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## Likely chaotic transitions of large-scale fluid flows using a stochastic transport model

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# Likely chaotic transitions of large-scale fluid flows using a stochastic transport model

Valentin Resseguier,  
Etienne Mémin,  
Bertrand Chapron

# Motivations for deriving random fluid dynamics models

- Rigorously identified subgrid dynamics effects
- Injecting likely small-scale dynamics
- Quantification of modeling errors:
  - especially for Data assimilation: ensemble forecasts
- **Predicting possible distinct scenarios**

# Contents

- Chaotic transitions
- Dynamics under location uncertainty
- Ensemble of simulations

# Chaotic transitions

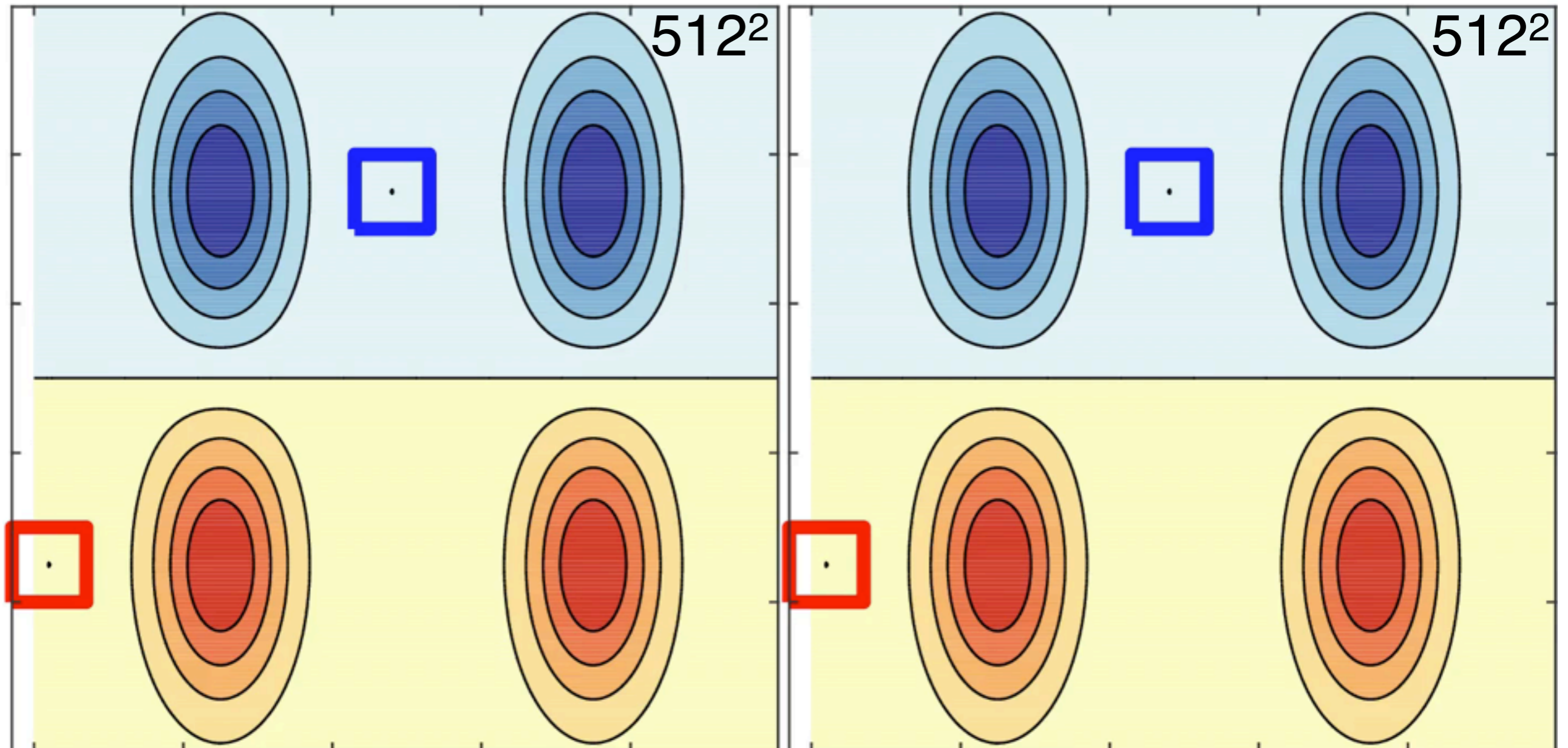
in SQG dynamics

# Reference flow: deterministic SQG $512^2$

Initial condition 1  Scenario 1

$t = 0$  days

$t = 0$  days

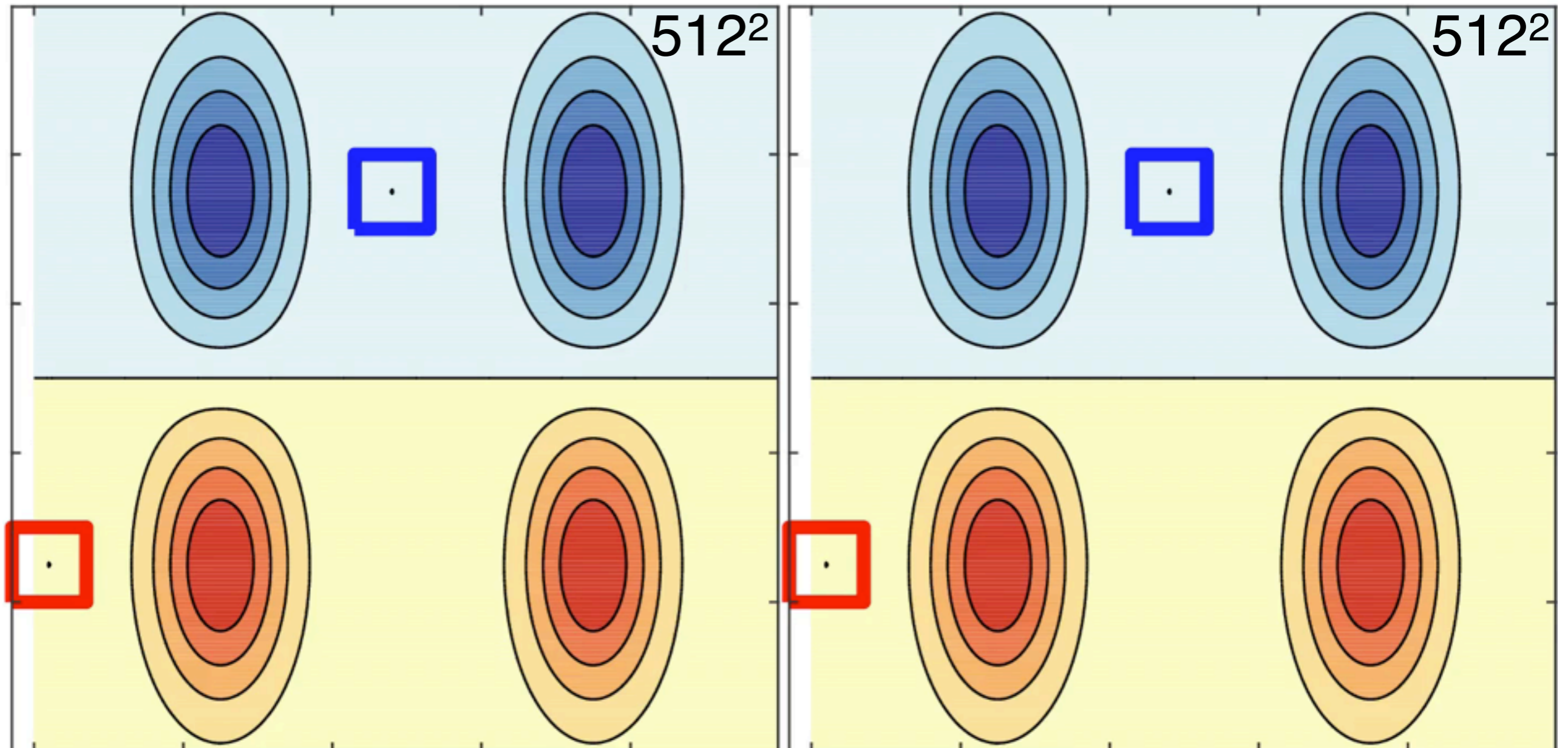


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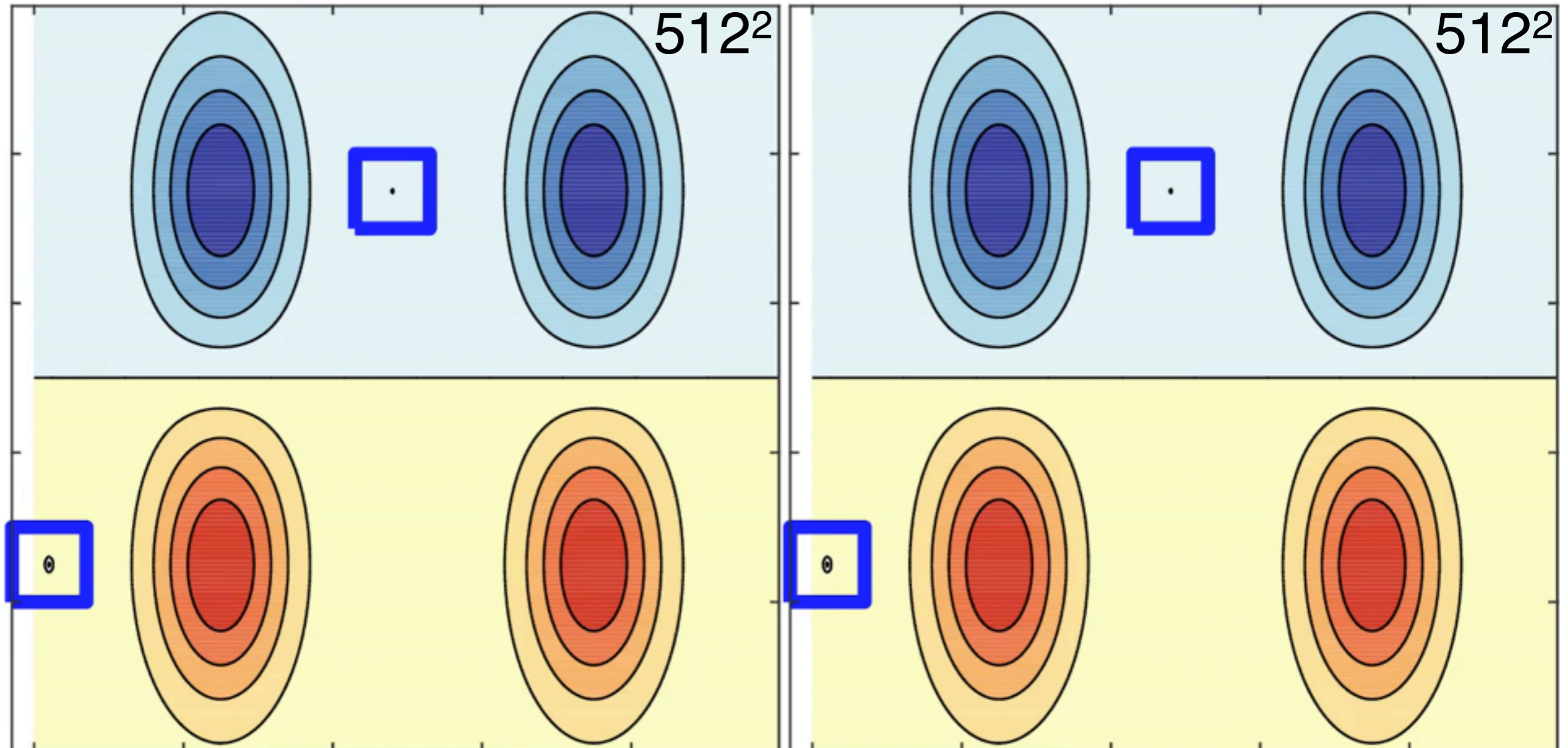


# Reference flow: deterministic SQG $512^2$

Initial condition 2  $\rightarrow$  Scenario 2

$t = 0$  days

$t = 0$  days



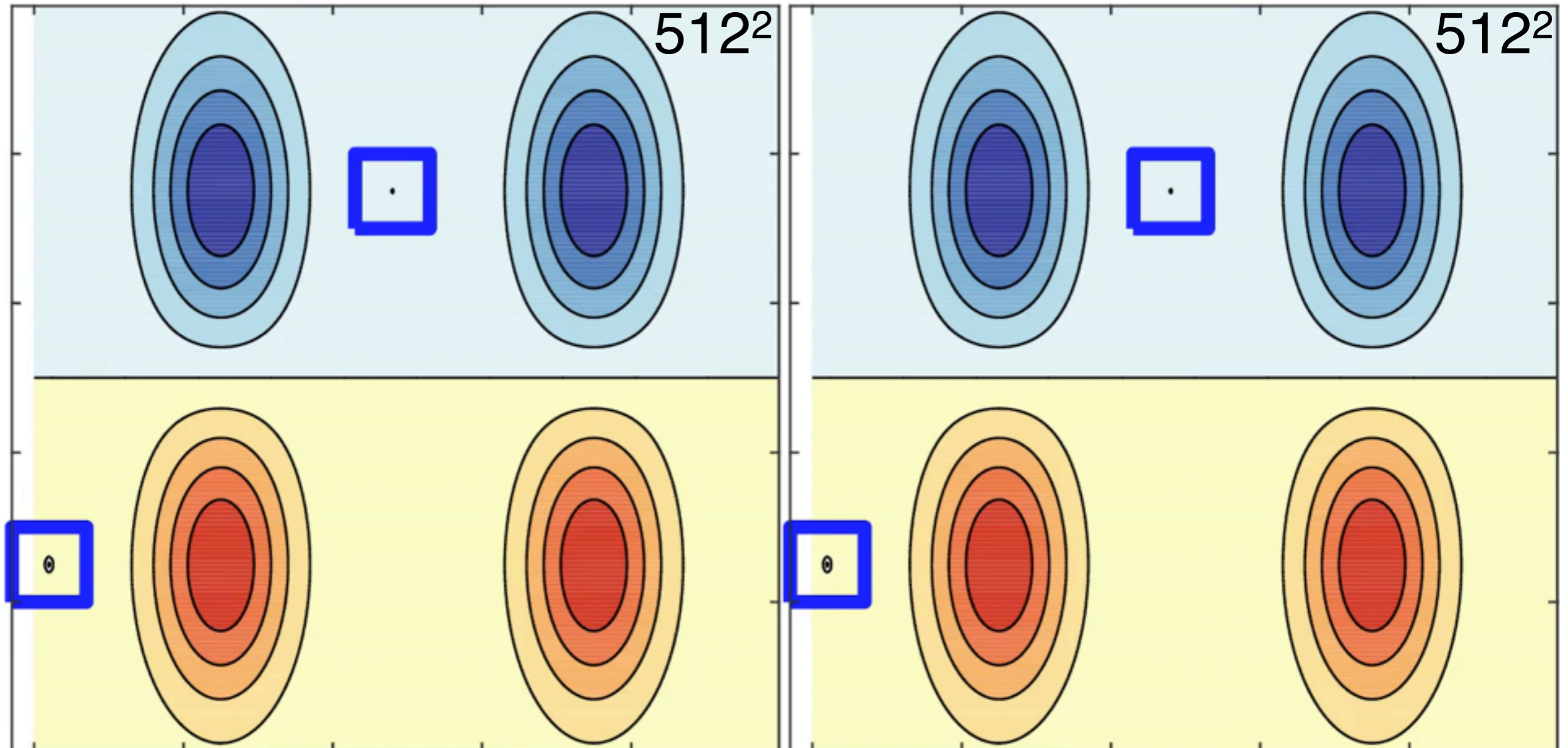


# Reference flow: deterministic SQG $512^2$

Initial condition 2  $\rightarrow$  Scenario 2

$t = 0$  days

$t = 0$  days



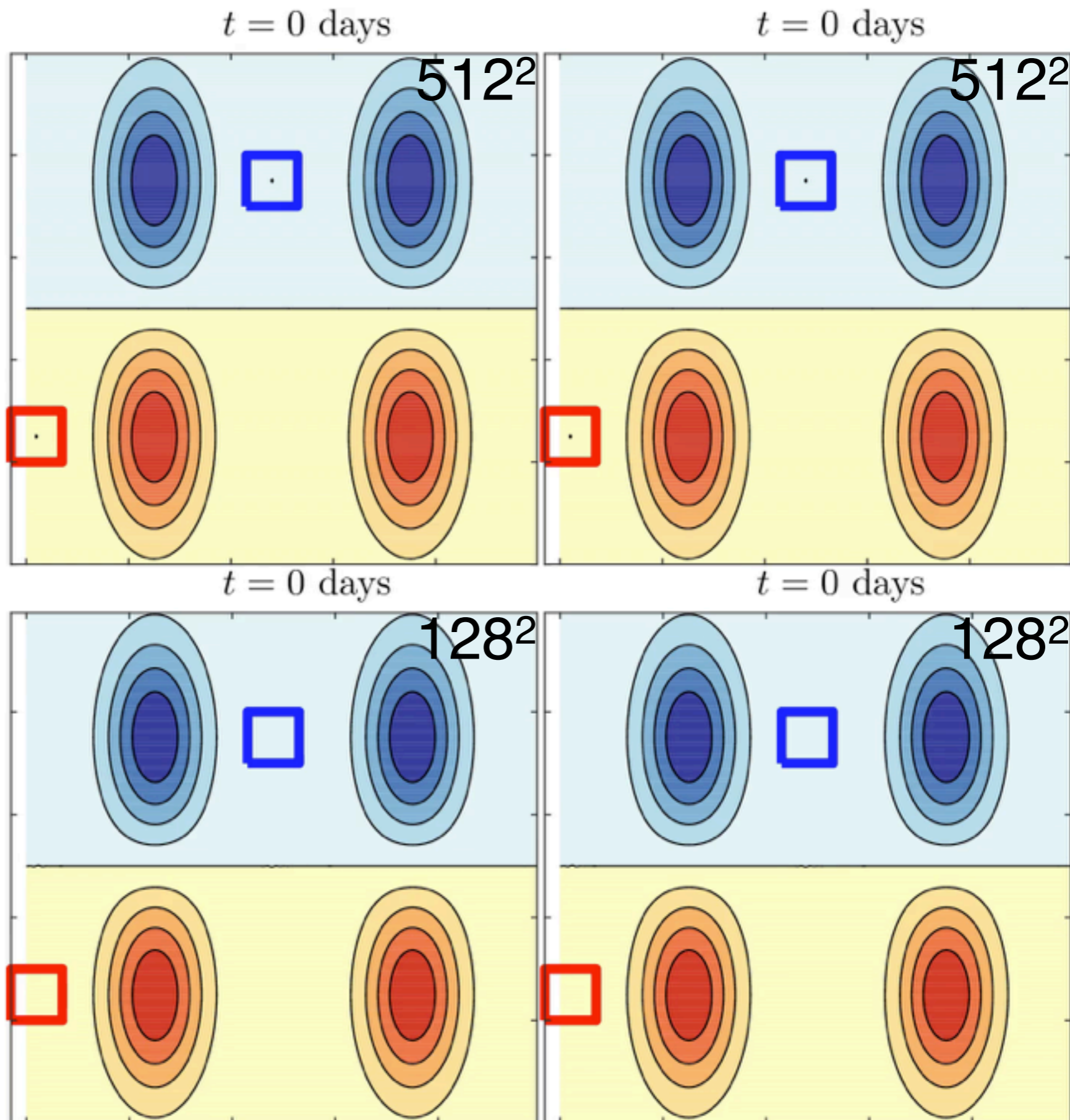
Reference flow:  
deterministic  
SQG

**512<sup>2</sup> versus 128<sup>2</sup>**

Initial condition 1



?





