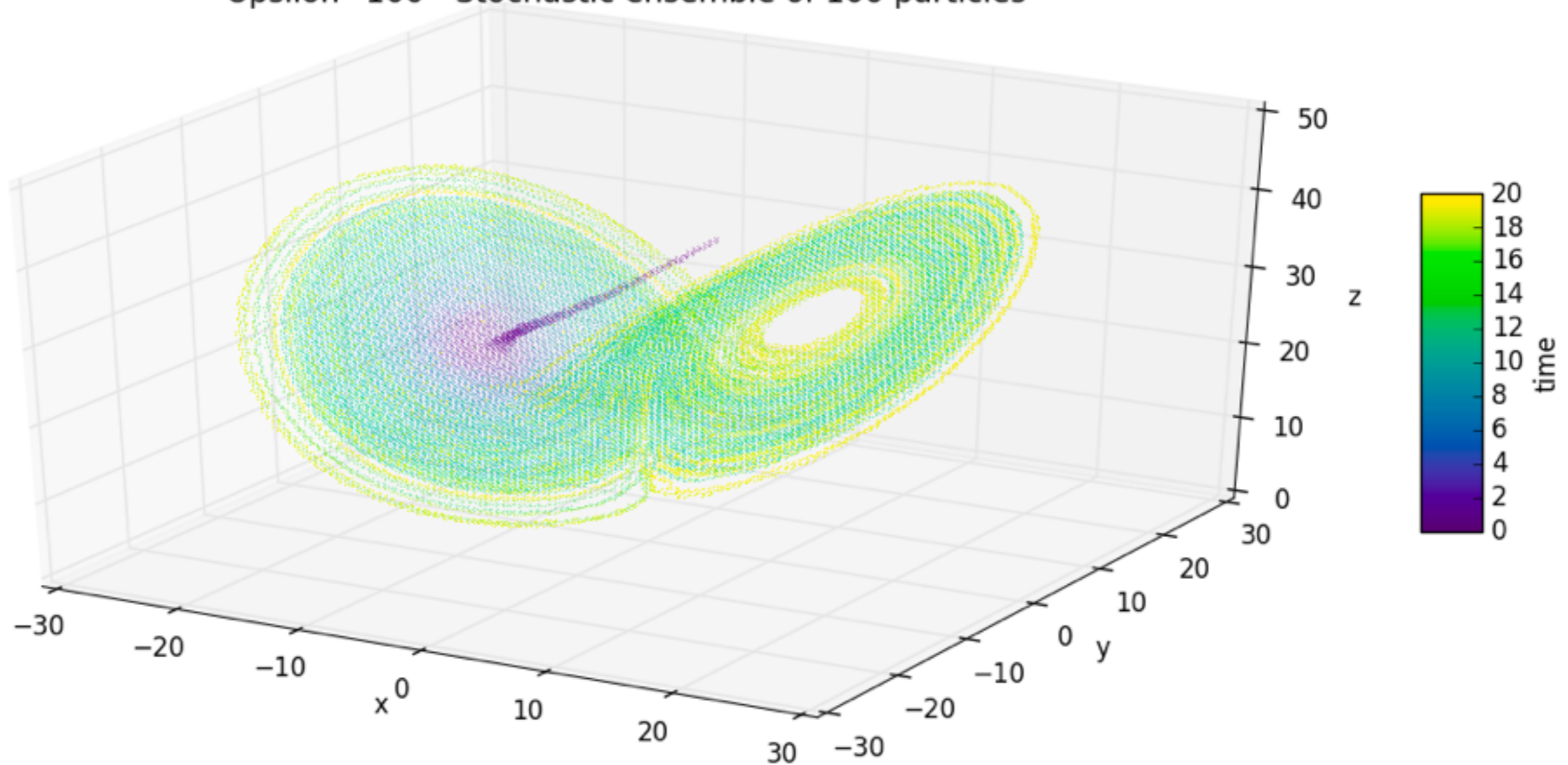
Upsilon=100 - Diffusive ensemble of 100 particles





