Deterministic gathering of anonymous agents in arbitrary networks

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Abstract

A team consisting of an unknown number of mobile agents, starting from different nodes of an unknown network, possibly at different times, have to meet at the same node. Agents are anonymous (identical), execute the same deterministic algorithm and move in synchronous rounds along links of the network. An initial configuration of agents is called *gatherable* if there exists a deterministic algorithm (even dedicated to this particular configuration) that achieves meeting of all agents in one node. Which configurations are gatherable and how to gather all of them deterministically by the same algorithm?

We give a complete solution of this gathering problem in arbitrary networks. We characterize all gatherable configurations and give two *universal* deterministic gathering algorithms, i.e., algorithms that gather all gatherable configurations. The first algorithm works under the assumption that an upper bound n on the size of the network is known. In this case our algorithm guarantees *gathering with detection*, i.e., the existence of a round for any gatherable configuration, such that all agents are at the same node and all declare that gathering is accomplished. If no upper bound on the size of the network is known, we show that a universal algorithm for gathering with detection does not exist. Hence, for this harder scenario, we construct a second universal gathering algorithm, which guarantees that, for any gatherable configuration, all agents eventually get to one node and stop, although they cannot tell if gathering is over. The time of the first algorithm is polynomial in the upper bound n on the size of the network, and the time of the second algorithm is polynomial in the (unknown) size itself.

Our results have an important consequence for the leader election problem for anonymous agents in arbitrary graphs. Leader election is a fundamental symmetry breaking problem in distributed computing. Its goal is to assign, in some common round, value 1 (leader) to one of the entities and value 0 (non-leader) to all others. For anonymous agents in graphs, leader election turns out to be equivalent to gathering with detection. Hence, as a by-product, we obtain a complete solution of the leader election problem for anonymous agents in arbitrary graphs.

Keywords: gathering, deterministic algorithm, anonymous mobile agent.

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1 Introduction

The background. A team of at least two mobile agents, starting from different nodes of a network, possibly at different times, have to meet at the same node. This basic task, known as *gathering* or *rendezvous*, has been thoroughly studied in the literature. This task has even applications in everyday life, e.g., when agents are people that have to meet in a city whose streets form a network. In computer science, mobile agents usually represent software agents in computer networks, or mobile robots, if the network is a labyrinth. The reason to meet may be to exchange data previously collected by the agents, or to coordinate some future task, such as network maintenance or finding a map of the network.

The model and the problem. The network is modeled as an undirected connected graph, referred to hereafter as a graph. We seek gathering algorithms that do not rely on the knowledge of node labels, and can work in anonymous graphs as well (cf. [4]). The importance of designing such algorithms is motivated by the fact that, even when nodes are equipped with distinct labels, agents may be unable to perceive them because of limited sensory capabilities, or nodes may refuse to reveal their labels, e.g., due to security or privacy reasons. Note that if nodes had distinct labels, then agents might explore the graph and meet in the smallest node, hence gathering would reduce to exploration. On the other hand, we assume that edges incident to a node v have distinct labels in $\{0, \ldots, d-1\}$, where d is the degree of v. Thus every undirected edge $\{u, v\}$ has two labels, which are called its *port numbers* at u and at v. Port numbering is *local*, i.e., there is no relation between port numbers at u and at v. Note that in the absence of port numbers, edges incident to a node would be undistinguishable for agents and thus gathering would be often impossible, as the adversary could prevent an agent from taking some edge incident to the current node.

There are at least two agents that start from different nodes of the graph and traverse its edges in synchronous rounds. They cannot mark visited nodes or traversed edges in any way. The adversary wakes up some of the agents at possibly different times. A dormant agent, not woken up by the adversary, is woken up by the first agent that visits its starting node, if such an agent exists. Agents are anonymous (identical) and they execute the same deterministic algorithm. Every agent starts executing the algorithm in the round of its wake-up. Agents do not know the topology of the graph or the size of the team. We consider two scenarios: one when agents know an upper bound on the size of the graph and another when no bound is known. In every round an agent may perform some local computations and move to an adjacent node by a chosen port, or stay at the current node. When an agent enters a node, it learns its degree and the port of entry. When several agents are at the same node in the same round, they can exchange all information they currently have. However, agents that cross each other on an edge, traversing it simultaneously in different directions, do not notice this fact. We assume that the memory of the agents is unlimited: from the computational point of view they are modeled as Turing machines.

An initial configuration of agents, i.e., their placement at some nodes of the graph, is called *gatherable* if there exists a deterministic algorithm (even only dedicated to this particular configuration) that achieves meeting of all agents in one node, regardless of the times at which some of the agents are woken up by the adversary. In this paper we study the following gathering problem:

> Which initial configurations are gatherable and how to gather all of them deterministically by the same algorithm?

In other words, we want to decide which initial configurations are possible to gather, even by an algorithm specifically designed for this particular configuration, and we want to find a *universal* gathering algorithm that gathers all such configurations. We are interested only in *terminating* algorithms, in which every agent eventually stops forever.

Our results. We give a complete solution of the gathering problem in arbitrary networks. We characterize all gatherable configurations and give two *universal* deterministic gathering algorithms, i.e., algorithms that gather all gatherable configurations. The first algorithm works under the assumption that an upper bound

n on the size of the network is known. In this case our algorithm guarantees *gathering with detection*, i.e., the existence of a round for any gatherable configuration, such that all agents are at the same node and all declare that gathering is accomplished. If no upper bound on the size of the network is known, we show that a universal algorithm for gathering with detection does not exist. Hence, for this harder scenario, we construct a second universal gathering algorithm, which guarantees that, for any gatherable configuration, all agents eventually get to one node and stop, although they cannot tell if gathering is over. The time of the first algorithm is polynomial in the upper bound n on the size of the network, and the time of the second algorithm is polynomial in the (unknown) size itself.

While gathering two anonymous agents is a relatively easy task (cf. [15]), our problem of gathering an unknown team of anonymous agents presents the following major difficulty. The asymmetry of the initial configuration because of which gathering is feasible, may be caused not only by non-similar locations of the agents in the graph, but by their different situation *with respect to other agents*. Hence a new algorithmic idea is needed in order to gather: agents that were initially identical, must make decisions based on the memories of other agents met to date, in order to distinguish their future behavior. In the beginning the memory of each agent is a blank slate and in the execution of the algorithm it records what the agent has seen in previous steps of the navigation and what it heard from other agents during meetings. Even a slight asymmetry occurring in a remote part of the graph must eventually influence the behavior of initially distant agents. Notice that agents in different initial situations may be unaware of this difference in early meetings, as the difference may be revealed only later on, after meeting other agents. Hence, for example, an agent may mistakenly "think" that two different agents that it met in different stages of the algorithm execution, are the same agent. Confusions due to this possibility are a significant challenge absent both in gathering two (even anonymous) agents and in gathering many labeled agents.

Our results have an important consequence for the leader election problem for anonymous agents in arbitrary graphs. Leader election [30] is a fundamental symmetry breaking problem in distributed computing. Its goal is to assign, in some common round, value 1 (leader) to one of the entities and value 0 (non-leader) to all others. For anonymous agents in graphs, leader election turns out to be equivalent to gathering with detection (see Section 5). Hence, as a by-product, we obtain a complete solution of the leader election problem for anonymous agents in arbitrary graphs.

Related work. Gathering has been mostly studied for two mobile agents and in this case it is usually called rendezvous. An extensive survey of randomized rendezvous in various scenarios can be found in [4], cf. also [2, 3, 6, 25]. Deterministic rendezvous in networks has been surveyed in [31]. Several authors considered the geometric scenario (rendezvous in an interval of the real line, see, e.g., [10, 24], or in the plane, see, e.g., [7, 8]). Gathering more than two agents has been studied, e.g., in [25, 29]. In [35] the authors considered rendezvous of many agents with unique labels, and gathering many labeled agents in the presence of Byzantine agents was studied in [19]. The problem was also studied in the context of multiple robot systems, cf. [13, 20], and fault tolerant gathering of robots in the plane was studied, e.g., in [1, 14].

For the deterministic setting a lot of effort has been dedicated to the study of the feasibility of rendezvous, and to the time required to achieve this task, when feasible. For instance, deterministic rendezvous with agents equipped with tokens used to mark nodes was considered, e.g., in [28]. Deterministic rendezvous of two agents that cannot mark nodes but have unique labels was discussed in [18, 26, 33]. These papers are concerned with the time of rendezvous in arbitrary graphs. In [18] the authors show a rendezvous algorithm polynomial in the size of the graph, in the length of the shorter label and in the delay between the starting time of the agents. In [26, 33] rendezvous time is polynomial in the first two of these parameters and independent of the delay.

Memory required by two anonymous agents to achieve deterministic rendezvous has been studied in [21, 22] for trees and in [15] for general graphs. Memory needed for randomized rendezvous in the ring is discussed, e.g., in [27].

Apart from the synchronous model used, e.g., in [15, 18, 33, 35] and in this paper, several authors have investigated asynchronous rendezvous in the plane [12, 20] and in network environments [9, 16, 17]. In the latter scenario the agent chooses the edge which it decides to traverse but the adversary controls the speed of the agent. Under this assumption rendezvous in a node cannot be guaranteed even in very simple graphs and hence the rendezvous requirement is relaxed to permit the agents to meet inside an edge.

2 Preliminaries

Throughout the paper, the number of nodes of a graph is called its size. In this section we recall four procedures known from the literature, that will be used as building blocks in our algorithms. The aim of the first two procedures is graph exploration, i.e., visiting all nodes of the graph (cf., e.g., [11, 32]). The first of these procedures assumes an upper bound n on the size of the graph and the second one makes no assumptions on the size but it is performed by an agent using a fixed token placed at the starting node of the agent. (It is well known that a terminating exploration even of an anonymous ring of unknown size by a single agent without a token is impossible.) In our applications the roles of the token and of the exploring agent will be played by agents or by groups of agents. The first procedure works in time polynomial in the known upper bound n on the size of the graph and the second in time polynomial in the size of the graph. Moreover, at the end of the second procedure the agent is with the token and has a complete map of the graph with all port numbers marked. We call the first procedure $EXPLO(n)$ and the second procedure EST, for *exploration with a stationary token*. We denote by $T(EXPLO(n))$ (respectively $T(EST(n))$) the maximum time of execution of the procedure $EXPLO(n)$ (respectively procedure EST) in a graph of size at most n.