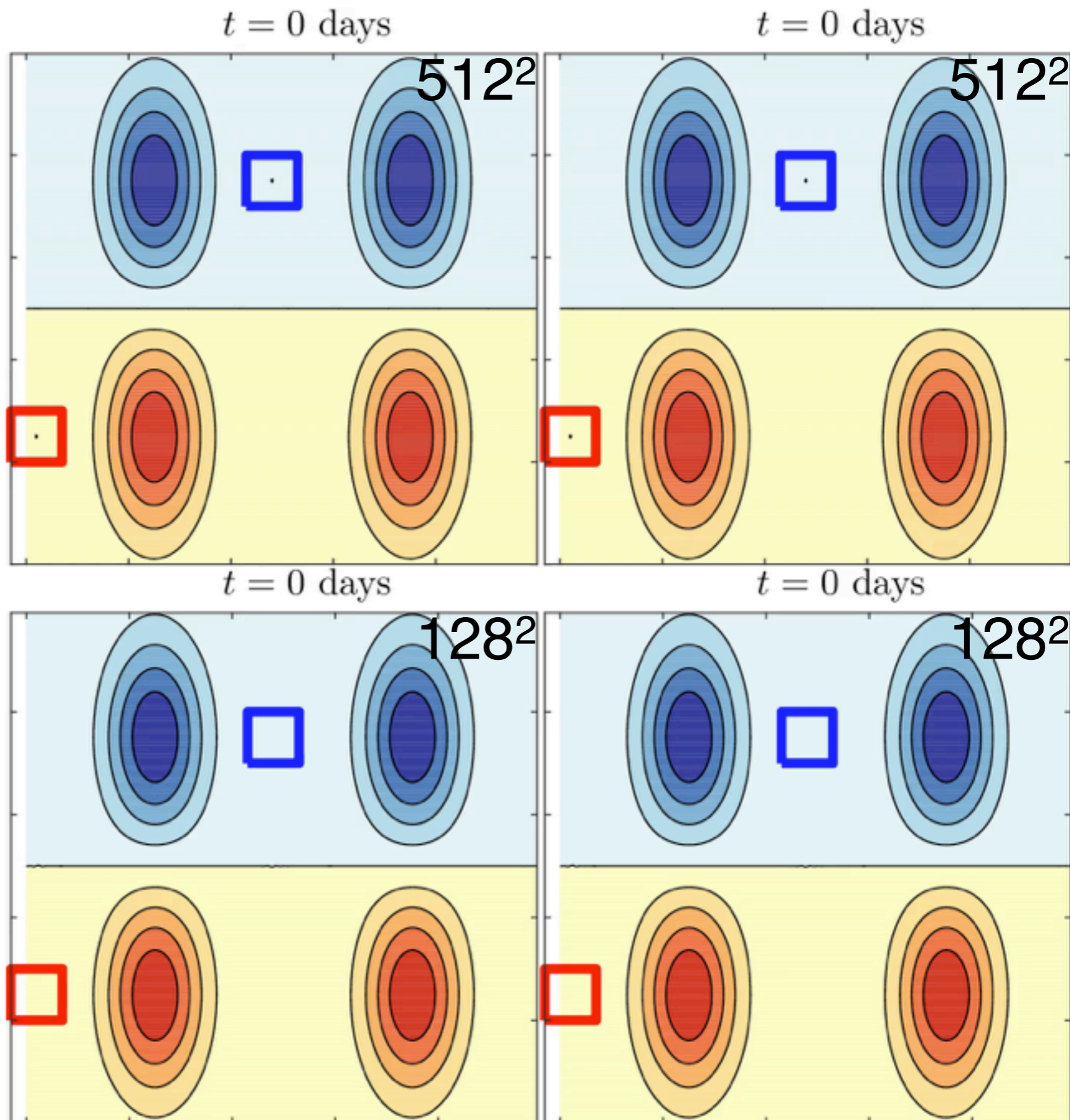
Reference flow:  
deterministic  
SQG

**512<sup>2</sup> versus 128<sup>2</sup>**

Initial condition 1

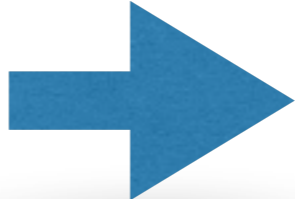
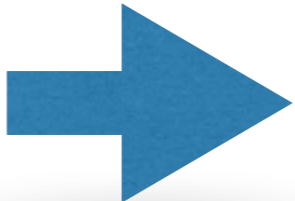
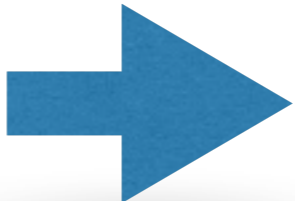



?



# Dynamics under location uncertainty

# Random equations

- Random initial conditions  Underdispersive + need large ensemble
- Arbitrary Gaussian forcing  Adding energy + wrong phase
- Averaging, homogenization  Assumptions and energy issues
- Adding white random velocity   $\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$

# Advection of tracer $\Theta$

$$\frac{D\Theta}{Dt} = 0$$



# Advection of tracer $\Theta$

# Advection of tracer $\Theta$

$$\partial_t \Theta + \boldsymbol{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \boldsymbol{a} \nabla \Theta \right)$$

# Advection of tracer $\Theta$

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$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Drift correction

# Advection of tracer $\Theta$

The diagram illustrates the advection of a tracer  $\Theta$ . The equation is presented as follows:

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta}_{\text{Drift correction}} + \underbrace{\sigma \dot{B} \cdot \nabla \Theta}_{\text{Multiplicative random forcing}} = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$

The left side of the equation is labeled "Advection" and is enclosed in a blue rounded rectangle. The right side is labeled "Diffusion" and is enclosed in a green rounded rectangle. An orange arrow points from the text "Drift correction" to the  $w^*$  term in the advection part. A purple arrow points from the text "Multiplicative random forcing" to the  $\sigma \dot{B} \cdot \nabla \Theta$  term in the advection part. The  $w^*$  term is also circled in orange.



# Advection of tracer $\Theta$

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta}_{\text{Drift correction}} + \underbrace{\sigma \dot{B} \cdot \nabla \Theta}_{\text{Multiplicative random forcing}} = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$

Advection

Diffusion

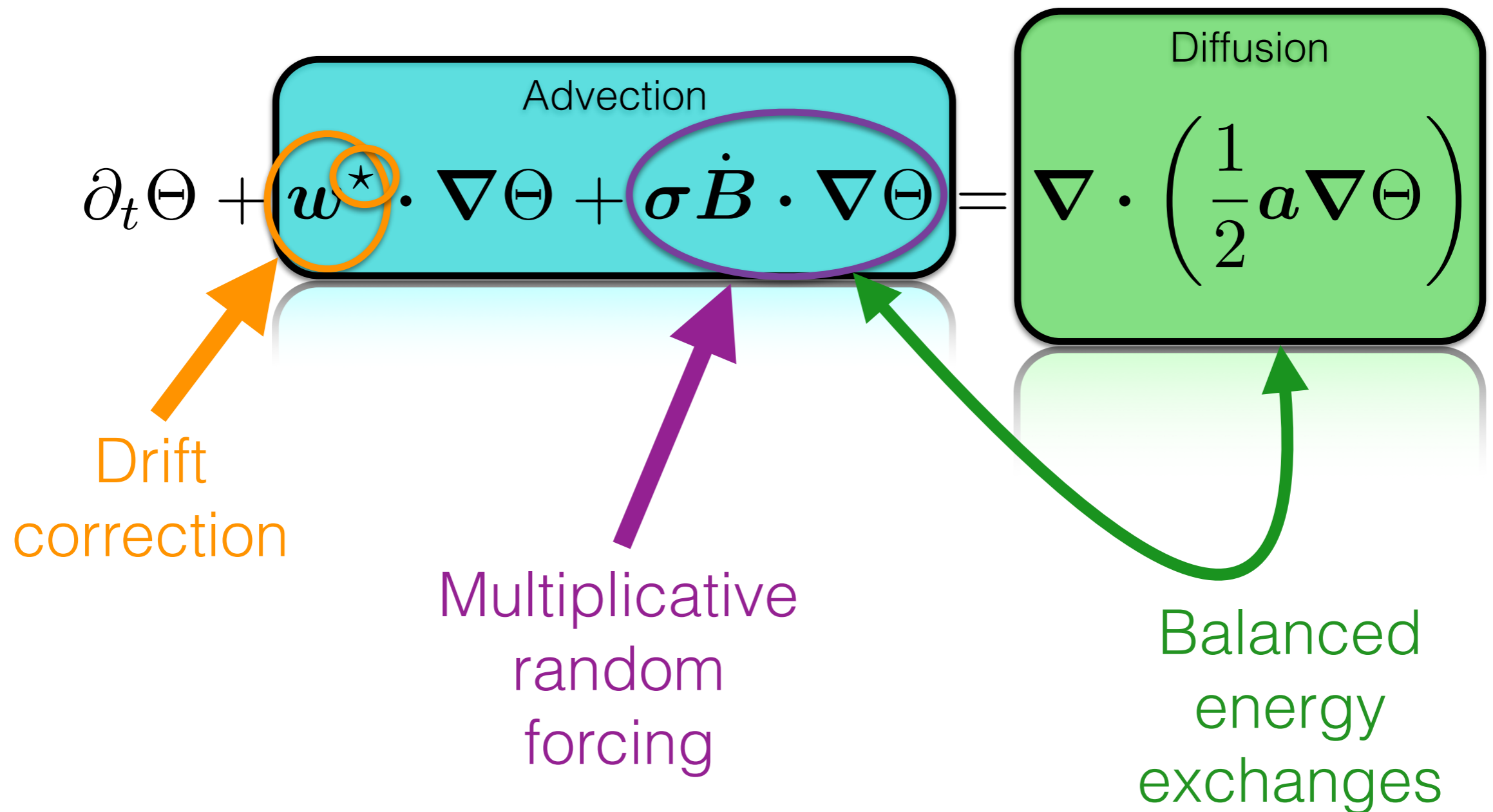
# Advection of tracer $\Theta$

The diagram illustrates the advection of a tracer  $\Theta$ . The equation is presented as follows:

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta}_{\text{Drift correction}} + \underbrace{\sigma \dot{B} \cdot \nabla \Theta}_{\text{Multiplicative random forcing}} = \underbrace{\nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

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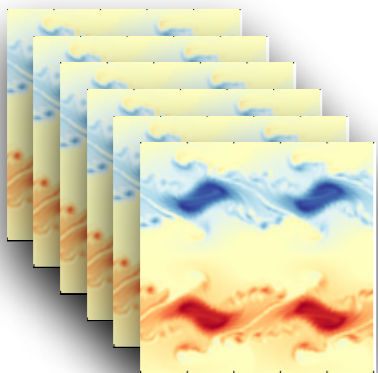
# Advection of tracer $\Theta$



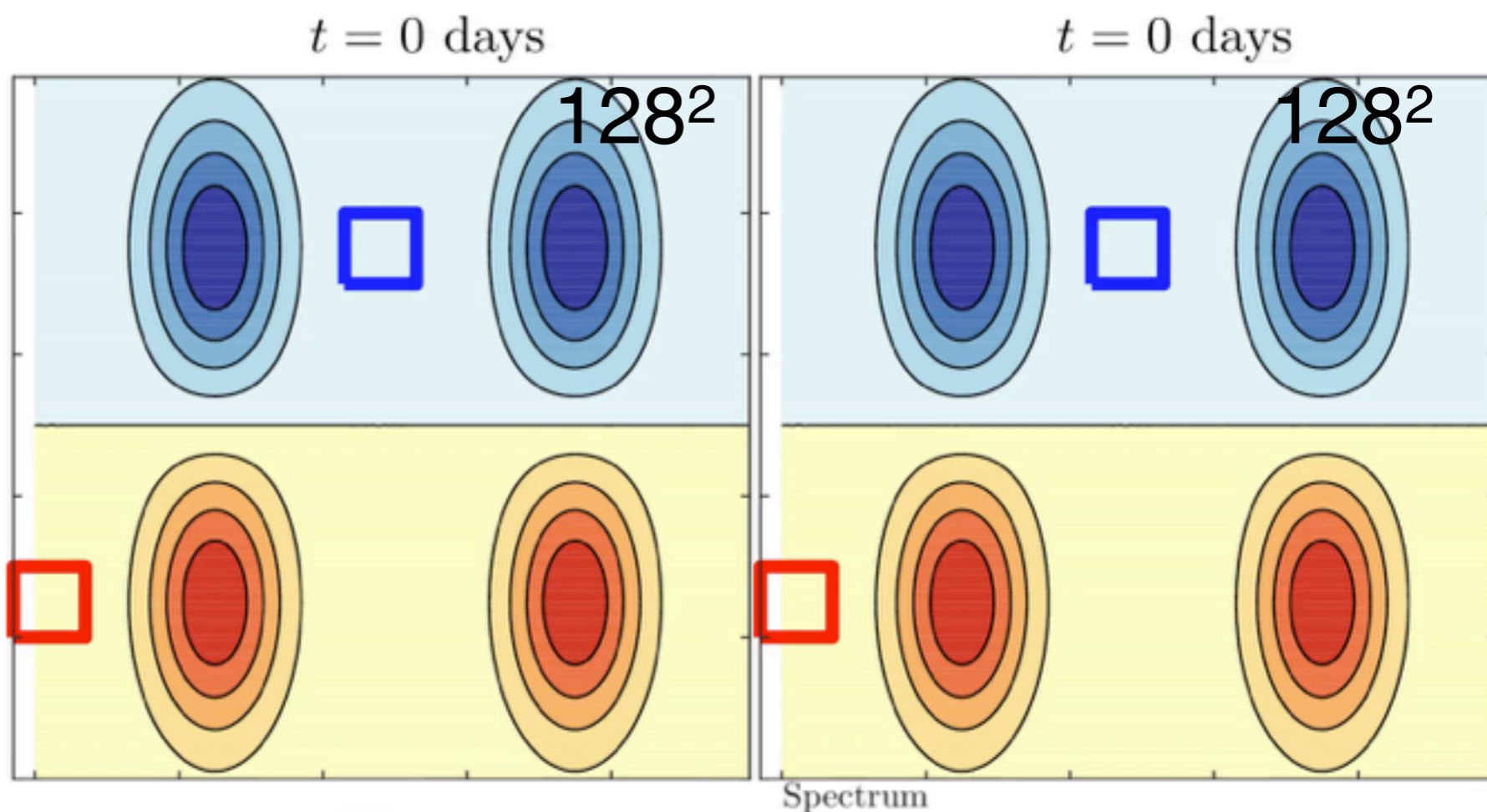
# Ensemble of simulations

with SQG MU

*Code available online*

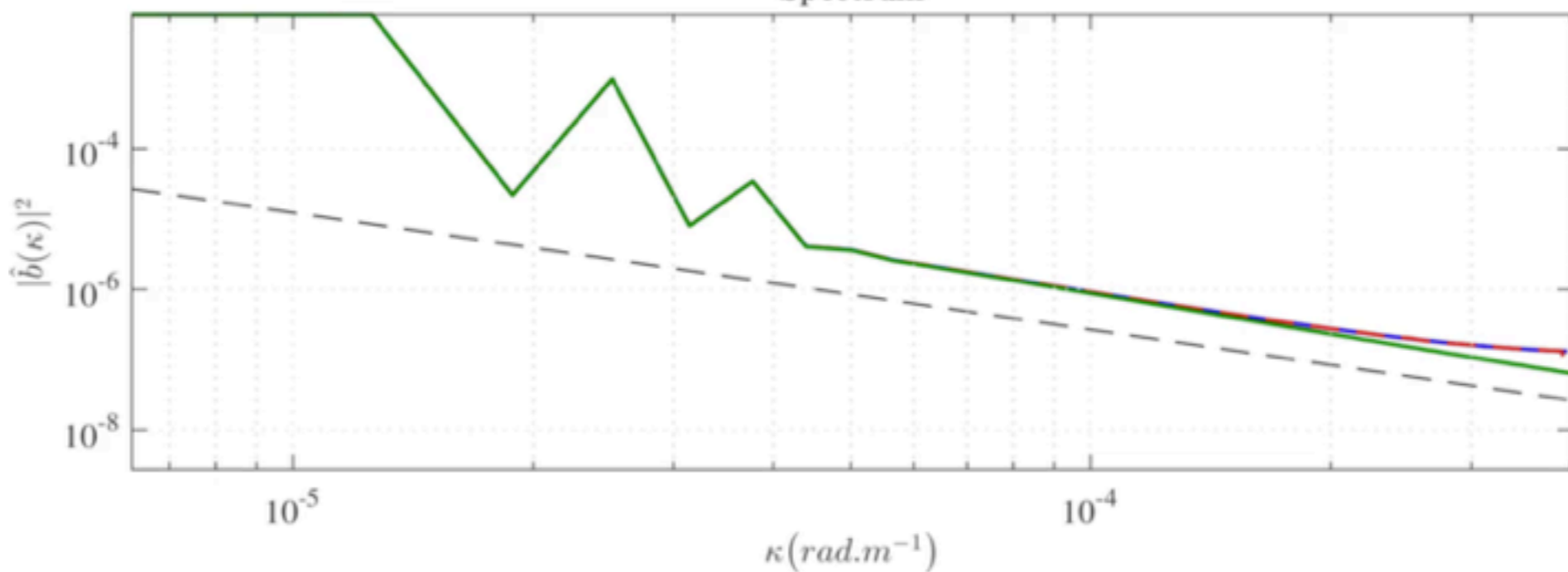


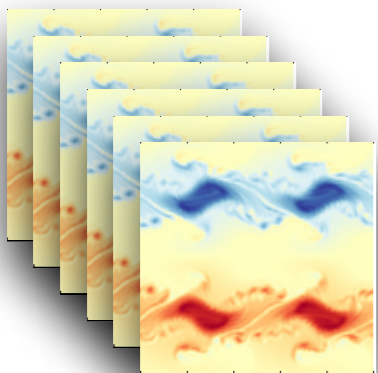
examples of realizations:



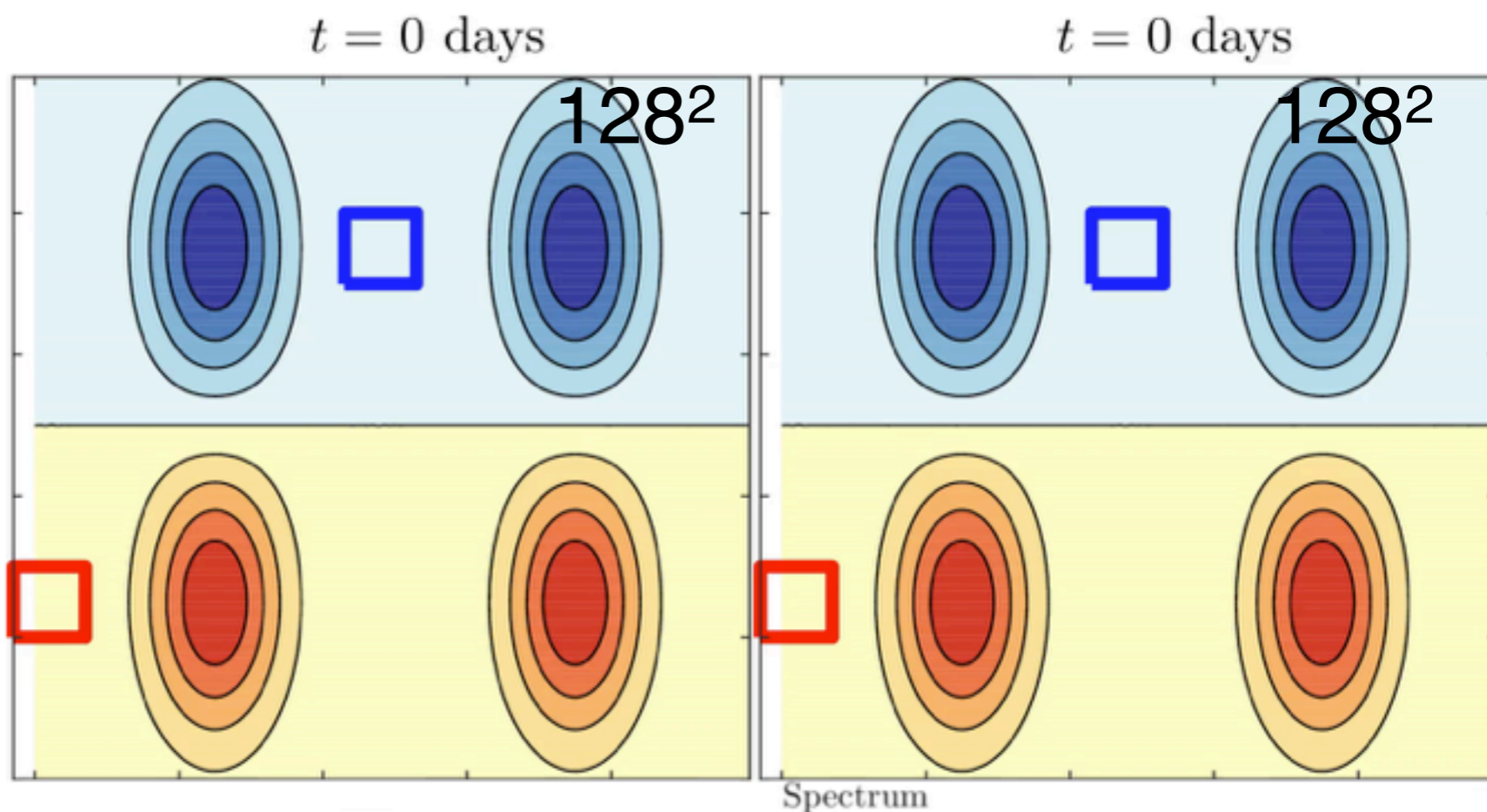
Spectrum

mean<sup>2</sup>/variance:



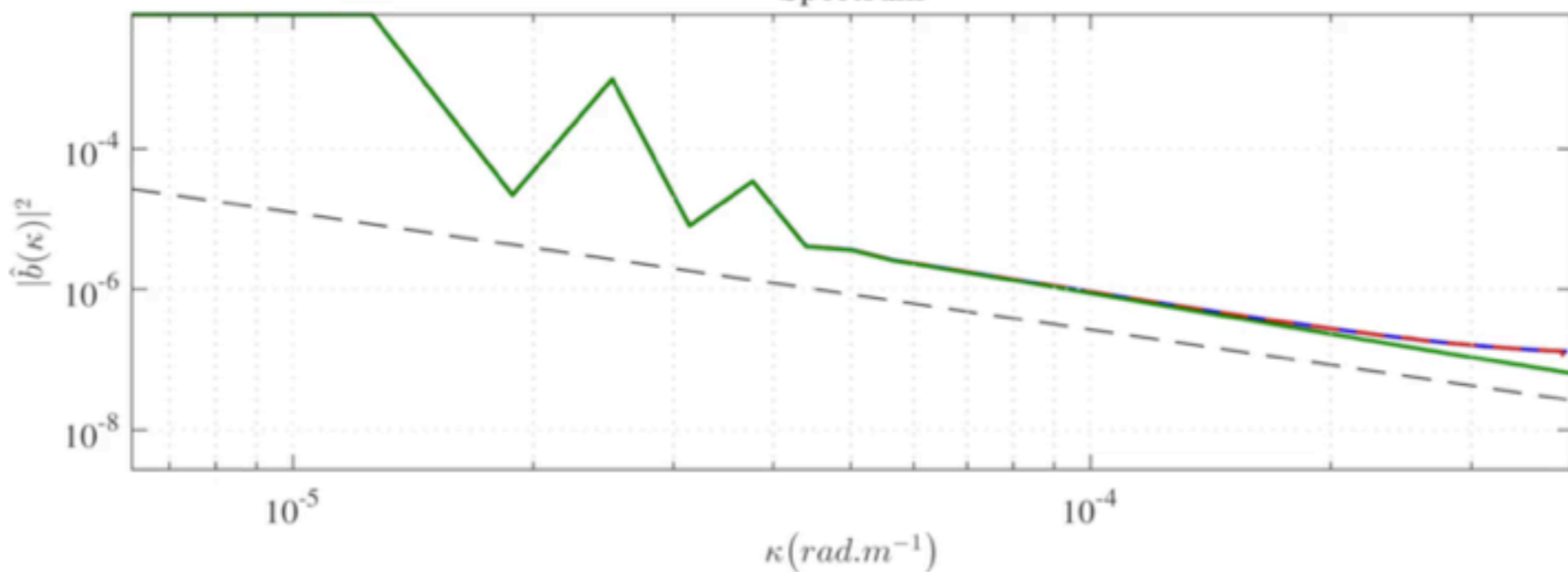


examples of realizations:

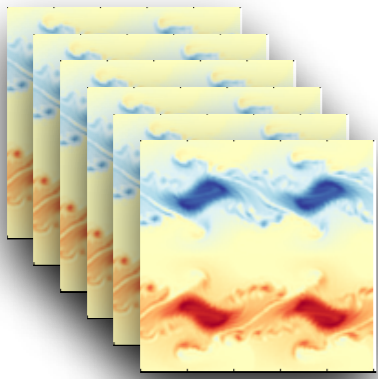


Spectrum

mean<sup>2</sup>/variance:





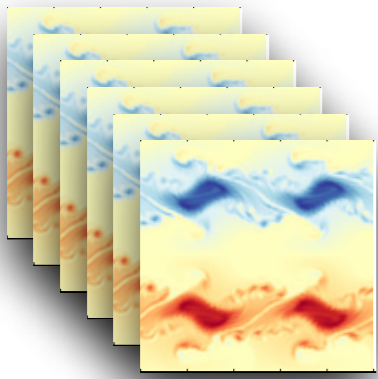


# Visualization of the ensemble

At a fixed time  $t$ ,

Principal Component Analysis (EOF)

$$\Theta^{(i)}(\mathbf{x}, t) \approx \hat{\mathbb{E}}(\Theta)(\mathbf{x}, t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t) \Psi_n(\mathbf{x}, t)$$



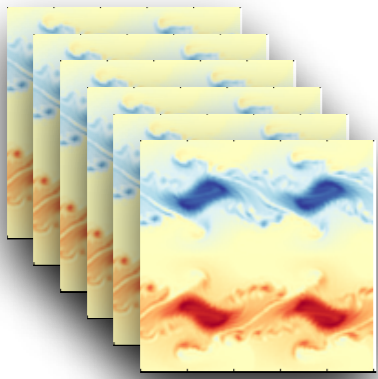
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$i^{\text{th}}$  realization



# Visualization of the ensemble

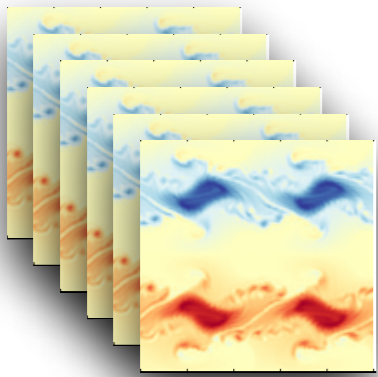
At a fixed time  $t$ ,

Principal Component Analysis (EOF)

$$\Theta^{(i)}(\mathbf{x}, t) \approx \hat{\mathbb{E}}(\Theta)(\mathbf{x}, t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t) \Psi_n(\mathbf{x}, t)$$

$i^{\text{th}}$  realization

Mean



# Visualization of the ensemble

At a fixed time  $t$ ,

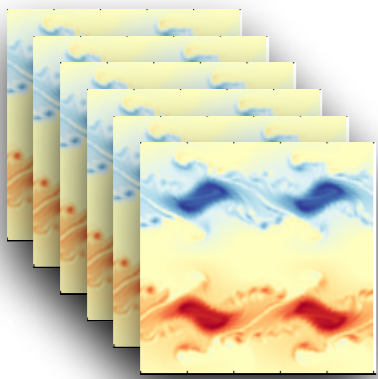
Principal Component Analysis (EOF)

$$\Theta^{(i)}(\mathbf{x}, t) \approx \hat{\mathbb{E}}(\Theta)(\mathbf{x}, t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t) \Psi_n(\mathbf{x}, t)$$