Long time: weak noise

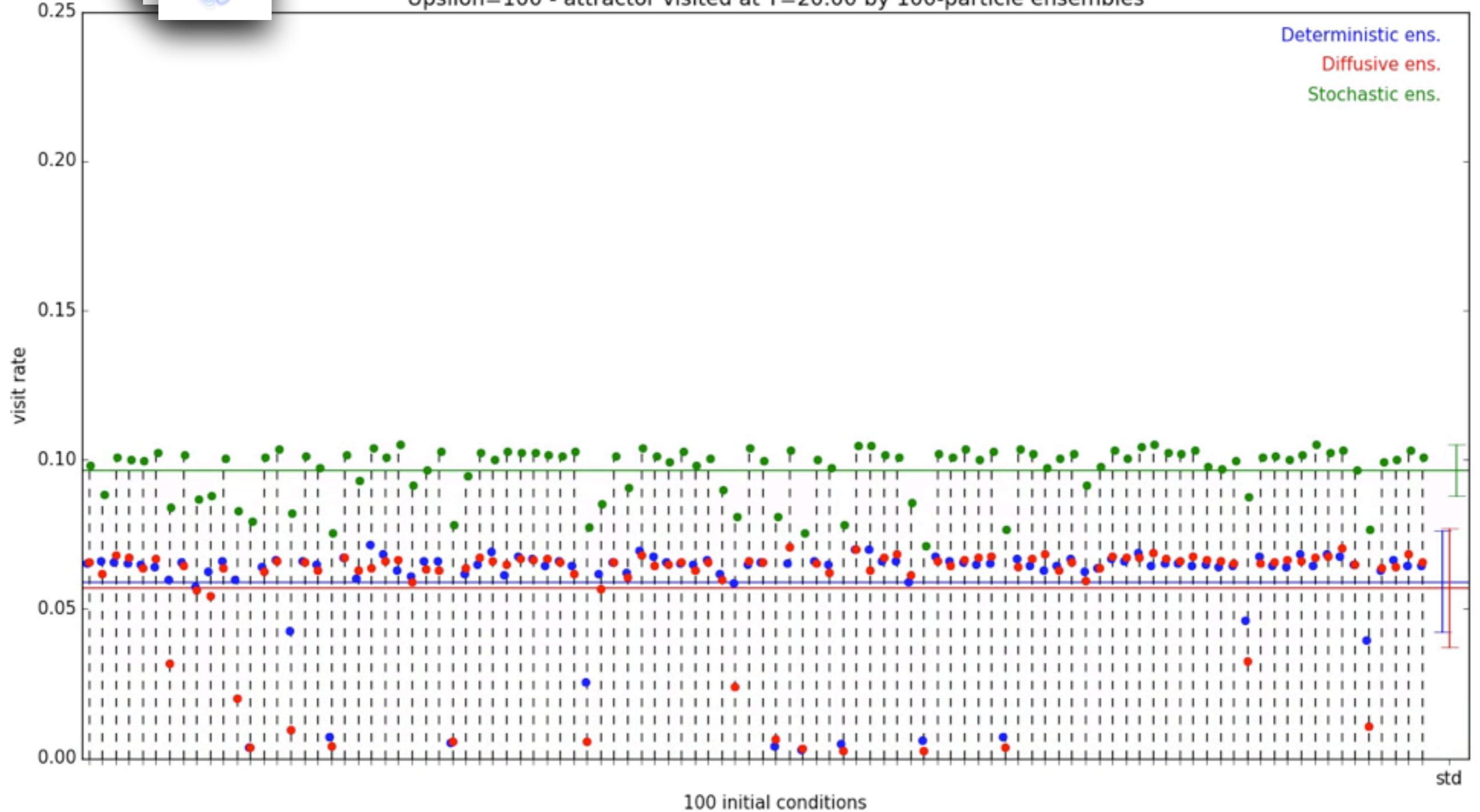
Upsilon=100 - Stochastic ensemble of 100 particles



