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# Mathematical formulation of REDEM algorithm for National soft landing CO<sub>2</sub> trajectories under global carbon budgets

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## Mathematical formulation of REDEM algorithm for National soft landing CO<sub>2</sub> trajectories under global carbon budgets

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Project-Teams STEEP

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**Abstract:** Global warming may be one of the greatest threats facing the human civilization. It is now widely shared that it is necessary to reduce quickly and significantly the greenhouse gas emissions to avoid uncontrolled and irreversible evolutions of climate. It has now become urgent to develop a legal instrument addressing the post-2020 period and to achieve a successful outcome in the international climate negotiations. In this paper we propose a new computational tool which provides elements of benchmarking for the climate negotiations. The model and algorithm we propose is designed on rationale elaborated by energy and climate policy experts. We detail how to estimate the parameters of the model and how this benchmarking tool could be used.

**Key-words:** global warming, climate change, international climate negotiations, benchmark, national GHG emission curves

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## Formulation mathématique de l'algorithme REDEM, pour des trajectoires de référence des émissions nationales de CO<sub>2</sub> sous contrainte de budget carbone global

**Résumé :** Le changement climatique est probablement l'une des plus grandes menaces à laquelle la civilisation humaine doit faire face. Il est désormais largement partagé qu'il est nécessaire de réduire rapidement et de manière très importante les émissions de gaz à effet de serre afin d'éviter un emballement irréversible du système climatique. Il est aujourd'hui urgent de développer les instruments politiques et juridiques pour la période post-2020 et de faire aboutir les négociations internationales. Dans ce rapport, nous proposons un nouvel outil permettant de calculer des courbes de références des émissions nationales de gaz à effet de serre. L'algorithme et le modèle proposés se basent sur une logique développée par des économistes spécialistes de l'énergie et du changement climatique. Nous montrons comment les paramètres du modèle peuvent être estimés et comment ces outils peuvent être utilisés.

**Mots-clés :** réchauffement climatique, négociations internationales, émission de gaz à effet de serre, courbes de référence

## 1 Introduction

Climate change and global warming may be one of the greatest threats facing the human well-being. The world is not on track to stay within the 2°C limit of temperature rise. The consequences would be catastrophic [5]. Howsoever bad the situation may seem, the latest scientific research indicates that keeping below the 2°C limit is challenging but feasible. Nevertheless only an internationally coordinated, goal oriented approach to operationalizing the 2°C limit will allow humanity to avoid dangerous climate change [3].

Compared to expectations, the 16th Conference of the Parties to the UNFCCC at Copenhagen was a failure. The main outcome of COP-17 is the decision to launch a new negotiation process in order to develop a “legal instrument, or agreed outcome” addressing the post-2020 period and “applicable to all Parties”. To break the deadlock in international climate negotiations is more than ever crucial and urgent.

The installation of an international climate regime is a complex process. One of the main difficulties of the next climate accords is that they must be able to combine a bottom-up process and a top-down rationale. In effect, since the beginning, the distinction made in the international negotiations –between the Annex I (developed countries) and non-Annex I (developing countries)– drives to a greater emphasis on differentiated responsibilities than on common responsibilities [1]. Then this drift has been significantly increasing when developing countries stepped forward to join the main emitters in the limelight. In particular, the conclusions of the COP-16 at Copenhagen clearly show that the will of most Parties is to propose their own pledges to emission reductions. Continuing to follow the bottom-up approach is feasible but not desirable : when added up, the actual national pledges lead to a 3-4°C warming of the temperature [6]. On the other side, the outcomes of COP-17 at Durban recognize this emission gap and mention the 2°C target which clearly illustrate the importance of the top-down rationale. Whatever the protocol will come from the negotiations by 2015, this new accord has to include a differentiation criterion that doesn’t violate basic equity principles in emission reduction and a regulatory approach which may provide a reasonable chance to reach the 2°C target. This top-down rationale is desirable but for the moment hardly feasible.