$i^{\text{th}}$  realization

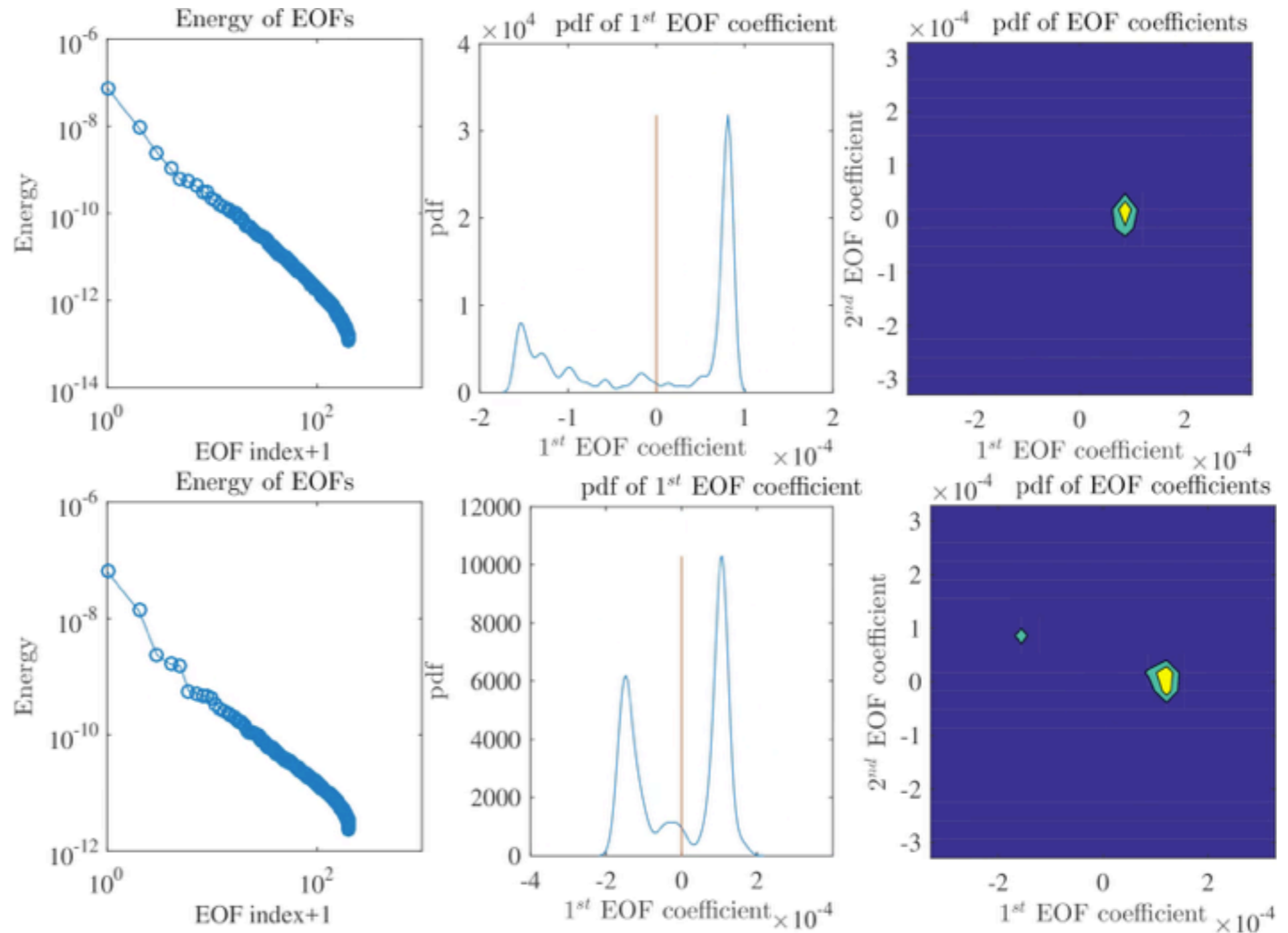
Mean

(random) EOF  
coefficient

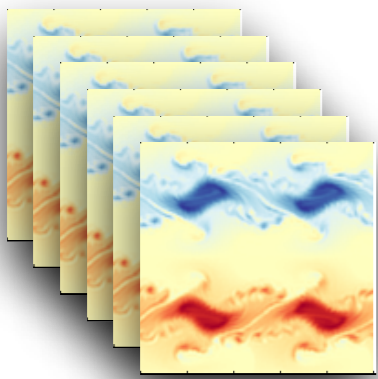


# Visualization of the ensemble (200 realizations)

Random initial conditions

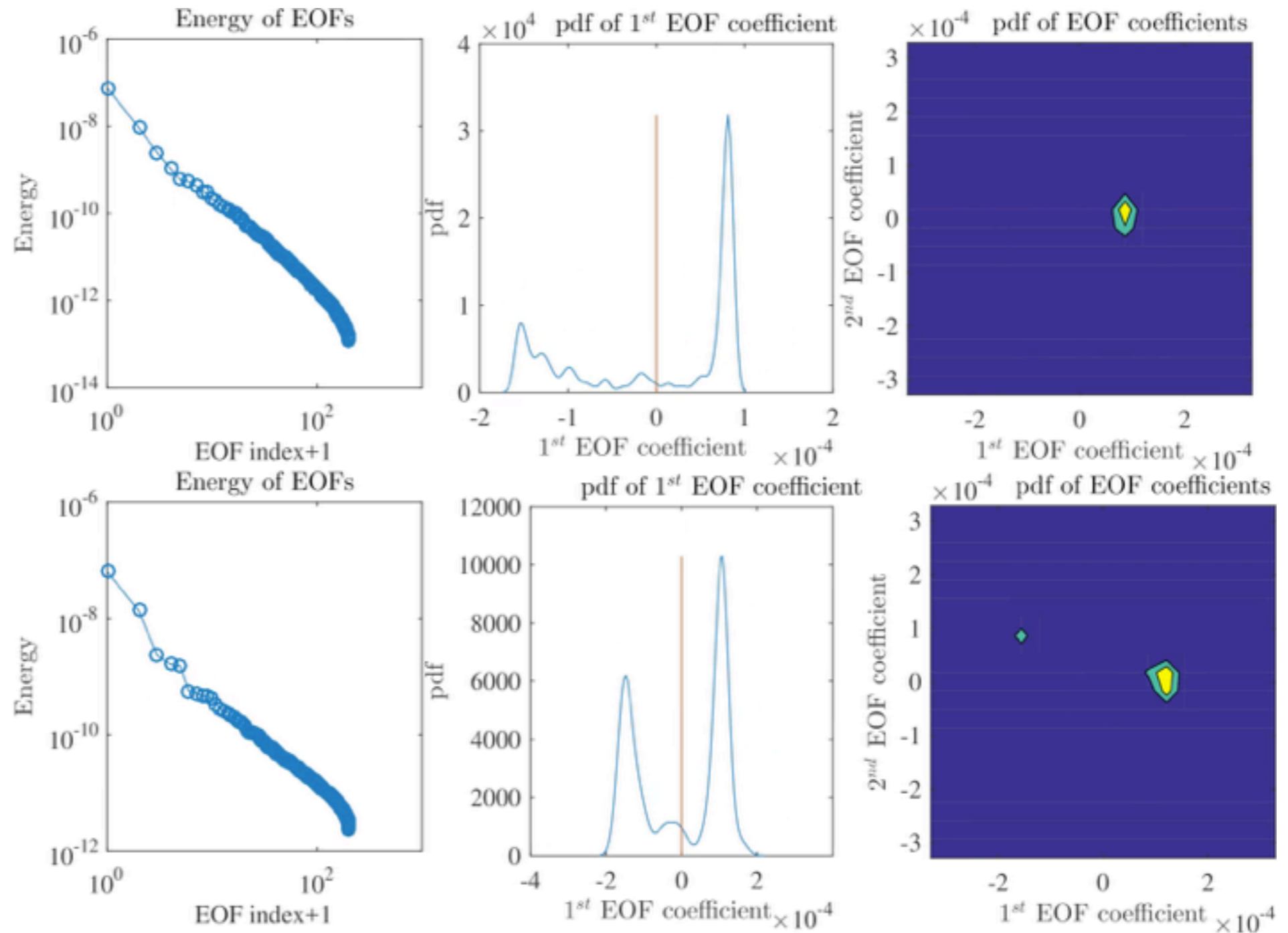


Under location uncertainty

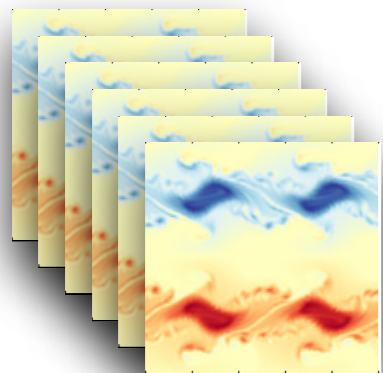


# Visualization of the ensemble (200 realizations)

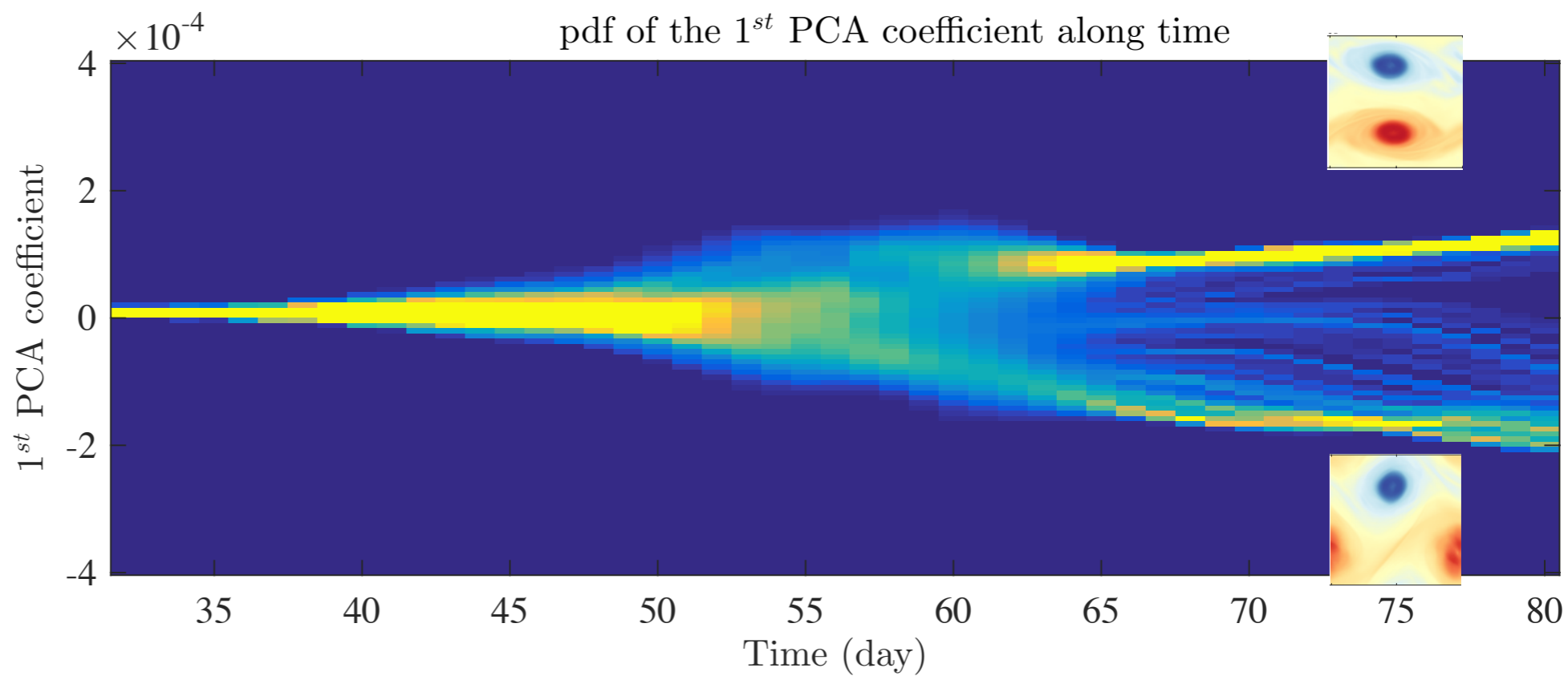
Random initial conditions



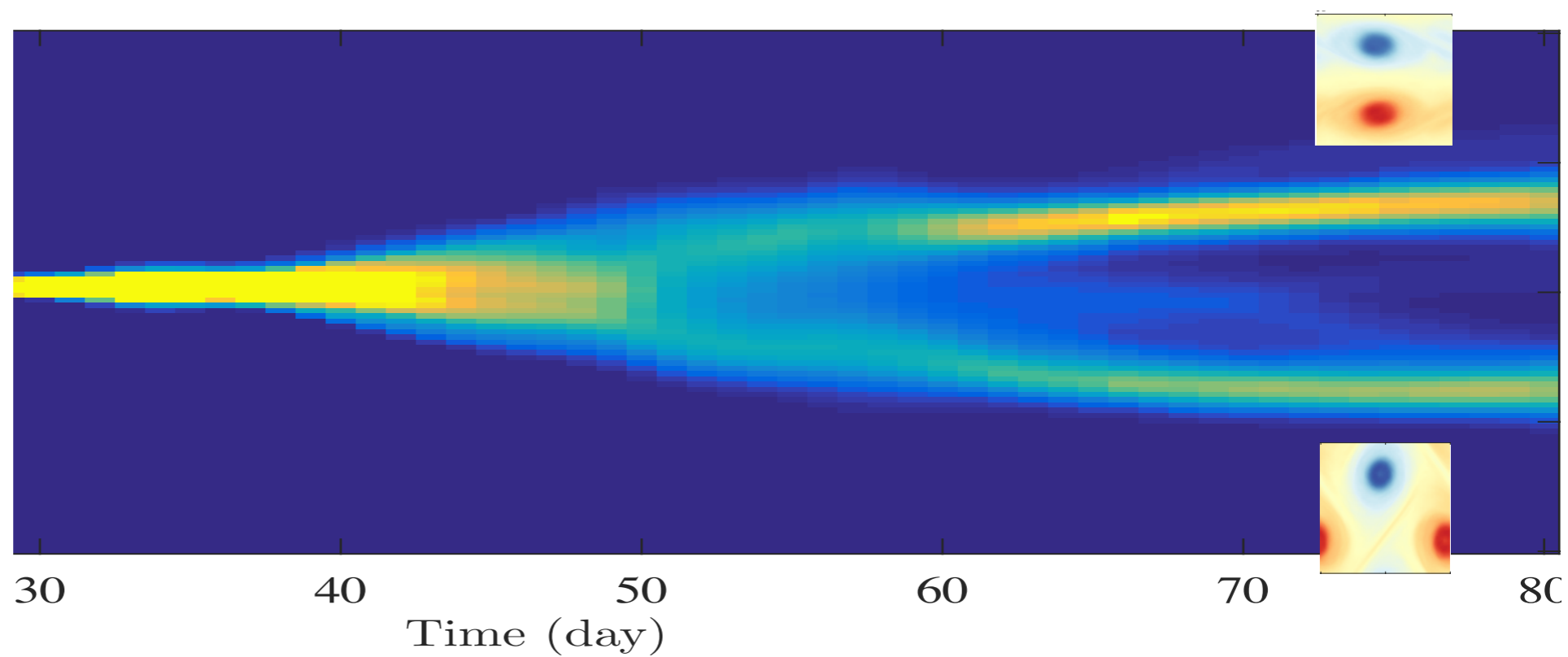
Under location uncertainty



Random  
initial  
conditions



Under  
location  
uncertainty



# Conclusion



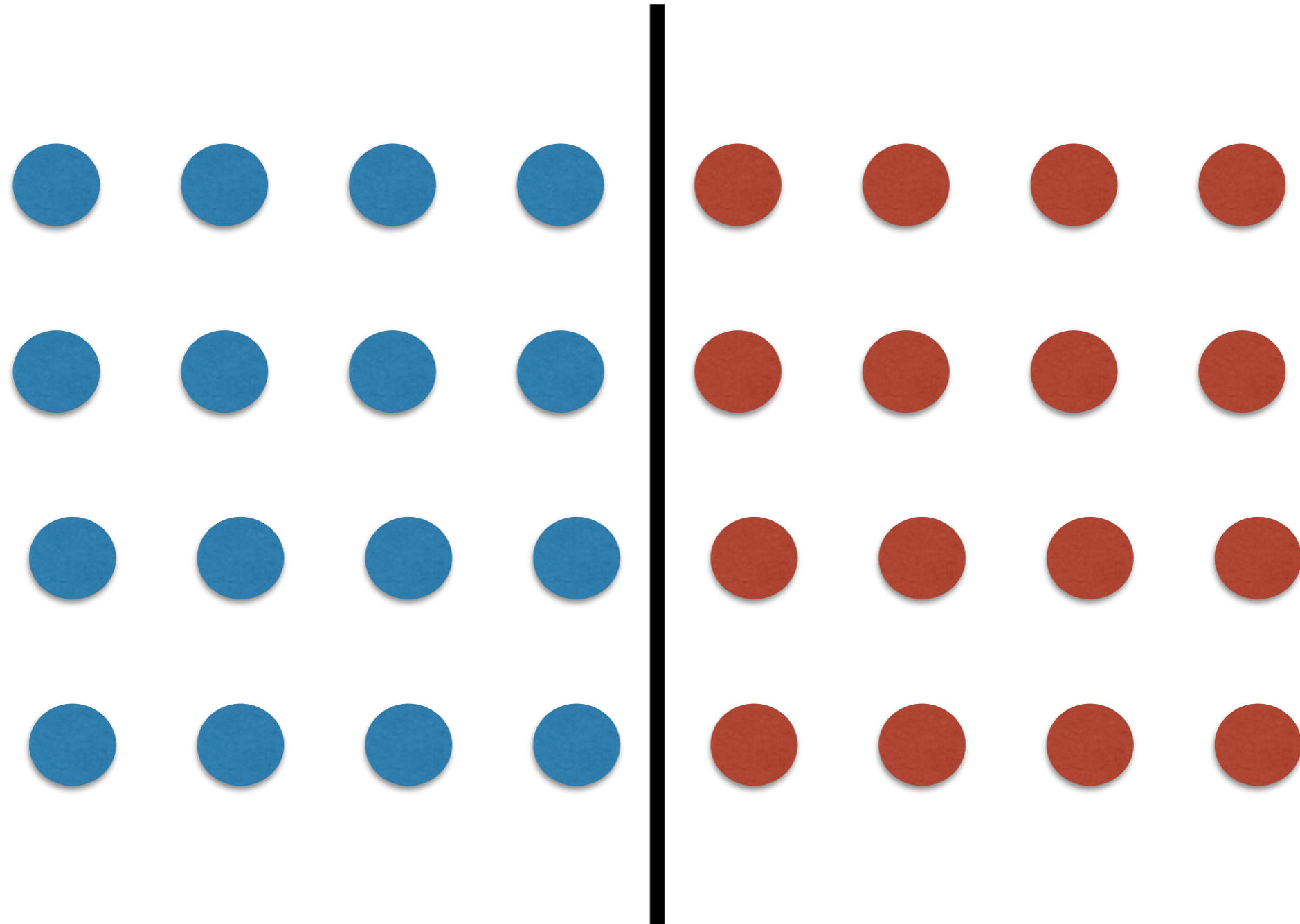
# Conclusion

- Random transport applicable to any dynamics
- Link (inhomogeneous) diffusion, drift correction and multiplicative noise
- Predict likely scenarios with few realizations
- Other results:
  - Better small scales
  - Estimate positions and amplitudes of errors
  - Extreme events
  - Additional physical information

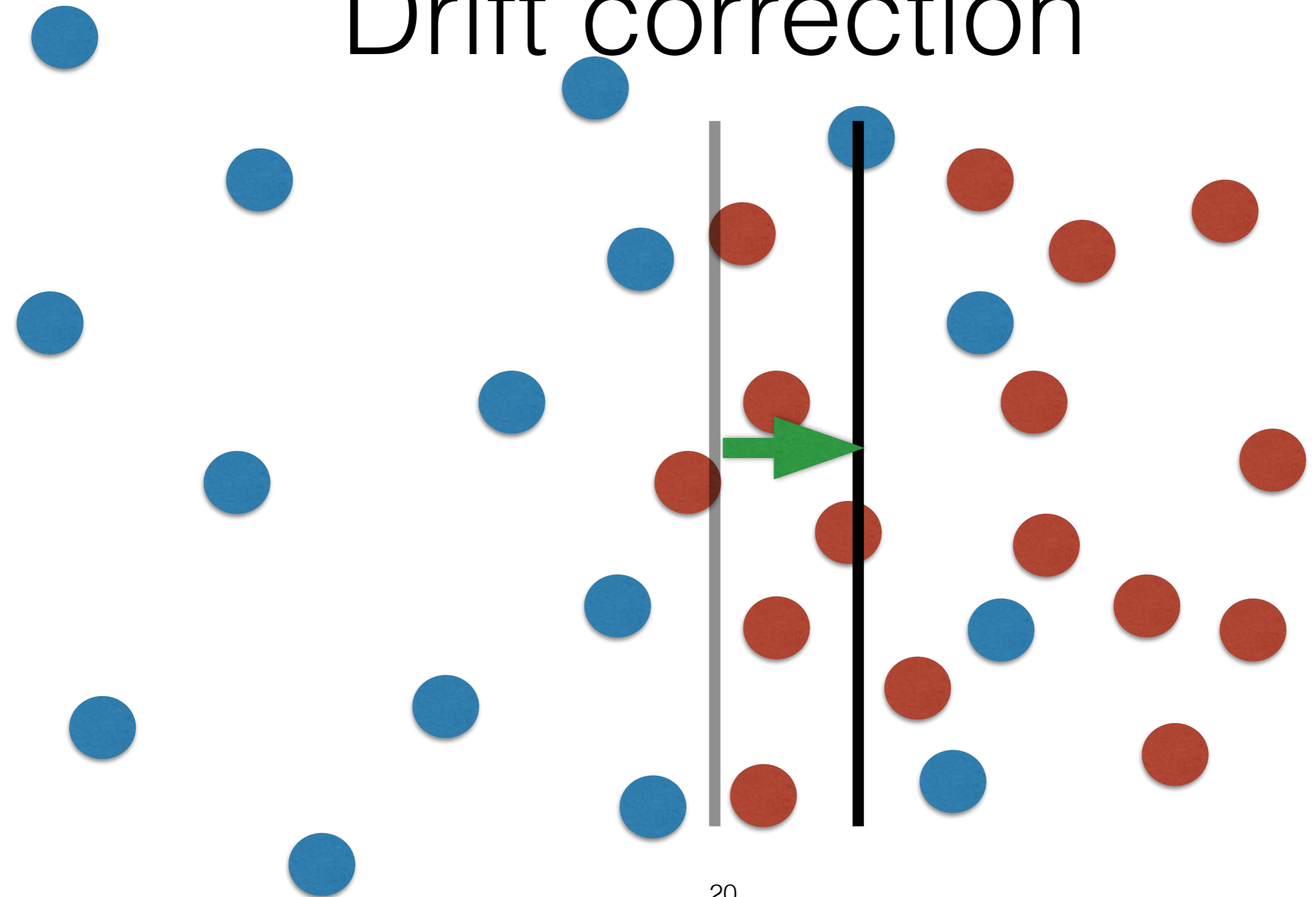
Thank you for your attention

Code SQG MU:  
link from Fluminance website - V. Resseguier

# Drift correction

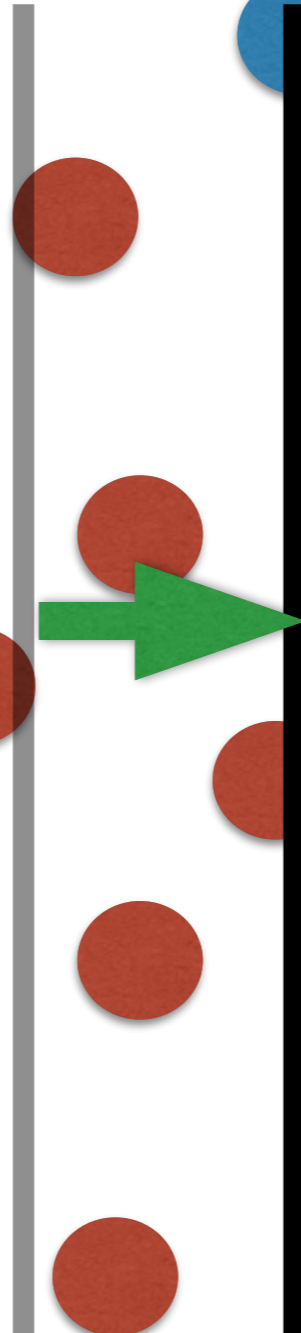


# Drift correction



# Drift correction

$$w^* = w - \frac{1}{2} (\nabla \cdot \mathbf{a})^T$$



# SQG under Moderate Uncertainty

SQG MU

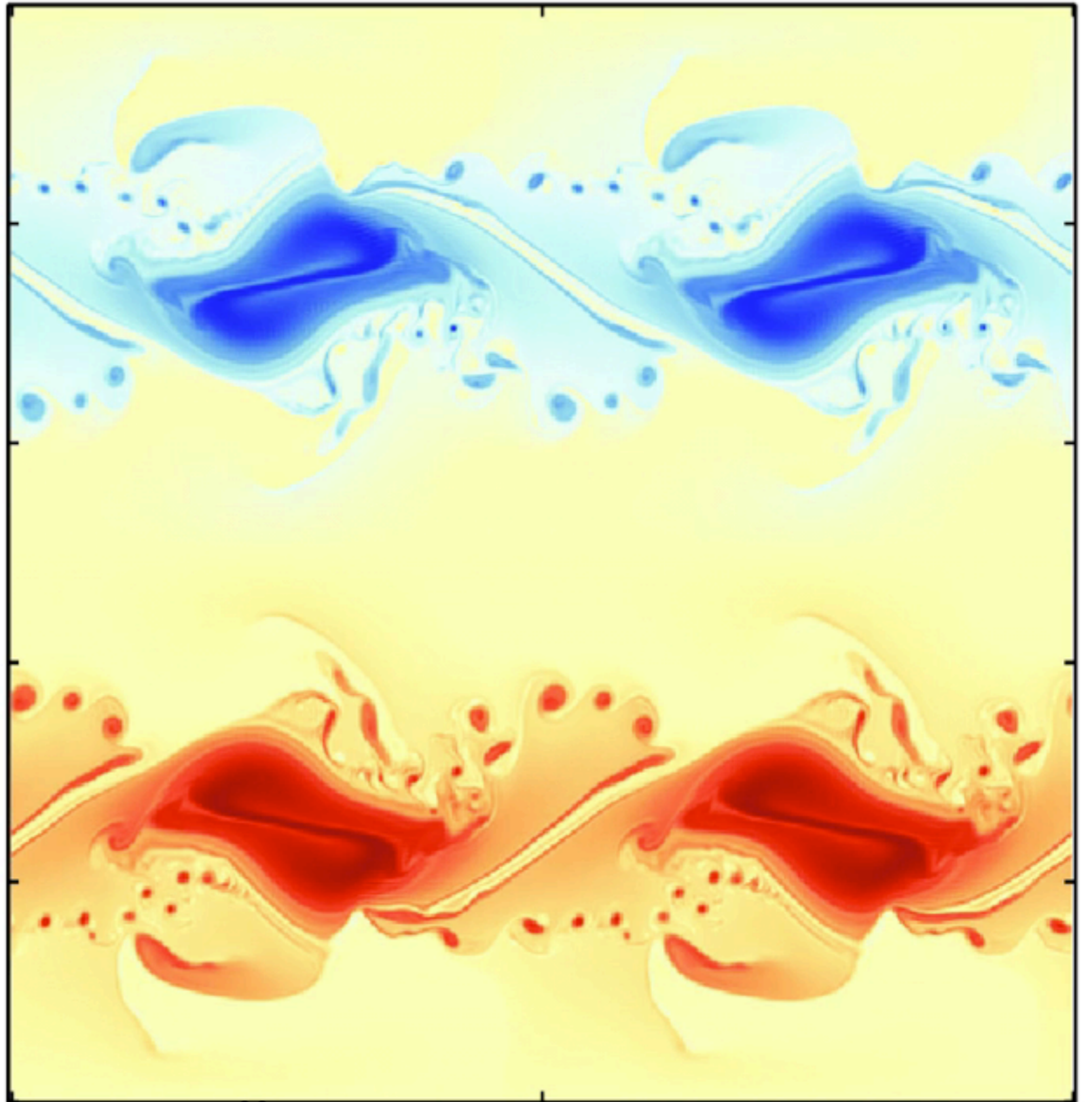
*Code available online*

$t = 17$  days

Reference flow:  
deterministic

SQG

512 x 512





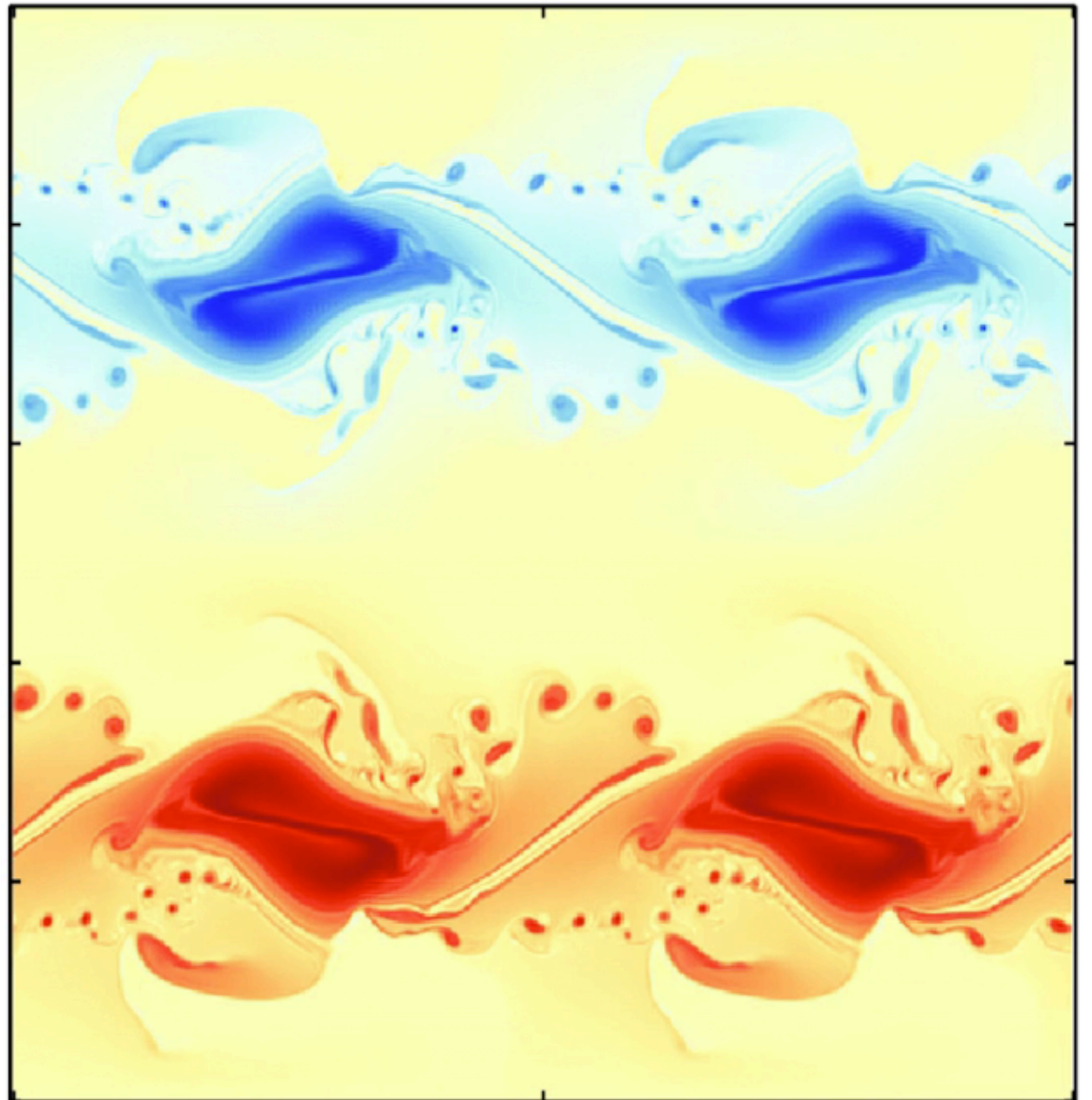
$t = 17$  days

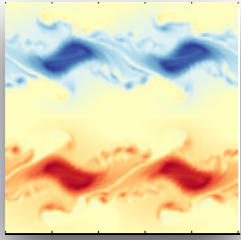
Reference flow:

deterministic

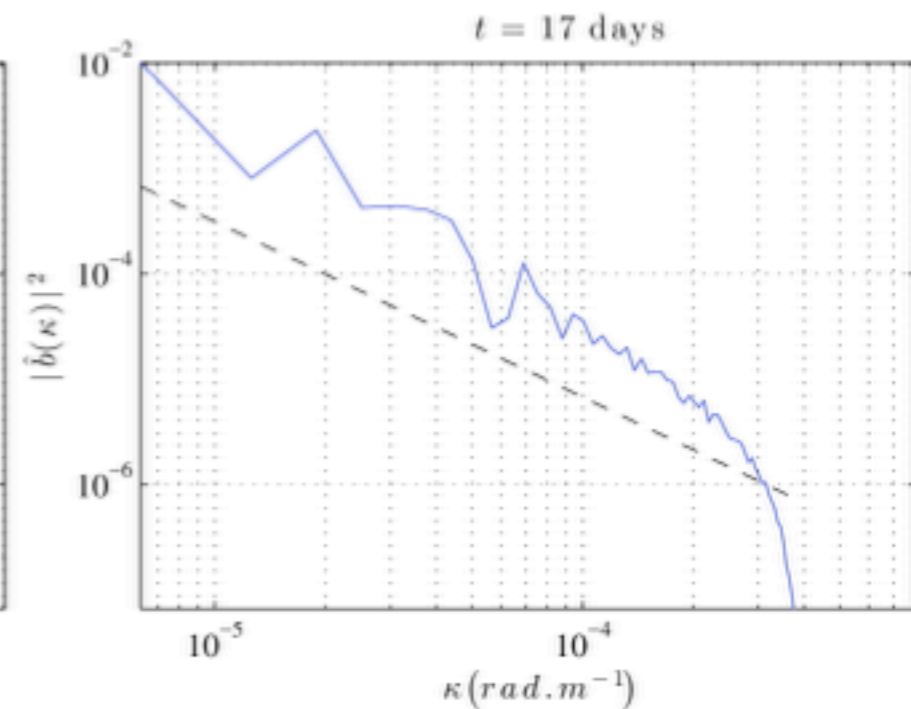
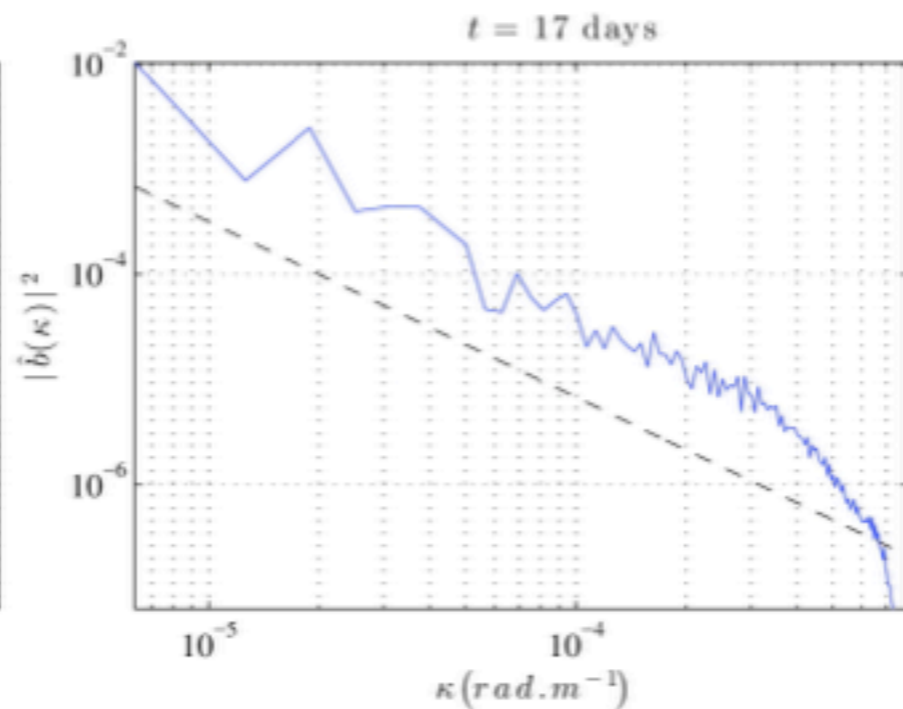
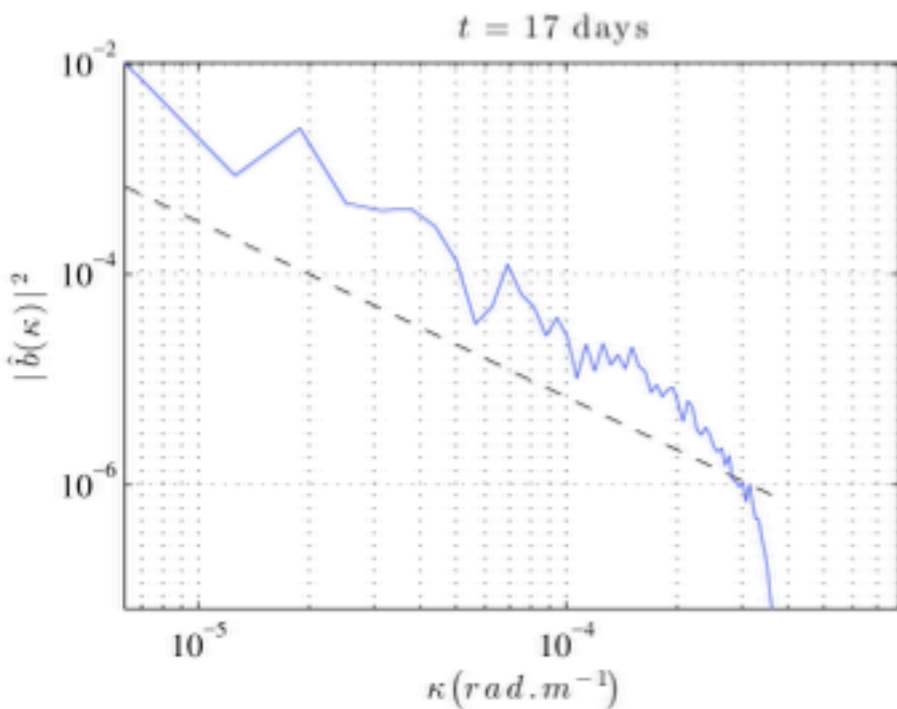
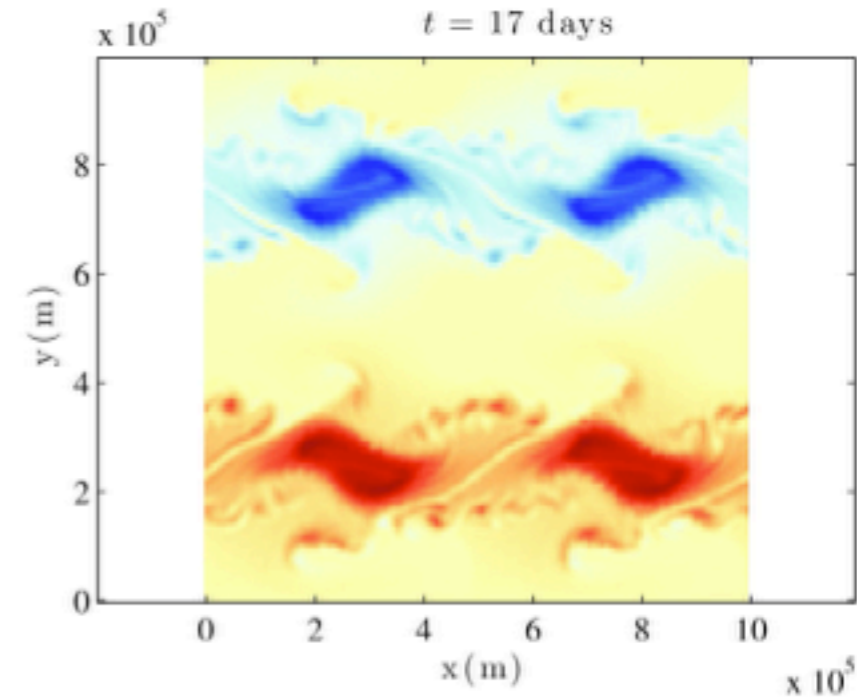
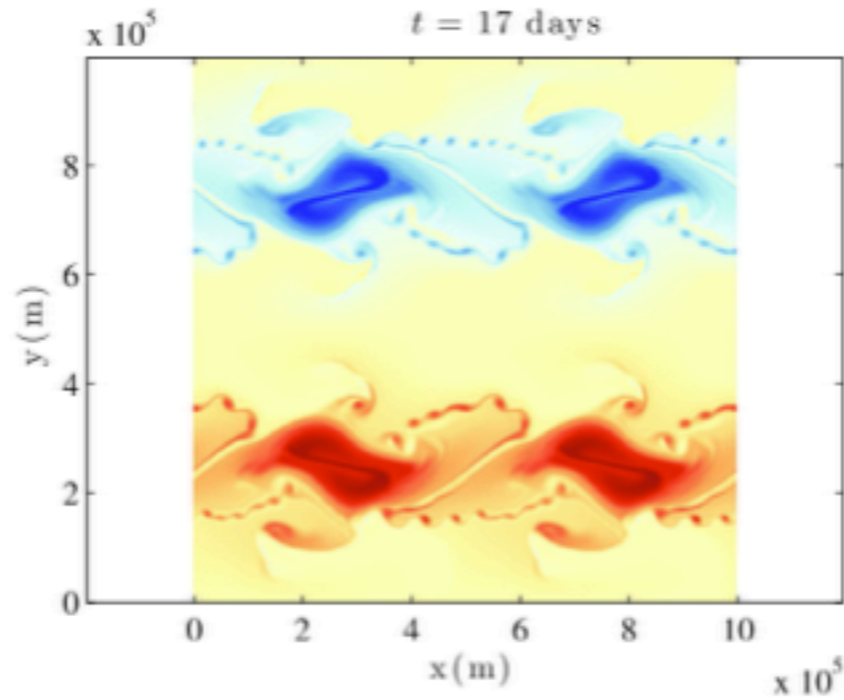
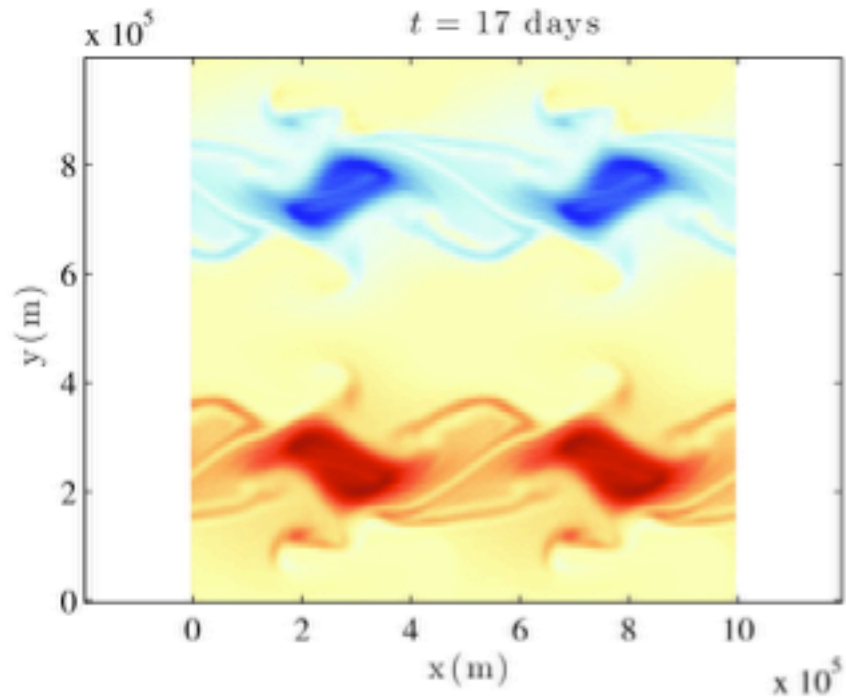
SQG

512 x 512





# One realization

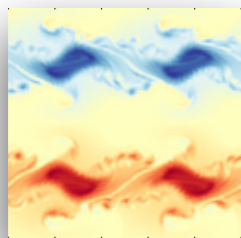


**Deterministic 128x128**

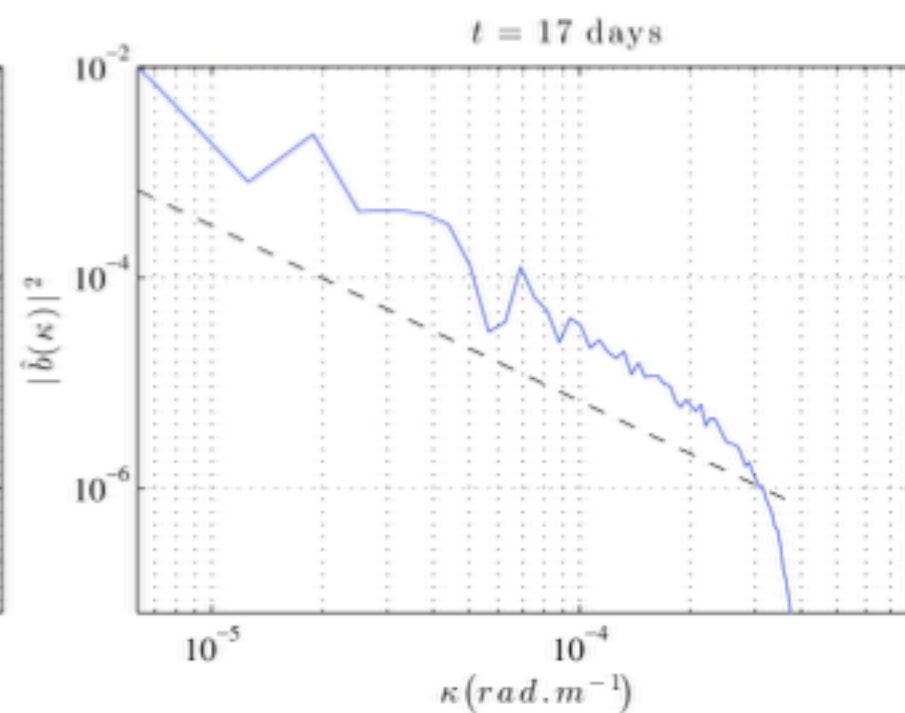
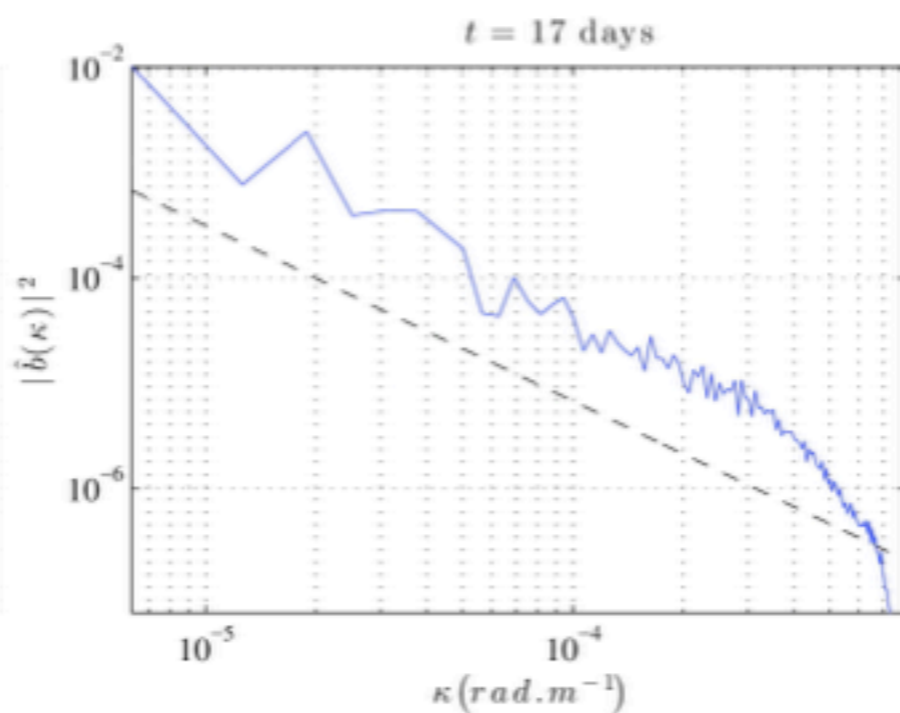
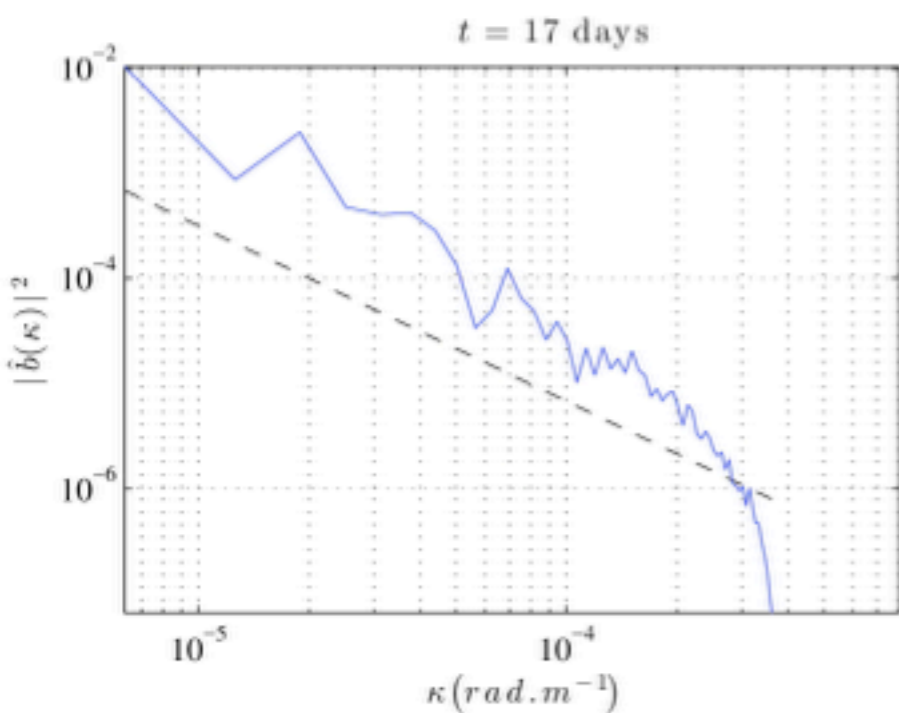
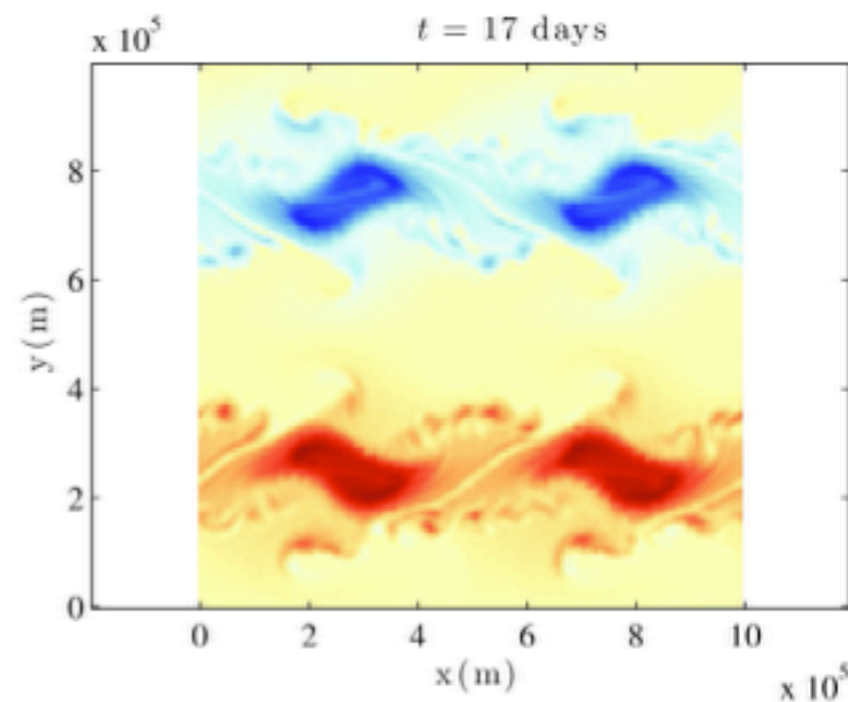
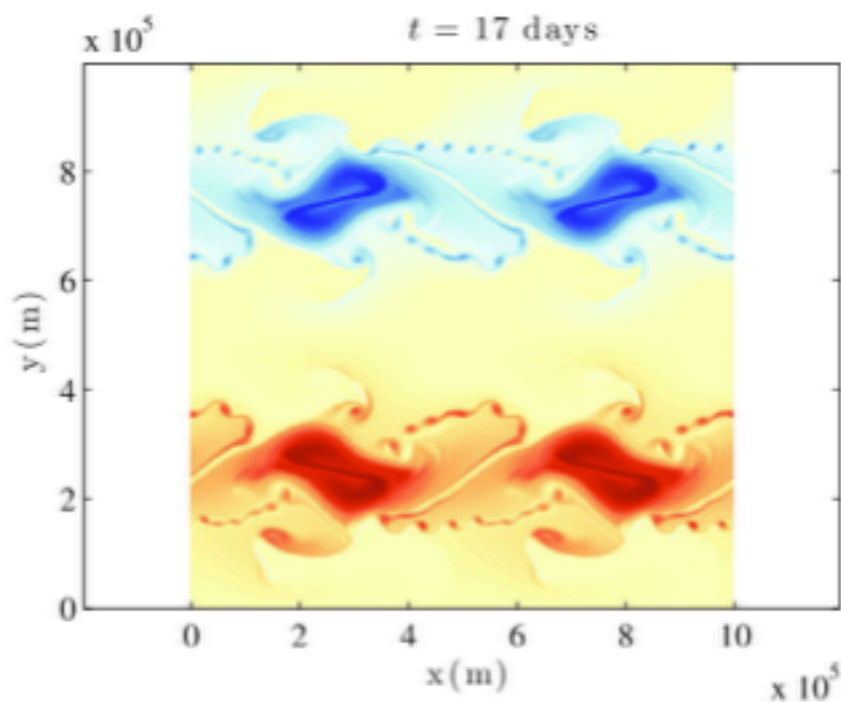
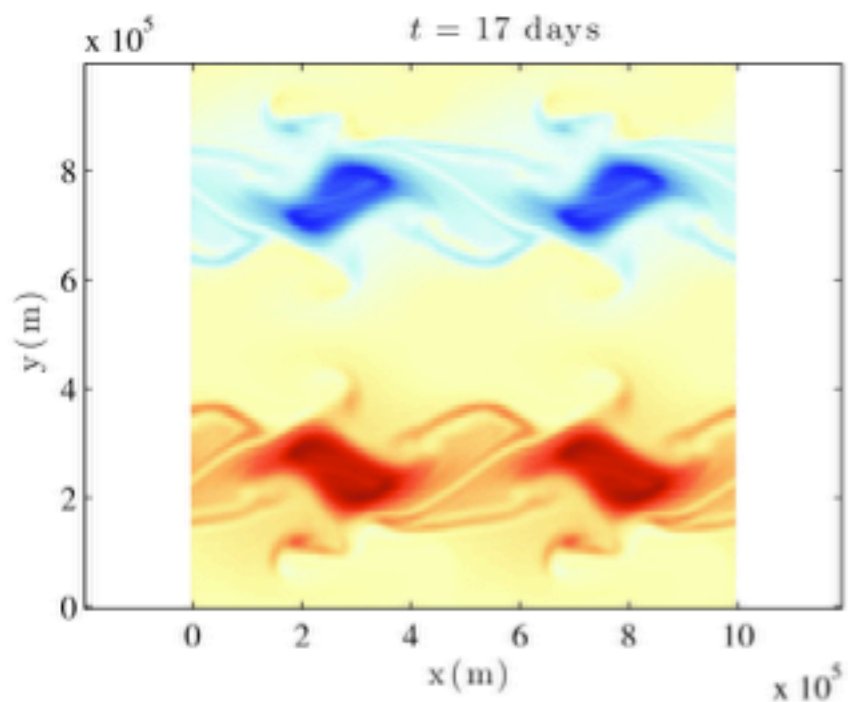
**Deterministic 512x512**