Attractor visit rates



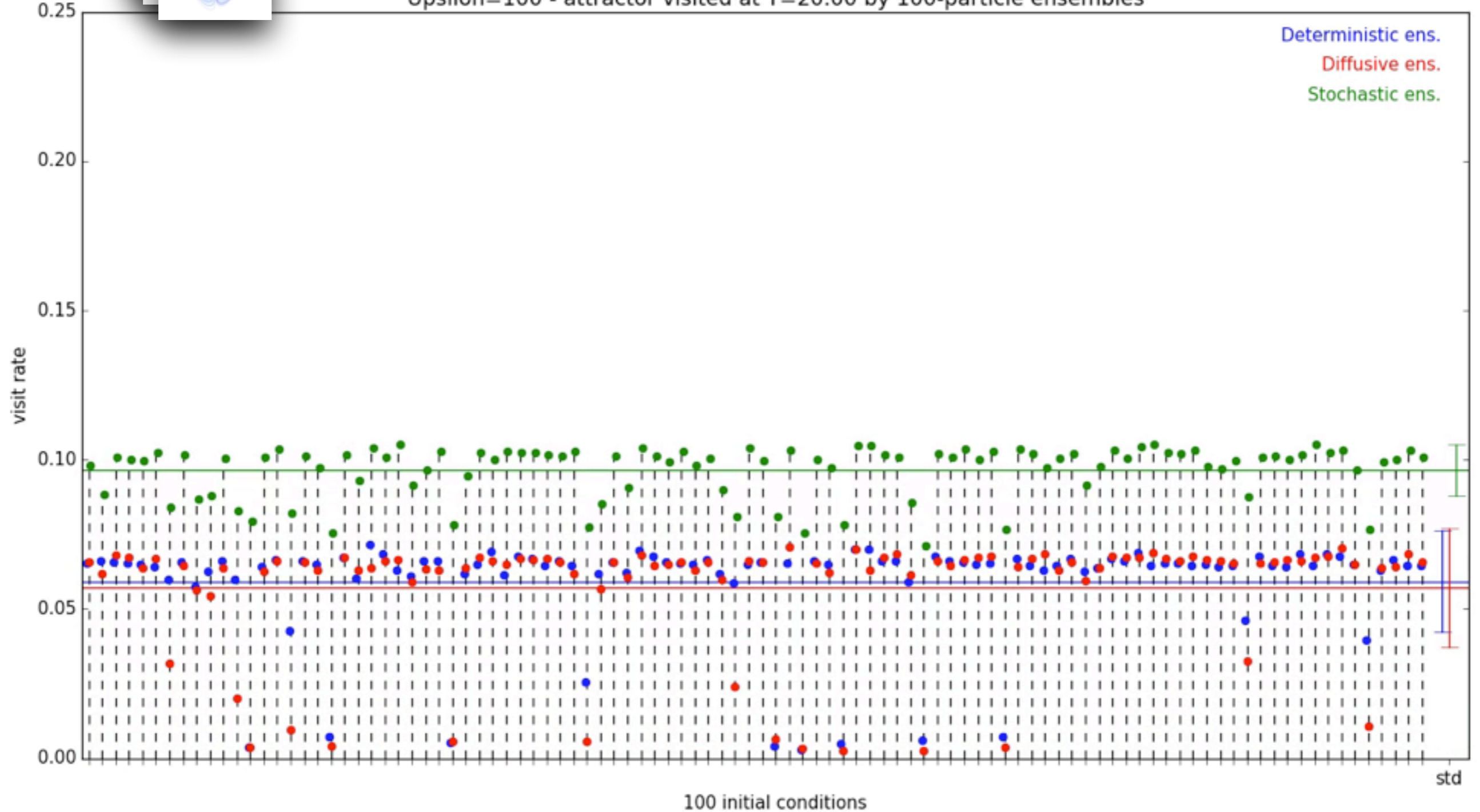
Upsilon=100 - attractor visited at T=20.00 by 100-particle ensembles



Attractor visit rates



Upsilon=100 - attractor visited at T=20.00 by 100-particle ensembles





Computing the visit rate

- Discrete covering of the Lorenz attractor (GAIO)
→ 611,550 cubic boxes of radius=0.15625
- For each ensemble, the visit rate:

$$\tau(T) = \frac{\#\{\text{unique boxes visited by ensemble over } [0; T]\}}{\text{total } \# \text{ of boxes}}$$