In this context, we propose a tool which attempt to reconcile bottom-up aspirations and top-down scheme. More precisely, this paper proposes a harmonized approach that aims at developing a common benchmarking tool for national emission reduction policies. Indeed, the decision to keep the 2°C temperature increase as a reference and the mention to the legal form need to create tools which allow to compare GHG emission reductions of the various countries and put into coherence.

This technical report describes the mathematical model and the associated algorithms of the computational tool we propose. For a more complete introduction, a comparison with the state of the art, and a deep analysis of the evolution and actual situation of international climate negotiations, we refer the reader to our previous report [2].

## 2 A benchmarking tool which reconciles bottom-up aspirations and top-down scheme

In that perspective, our goal is to identify practical solutions for reaching the Convention’s main objectives. In order to reconcile bottom-up aspirations and top-down scheme, the solutions have to verify the features expressed in the Parties’ positions as well as in the latest developments of the discussions within the ADP<sup>1</sup> process. The key features, that the benchmarking system for

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<sup>1</sup>Ad Hoc Working Group on the Durban Platform for Enhanced Action

national pledges has to verify, can be synthesized as follows:

1. The objectives and the benchmarking profiles must be differentiated, based on the countries' responsibility and capability (and not on the affiliation to different "clubs" or stages);
2. The reasoning should be based on the simplest, yet the most uncontested variables (population, GDP, level of emissions) in order to develop differentiation indicators which could be least subject to controversy;
3. The national pledges, formulated in a bottom-up manner, should not be leading to an imposed emission level. Transparency, visibility and common accounting rules could be provided through an international benchmark system.
4. The scheme should be submitted to a measurement and verification process in order to insure the reference objectives and the convergence of the emission reduction efforts.
5. The benchmark process should be able to ensure the connection between the two approaches (top-down and bottom-up). More precisely, it should show either the coherence or the discrepancy between the global and the national emission profiles.
6. The differentiating system should be based on an economical perspective, ensuring that no shock is imposed to the national economies through emission reduction targets.
7. The scheme should be submitted to a transparent measurement and verification process in order to ensure the convergence of the emission reduction efforts towards the 2°C target.

In the sequel of this report, we propose a tool that is designed for the benchmarking of national emission reduction trajectories and which fulfills these underlying principles. The idea is to develop a tool which generates curves which allow the countries to evaluate and compare themselves to others. The comparisons must be done in a framework ensuring the consistency with the global objective of GHG emissions. This framework must verify some common equity rules which are the simplest and the most objective possible and which allow to differentiate the countries according to their capacity and responsibility. The obtained curves are benchmark curves (of future GHG emissions until 2100). They are not forecasting curves based on simulations of complex economic phenomena.

To differentiate the countries, we propose here to use a differentiation indicator which is a simple combination of each country's per capita emission and per capita GDP. The per capita emission can be considered as a reasonable proxy of a country's responsibility, and the per capita GDP a proxy of its capacity for action. In the following, we call this indicator, the CRI (Capacity-Responsibility Indicator).

One of the characteristics of our method is that it principally focuses on the rate of variation of GHG emissions and on the total emission budget for the period considered. In addition, it is based on the fundamental observation that when emission curves follow a pattern of type "Peak, plateau and decline" [7] with a final stabilization at a low level, the rates of emission growth 1) decrease, then 2) reach a minimum and then 3) slowly increase and converge to zero.

More precisely, in our model, we focus here on the peak in the rate of reduction of emissions, or decarbonization rate. This peak – indeed rather a trough – takes place later than the emission peak, and can be interpreted as a peak of effort. We propose to parameterize the maximum effort for each country according to the above-mentioned capacity-responsibility indicator in a "proportional" way. By doing so, we then obtain a common benchmarking method which proposes differentiated objectives based on a simple indicator. In addition, the algorithm is designed so as to compute emission curves for all the countries in such a way that the global aggregated

emission budget is consistent with the chosen temperature target (for example, the 2°C climate target).