**Stochastic 128x128**





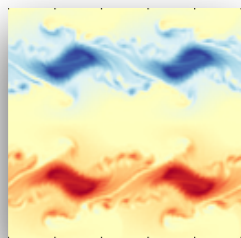
# One realization



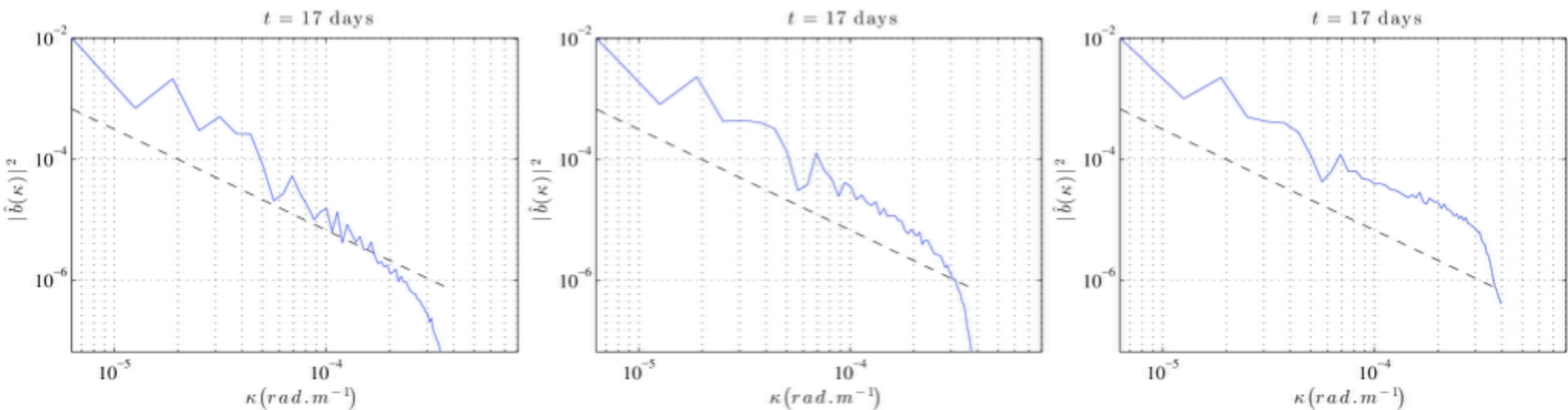
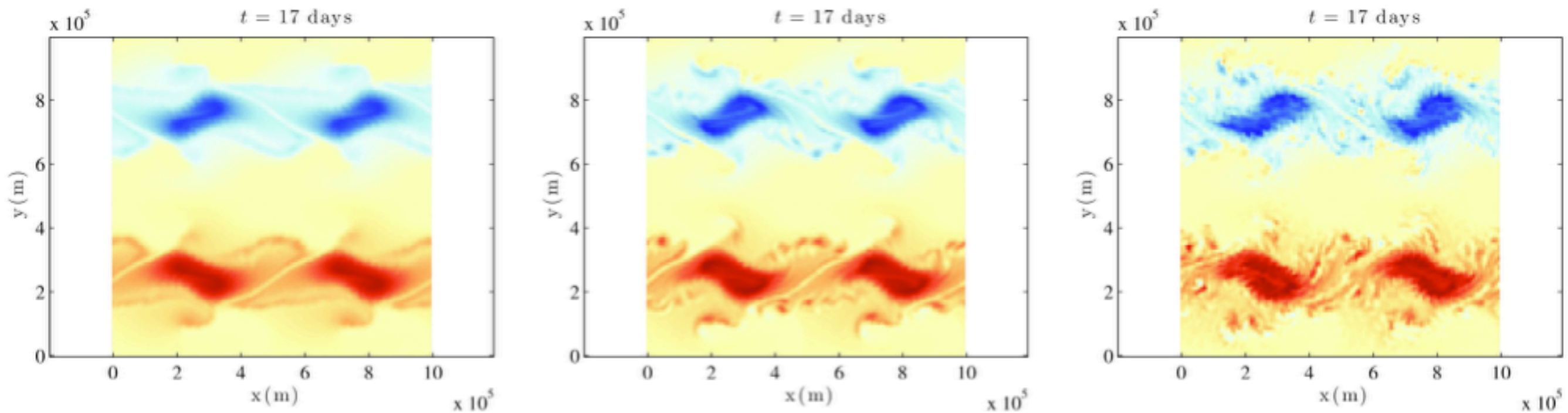
**Deterministic 128x128**

**Deterministic 512x512**

**Stochastic 128x128**



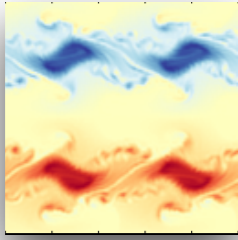
# One realization



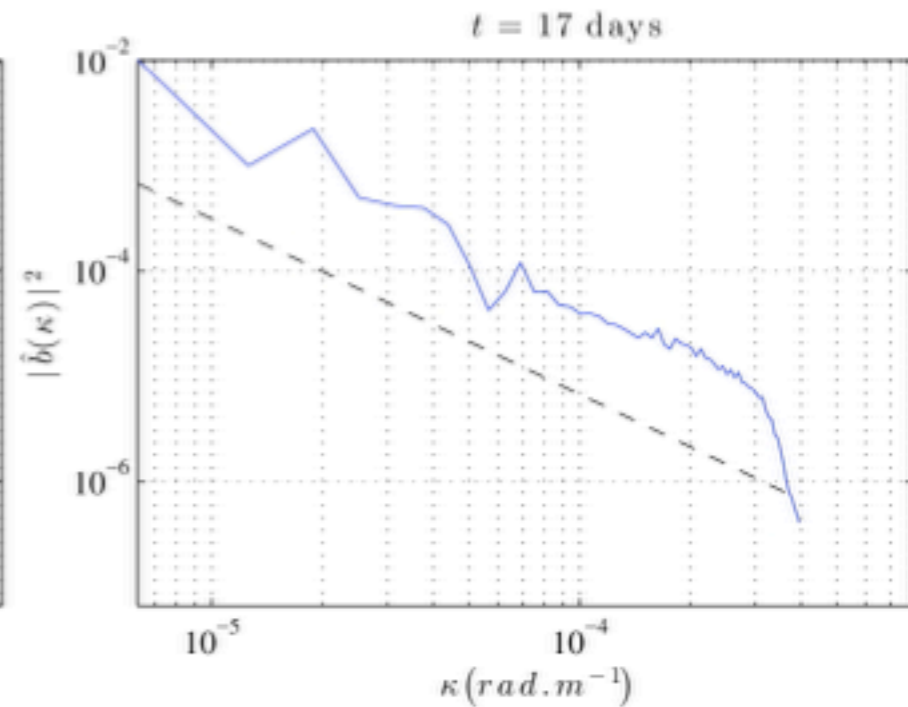
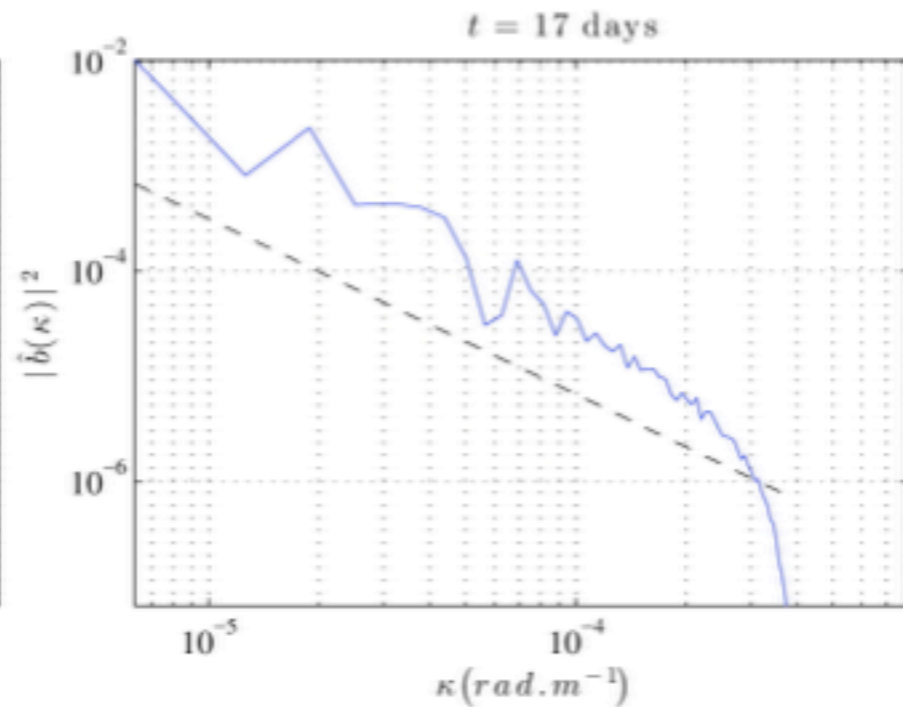
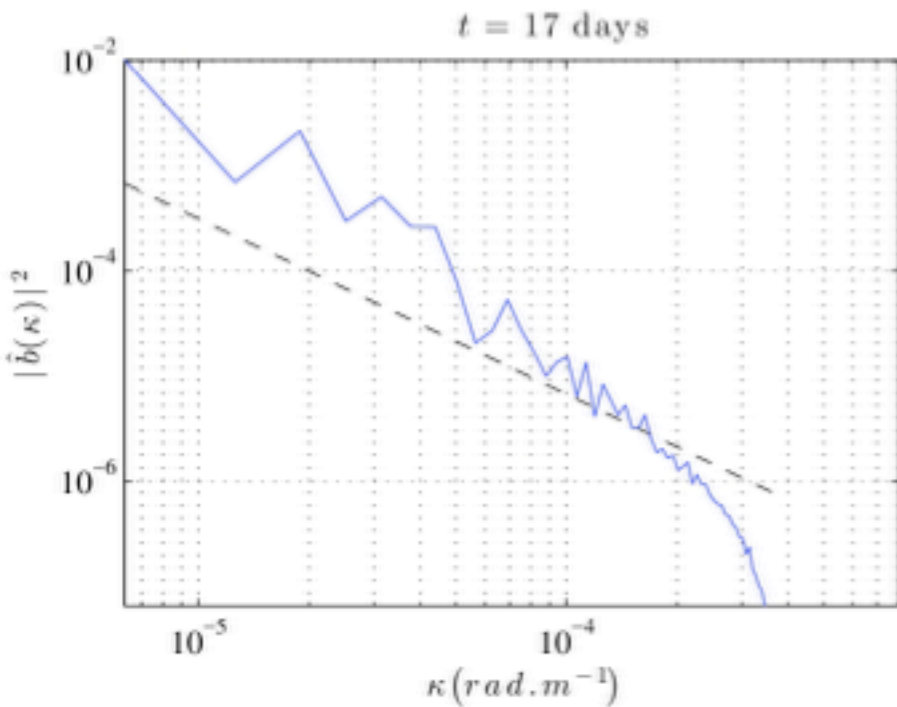
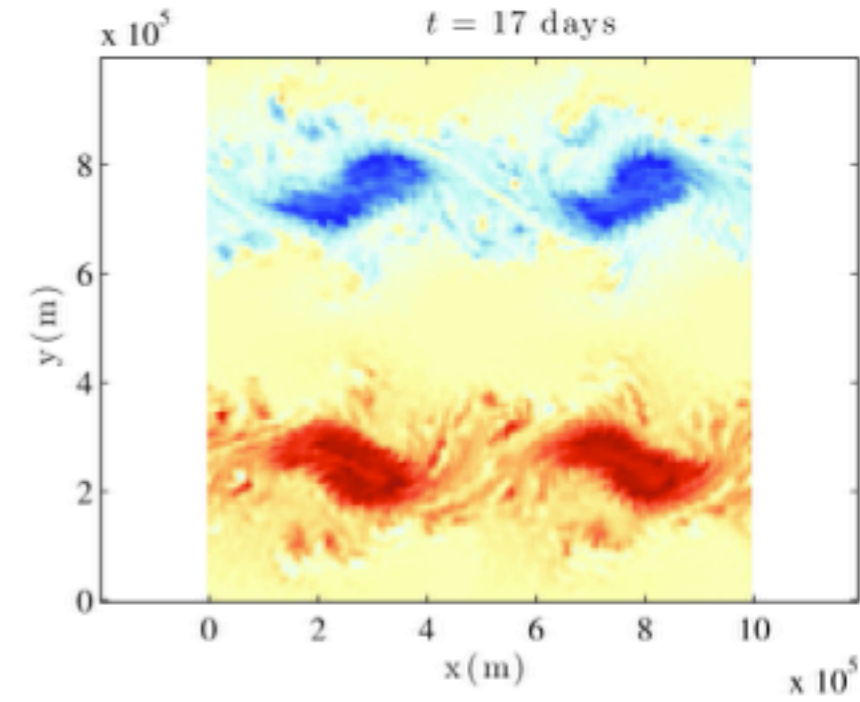
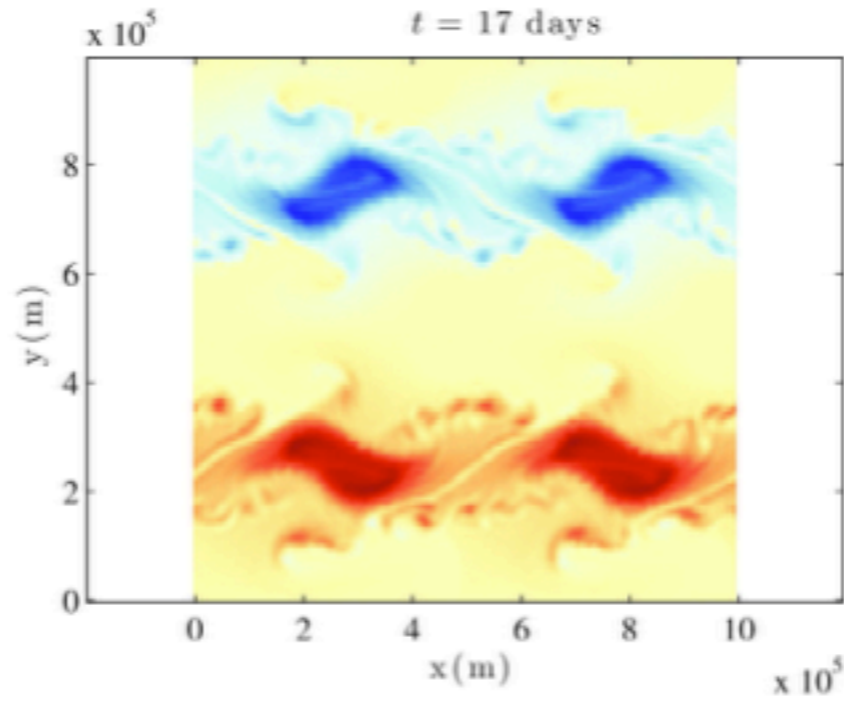
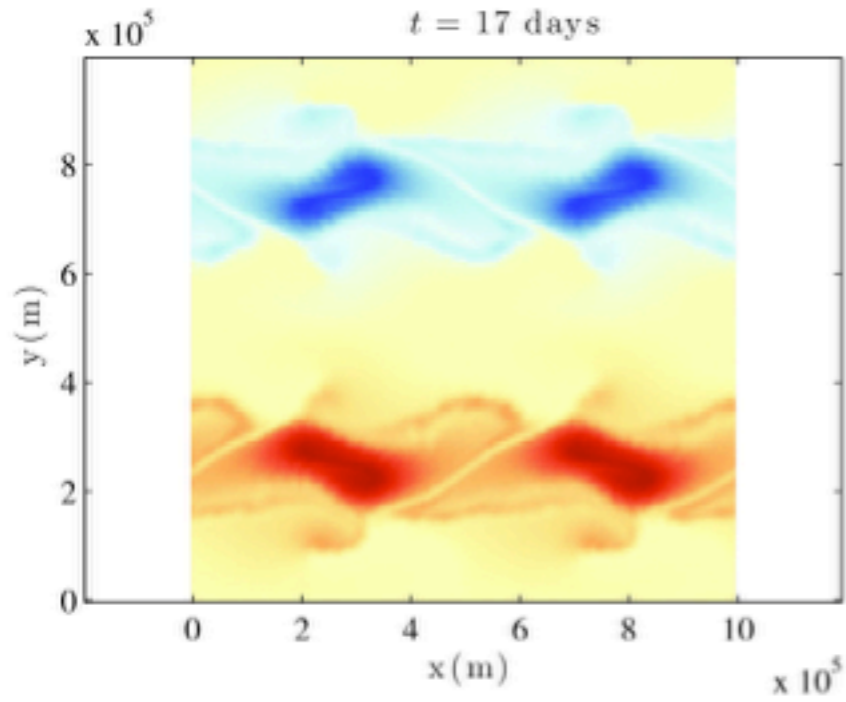
**Lower noise**