Before describing the third procedure we define the following notion from [34]. Let G be a graph and v a node of G, of degree k. The *view* from v is the infinite rooted tree $\mathcal{V}(v)$ with labeled ports, defined recursively as follows. $\mathcal{V}(v)$ has the root x_0 corresponding to v. For every node v_i, $i = 1, \ldots, k$, adjacent to v, there is a neighbor x_i in $\mathcal{V}(v)$ such that the port number at v corresponding to edge $\{v, v_i\}$ is the same as the port number at x_0 corresponding to edge $\{x_0, x_i\}$, and the port number at v_i corresponding to edge $\{v, v_i\}$ is the same as the port number at x_i corresponding to edge $\{x_0, x_i\}$. Node x_i, for $i = 1, \ldots, k$, is now the root of the view from v_i.

The third procedure, described in [15], permits a single anonymous agent starting at node v to find a positive integer $S(v)$, called the signature of the agent, such that $\mathcal{V}(v) = \mathcal{V}(w)$ if and only if $S(v) = S(w)$. This procedure, called $SIGN(n)$ works for any graph of known upper bound n on its size and its running time is polynomial in n. After the completion of $SIGN(n)$ the agent is back at its starting node. We denote by $T(SIGN(n))$ the maximum time of execution of the procedure $SIGN(n)$ in a graph of size at most n.

Finally, the fourth procedure is for gathering two agents in a graph of unknown size. It is due to Ta-Shma and Zwick [33] and relies on the fact that agents have distinct labels. (Using it as a building block in our scenario of anonymous agents is one of the difficulties that we need to overcome.) Each agent knows its own label (which is a parameter of the algorithm) but not the label of the other agent. We will call this procedure $TZ(\ell)$, where ℓ is the label of the executing agent. In [33], the authors give a polynomial P in two variables, increasing in each of the variables, such that, if there are agents with distinct labels ℓ_1 and ℓ_2 operating in a graph of size n, appearing at their starting positions in possibly different times, then they will meet after at most $P(n, |\ell|)$ rounds since the appearance of the later agent, where ℓ is the smaller label. Also, if an agent with label ℓ_i performs $TZ(\ell_i)$ for $P(n, |\ell_i|)$ rounds and the other agent is inert during this time, the meeting is guaranteed.

We will also use a notion similar to that of the view but reflecting the positions of agents in an initial configuration. Consider a graph G and an initial configuration of agents in this graph. Let v be a node occupied by an agent. The *enhanced view* from v is the couple $(\mathcal{V}(v), f)$, where f is a binary valued function defined on the set of nodes of $\mathcal{V}(v)$, such that $f(w) = 1$ if there is an agent at w and $f(w) = 0$ otherwise. Thus the enhanced view of an agent additionally marks in its view the positions of other agents

in the initial configuration.

An important notion used throughout the paper is the *memory* of an agent. Intuitively, the memory of an agent in a given round is the total information the agent collected since its wake-up, both by navigating in the graph and by exchanging information with other agents met until this round. This is formalized as follows. The memory of an agent A at the end of some round which is the t-th round since its wake-up, is the sequence $(M_0, M_1, \ldots, M_t)$, where M_0 is the information of the agent at its start and M_i is the information acquired in round i. The terms M_i are defined as follows. Suppose that in round i the agent A met agents $A_1, \ldots, A_k$ at node v of degree d. Suppose that agent A entered node v in round i leaving an adjacent node w by port p and entering v by port q. Suppose that agent A_j entered node v in round i leaving an adjacent node w_j by port p_j and entering v by port q_j. If some agent did not move in round i then the respective ports are replaced by -1. The term M_i for agent A, called its i-th *memory box* is defined as the sequence $(d, p, q, \{(p_1, q_1, R_1), \ldots, (p_k, q_k, R_k)\})$, where R_j is the memory of the agent A_j at the end of round $i-1$. This definition is recursive with respect to global time (unknown to agents), starting at the wake-up of the earliest agent by the adversary. Note that if an agent is woken up by the adversary at a node of degree d and no other agents are at this node at this time then $M_0 = (d, -1, -1)$ for this agent (at its wake-up the agent sees only the degree of its starting position). If an agent is woken up by some other agents, it also learns their memories to date. Note also that, since the memory of the agent is the total information it acquired to date, any deterministic algorithm used by agents to navigate (including any deterministic gathering algorithm) may be viewed as a function from the set of all memories into the set of integers greater or equal to -1 telling the agent to take a given port p (or stay idle, in which case the output is -1), if it has a given memory. For a memory $\mathcal{M} = (M_0, M_1, \ldots, M_t)$ and for $k \leq t$ we denote $Pref_k(\mathcal{M}) = (M_0, M_1, \ldots, M_k)$.

Notice that if two agents meet, they must necessarily have different memories. Indeed, if they had identical memories, then they would be woken up in the same round and they would traverse identical paths to the meeting node. This contradicts the assumption that agents start at different nodes.

We will use the following order on the set $\mathcal{M}$ of all possible memories. Let $<$ be any linear order on the set of all memory boxes (one such simple order is to code all possible memory boxes as binary sequences in some canonical way and use lexicographic order on binary sequences). Now the linear order $\prec$ on the set $\mathcal{M}$ of sequences of memory boxes is defined as the lexicographic order based on the order $<$. The order $\prec$ has the property that if the memory of agent A is smaller than the memory of agent B in some round then it will remain smaller in all subsequent rounds. When $\mathcal{M}_1 \prec \mathcal{M}_2$, we say that memory $\mathcal{M}_2$ is *larger* than $\mathcal{M}_1$.

3 Known upper bound on the size of the graph

In this section we assume that an upper bound n on the size of the graph is known to all agents at the beginning. The aim of this section is to characterize gatherable configurations and give a *universal* gathering algorithm that gathers *with detection* all gatherable configurations. The time of such an algorithm is the number of rounds between the wake-up of the first agent and the round when all agents declare that gathering is accomplished. Consider the following condition on an initial configuration in an arbitrary graph.

G: There exist agents with different views, and each agent has a unique enhanced view.

We will prove the following result.

Theorem 3.1 *An initial configuration is gatherable if and only if it satisfies the condition* **G**. *If an upper bound n on the size of the graph is known then there exists an algorithm for gathering with detection all gatherable configurations. This algorithm works in time polynomial in n.*

In order to appreciate the full strength of Theorem 3.1 notice that it can be rephrased as follows. If an initial configuration does not satisfy condition **G** then there is no algorithm (even no algorithm dedicated

to this specific configuration, knowing it entirely) that permits to gather this configuration, even gather it without detection: simply no algorithm can bring all agents simultaneously to one node. On the other hand, assuming an upper bound on the size of the graph, there is a *universal* algorithm that gathers *with detection* all initial configurations satisfying condition **G**. Our algorithm works in time polynomial in any known upper bound n on the size of the graph. Hence, if this upper bound is polynomial in the size of the graph, the algorithm is polynomial in the size as well. In the next section we will show how the positive part of the result changes when no upper bound on the size of the graph is known.

The rest of the section is devoted to the proof of Theorem 3.1. We start with the following lemma.

Lemma 3.1 *If an initial configuration does not satisfy condition* **G***, then it is not gatherable.*

Proof. Suppose that an initial configuration C does not satisfy condition **G** and that agents execute the same deterministic algorithm. First suppose that the configuration C does not satisfy the first part of condition **G**, i.e., that the views of all agents are identical. Suppose that the adversary wakes up all agents in the same round. We show that no pair of agents can meet. Suppose, for contradiction, that agents A_1 and A_2 are the first to meet and that this occurs in round t from the common start. Since the initial views of the agents are identical and they execute the same deterministic algorithm, the sequences of port numbers encountered by both agents are identical and hence they both enter the node at which they first meet by the same port. This is a contradiction.

Now suppose that the configuration C does not satisfy the second part of condition **G**, i.e., that there exists an agent whose enhanced view is not unique. Consider distinct agents A and A' that have identical enhanced views. Let B be any agent and suppose that q is a sequence of port numbers that leads from agent A to agent B in the enhanced view of agent A. (Notice that there may be many such sequences, corresponding to different paths leading from A to B.) Then there exists an agent B' such that q is a sequence of port numbers that leads from agent A' to agent B' in the enhanced view of agent A'. Agent B' has the same enhanced view as agent B. Since agents A and A' were different, agents B and B' are different as well. Hence for every agent there exists another agent whose enhanced view is identical. Call such agents *homologs*.

Suppose again that the adversary wakes up all agents in the same round. We will show that, although now some meetings of agents are possible, homologs will never meet. Suppose that agents A and A' are homologs. Since A and A' have the same enhanced view at the beginning, it follows by induction on the round number that their memory will be identical in every round. Indeed, whenever A meets an agent B in some round, its homolog A' meets a homolog B' of B in the same round and hence the memories of A and A' evolve identically. In particular, since all agents execute the same deterministic algorithm, agents A and A' follow identical sequences of port numbers on their paths. As before, if they met for the first time at some node, they would have to enter this node by the same port. This is a contradiction. □

The other (much more difficult) direction of the equivalence from Theorem 3.1 will be shown by constructing an algorithm which, executed by agents starting from any initial configuration satisfying condition **G**, accomplishes gathering with detection of all agents. (Our algorithm uses the knowledge of an upper bound n of the size of the graph: this is enough to prove the other direction of the equivalence, as for any specific configuration satisfying condition **G** even a gathering algorithm dedicated to this specific configuration is enough to show that the configuration is gatherable, and such a dedicated algorithm knows the exact size of the graph. Of course, our algorithm accomplishes much more: knowing just an upper bound on the size of the graph it gathers *all* configurations satisfying condition **G** and does this *with detection*.) We first give a high-level idea of the algorithm, then describe it in detail and prove its correctness. From now on we assume that the initial configuration satisfies condition **G**.