In [2], the reader can find much more details and discussions about the underlying logic to our proposition. We also describe all interface parameters in a simple and intuitive manner. In the following, we detail the mathematical model we propose and associated algorithms which allow to compute the parameters.



### 3 Mathematical formulation

#### 3.1 Capacity-Responsibility Index

To be consistent with the first two rules mentioned above, we then propose to enforce the effort of reduction of GHG of states to directly depend on a Capacity-Responsibility Index (CRI) which combines: 1) the per capita emissions of GHG and 2) the per capita PIB for each state. These indicators are fixed once and for all (for the whole considered period). They are computed at a reference year that has to be negotiated. Here we fix this reference year to 2010.

Let  $i \in \{1 \dots n\}$  be the indices corresponding to the states or groups of states. Let  $p_i \geq 0$  be the per capita PIB for the state  $i$  (or group of states  $i$ ), expressed in k\$(in thousands of US Dollars). Let  $e_i \geq 0$  be the per capita emissions of GHG for the state  $i$  (or group of states  $i$ ), expressed in tons ( $CO_2$  and other GHG).

We denote  $N_i$  the CRI of state  $i$  (or region  $i$ ). We propose to define the CRI indexes by using the classical 1-norms used in  $\mathbb{R}^2$ :

$$N_i = \frac{rp_i + e_i}{\delta},$$

where  $\delta = \max_i (rp_i + e_i)$ . For each state  $i$ , we have then  $0 \leq N_i \leq 1$ . Let us also note that, by construction, there exists a specific  $i$  such that  $N_i = 1$ .

In some sense, the CRI allows to project the 2-dimensional distribution of country indicators  $(p_i, e_i)$  to a 1-dimensional distribution included in the range  $[0, 1]$ . The parameter  $r$  is an important parameter and must be carefully considered. It directly and significantly impacts the repartition of the indicator.

When the isolines are horizontal lines, then they are orthogonal to the trajectories (in the space  $(p, e)$ ) that would follow a state which would only reduce his GHG emission (without changing his GDP per cap.). Those trajectories would then be collinear to the gradient of the CRI. For such states, a reduction of the GHG emissions would be widely rewarded, when a variation of their GDP per capita would not be penalized. Inversely, when the isolines are vertical then variations in the GHG emissions do not change CRI whereas GDP variations modify in a maximal way this index.

#### 3.2 Model formulation and constraint specification

Let  $E_i$  be the total emissions of GHG for the state or region  $i$  (in million of tons). For all states and regions, the emissions  $E_i$  of GHG evolve over time; in other words,  $E_i$  is a function of time  $E_i : \mathbb{R} \rightarrow \mathbb{R} : t \mapsto E_i(t)$ . Their derivatives  $E_i'(t)$  measure the variations of the emissions  $E_i(t)$ ; more exactly they measure their growing. Now let us consider the decarbonization rate  $R_i(t) = -100 \frac{E_i'(t)}{E_i(t)}$ . This ratio is a percentage that can be interpreted as an indicator of efforts to make. The effort is maximal, when  $R_i(t)$  is maximal.

We suggest to use the CRI indexes to introduce equity in the reduction efforts of GHG emissions. We propose to perform this by introducing the CRI indexes in the peak of effort. To simplify, we propose that the date of peak of effort is the same for all the states and regions (if it is relevant and useful, an eventual relaxation of this constraint will be the subject of a future paper). We call  $t_{max}$  this date that has to be negotiated. A reasonable choice for  $t_{max}$  could be 2050. At  $t_{max}$ , we propose to force