**Our model**

**Larger noise**



# One realization

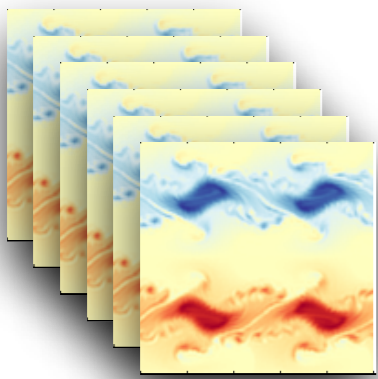


**Lower noise**

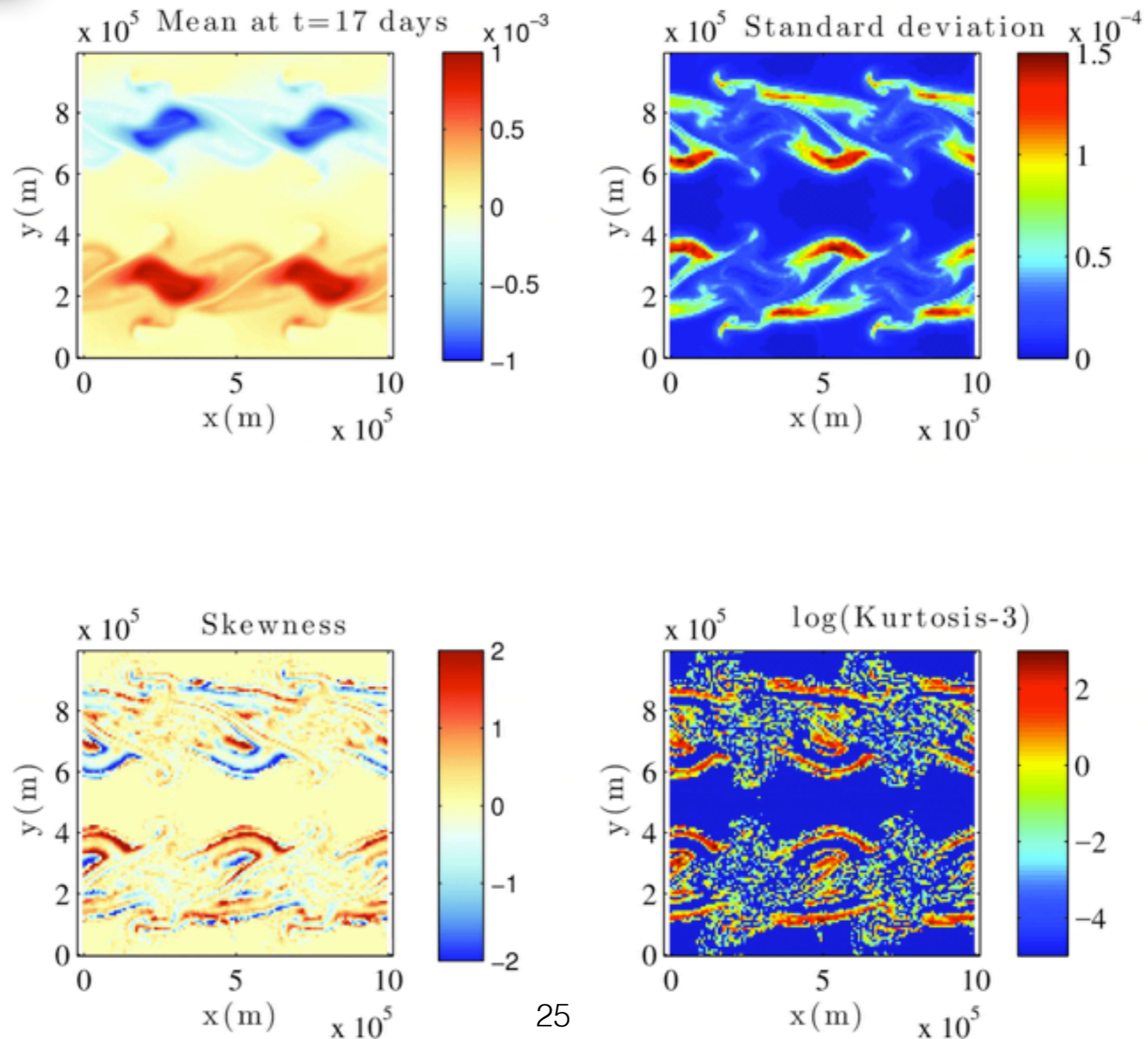
**Our model**

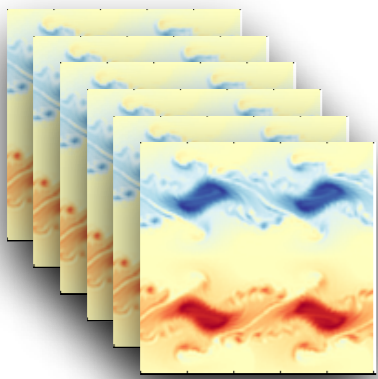
**Larger noise**



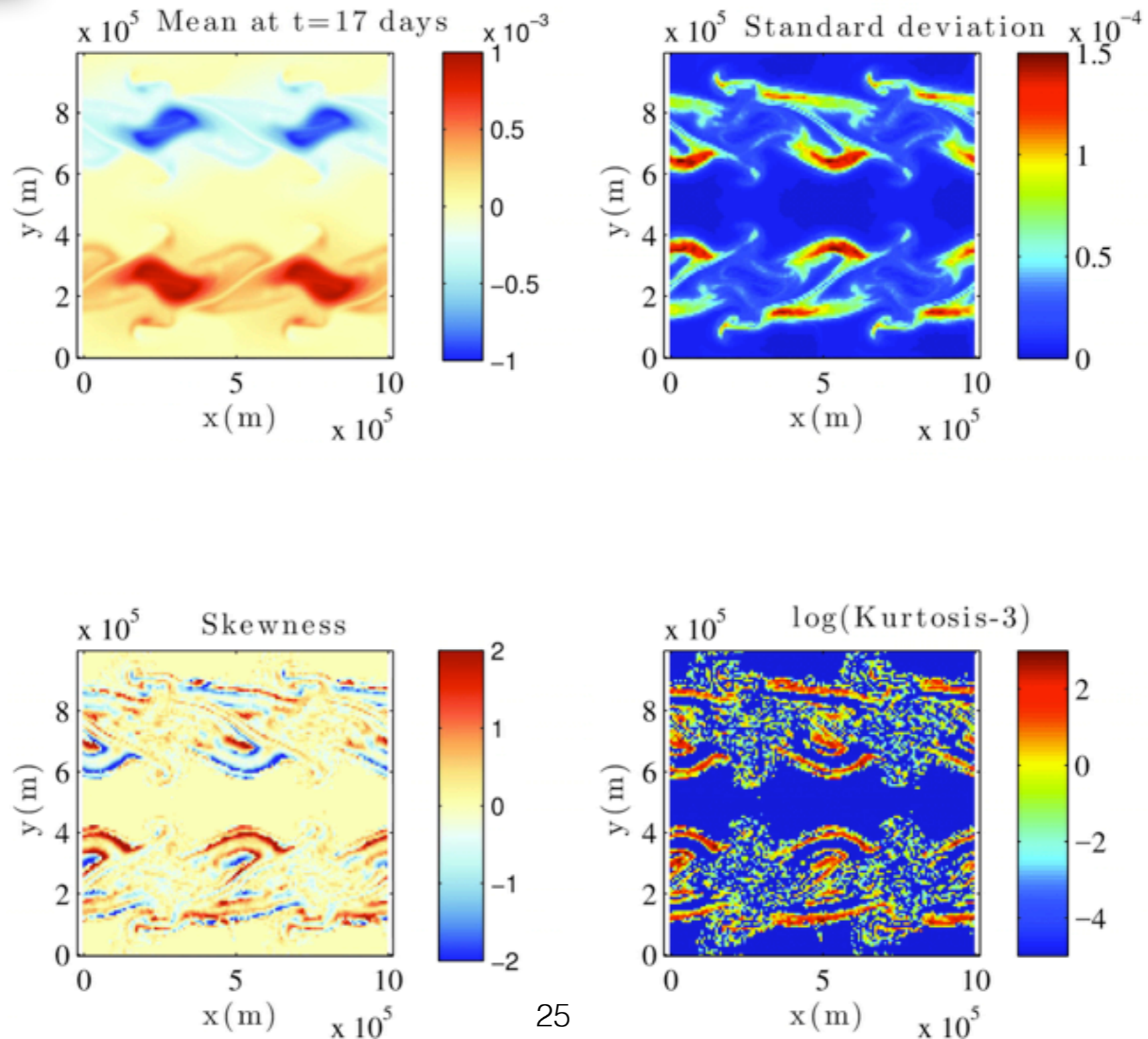


# Ensemble

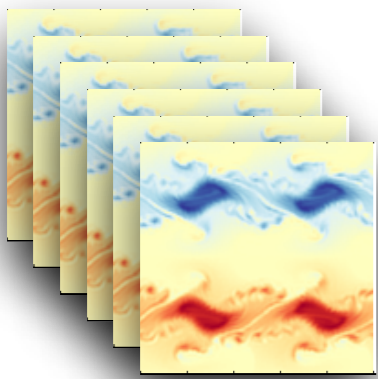




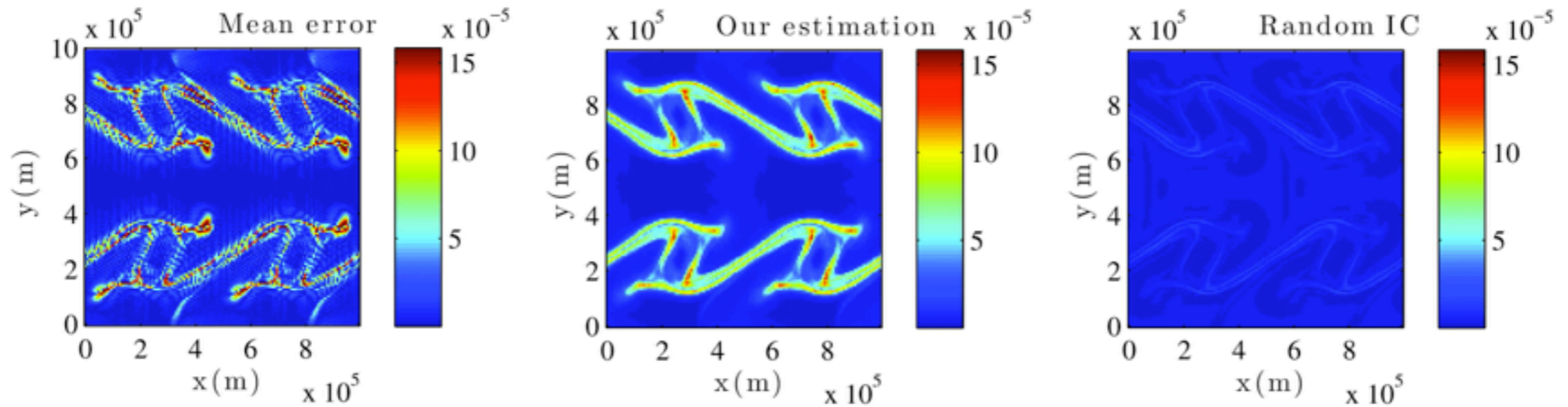
# Ensemble



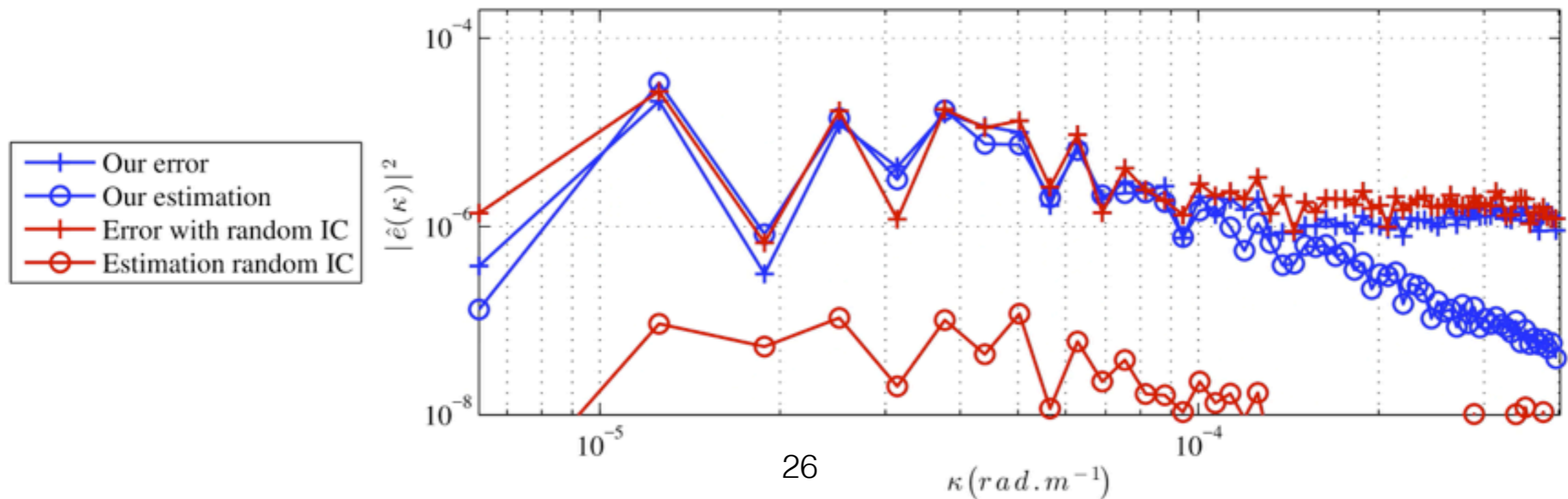


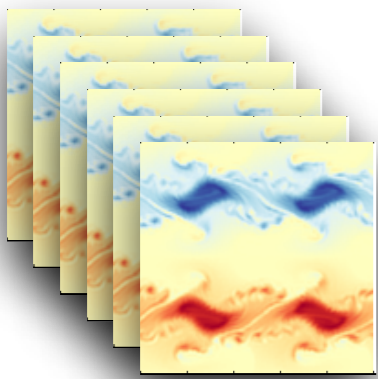


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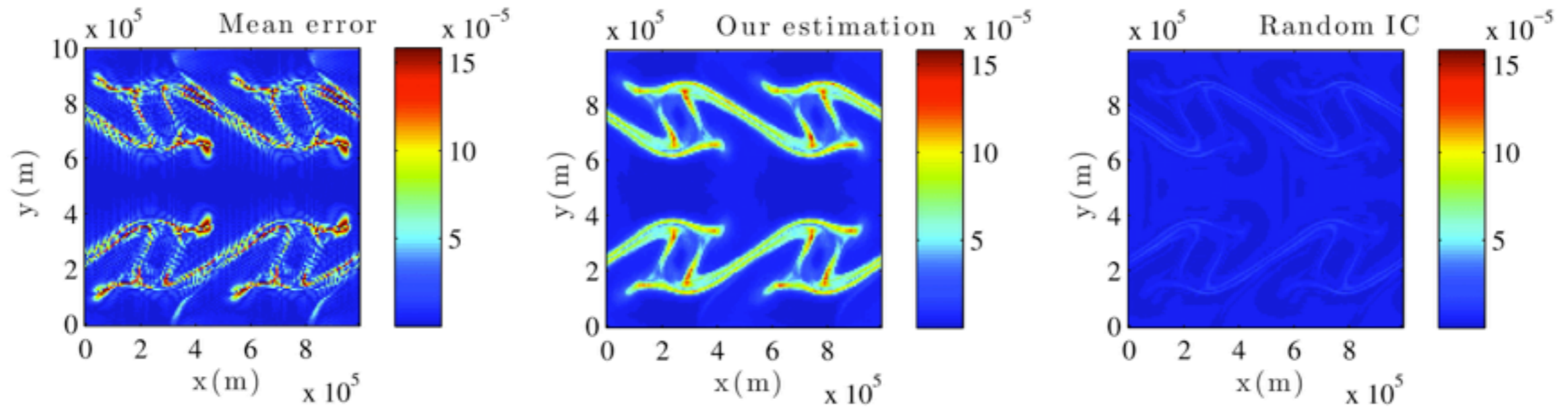