Idea of the algorithm. At high level the algorithm works in two stages. The aim of the first stage for any agent is to meet another agent, in order to perform later an exploration in which one of the agents will play

the role of the token and the other the role of the explorer (roles will be decided comparing memories of the agents). To this end the agent starts an exploration of the graph using the known upper bound n on its size. The aim of this exploration is to wake up all, possibly still dormant agents. Afterwards, in order to meet, agents use the procedure TZ mentioned in Section 2. However, this procedure requires a label for each agent and our agents are anonymous. The only way to differentiate agents and give them labels is to find their views: a view (truncated to n) from the initial position could serve as a label because the first part of condition **G** guarantees that there are at least two distinct views. However, finding the view of an agent (truncated to n) would require exponential time. Hence we use procedure $SIGN(n)$ mentioned in Section 2, which is polynomial in n and can still assign labels to agents, producing at least two different labels. Then, performing procedure TZ for a sufficiently long time guarantees a meeting for every agent.

In the second stage each explorer explores the graph using procedure $EXPLO(n)$ and then backtracks to its token left at the starting node of the exploration. After the backtrack, memories of the token and of the explorer are updated to check for anomalies caused by other agents meeting in the meantime either the token or the explorer. Note that, due to the fact that some agents may have identical memories at this stage of the algorithm, an explorer may sometimes falsely consider another token as its own, due to their identical memories. By contrast, an end of each backtrack is a time when an explorer can be sure that it is on its token and the token can be sure that its explorer is with it.

Explorers repeat these explorations with backtrack again and again, with the aim of creating meetings with other agents and detecting anomalies. As a consequence of these anomalies some agents merge with others, mergers being decided on the basis of the memories of the agents. Each explorer eventually either merges with some token or performs an exploration without anomalies. In the latter case it waits a prescribed amount of time with its token: if no new agent comes during the waiting time, the end of the gathering is declared, otherwise another exploration is launched. It will be proved that eventually, due to the second part of condition **G**, all agents merge with the same token B and then, after the last exploration made by the explorer A of B and after undisturbed waiting time, the end of the gathering is correctly declared.

We now give a detailed description of the algorithm.

Algorithm Gathering-with-Detection with parameter n (upper bound on the size of the graph)

During the execution of the algorithm an agent can be in one of the following six states: `setup`, `cruiser`, `shadow`, `explorer`, `token`, `searcher`. For every agent A in state `shadow` there is exactly one agent B in some state different from `shadow`, called the *guide* of A. We will also say that A is a shadow of B. Below we describe the actions of an agent A in each of the states and the transitions between the states. At wake-up agent A enters the state `setup`.

State `setup`.

Agent A performs $EXPLO(n)$ visiting all nodes and waking up all still dormant agents. Then agent A performs the procedure $SIGN(n)$ finding the signature of its initial position v, called the *label* of agent A. Agent A transits to state `cruiser`.

State `cruiser`.

Agent A performs $TZ(\ell)$, where ℓ is its label, until meeting an agent in state `cruiser` or `token` at a node v.

Case 1. Agent A meets an agent B in state `token`.

Then it transits to state `shadow` of B.

Case 2. Agent A does not meet an agent in state `token`.

Then there is at least one other agent in state `cruiser` at node v.

Subcase 2.1. Agent A has the largest memory among all agents in state `cruiser` at node v.

Then agent A transits to state `explorer`.

Subcase 2.2. Agent A does not have the largest memory among all agents in state `cruiser` at node v. If there is exactly one agent B in state `cruiser` with memory larger than A at node v, then agent A transits to state `token`. Otherwise, it transits to state `shadow` of the agent in state `cruiser` at node v with largest memory.

State `shadow`.

Agent A has exactly one guide and is at the same node as the guide in every round. In every round it makes the same move as the guide. If the guide B transits itself to state `shadow` and gets agent C as its guide, then agent A changes its guide to C as well. Agent A declares that gathering is over if the unique agent in state `explorer` collocated with it makes this declaration.

Before describing the actions in the three remaining states, we define the notion of *seniority* of an agent in state `token` (respectively `explorer`). The seniority in a given round is the number of rounds from the time when the agent became `token` (respectively `explorer`).

State `explorer`

When agent A transits to state `explorer`, there is another agent B that transits to state `token` in the same round at the same node v. Agent B is called the token of A. Agent A has a variable $recent - token$ that it initializes to the memory of B in this round. Denote by $EXPLO^*(n)$ the procedure $EXPLO(n)$ followed by a complete backtrack in which the agent traverses all edges traversed in $EXPLO(n)$ in the reverse order and the reverse direction. The variable $recent - token$ is updated in the beginning of each execution of $EXPLO^*(n)$. An execution of $EXPLO^*(n)$ is called *clean* if the following condition is satisfied: in each round during this execution, in which A met an agent C, the memory of C is equal to that of B, and in each round during this execution, in which the token B was met by an agent D, the memory of D was equal to that of A. Notice that after the execution of $EXPLO^*(n)$, agent A is together with its token B and thus they can verify if the execution was clean, by inspecting their memories. The execution time of $EXPLO^*(n)$ is at most $2T(EXPLO(n))$.

After transiting to state `explorer`, agent A waits for $T(EXPLO(n)) + T(SIGN(n)) + P(n, L)$ rounds, where L is the largest possible label (it is polynomial in n). Then it executes the following protocol:

while A has not declared that gathering is over **do**
do
$EXPLO^*(n)$
/*now agent A is with its token.*/

if A met an agent C in state `token` of higher seniority than that of A or of equal seniority but such that $recent - token \prec Pref_t(\mathcal{M}_C)$ where $\mathcal{M}_C$ is the memory of agent C and t is the last round when agent A updated its variable $recent - token$ **then** A transits to state `searcher`

if B was visited in round t' by an agent C in state `explorer` of higher seniority than that of B or of equal seniority but such that $\mathcal{M}_B \prec R$ where $\mathcal{M}_B$ is the memory of agent B and R is the variable $recent - token$ of agent C in round t' **then** A transits to state `searcher`

until the execution of $EXPLO^*(n)$ is clean agent A waits $2 \cdot T(EXPLO(n))$ rounds;

if during this time A has not been met by any new agent **then** A declares that gathering is over.

State `token`

When agent A transits to state `token`, there is another agent B that transits to state `explorer` in the same round at the same node v. Agent B is called the explorer of A. Agent A remains idle at a node v and does not change its state, except when its explorer B transits to state `searcher`. In this case it transits to state `shadow` and B becomes its guide. Agent A declares that gathering is over if the unique agent in state

`explorer` collocated with it makes this declaration.

State `searcher`

Agent A performs an entire execution of $EXPLO^*(n)$ and then another execution of $EXPLO^*(n)$ until meeting an agent B in state `token`. Then agent A transits to state `shadow` and B becomes its guide.

The proof of the correctness of the algorithm is split into the following lemmas.

Lemma 3.2 *In Algorithm Gathering-with-Detection every agent eventually stops after time polynomial in n and declares that gathering is over.*

Proof. At its wake-up an agent A enters state `setup` and remains in it for at most $T(EXPLO(n)) + T(SIGN(n))$ rounds (the time to complete an exploration and find its label ℓ) and then transits to state `cruiser`. We will prove that in state `cruiser` agent A can spend at most $T(EXPLO(n))+T(SIGN(n))+2P(n,\ell)$ rounds. We will use the following claim.

Claim 1. Let t be the first round in which an agent transits to state `token`. Then there exists an agent B that remains in state `token` and is idle from round t on.

To prove the claim, let Z be the set of agents that transitted to state `token` in round t. In every round $t' \geq t$, the agent from Z with the current largest memory remains in state `token` and stays idle. Since an agent with the largest memory in a given round must have had the largest memory in all previous rounds, the claim follows.

In order to prove our upper bound on the time spent by A in state `cruiser`, observe that after at most $T(EXPLO(n))+T(SIGN(n))$ rounds since A transits to state `cruiser`, all other agents have quit state `setup`. Consider the additional $2P(n,\ell)$ rounds during which agent A performs $TZ(\ell)$. Let round τ be the end of the first half of this segment S of $2P(n,\ell)$ rounds. Some meeting must have occurred on or before round τ, due to the properties of TZ. If agent A was involved in one of those meetings, it left state `cruiser` by round τ. Otherwise, it must have met some other agent in state either `cruiser` or `token` during the second half of the segment S. Indeed, if it does not meet another agent in state `cruiser`, it must meet another agent in state `token`, which transited to this state by round τ. (Claim 1 guarantees the existence of such an agent after round τ.) This proves our upper bound on the time spent by A in state `cruiser`.

From state `cruiser` agent A can transit to one of the three states: `shadow`, `explorer` or `token`. Since the termination conditions for an agent in state `shadow` are the same as of its guide, we may eliminate the case of state `shadow` from our analysis. Consider an agent in state `explorer`. After the waiting time of $T(EXPLO(n))+T(SIGN(n))+P(n,L)$ rounds (where L is the largest possible label (it is polynomial in n) agent A knows that all other agents have already transited from the state `cruiser` (they used at most $T(EXPLO(n)) + T(SIGN(n))$ rounds in state `setup` and at most $P(n,L)$ rounds in state `cruiser`, as their labels are at most L and at least one token is already present in the graph).

Either agent A never leaves state `explorer`, in which case we will prove that it declares that gathering is over after polynomial time, in some round ρ, or it transits to state `searcher` before round ρ, in which case it uses at most $2 \cdot T(EXPLO(n))$ rounds for one execution of $EXPLO^*(n)$ and after additional at most $2 \cdot T(EXPLO(n))$ rounds it finds an idle agent in state `token` and becomes its shadow (claim 1 guarantees the existence of such an agent).