$$R_i(t_{max}) = \bar{\mu} + \frac{\bar{\sigma}}{\sigma}(N_i - \mu),$$

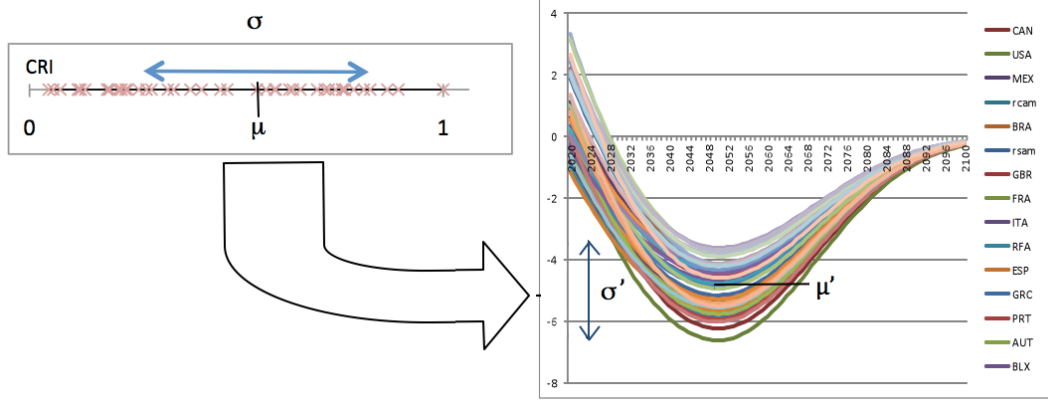


Figure 1: Parameterization the maximum effort for each of the countries with its CRI.

where  $\mu = \frac{1}{n} \sum_i N_i$  is the mean of the CRIs (of all the states and regions) and  $\sigma$  their standard deviation.  $\bar{\sigma}$  is a parameter which has to be negotiated. In a sense, here we have rescaled and positioned the CRIs in an “effort” range via a simple affine transformation of the CRIs; by changing their mean and their standard deviation: see Figure 1. Parameter  $\bar{\sigma}$  gives more or less weight to this equity desire. It allows to weaken or to magnify the differences of treatment imposed to the states and regions. It gives more or less importance and effects to the CRIs. When  $\bar{\sigma} = 0$ , there is no differentiation ; this would correspond to a one-size-fits-all approach. In the following, we are going to denote

$$\tilde{N}_i = \bar{\mu} + \frac{\bar{\sigma}}{\sigma}(N_i - \mu).$$

In the sequel we will explain how to get a relevant value for  $\bar{\mu}$ .

The reduction efforts being assumed to be maximal at  $t_{max}$ , we have then

$$R'_i(t_{max}) = 0.$$

We can also have at our disposal some approximations of the actual efforts  $D_i$  made by all the states. So we have to enforce  $R_i$  to verify

$$R_i(t_0) = D_i ;$$

i.e.  $R_i(t_0)$  is fixed and known for a reference year  $t_0$ . In our experiments, we have fixed  $t_0 = 2010$ .

In other respects, to ensure the 2°C global objective, we have to force the global GHG emission budget to do not exceed a certain bound. In other words, we have to enforce the following constraint:

$$\sum_i \int_{t_0}^{t_{budget}} E_i(t) dt \leq B_{budget}, \quad (1)$$

where  $B_{budget}$  is an adequate bound (in million of tons) and  $[t_0, t_{budget}]$  is the budget period. In our experiments, we have chosen  $t_{budget} = 2100$ .

### 3.3 Mathematical formulation in brief

The reference emission curves  $E_i$  of the benchmarking tools have then to verify the following equation:

$$100 E'_i(t) + R_i(t)E_i(t) = 0, \quad (2)$$

with  $E_i(t_0) = E_{0i}$  which are the (known) emissions of GHG at the reference year  $t_0$ . Equation (2) is a classical homogeneous linear differential equation of order 1. In other respects, the  $R_i$  functions have to verify

$$\begin{aligned} R_i(t_0) &= D_i, \\ R'_i(t_{max}) &= 0. \end{aligned} \quad (3)$$

It is also natural to impose that the  $R_i$  are regularly increasing between  $t_0$  and  $t_{max}$ , and that they are regular and monotonic after  $t_{max}$ .

In addition, the global emission curves  $E_i$  have to verify

$$\sum_i \int_{t_0}^{t_{budget}} E_i(t) dt \leq B_{budget}. \quad (4)$$

In the following we show how to compute emission curves  $E_i$  that verify these equations and constraints.