Spectrum of the errors and its estimation at  $t=12$  days

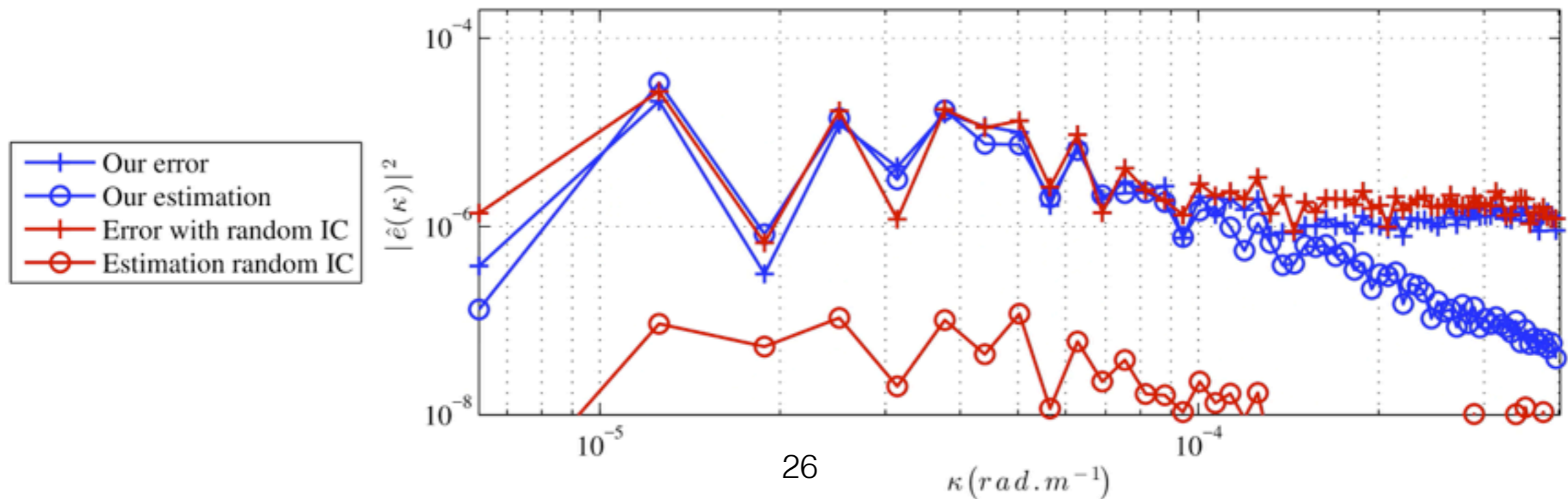




# Ensemble



Spectrum of the errors and its estimation at  $t=12$  days



# Likely SQG scenarios

tracked by SQG MU

# SQG under Strong Uncertainty

SQG SU

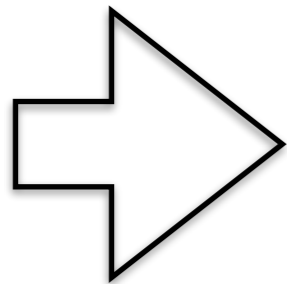
# Mesoscale divergence

Geostrophic balance

$$\mathbf{f} \times \mathbf{u} = -\frac{1}{\rho_b} \nabla p'$$

Horizontal  
Diffusion

$$+ \frac{a}{2} \Delta \mathbf{u}$$

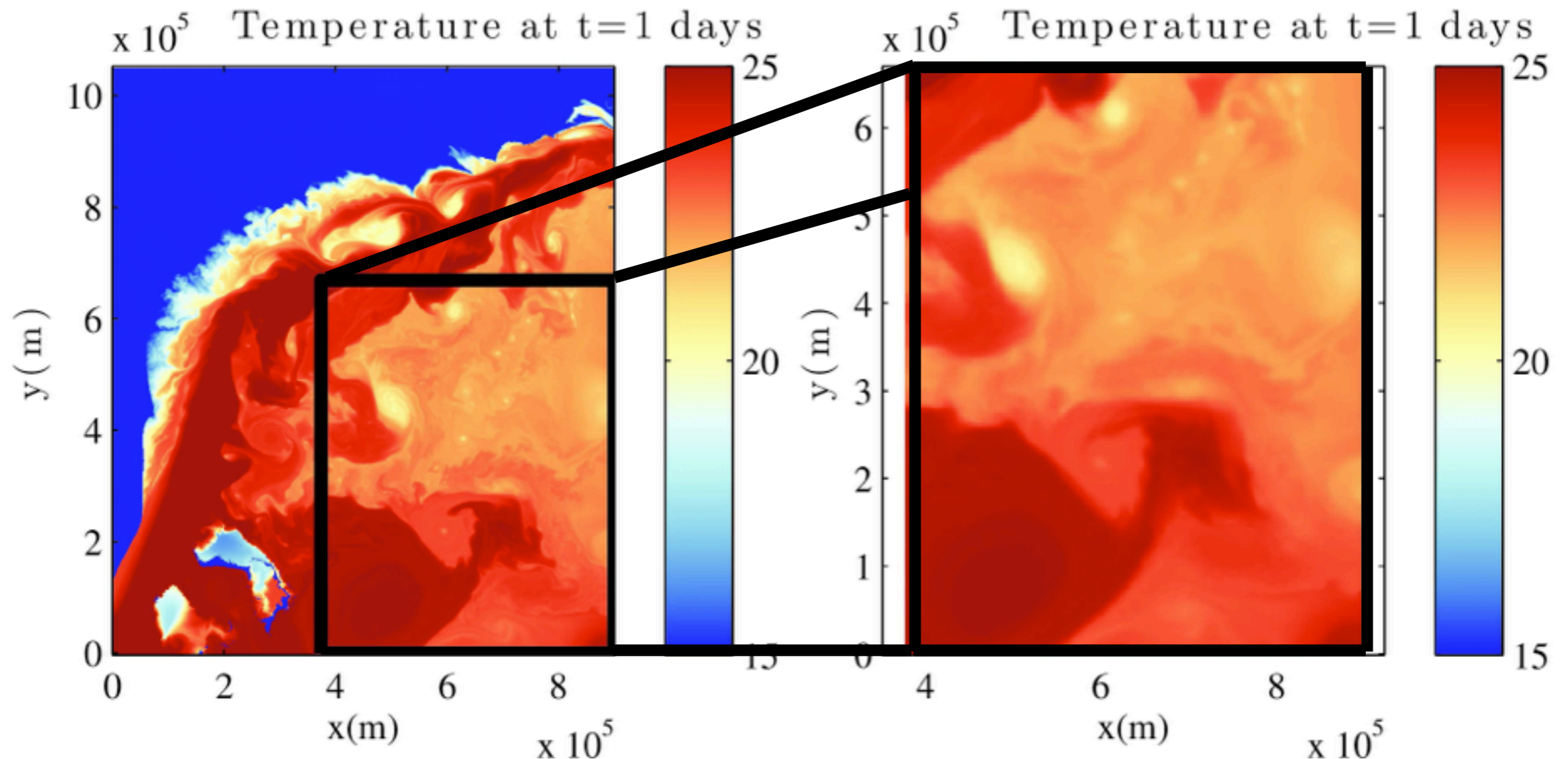


$$\nabla \cdot \mathbf{u} \propto \Delta \nabla^\perp \cdot \mathbf{u}$$



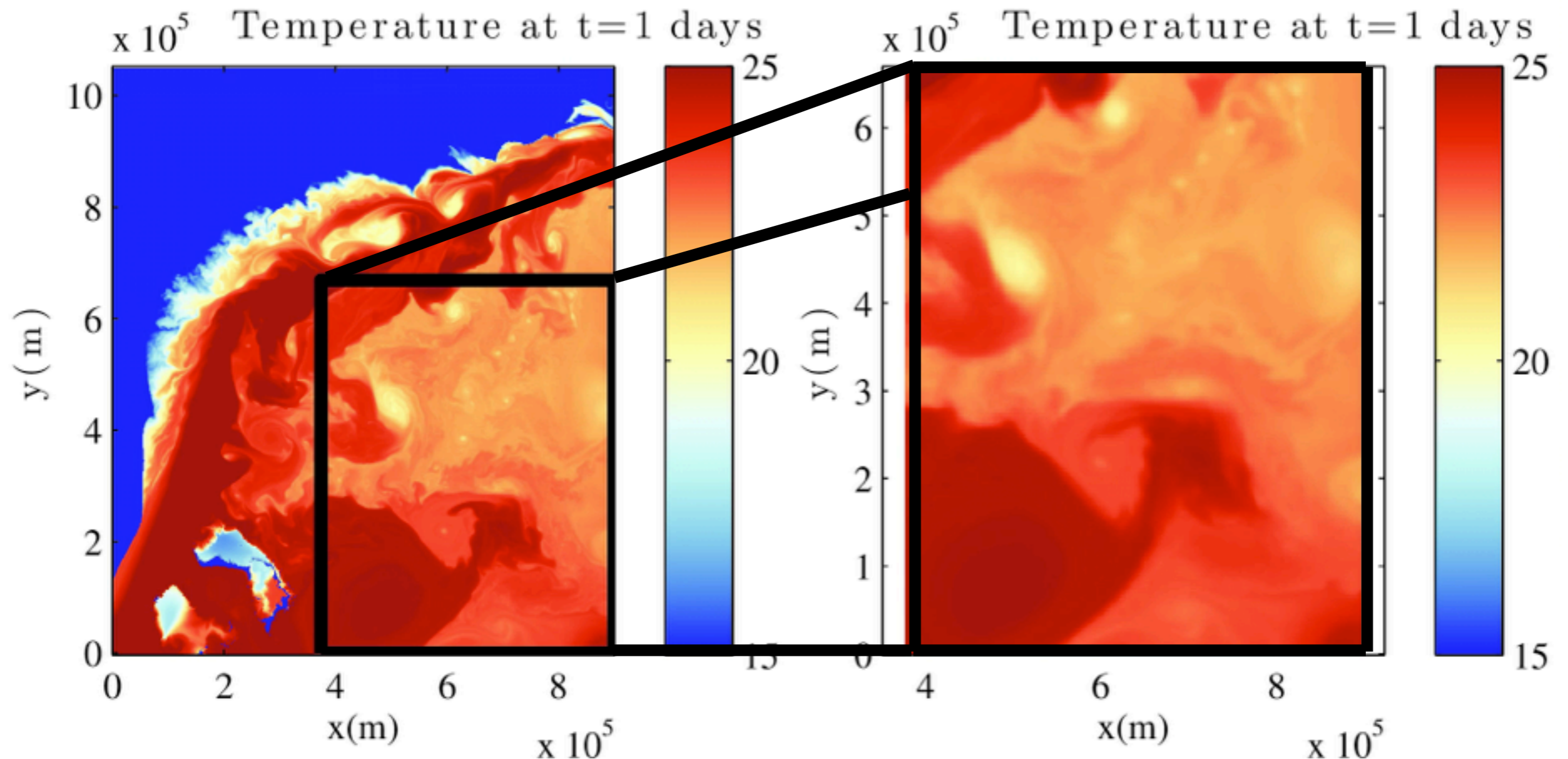
# Filtering of model outputs:

Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams  
"Gulf Stream dynamics along the southeastern US seaboard."  
*Journal of Physical Oceanography* 45.3 (2015): 690-715.



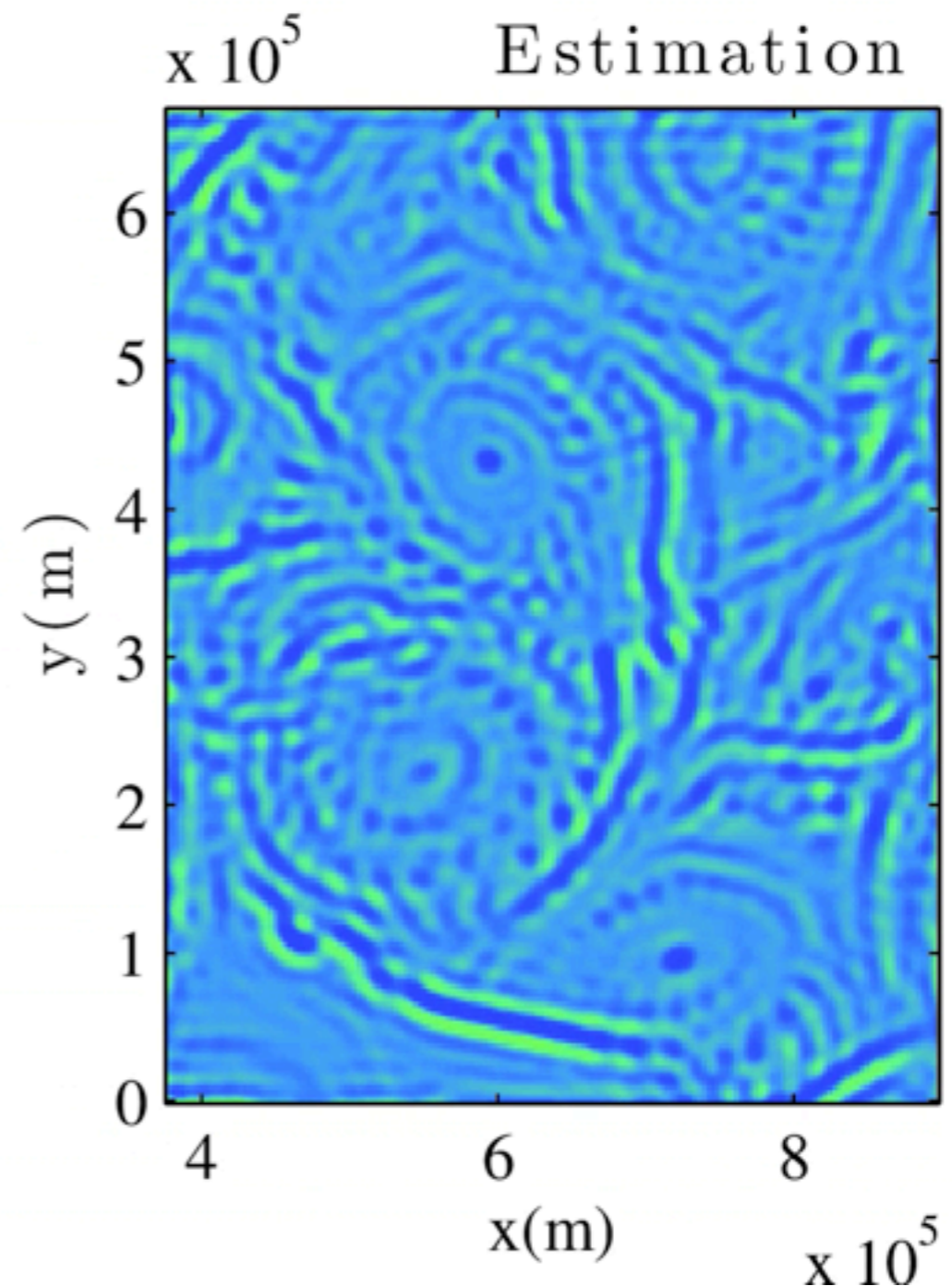
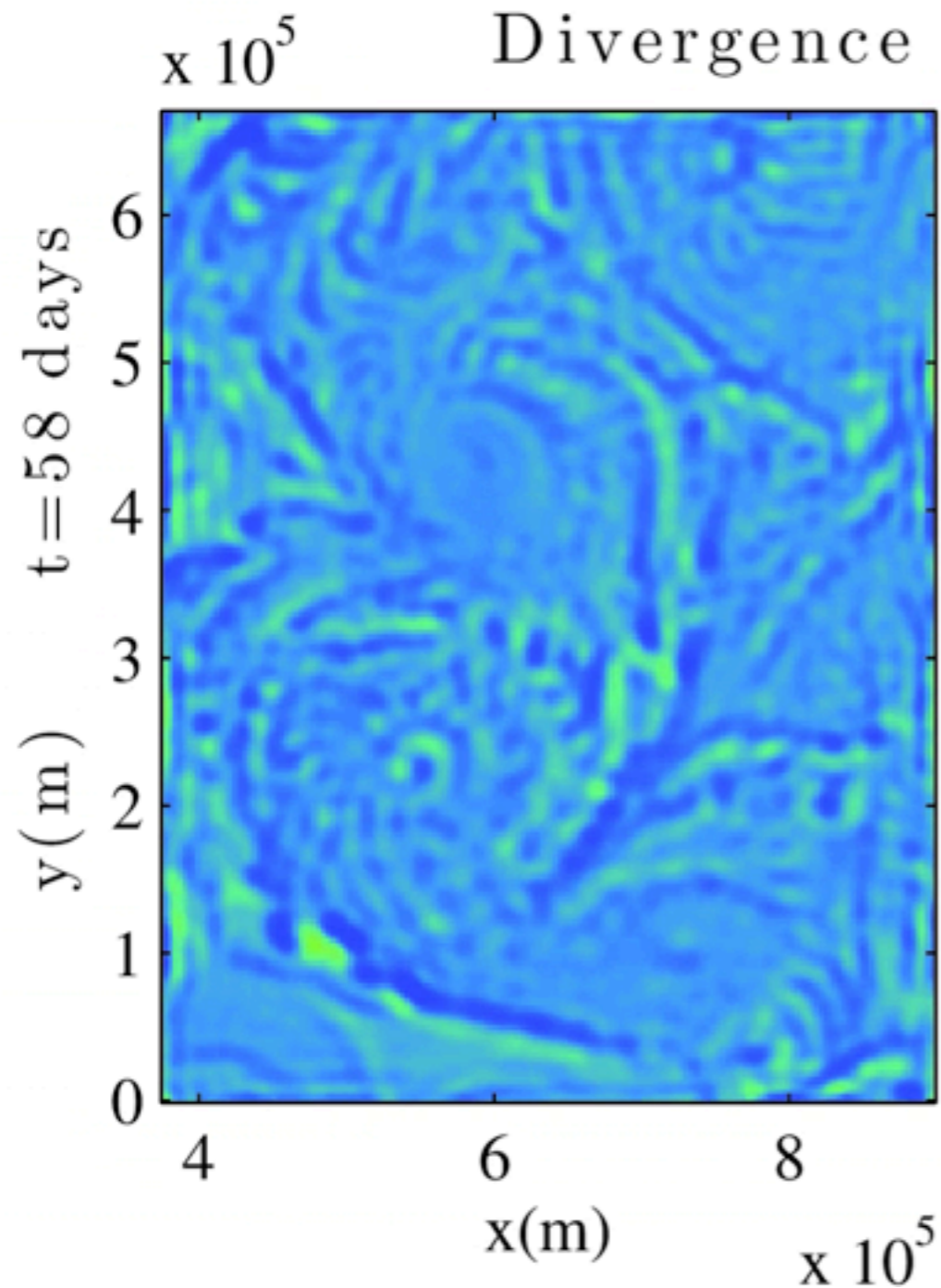
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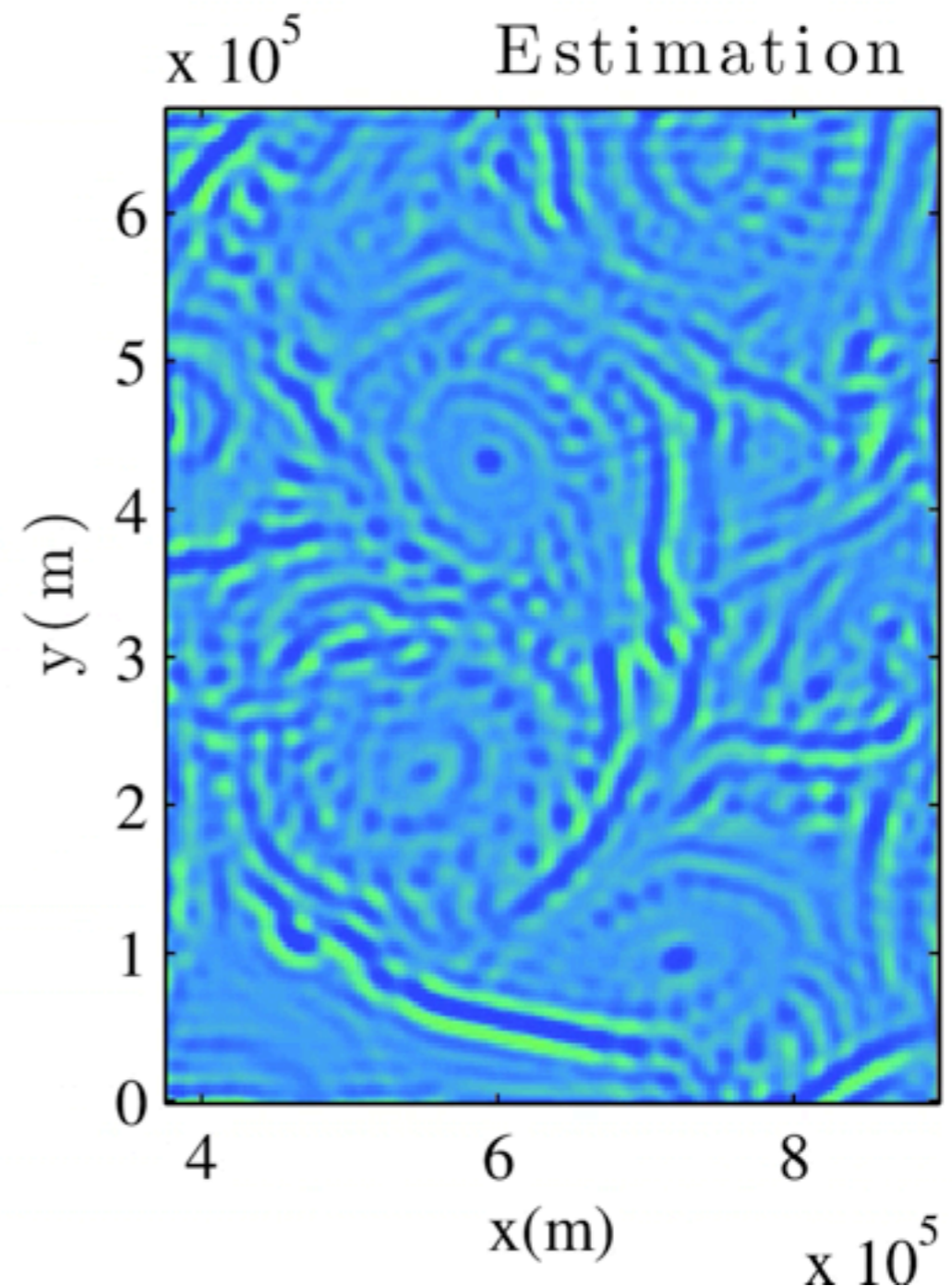
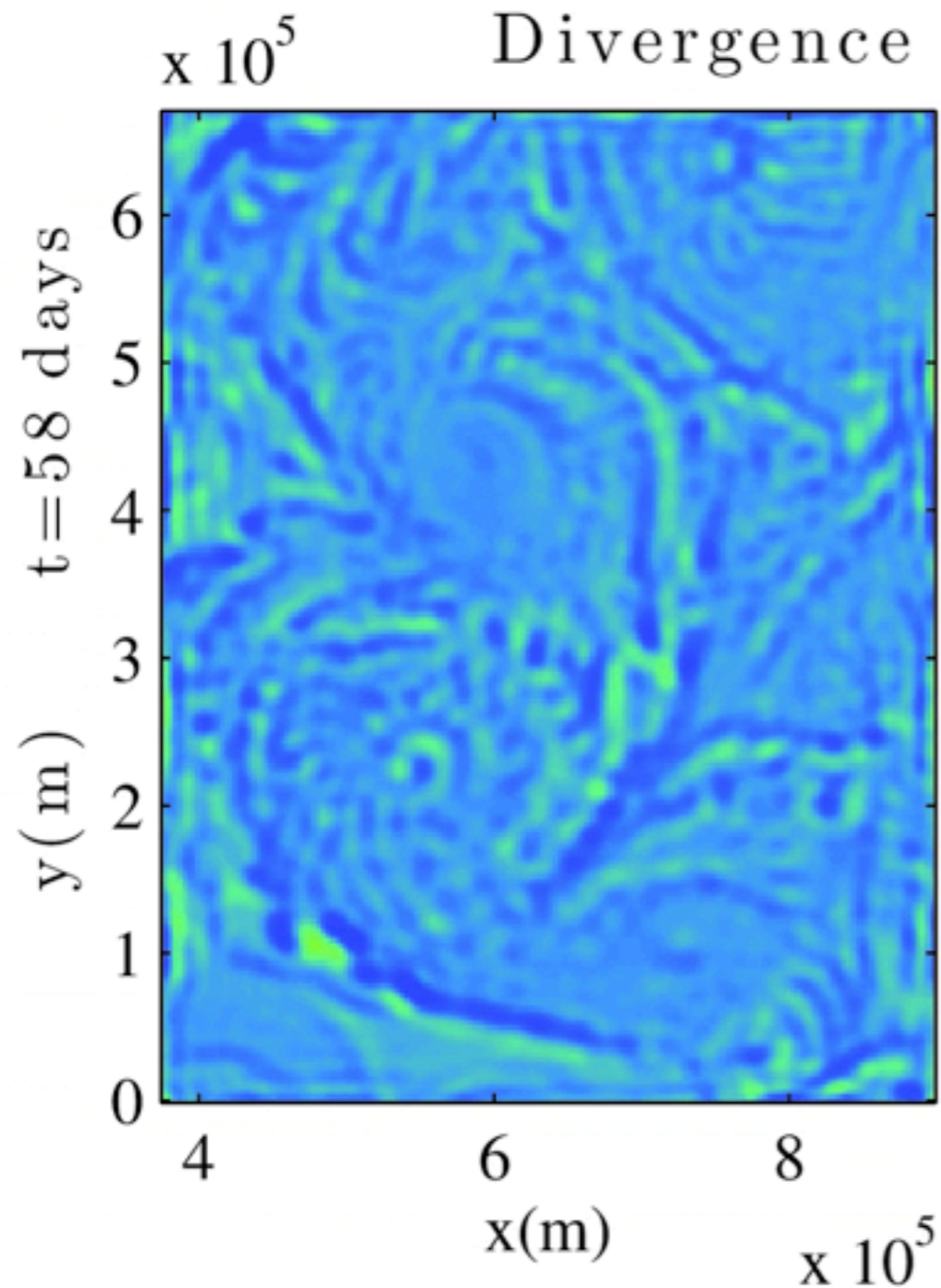




# Spatial test



# Spatial test





# Spectral test