Consider an agent A that remains in state `explorer` till the end of the algorithm. In order to estimate the time before which it must declare that gathering is over, we first compute an upper bound on the number of non-clean explorations $EXPLO^*(n)$ performed by it. An exploration could be non-clean due to several reasons, according to the description of the algorithm.

- A met an agent C in state `token` of higher seniority than that of A or of equal seniority but such that $recent - token \prec Pref_t(\mathcal{M}_C)$, or the token B of A was visited by an agent C in state `explorer`

of higher seniority than that of B or of equal seniority but such that $\mathcal{M}_B \prec R$, where R is the variable $recent - token$ of agent C. This case is impossible, as A would not remain in state `explorer` till the end of the algorithm.

- Either agent A or its token B met an agent in state `searcher`. Since the lifespan of a searcher is at most the time of two consecutive executions of $EXPLO^*(n)$, it can overlap at most three consecutive executions of this procedure. Hence one searcher can make non-clean at most 6 explorations (3 by meeting A and 3 by meeting B). Since there are at most n searchers, this gives at most $6n$ non-clean explorations.

- A met an agent C in state `token` of lower seniority than that of A or of equal seniority but such that $\mathcal{M}_C \prec recent - token$. After this meeting, the remaining time when agent C remains in state `token` is at most the duration of one execution of $EXPLO^*(n)$ (after at most this time the explorer of C becomes searcher and hence C transits to state `shadow`). This time can overlap at most two consecutive executions of $EXPLO^*(n)$, hence such meetings can make at most $2n$ non-clean explorations.

- The token B of A met an agent C in state `explorer` of lower seniority than that of B or of equal seniority but such that $recent - token \prec Pref_t(\mathcal{M}_B)$. A similar analysis as in the previous case shows that such meetings can make at most $2n$ non-clean explorations.

- Agent A met an agent C in state `explorer`. The memories of the two agents at this time are different. After this meeting, the remaining time when agent C remains in state `explorer` is at most the duration of two consecutive executions of $EXPLO^*(n)$ because after the return of C on its token, the tokens of A and C have different memories and hence after another exploration, C must become a searcher. Indeed, since by assumption A remains in state `explorer` till the end of the algorithm, we must have $R \prec Pref_t(\mathcal{M}_B)$, where R is the variable $recent - token$ of C at the time t, where t is the first round after the meeting of A and C, in which agent C updated its variable $recent - token$. This gives at most $3n$ non-clean explorations.

- A met an agent C in state `token` in round s, that looked like its token B at this time, but that turned out not to be the token B after the backtrack of A on B. More precisely, $recent - token = Pref_t(\mathcal{M}_c)$ in round s (where t is the last round when the variable $recent - token$ of A was updated) but $Pref_s(\mathcal{M}_B) \neq Pref_s(\mathcal{M}_C)$. After round s agent C remains in state `token` for at most the duration of two executions of $EXPLO^*(n)$. This gives at most $3n$ non-clean explorations.

- The token B was visted by an agent C in state `explorer`, that looked like its explorer A at this time, but that turned out not to be A after the backtrack of A on B. Similarly as before, this gives at most $3n$ non-clean explorations.

Hence there can be at most $19n$ non-clean executions of procedure $EXPLO^*(n)$ for agent A (notice that, e.g., an agent can make non-clean one exploration in the state `explorer` and then in the state `searcher`, hence for simplicity we add all the above upper bounds). A similar analysis shows that during at most $19n$ waiting periods of a duration $2T(EXPLO(n))$ agent A can be met by a new agent. Recall that before performing the first execution of $EXPLO^*(n)$ agent A has been waiting for $T(EXPLO(n)) + T(SIGN(n)) + P(n, L)$ rounds. Hence after at most $T(EXPLO(n)) + T(SIGN(n)) + P(n, L) + (2 + 38n)(2T(EXPLO(n))$ rounds since agent A transited to state `explorer` there has been a clean execution of $EXPLO^*(n)$ followed by a waiting period without any new agent coming. Hence agent A declares that gathering is over.

It remains to consider an agent A in state `token`. From this state, either at some point the agent transits to state `shadow` or it remains in state `token` till the end of the algorithm. In this latter case, its explorer declares that gathering is over after at most $T(EXPLO(n)) + T(SIGN(n)) + P(n, L) + (2 + 38n)(2T(EXPLO(n))$ rounds since it transited to state `explorer`. However, as soon as an explorer declares that gathering is over, its token does the same. So, agent A declares that gathering is over after at most $T(EXPLO(n)) + T(SIGN(n)) + P(n, L) + (2 + 38n)(2T(EXPLO(n))$ rounds since it transited to state `token` (recall that, according to the algorithm, agent A and its explorer have reached their current state at the same time).

Hence every agent eventually terminates. We conclude by observing that the execution time of the entire algorithm is upper bounded by the sum of the following upper bounds:

- the time between the wake up of the first agent and the time of the wake up of an agent A that will be in state `explorer` when declaring that gathering is over; this time is upper bounded by $T(EXPLO(n))$.
- the time that such an agent A spends in state `setup` and `cruiser`
- the time that such an agent A spends in state `explorer`

We have shown above that each of these upper bounds is $O(T(EXPLO(n)) + T(SIGN(n)) + P(n, L))$, where L is polynomial in n. Since $T(EXPLO(n))$, $T(SIGN(n))$ and $P(n, L)$ are all polynomial in n, this proves that the running time of Algorithm Gathering-with-Detection is polynomial in n. □

In the sequel we will use the following notion, which is a generalization of the enhanced view of a node. Consider a configuration of agents in any round. Color nodes v and w with the same color if and only if they are occupied by agents $A_1, \ldots, A_r$ and $B_1, \ldots, B_r$, respectively, where A_i and B_i have the same memory in this round. A *colored* view from node v is the view from v in which nodes are colored according to the above rule.

In view of Lemma 3.2, all agents eventually declare that gathering is over. Hence the final configuration must consist of agents in states `explorer` , `token` and `shadow`, all situated in nodes $v_1, \ldots, v_k$, such that in each node v_i there is exactly one agent E_i in state `explorer`, exactly one agent T_i in state `token` and possibly some agents in state `shadow`. Call such a final configuration a *clone* configuration if there are at least two distinct nodes v_i, v_j which have identical colored views. We will first show that the final configuration cannot be a clone configuration and then that it must consist of all agents gathered in a unique node and hence our algorithm is correct.

Lemma 3.3 *The final configuration cannot be a clone configuration.*

Proof. Suppose for contradiction that the final configuration in round f contains distinct nodes which have identical colored views. Let A be one of the agents woken up earliest by the adversary. There exists an agent A' (also woken up earliest by the adversary) which has an identical memory as A and an identical colored view. Notice that if two agents have the same memory at time t they must have had the same memory at time $t - 1$. Since colors in a colored view are decided by memories of agents, this implies (by a backward induction on the round number) that the colored views of A and A' are the same in each round after their wake-up, and in particular *in* the round of their wake-up. In this round no agent has moved yet and hence each agent is in a different node. Hence colored views in this round correspond to enhanced views. Thus we can conclude that the enhanced views from the initial positions of agents A and A' were identical, which contradicts the assumption that in the initial configuration every agent has a unique enhanced view. □

Lemma 3.4 *In the final configuration all agents must be at the same node.*

Proof. Let B be the first (or one of the first) agents in state `token` that declares that gathering is over. Let A be its explorer. Let τ_0 be the round in which agent A starts its last exploration $EXPLO^*(n)$. Let $\tau_{1/2}$ be the round in which backtrack begins during this execution. Let τ_1 be the round in which this backtrack (and hence the execution of $EXPLO^*(n)$) is finished, and let τ_2 be the round in which A declares that gathering is over.

Claim 1. In round τ_0 all agents in state `token` have the same memory.

In order to prove the claim we first show that all agents in state `token` in round τ_0 have the same seniority. Observe that there cannot be any agent in state `token` of higher seniority than B: any such agent would be seen by A during its last clean exploration $EXPLO^*(n)$ between rounds τ_0 and τ_1 contradicting its cleanliness. Also there cannot be any agent C in state `token` of lower seniority than B. Indeed, let D be the explorer of C. Either D becomes a `searcher` between τ_0 and τ_1 and thus it meets the token B before time τ_2 which contradicts the declaration of A and B at time τ_2 or it remains an `explorer`, in which case C remains a `token` between τ_0 and τ_1 and thus C is visited by A during its last clean exploration, contradicting its cleanliness. This shows that all agents in state `token` in round τ_0 have the same seniority. Hence their explorers start and finish $EXPLO^*(n)$ at the same time. Consequently no token existing in round τ_0 can transit to state `shadow` before round τ_1. Agent A must have seen all these tokens during its last exploration. It follows that the memory of each such token in round τ_0 must be equal to the memory of B at this time: otherwise, agent A would detect such a discrepancy during its last exploration, which would contradict the cleanliness of this exploration. This proves Claim 1.

Claim 1 implies that in time τ_0 all agents in state `explorer` have the same memory. Indeed, since at time τ_0 agent A is together with B, each explorer must be with its token, since tokens have the same memory.

Claim 2. In round τ_0 there are no agents in state `searcher`.

Suppose for contradiction that there is a searcher S in round τ_0. Recall that S performs two explorations: one entire exploration $EXPLO^*(n)$ and another partial exploration $EXPLO^*(n)$ until meeting a token or an explorer.

Case 1. S finished its first exploration $EXPLO^*(n)$ by round τ_0.
Hence its second exploration ends by round τ_1. It could not end by round τ_0 because S would not be a searcher in this round anymore. If it ended between τ_0 and τ_1, it must have met a token C. By Claim 1, all explorers have the same seniority and hence at time τ_1 the explorer D of C backtracked to C. This exploration is not clean for D. Either D becomes a `searcher` at time τ_1 and thus meets A and B before time τ_2, contradicting their declaration at time τ_2, or D starts another $EXPLO^*(n)$ and it meets itself A and B before time τ_2, contradicting their declaration at time τ_2. This shows that the second exploration of S cannot end between τ_0 and τ_1, hence Case 1 is impossible.