## 4 Computation of emission curves

### 4.1 Resolution of the differential equation (2)

While the whole problem formulated by the set of the equations (2), (3) and (4) is not well-posed (in the sense that it has in general an infinite number of solutions as it will be illustrated in the following), the ordinary differential equation (2)

$$\begin{cases} 100 E'_i(t) + R_i(t)E_i(t) = 0, \\ E_i(t_0) = E_{0i}, \end{cases} \quad (5)$$

is completely well-posed. Also it is quite well-known that this differential equation has a unique solution:

$$E_i(t) = E_{0i} e^{-\frac{1}{100} \int_{t_0}^t R_i(s) ds}, \quad (6)$$

see for example [4]. The issue here is then to find some adequate decarbonization ratio curves  $R_i$ .

### 4.2 Decarbonization ratio curves $R_i$

The goal here is then to propose some adequate decarbonization ratio curves  $R_i$ . Clearly some ambiguities appear because the three conditions (3) are not sufficient to completely determine these curves. We then propose to select some simple and reasonable solutions.

On the interval  $[t_0, t_{max}]$ , we then pose

$$R_i(t) = \left( D_i - \tilde{N}_i \right) \left( \frac{t_{max} - t}{t_{max} - t_0} \right)^\gamma + \tilde{N}_i,$$

where  $c$  can be interpreted as an acceleration coefficient. This last parameter also indicates the degree of convexity of the curves of the rate of variation. We fix this parameter to be identical for all countries. The value of  $\gamma$  is exogenous and have to be negotiated between all the Parties. By default, we fix  $\gamma = 2$  in our experiments. To simplify notations, we pose

$$a_i := \frac{D_i - \tilde{N}_i}{(t_{max} - t_0)^\gamma}$$

and

$$b_i := \tilde{N}_i.$$

So

$$R_i(t) = a_i (t_{max} - t)^\gamma + b_i.$$

In the long term, emissions have to be stabilized. It is then necessary that  $R_i(t)$  regularly converges towards zero when  $t \geq 0$ . We then propose the following function for all  $t \geq t_{max}$ :

$$R_i(t) = \tilde{N}_i e^{-\frac{(t-t_{max})^2}{2\theta^2}}.$$

$\theta$  is another acceleration term, which plays a similar role as  $\gamma$ .

### 4.3 Emission curves $E_i$ before the peak of effort

The reader can easily verify that for  $t$  in the interval  $[t_0, t_{max}]$ , we have

$$\int_{t_0}^t R_i(s) ds = -a_i \frac{(t_{max} - t)^{\gamma+1}}{\gamma + 1} + b_i t + c_i,$$

where

$$c_i = a_i \frac{(t_{max} - t_0)^{\gamma+1}}{\gamma + 1} - b_i t_0 = \left( D_i - \tilde{N}_i \right) \frac{t_{max} - t_0}{\gamma + 1} - \tilde{N}_i t_0. \quad (7)$$

Then, for  $t \in [t_0, t_{max}]$ , we have

$$E_i(t) = E_{0i} e^{\frac{1}{100} [a_i \frac{(t_{max}-t)^{\gamma+1}}{\gamma+1} - b_i t - c_i]}. \quad (8)$$

### 4.4 Emission curves $E_i$ after the peak of effort

We have:

$$\int_{t_{max}}^t R_i(s) ds = \int_{t_{max}}^t \tilde{N}_i e^{-\frac{(s-t_{max})^2}{2\theta^2}} ds = \theta \sqrt{\frac{\pi}{2}} \tilde{N}_i \operatorname{erf} \left( \frac{t - t_{max}}{\sqrt{2}\theta} \right), \quad (9)$$

where the Gauss error function is the (non-elementary) sigmoid function

$$\operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-s^2} ds.$$

So, for  $t \geq t_{max}$ , we have

$$E_i(t) = E_{0i} e^{-\frac{1}{100} [b_i t_{max} + c_i + \theta \sqrt{\frac{\pi}{2}} \tilde{N}_i \operatorname{erf} \left( \frac{t - t_{max}}{\sqrt{2}\theta} \right)]}. \quad (10)$$

## 5 Enforcing the 2°C global objective

The issue here is to find a value for the parameter  $\bar{\mu}$  such that constraint (4) reminded below is verified

$$\sum_{i=1}^n \int_{t_0}^{t_{budget}} E_i(t) dt \leq B_{budget}.$$

In practice the states will always try to minimize their efforts, so the constraint is *de facto*

$$\sum_{i=1}^n \int_{t_0}^{t_{budget}} E_i(t) dt = B_{budget} \quad (11)$$

instead of equation (4).

Since  $\tilde{N}_i$  are increasing with respect to  $\bar{\mu}$  and  $R_i$  are increasing with respects to  $\tilde{N}_i$ , we obtain  $R_i$  increasing with respects to  $\bar{\mu}$ . With similar monotonic arguments we can conclude that  $\sum_i \int_{t_0}^{t_{budget}} E_i(t) dt$  is decreasing with respect to  $\bar{\mu}$ . So there exists at most one  $\bar{\mu}$  which verifies equation (11). Let us remark that  $\int_{t_0}^{t_{budget}} E_i(t) dt$  is an exponential integral that cannot be written as a combination of elementary functions. So we propose to solve equation (11) by using the Newton method. Let us note

$$f(\bar{\mu}) = \sum_{i=1}^n \int_{t_0}^{t_{budget}} E_i(t, \bar{\mu}) dt - B_{budget}.$$

Starting from a first approximation  $\bar{\mu}_0$  of the solution, Newton method consists in computing a sequence of approximations of the solution by performing the following iterations until the convergence:

$$\bar{\mu}_{k+1} = \bar{\mu}_k - \frac{f(\bar{\mu}_k)}{f'(\bar{\mu}_k)}.$$

To apply this method, we need to calculate the derivative of  $f$  with respect to  $\bar{\mu}$ . Here we have:

$$f'(\bar{\mu}) = \sum_{i=1}^n \left[ \int_{t_0}^{t_{max}} \partial_{\bar{\mu}} E_i(t, \bar{\mu}) dt + \int_{t_{max}}^{t_{budget}} \partial_{\bar{\mu}} E_i(t, \bar{\mu}) dt \right]$$

where, for  $t \leq t_{max}$ , we have

$$\partial_{\bar{\mu}} E_i(t, \bar{\mu}) = \frac{E_i(t, \bar{\mu})}{100} \left[ a'_i(\bar{\mu}) \frac{(t_{max} - t)^{\gamma+1}}{\gamma + 1} - b'_i(\bar{\mu}) t - c'_i(\bar{\mu}) \right].$$

Since

$$\begin{aligned} a'_i(\bar{\mu}) &= -\frac{1}{(t_{max} - t_0)^\gamma}, \\ b'_i(\bar{\mu}) &= 1, \\ c'_i(\bar{\mu}) &= -\frac{t_{max} + \gamma t_0}{\gamma + 1}, \end{aligned}$$

then, for  $t \leq t_{max}$ , we have

$$\partial_{\bar{\mu}} E_i(t, \bar{\mu}) = \frac{1}{100} E_i(t, \bar{\mu}) \left[ -\frac{(t_{max} - t)^{\gamma+1}}{(\gamma + 1)(t_{max} - t_0)^\gamma} - t + \frac{t_{max} + \gamma t_0}{\gamma + 1} \right].$$

For  $t \geq t_{max}$ , we have

$$\partial_{\bar{\mu}} E_i(t, \bar{\mu}) = -\frac{1}{100} E_i(t, \bar{\mu}) \left[ t_{max} - \frac{t_{max} + \gamma t_0}{\gamma + 1} + \theta \sqrt{\frac{\pi}{2}} \operatorname{erf} \left( \frac{t - t_{max}}{\sqrt{2}\theta} \right) \right].$$

## 6 Examples of use of the algorithm

Our tool aims at illustrating the conditions of convergence between the national pledges and the 2°C target.