Case 2. S finished its first exploration $EXPLO^*(n)$ between τ_0 and $\tau_{1/2}$.
Hence it must visit some token C during its second exploration (and before starting the backtrack) by round τ_1. As before, this contradicts the declaration of A and B at time τ_2.

Case 3. S finished its first exploration $EXPLO^*(n)$ between $\tau_{1/2}$ and τ_1.
Hence the entire backtrack during this first exploration took place between rounds τ_0 and τ_1. During this backtrack, S visited some token. As before, this contradicts the declaration of A and B at time τ_2.

Case 4. S finished its first exploration $EXPLO^*(n)$ after round τ_1.
This is impossible, as it would not be in state `searcher` in round τ_0.

This concludes the proof of Claim 2.

Claim 3. Let $\mathcal{E}$ be the set of agents in state `explorer` in round τ_0. In round $\tau_{1/2}$ every agent from $\mathcal{E}$ can reconstruct its colored view in round τ_0.

To prove the claim first note that since agent A starts its last exploration in round τ_0 and all agents from $\mathcal{E}$ have the same memory in round τ_0, they all start an exploration $EXPLO^*(n)$ in this round. In round $\tau_{1/2}$ every agent from $\mathcal{E}$ has visited all nodes of the graph and starts its backtrack. In round τ_0 there are no agents in state `setup` or `cruiser`, in view of the waiting time when A transited to state `explorer`, and there are no agents in state `searcher` by Claim 2. Hence the visit of all nodes between rounds τ_0 and $\tau_{1/2}$ permits to see all agents that were tokens at time τ_0. Since at this time every explorer were with its token, this permits to reconstruct the memories and the positions of all agents in round τ_0. This is enough to reconstruct the colored views of all agents in round τ_0, which proves the claim.

To conclude the proof of the lemma it is enough to show that in round τ_0 only one node is occupied by agents, since this will be the final configuration. Suppose that nodes $v \neq v'$ are occupied in this round. Let A be the explorer at v and A' the explorer at v'. Note that the colored views of A and A' in round τ_0 must be different, for otherwise the configuration in round τ_0 would be a clone configuration, and consequently the final configuration would also be clone, contradicting Lemma 3.3. Since, by Claim 3, in round $\tau_{1/2}$ each of the agents A and A' has reconstructed its colored view in round τ_0, their memories in round $\tau_{1/2}$ are different. Between rounds $\tau_{1/2}$ and τ_1, during its backtrack, agent A' has visited again all tokens, in particular the token of A. Hence A, after backtracking to its token in round τ_1, realizes that another explorer has visited its token, which contradicts the cleanliness of the last exploration of A. This contradiction shows that in round τ_0 only one node is occupied and hence the same is true in the final configuration. This concludes the proof of the lemma. □

Now the proof of Theorem 3.1 follows directly from Lemmas 3.1, 3.2, and 3.4.

4 Unknown upper bound on the size of the graph

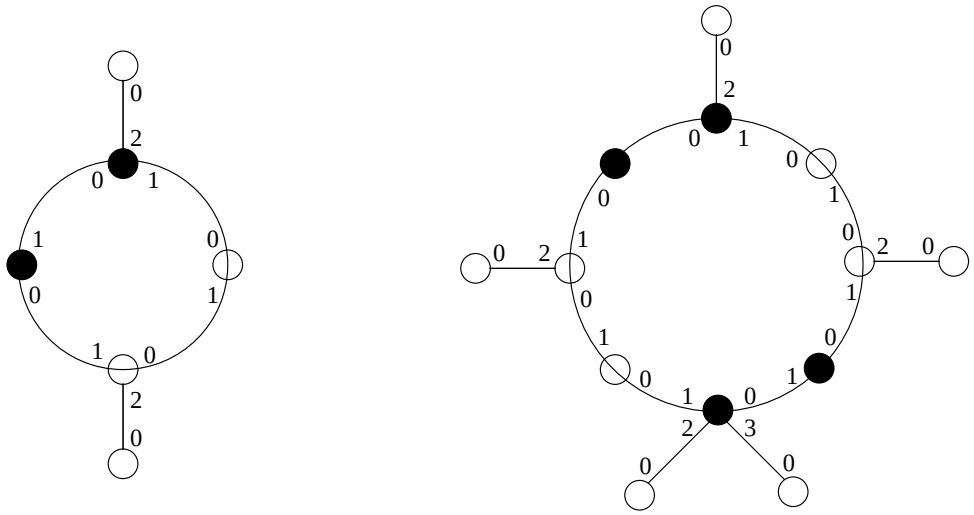
In this section we show that, if no upper bound on the size of the graph is known, then there is no universal algorithm for gathering *with detection* all gatherable configurations. Nevertheless, we still show in this case a universal algorithm that gathers all gatherable configurations: upon completion of this algorithm all agents from any gatherable configuration eventually stop forever at the same node (although no agent is ever sure that gathering is over). The time of such an algorithm is the number of rounds between the wake-up of the first agent and the last round in which some agent moves. Our algorithm is polynomial in the (unknown) size of the graph.

We first prove the following negative result.

Theorem 4.1 *There is no universal algorithm for gathering with detection all gatherable configurations in all graphs.*

Proof. Consider the following initial configurations. In configuration C the graph is a 4-cycle with clockwise oriented ports 0,1 at each node, and with additional nodes of degree 1 attached to two non-consecutive nodes. There are two agents starting at a node of degree 2 and at its clockwise neighbor, cf. Fig. 1 (a). In configuration D_n, for $n = 4k$, the graph is constructed as follows. Take a cycle of size n with clockwise oriented ports 0,1 at each node. Call clockwise consecutive nodes of the cycle $v_0, \ldots, v_{n-1}$ (names are used only to explain the construction) and attach two nodes of degree 1 to v_0 and one node of degree 1 to every other node with even index. Initial positions of agents are at nodes v_i, where $i = 4j$ or $i = 4j - 1$, for some j, cf. Fig. 1 (b).

Each of the configurations C and D_n, for $n \geq 8$, is gatherable. Indeed, in each of these configurations there exist agents with different views (agents starting at nodes of degree 2 and of degree 3) and each agent has a unique enhanced view (this is obvious for configuration C and follows from the existence of a unique node of degree 4 for configurations D_n). Hence each of these configurations satisfies condition **G** and consequently, by Theorem 3.1, there is an algorithm for gathering with detection each specific configuration,



(a) CONFIGURATION C (b) CONFIGURATION D

Figure 1: Configurations C and D_8 in the proof of Theorem 4.1. Black nodes are occupied by agents

as such a dedicated algorithm knows the configuration and hence may use the knowledge of the size of the graph.

It remains to show that there is no *universal* algorithm that gathers with detection all configurations C and D_n. Suppose, for contradiction, that $\mathcal{A}$ is such an algorithm. Suppose that the adversary wakes up all agents simultaneously and let t be the time after which agents in configuration C stop at the same node and declare that gathering is over. Consider the configuration D_{8t} and two consecutive agents antipodal to the unique node of degree 4, i.e., starting from nodes v_{4t} and v_{4t-1}. Call X the agent starting at a node of degree 2 in configuration C and call Y the agent starting at its clockwise neighbor (of degree 3) in this configuration. Call X' the agent starting at node v_{4t-1} and call Y' the agent starting at node v_{4t} in configuration D_{8t}. (Again names are used only to explain the construction.)

In the first t rounds of the executions of algorithm $\mathcal{A}$ starting from configurations C and D_{8t} the memories of the agents X and X' and of the agents Y and Y' are the same. This easily follows by induction on the round number. Hence after t rounds agents X' and Y' starting from configuration D_{8t} stop and (falsely) declare that gathering is over. This contradicts universality of algorithm $\mathcal{A}$. □

Our final result is a universal algorithm gathering all gatherable configurations, working without any additional knowledge. It accomplishes correct gathering and always terminates but (as opposed to Algorithm Gathering-with-Detection which used an upper bound on the size of the graph), this algorithm does not have the feature of detecting that gathering is over. We first present a high-level idea of the algorithm, then describe it in detail and prove its correctness. Recall that we assume that the initial configuration satisfies condition **G** (otherwise gathering, even without detection, is impossible by Lemma 3.1).

Idea of the algorithm.

Since in our present scenario no upper bound on the size of the graph is known, already guaranteeing any meeting between agents must be done differently than in Algorithm Gathering-with-Detection. After wake-up each agent proceeds in phases $n = 1, 2, \ldots$, where in phase n it "supposes" that the graph has size n. In each phase an appropriate label based on procedure $SIGN(n)$ is computed and procedure TZ is

performed sufficiently long to guarantee a meeting at most at the end of phase m, where m is the real size of the graph. If no meeting occurs in some phase sufficiently long, the agent passes to the next phase.

Another important difference occurs after the meeting, when one of the agents becomes an explorer and the other its token. Unlike in the case of known upper bound on the size of the graph, there is no way for any explorer to be sure at any point of the execution that it has already visited the entire graph. Clearly procedure $EXPLO(n)$ cannot give this guarantee, as n is unknown, and procedure EST of exploration with a stationary token, which does not require the knowledge of an upper bound, cannot give this guarantee either, as an explorer cannot be always sure that it visits its own token, because memories of several agents playing the role of the token can be identical at various stages of the execution, and hence these "tokens" may be undistinguishable for the explorer.

Nevertheless, our algorithm succeeds in accomplishing the task by using a mechanism which is analogous to the "butterfly effect". Even a slight asymmetry in a remote part of the graph is eventually communicated to all agents and guarantees that at some point some explorer will visit the entire graph (although in some graphs no explorer can ever be sure of it at any point of an execution) and then all agents will eventually gather at the token of one of these explorers. Making all agents decide on the same token uses property **G** and is one of the main technical difficulties of the algorithm.