In practice, the parameters  $r$ ,  $t_0$ ,  $t_{max}$ ,  $\bar{\sigma}$ ,  $\theta$  and  $\gamma$  are fixed by users. They result of a negotiation by all the Parties. Then the algorithm computes the emission curves  $E_i$  for all countries (whose curves of rate of variation verify the parameterization defined above) such that the objective for the expected increase in temperature is verified (the temperature target being also indicated as parameter). The actual version of the program produces a set of curves: first it displays the curves of the rates of variation of emissions (in % per year), the emission trajectories, the emissions per capita, and a chart illustrating the distribution of the budget of emissions (budget between 2000-2100) for all countries (see Figure 2). Finally, it is also possible to get the rate of variation curves and emission curves for one given country as obtained for several budgets on the same graph, as illustrated in Figure 3 (all the other parameters being fixed). This tool would thus allow to easily visualize “national decarbonisation corridors” which are globally consistent for all the countries and verifying the rules mentioned in the previous sections.

This program can be used to examine different “effort profiles”. For example, various scenarios can allow to compare an “early” and a “delayed” action, the impacts of low and high levels of probability of attaining the 2°C target and finally low and high levels of differentiation across countries. Such scenarios are characterized by changes in the values of the parameter  $t_{peak}$ , of the emission budget and of the differentiation parameter  $\sigma'$ . The results obtained with several examples of scenario are detailed in our technical report [2]. We refer interested reader to this report because of space constraint.