Algorithm Gathering-without-Detection

Similarly as in Algorithm Gathering-with-Detection, an agent can be in one of the following five states: `traveler`, `shadow`, `explorer`, `token`, `searcher`. State `traveler` partly combines the roles of previous states `setup` and `cruiser`. For every agent A in state `shadow` the notion of guide is defined as before. Below we describe the actions of an agent A in each of the states and the transitions between the states. At wake-up agent A enters the state `traveler`.

State `traveler`.

In this state agent A works in phases numbered $1, 2, \ldots$. In phase n the agent supposes that the graph has size n. Agent A performs $EXPLO(n)$ in order to visit all nodes and wake up all still dormant agents, if the assumption was correct. Then agent A performs the procedure $SIGN(n)$ finding the current signature of its initial position v, called the *label* ℓ_n of agent A. Let L_n be the maximum possible label of an agent in phase n (Note that L_n is polynomial in n.) Then agent A performs $TZ(\ell_n)$ for Δ_n rounds, where $\Delta_n = T(EXPLO(n)) + T(SIGN(n)) + 2P(n, L_n) + \Delta_{n-1}$, for $n \geq 2$, and $\Delta_1 = 0$. If no agent has been met during phase n, agent A starts phase $n+1$. As soon as another agent is met in some phase k, agent A interrupts this phase and transits either to state `shadow` or `token` or `explorer`. Suppose that the first meeting of agent A occurs in round t at node v.

Case 1. There are some agents in round t at node v which are either in state `searcher`, or `explorer` or `token` .

Let $\mathcal{H}$ be the set of these agents. Let H be the agent of largest memory in set $\mathcal{H}$. Agent A transits to state `shadow` and its guide is H.

Case 2. There are only agents in state `traveler` in round t at node v.

Subcase 2.1. Agent A has the largest memory among all agents in round t at node v.

Then agent A transits to state `explorer`.

Subcase 2.2. Agent A does not have the largest memory among all agents in round t at node v.

If there is exactly one agent B with memory larger than A, then agent A transits to state `token`. Otherwise, it transits to state `shadow` of the agent with largest memory.

(Note that cases 1 and 2 cover all possibilities because an agent in state `shadow` always accompanies its guide and this guide cannot be an agent in state `traveler`.)

State `shadow`.

Agent A has exactly one guide and is at the same node as the guide in every round. In every round it

makes the same move as the guide. If the guide B transits itself to state `shadow` and gets agent C as its guide, then agent A changes its guide to C as well.

In the description of the actions in the three remaining states, we will use the notion of seniority defined for Algorithm Gathering-with-Detection.

State `explorer`.

When agent A transits to state `explorer`, there is another agent B that transits to state `token` in the same round at the same node v. Agent B is called the token of A. Agent A has a variable $recent - token$ that it initializes to the memory of B in this round.

We first define the notion of a *consistent meeting* for agent A. Let t be the last round when agent A updated its variable $recent - token$. A consistent meeting for A is a meeting in round $t' > t$ with an agent C in state `token` of the same seniority as A, such that $\mathcal{M}$ is the current memory of C and $Pref_t(\mathcal{M}) = recent - token$. Intuitively, a consistent meeting is a meeting of an agent that A can plausibly consider to be its token B. Note that, according to this definition, a meeting in the round when the variable $recent - token$ is updated, is not a consistent meeting.

We now briefly describe the procedure EST based on [11] that will be subsequently adapted to our needs. The agent constructs a BFS tree rooted at its starting node r marked by the stationary token. In this tree it marks port numbers at all nodes. During the BFS traversal, some nodes are added to the BFS tree. In the beginning the root is added. Whenever a node w is added to the BFS tree, all its neighbors are visited by the agent. For each neighbor v of w, the agent verifies if v is equal to some node u previously added to the tree. To do this, the agent travels from v using the reversal $\overline{q}$ of the path q from r to u (the path q is a sequence of port numbers). If at the end of this backtrack it meets the token, then $v = u$. In this case v is not added to the tree as a neighbor of w and is called *w-rejected*. If not, then $v \neq u$. This verification is done for all nodes that are already in the tree. If v is different from all these nodes, then it is added to the tree.

The procedure EST^* is a simulation of EST with the following two changes. The first change is as follows. Suppose that the execution of EST produced the route α of the agent. In procedure EST^*, upon completing procedure EST, the agent traverses the reverse route $\overline{\alpha}$ and then again α and $\overline{\alpha}$. Hence in procedure EST^* the agent traverses the concatenation of routes $\alpha, \overline{\alpha}, \alpha, \overline{\alpha}$. These parts of the trajectory will be called, respectively, the first, second, third and fourth segment of EST^*. The variable $recent - token$ is updated in the beginning of the first and third segment of EST^*. Note that in these rounds agent A is certain to be with its token.

The second change concerns meetings with the token. Consider a verification if a newly reached node w in EST is equal to some previously constructed node u. This verification consists in traveling from w using the reverse path $\overline{q}$, where q is the path from the root r to u in the BFS tree and checking the presence of the token. If at the end of the simulation of path $\overline{q}$ in the first segment of EST^* agent A makes a consistent meeting, then it acts as if it saw the token in EST; otherwise it acts as if it did not see the token in EST.

Similarly as for $EXPLO^*(n)$, an execution of EST^* is called *clean* if the following condition is satisfied: in each round during this execution, in which A met an agent C, the memory of C is equal to that of B, and in each round during this execution, in which the token B was met by an agent D, the memory of D was equal to that of A. Notice that after the execution of EST^*, agent A is together with its token B and thus they can verify if the execution was clean, by inspecting their memories. The execution time of EST^* in a graph of size m (unknown to the agents) is at most $4T(EST(m))$.

After transiting to state `explorer`, agent A executes the following protocol:

repeat forever
/*Before the first turn of the loop agent A has just entered state `explorer` and is with its token. After each turn of the loop, agent A is with its token, waiting after a clean exploration.*/

if A has just transited to state `explorer` **or** A has just been visited by another agent **then**

do

EST^*

if A met an agent C in state `token` of higher seniority than that of A or of equal seniority but such that $recent - token \prec Pref_t(\mathcal{M}_C)$ where $\mathcal{M}_C$ is the memory of agent C and t is the last round when agent A updated its variable $recent - token$ **then** A transits to state `searcher`

if A met another agent C in state `explorer`, such that either the seniority of C is higher than that of A, or these seniorities are equal but $R_A \prec R_C$, where R_A is the value of the variable $recent - token$ of A and R_B is the value of the variable $recent - token$ of B at the time of the meeting, or the seniorities are equal and $R_A = R_C$, but $\mathcal{M}_A \prec \mathcal{M}_C$, where $\mathcal{M}_A$ (resp. $\mathcal{M}_C$) is the memory of A (resp. of C) at the time of the meeting **then** A transits to state `searcher`

if B was visited in round t' by an agent C in state `explorer` of higher seniority than that of B or of equal seniority but such that $\mathcal{M}_B \prec R$ where $\mathcal{M}_B$ is the memory of agent B and R is the variable $recent - token$ of agent C in round t' **then** A transits to state `searcher`

until the execution of EST^* is clean

State `token`.

When agent A transits to state `token`, there is another agent B that transits to state `explorer` in the same round at the same node v. Agent B is called the explorer of A. Agent A remains idle at a node v and does not change its state, except when its explorer B transits to state `searcher`. In this case it transits to state `shadow` and B becomes its guide.

State `searcher`

After transiting to state `searcher` agent A performs the sequence of explorations $EXPLO(n)$ for $n = 1, 2, \ldots$ until it meets an agent in state `token` or `explorer` in round t. Let $\mathcal{S}$ be the set of these agents met by A in round t. Agent A transits to state `shadow` and its guide is the agent from $\mathcal{S}$ with largest memory.

The analysis of the algorithm is split into the following lemmas.

Lemma 4.1 *In Algorithm Gathering-without-Detection every agent eventually stops after time polynomial in the size of the graph.*

Proof. Let m be the size of the graph (unknown to the agents). Let A be any agent. We may assume that at some point A is woken up (otherwise it would be idle all the time). Similarly as in the proof of Lemma 3.2 we show that A must meet some other agent at the end of phase m at the latest. Indeed, the time Δ_m during which A performs the procedure TZ in phase m is sufficiently long for meeting some agent, as all agents have to be woken up by the round when A starts performing TZ in phase m. Hence after time at most Δ_m agent A transits from state `traveler` either to state `shadow` or `token` or `explorer`. As before we may exclude the state `shadow` from our analysis.

Consider an agent in state `explorer`. Either at some point it transits to state `searcher`, in which case, after this transition, it uses at most $\Sigma_{i=1}^{m} T(EXPLO(i))$ rounds to perform procedures $EXPLO(i)$ for $i = 1, 2, \ldots, m$, by which time it must have met some token or explorer (because at least one token is idle all the time, cf. Claim 1 in the proof of Lemma 3.2) and hence must have transited to state `shadow`, or it remains in state `explorer` till the end of the algorithm.

Consider an agent A that remains in state `explorer` till the end of the algorithm. We will first show that the total number of rounds when this agent moves is polynomial in m. This is not enough to show that A remains idle forever after polynomial time, since we still need to bound the duration of each period of

idleness between any consecutive periods of moving. This will be addressed later.

Two events can trigger further moves of the agent: a meeting causing a non-clean exploration EST^* or a visit of A by some agent, when A stays with its token after a clean exploration.

We first treat the first of these two types of events and bound the total time of explorations caused by them. An exploration could be non-clean due to several reasons, according to the description of the algorithm.

- A met an agent C in state `token` of higher seniority than that of A or of equal seniority but such that $recent - token \prec Pref_t(\mathcal{M}_C)$, or the token B of A was visited by an agent C in state `explorer` of higher seniority than that of B or of equal seniority but such that $\mathcal{M}_B \prec R$, where R is the variable $recent - token$ of agent C. This case is impossible, as A would not remain in state `explorer` till the end of the algorithm.

- Either agent A or its token B met an agent C in state `traveler`. Since C transits immediately to state `shadow`, all agents in state `traveler` can cause at most $m \cdot 4T(EST(m))$ rounds of motion of A. Indeed, each agent in state `traveler` can cause at most one exploration EST^* to be non-clean.

- Either agent A or its token B met an agent C in state `searcher`. Since C transits immediately to state `shadow`, all agents in state `searcher` can cause at most $m \cdot 4T(EST(m))$ rounds of motion of A. Indeed, each agent in state `searcher` can cause at most one exploration EST^* to be non-clean.

- A met an agent C in state `token` of lower seniority than that of A or of equal seniority but such that $\mathcal{M}_C \prec recent - token$. After this meeting, the remaining time when agent C remains in state `token` is at most the longest duration of one execution of EST^* (after at most this time the explorer of C becomes searcher and hence C transits to state `shadow`). Let t be the round of the meeting. Agent C remains in state `token` till round t', where $t' \leq t + 4T(EST(m))$. So after t' there can be at most one exploration EST^* of A non-clean because of C, finishing in round at most $t + 8T(EST(m))$ and then a possibly clean exploration till round $t + 12T(EST(m))$. Hence all such meetings can cause at most $m \cdot 12T(EST(m))$ rounds of motion of A, apart from the current exploration.

- The token B of A met an agent C in state `explorer` of lower seniority than that of B or of equal seniority but such that $recent - token \prec Pref_t(\mathcal{M}_B)$. A similar analysis as in the previous case shows that such meetings can cause at most $m \cdot 12T(EST(m))$ rounds of motion of A, apart from the current exploration.

- Agent A met an agent C in state `explorer`. After the meeting one of these agents "loses", i.e., it will transit to state `searcher` after backtracking to its token. This loser cannot be A because, by definition A is an agent that remains in state `explorer` till the end. Hence agent C remains in state `explorer` for at most $4T(EST(m))$ rounds after the meeting. Similarly as before such meetings can cause at most $m \cdot 12T(EST(m))$ rounds of motion of A, apart from the current exploration.

- A met an agent C in state `token` in round s, that looked like its token B at this time, but that turned out not to be the token B after the backtrack of A on B. More precisely, $recent - token = Pref_t(\mathcal{M}_c)$ in round s (where t is the last round when the variable $recent - token$ of A was updated) but $Pref_s(\mathcal{M}_B) \neq Pref_s(\mathcal{M}_C)$. After round s agent C may look like token B of A for at most $4T(EST(m))$ rounds because after at most this time A backtracks to its token B and, from this time on, it can see the difference between B and C. Similarly as before such meetings can cause at most $m \cdot 12T(EST(m))$ rounds of motion of A, apart from the current exploration.

- The token B was visited by an agent C in state `explorer`, that looked like its explorer A at this time, but that turned out not to be A after the backtrack of A on B. Similarly as before such meetings can cause at most $m \cdot 12T(EST(m))$ rounds of motion of A, apart from the current exploration.

Hence the first of the two types of events (meeting causing a non-clean exploration) can cause at most $68m \cdot T(EST(m))$ rounds of motion of A, apart from the first exploration. (As before we add all upper bounds for simplicity). The second type of events (a visit of A by some agent, when A stays with its token after a clean exploration) can cause at most the above time of moving of A. Adding one clean exploration that must be made by A even without any meetings and adding the first exploration, we get an upper bound of $(136m + 8) \cdot T(EST(m))$ rounds during which agent A moves in state `explorer`.

It remains to consider an agent in state `token`. It may either transit to state `shadow` or remain in state `token` forever. In the latter case it is idle all the time.

Since $\Delta(n)$, $T(EST(n))$ and $T(EXPLO(n))$ are all polynomial in n, the above analysis shows that there exists a polynomial Q, such that, for each agent A executing Algorithm Gathering-without-Detection in any graph of size m, the number of rounds during which this agent moves is at most $Q(m)$. In order to finish the proof, we need to bound the number of rounds during which an agent A can be idle before moving again. To do this we will use the following claim.

Claim. If in round t of the execution of Algorithm Gathering-without-Detection no agent moves, then no agent moves in any later round of this execution.

To prove the claim notice that if no agent moves in round t, then in this round no agent is in state `traveler` or `searcher`. Moreover each agent in state `explorer` must be idle and stay with its token in this round. (all other nodes must be in state `shadow`). In order for some agent to move in round $t + 1$, some explorer would have to visit some other token in round t, contradicting the definition of t. Hence all agents are idle in round $t + 1$. By induction, all agents are idle from round t on. This proves the claim.

Since for each agent executing Algorithm Gathering-without-Detection in a graph of size m, the number of rounds in which it moves is at most $Q(m)$ and there are at most m agents, the claim implies that after time at most $m \cdot Q(m)$ since the wake up of the first agent, all agents must stop forever. □

By Lemma 4.1 there exists a round after which, according to Algorithm Gathering-without-Detection, no agent moves. Call the resulting configuration *final*. The following lemma implies that Algorithm Gathering-without-Detection is correct.

Lemma 4.2 *In every final configuration exactly one node is occupied by agents.*

Proof. A final configuration must consist of agents in states `explorer` , `token` and `shadow`, all situated in nodes $v_1, \ldots, v_k$, such that in each node v_i there is exactly one agent E_i in state `explorer`, exactly one agent T_i in state `token` and possibly some agents in state `shadow`. As before we call such a final configuration a *clone* configuration if there are at least two distinct nodes v_i, v_j which have identical colored views. The same argument as in the proof of Lemma 3.3 shows that a final configuration cannot be a clone configuration.

It is enough to prove that $k = 1$. Suppose for contradiction that $k > 1$. We will consider two cases. In the first case the memories of all explorers E_i are identical and in the second case they are not. In both cases we will derive a contradiction.

Case 1. All explorers E_i in the final configuration have identical memory.

In this case all these explorers performed the last exploration EST^* simultaneously.

We start with the following claim.

Claim 1. If a node w has been rejected by the explorer E_j in the construction of its BFS tree during its last exploration EST^*, then node w must have been either added previously by E_j to its BFS tree, or added by another explorer E_s in the construction of its BFS tree during its last exploration EST^*.

The node w was rejected by E_j for the following reason. E_j traveled from w using the reversal $\overline{q}$ of the path q, where q is a path (coded as a sequence of ports) from v_j to some node u already in the BFS tree of E_j, and at the end of this path $\overline{q}$, E_j met a token with memory $\mathcal{M}$, such that $Pref_t(\mathcal{M}) = recent - token$, where t is the last round when E_j updated its variable $recent - token$.

There are two possible cases. If the token met by E_j is its own token (residing at v_j), then w is equal to some node u already added previously to the BFS tree of E_j. If, on the other hand, the token met by E_j is the token of some other explorer E_s, then we will show that w is added by E_s to its BFS tree. Indeed, since E_j has added a node u to its BFS tree, such that the path from v_j to u is q, the explorer E_s must have added a node u' to its BFS tree, such that the path from v_s to u' is q as well, because both E_s and E_j have identical memories. However, this node u' must be equal to w, since the path from w to v_s is $\overline{q}$. This proves the claim.

The contradiction in Case 1 will obtained in the following way. Using BFS trees produced by explorers E_i during their last exploration EST^* (recall that these trees are isomorphic, since memories of the explorers are identical), we will construct the colored view for each explorer. Using the fact that memories of the explorers are identical, these colored views will be identical. This will imply that the final configuration is a clone configuration, which is impossible.

The construction proceeds as follows (we will show it for explorer E_1). Let T_i be the BFS tree produced by E_i. Each tree T_i has its root v_i colored black and all other nodes colored white. We will gradually attach various trees to T_1 in order to obtain the colored view from v_1. First attach to every node of T_1 its neighbors that have been rejected by E_1 during the construction of T_1. Explorer E_1 has visited these nodes, hence the respective port numbers can be faithfully added. Consider any such rejected node w. By Claim 1, there are two possibilities. If node w was previously added by E_1 to T_1 as some node u, then we proceed as follows. Let T_1' be the tree T_1 but rooted at u instead of v_1. We attach tree T_1' at w, identifying its root u with w. If node w was added by another explorer E_s in the construction of its BFS tree T_s, we proceed as follows. As mentioned in the proof of Claim 1, the explorer E_s must have added a node u' to its BFS tree, such that the path from v_s to u' is q. Let T_s' be the tree T_s but rooted at u' instead of v_s. We attach tree T_s' at w, identifying its root u' with w.

After processing all nodes rejected by E_1 and adding the appropriate trees, we attach all rejected neighbors of nodes in the newly obtained increased tree. These nodes could have been rejected either by E_1 itself or by another explorer E_j whose (re-rooted) tree T_j' has been attached. For each newly attached node rejected by E_j, the construction continues as before, replacing the role of T_1 by T_j.

The above construction proceeds infinitely, producing the view from v_1, which is an infinite tree. (This infinite tree is indeed the view from v_1 because whenever a node is added to the tree, all its neighbors are eventually added as well, with the correct port numbers.) To produce the colored view, notice that there are only two colors in this colored view: white corresponding to empty nodes in the final configuration and black corresponding to nodes v_1,..., v_k (all these nodes get identical colors: since memories of explorers are the same, memories of their tokens are also the same and memories of corresponding nodes in state `shadow` are also identical). It remains to indicate how the colors are distributed in the constructed view. This is done as follows. When a tree T_j' is attached, exactly one of its nodes (namely the node corresponding to v_j) is black. Exactly these nodes become black in the obtained colored view.

This construction of colored views is done for all explorers E_i. Consider two explorers E_i and E_j. Since these explorers have the same memory, the trees T_s' attached at a given stage of the construction of the views of E_i and E_j are isomorphic. They are also attached in the same places of the view. Hence by induction of the level of the view it follows that both colored views are identical. This implies that the final configuration is a clone configuration which gives a contradiction in Case 1.

Case 2. There are at least two explorers E_i and E_j with different memories in the final configuration.