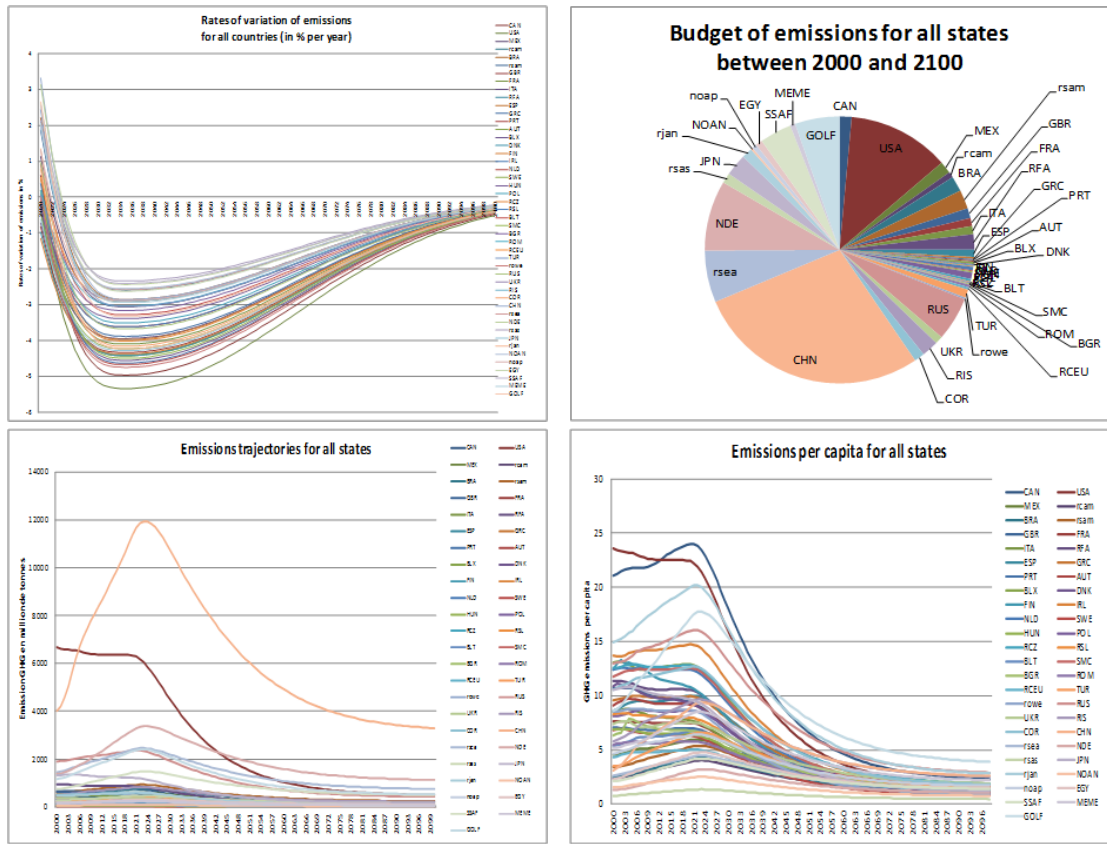


Figure 2: Examples of output curves for all countries: rate of variation of emissions, emission trajectories, emissions per capita, distribution of the budget

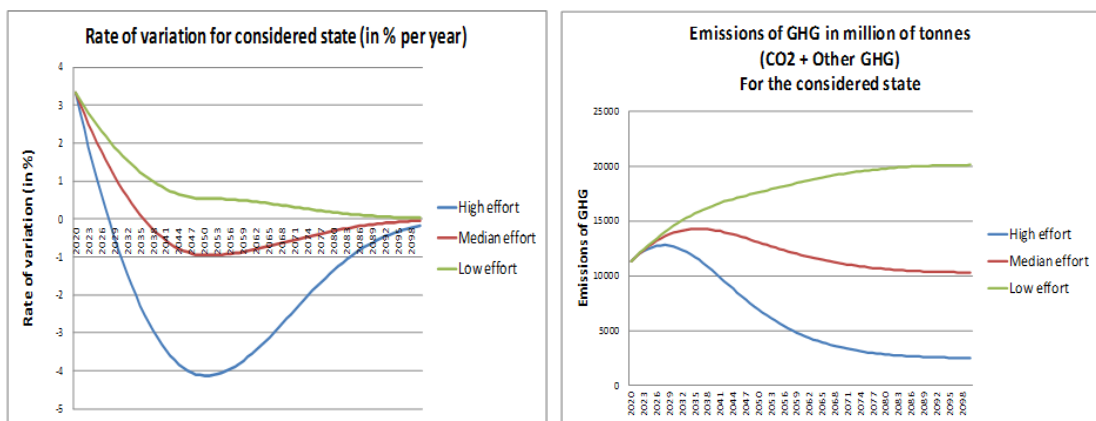


Figure 3: Example of national decarbonisation corridors

## 7 Conclusion

The current process of climate negotiation is focused on the framing of an outcome supposed to be applicable to all the countries. Meanwhile, the sum of the mitigation contributions – which are to be based on national circumstances and policies, should be compatible with the 2°C target. In this context our approach provides a rationale to produce mitigation trajectories that are consistent in a cross-country analysis with the control of the increase in world temperature.

Our tool shows a practical way to guide the national trajectories or pledges through a convergence mechanism into a comprehensible framework. The algorithm starts from the consideration of a generic emission profile (peak, plateau and decline) and a consistent way to relate the carbon budget in 2100 to the temperature increase. At the same time, it considers the equity matter, which is provided through the Capacity Responsibility Indicator and afterwards by the use of a standard deviation for the differentiation of the national decarbonisation trajectories. The main feature of the algorithm consists in the simulation of the decarbonisation rate for each individual country while it also fully accounts for total emissions and emission budgets over the period. We detail in this report the mathematical model we propose and how we compute the benchmarking emission curves and the associated parameters.

We finally rapidly illustrate how the tools could be used. The simulations describe different settings for national and global emission profiles. They illustrate different options and tradeoffs for the international community so as their potential consequences in terms of national trajectories. The logic of providing national decarbonisation corridors, representing different stringency efforts, may provide a help for implementing a benchmarking system for national decarbonisation policies. Thus the use of our tool or of its results may be a help in designing a better alignment of national contributions towards the 2°C target.



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