Consider the equivalence relation on the set of explorers $E_1, \ldots, E_k$, such that two explorers are equiv-

alent if their memories in the final configuration are identical. Let $\mathcal{C}_1, \ldots, \mathcal{C}_h$, where $h > 1$, be the equivalence classes of this relation. Suppose w.l.o.g. that $\mathcal{C}_1$ is a class of explorers with smallest seniority. We will use the following claim.

Claim 2. During the last exploration EST^* of explorers in $\mathcal{C}_1$, at least one of the following statements holds:

- an explorer from $\mathcal{C}_1$ has visited a token of an explorer not belonging to $\mathcal{C}_1$;
- a token of an explorer from $\mathcal{C}_1$ has been visited by an explorer not belonging to $\mathcal{C}_1$.

In order to prove the claim consider two cases. If every node of the graph has been visited by some explorer from $\mathcal{C}_1$ we will show that the first statement holds. Indeed, since explorers from $\mathcal{C}_1$ have the smallest seniority, during their last execution of EST^* all tokens are already at their respective nodes. Hence some explorers from $\mathcal{C}_1$ must visit the tokens of explorers outside of $\mathcal{C}_1$. Hence we can restrict attention to the second case, when some nodes of the graph have not been visited by any explorer from $\mathcal{C}_1$. Notice that if there were no other classes than $\mathcal{C}_1$, this could not occur. Indeed, we would be then in Case 1 (in which all explorers have identical memory). Thus Claim 1 would hold, which implies that all nodes must be visited by some explorer, in view of the graph connectivity.

Hence the fact that some node is not visited by explorers from $\mathcal{C}_1$ must be due to a meeting of some other agent (which is neither an explorer from $\mathcal{C}_1$ nor a token of such an explorer) during their last exploration EST^*. What kind of a meeting can it be? It cannot be a meeting with an agent in state `traveler` or `searcher` because this would contradict that the last exploration was clean. For the same reason it cannot be a meeting of an explorer from $\mathcal{C}_1$ with another explorer. This leaves only the two types of meetings specified in the claim, which finishes the proof of the claim.

Let (Ex, Tok) be a couple of an explorer outside of $\mathcal{C}_1$ and of its token, such that either an explorer from $\mathcal{C}_1$ visited Tok or a token of an explorer from $\mathcal{C}_1$ has been visited by Ex during the last exploration of explorers in the class $\mathcal{C}_1$. Such a couple exists by Claim 2. The seniority of Ex and Tok must be the same as that of explorers from $\mathcal{C}_1$, for otherwise their last exploration would not be clean. For the same reason, when explorers from $\mathcal{C}_1$ started their last exploration, the explorer Ex must have started an exploration as well (possibly not its final exploration): otherwise the exploration of explorers from $\mathcal{C}_1$ would not be clean. Moreover we show that when explorers from $\mathcal{C}_1$ finished their last exploration, explorer Ex must have finished an exploration as well. To prove this, consider two cases, corresponding to two possibilities in Claim 2. Suppose that an explorer E_j from $\mathcal{C}_1$ has visited Tok and that its exploration did not finish simultaneously with the exploration of Ex. Consider the consecutive segments S_1, S_2, S_3, S_4 of the last exploration EST^* of E_j. (Recall that these segments were specified in the definition of EST^*.) Since E_j has visited Tok during EST^*, it must have visited it during each segment S_i. At the end of S_1, explorer E_j knows how long EST^* will take. At the end of S_2 its token learns it as well. When E_j visits Tok again in segment S_3, there are two possibilities. Either Tok does not know when the exploration of Ex finishes, or it does know that it finishes at a different time than the exploration of E_j. In both cases the explorer E_j that updated its variable $recent-token$ at the end of S_2 can see that $recent-token \neq Pref_t(\mathcal{M})$, where $\mathcal{M}$ is the memory of Tok and t is the end of S_2. This makes the last exploration of E_j non clean, which is a contradiction. This proves that Ex and E_j finish their exploration simultaneously, if E_j has visited Tok. The other case, when Ex has visited the token of E_j is similar. Hence we conclude that explorations of E_j and of Ex started and finished simultaneously.

Let τ be the round in which the last exploration of E_j (and hence of all explorers in $\mathcal{C}_1$) finished. The exploration of Ex that finished in round τ cannot be its final exploration because then it would have the same memory as E_j in the final configuration and thus it would be in the class $\mathcal{C}_1$ contrary to the choice of Ex. Hence Ex must move after round τ. It follows that there exists a class $\mathcal{C}_i$ (w.l.o.g. let it be $\mathcal{C}_2$) such that explorers from this class started their last exploration after round τ. Note that during this last exploration,

explorers from $\mathcal{C}_2$ could not visit all nodes of the graph, for otherwise they would meet explorers from $\mathcal{C}_1$ after round τ, inducing them to move after this round, contradicting the fact that explorers from $\mathcal{C}_1$ do not move after round τ.

The fact that some node is not visited by explorers from $\mathcal{C}_2$ must be due to a meeting of some other agent (which is neither an explorer from $\mathcal{C}_2$ nor a token of such an explorer) during their last exploration EST^*. Otherwise, for explorers in $\mathcal{C}_2$ the situation would be identical as if their equivalence class were the only one, and hence, as in Case 1, they would visit all nodes. Moreover, the fact that some node is not visited by explorers from $\mathcal{C}_2$ must be due to a meeting of some explorer outside of $\mathcal{C}_1$ or of its token (if not, explorers from $\mathcal{C}_1$ would move after round τ, which is a contradiction). An argument similar to that used in the proof of Claim 2 shows that there exists a couple (Ex', Tok'), such that Ex' is an explorer outside of $\mathcal{C}_1 \cup \mathcal{C}_2$, Tok' is its token, and either an explorer from $\mathcal{C}_2$ visited Tok' or a token of an explorer from $\mathcal{C}_2$ has been visited by Ex' during the last exploration of explorers in the class $\mathcal{C}_2$. Let τ' be the round in which the last exploration of explorers from $\mathcal{C}_2$ is finished. Similarly as before, the explorer Ex' terminates some exploration in round τ' but continues to move afterwards.

Repeating the same argument $h-1$ times we conclude that there exists a round τ^* after which all explorers from $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_{h-1}$ never move again, but the last exploration of explorers from $\mathcal{C}_h$ starts on or after τ^*. During this last exploration there must be a node not visited by any explorer from $\mathcal{C}_h$, otherwise some explorers from $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_{h-1}$ would move after τ^*. This is due to a meeting. It cannot be a meeting with an agent in state `traveler` or `searcher` because this would contradict that the last exploration was clean. For the same reason it cannot be a meeting of an explorer from $\mathcal{C}_h$ with another explorer. Hence two possibilities remain. Either an explorer from $\mathcal{C}_h$ visits a token of an explorer from $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_{h-1}$ or a token of an explorer from $\mathcal{C}_h$ is visited by an explorer from $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_{h-1}$. The first situation is impossible because it would contradict the cleanliness of the last exploration of explorers from $\mathcal{C}_h$ and the second situation is impossible because explorers from $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_{h-1}$ do not move after τ^*. Hence in Case 2 we obtain a contradiction as well, which completes the proof. □

Lemmas 4.1 and 4.2 imply the following result.

Theorem 4.2 *Algorithm Gathering-without-Detection performs a correct gathering of all gatherable configurations and terminates in time polynomial in the size of the graph.*

5 Consequences for leader election

Leader election [30] is a fundamental symmetry breaking problem in distributed computing. Its goal is to assign, in some common round, value 1 (leader) to one of the entities and value 0 (non-leader) to all others. In the context of anonymous agents in graphs, leader election can be formulated as follows:

- There exists a common round in which one of the agents assigns itself value 1 (i.e., it declares itself a leader) and each other agent assigns itself value 0 (i.e., it declares itself non-leader).

The following proposition implies that Algorithm Gathering-with-Detection solves the leader election problem for anonymous agents in arbitrary graphs, assuming a known upper bound on the size of the graph.

Proposition 5.1 *Leader election is equivalent to gathering with detection.*

Proof. Suppose that gathering with detection is accomplished and let t be the round when all agents are together and declare that gathering is over. As mentioned in the Preliminaries, all agents must have different memories, since they are at the same node, and, being together, they can compare these memories. Since, in view of detection, the round t is known to all agents, in round $t+1$ the agent with the largest memory assigns itself value 1 and all other agents assign themselves value 0.

Conversely, suppose that leader election is accomplished and let t be the round in which one of the agents assigns itself value 1 and all other agents assign themselves value 0. Starting from round t the agent with value 1 stops forever and plays the role of the token, all other agents playing the role of explorers. First, every explorer finds the token by executing procedure $EXPLO(n)$ for $T(EXPLO(n))$ rounds in phases $n = 1, 2, ...$, until it finds the token in round t' (this round may be different for every explorer). Then every explorer executes procedure EST (using the token) and finds the map of the graph and hence its size m. Then it waits with the token until round $t^* = t + \sum_{i=1}^{m} T(EXPLO(i)) + T(EST(m))$. By this round all explorers must have found the token and executed procedure EST, i.e., they are all together with the token. In round t^* all agents declare that gathering is over. □

Proposition 5.1 implies that the class of initial configurations for which leader election is at all possible (even only using an algorithm dedicated to this specific configuration) is equal to the class of initial configurations for which gathering with detection is possible, i.e., the class of configurations satisfying property **G**. Similarly as for gathering, we will say that a leader election algorithm is *universal* if it performs leader election for all such configurations.

The following corollary gives a complete solution of the leader election problem for anonymous agents in arbitrary graphs.

Corollary 5.1 *If an upper bound on the size of the graph is known, then leader election is possible if and only if the initial configuration satisfies condition* **G**. *Algorithm Gathering-with-Detection accomplishes leader election for all these configurations (by electing as leader the agent that has the largest memory in the round of the gathering declaration). If no upper bound on the size of the graph is known, then a universal algorithm for leader election does not exist.*

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