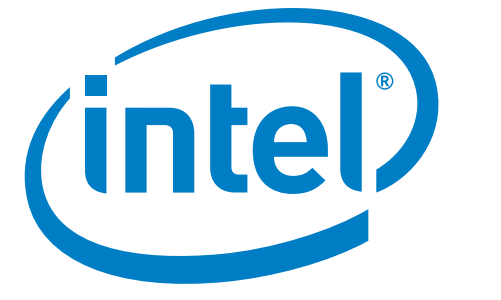


# Highlighting the Influence of Artifact Signals on the Equilibrium State of the Feedback Structure

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## INTRODUCTION

This work aims at providing useful relations and a new vision based on experimental observations for further analyzing the convergence and steady-state properties of the adaptive noise cancelling feedback scheme in the presence of crosstalk.

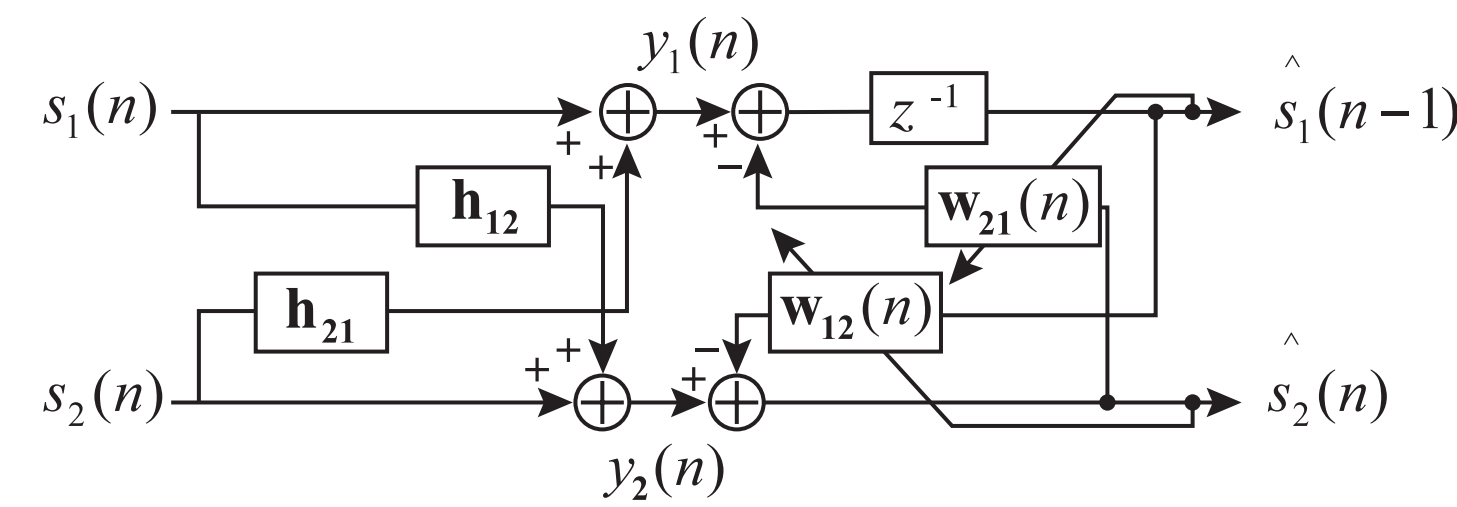


Fig. 1: Adaptive feedback structure

Our approach relies on the analysis of the relationships between the useful signals  $s_i(n)$  and their artifacts  $\Delta s_i(n)$  at the system outputs. Then, based on energy conservation arguments [1], we define the equilibrium point observed experimentally.

## RELEVANCE OF THE PROBLEM

Inherent cross-coupling of the feedback structure

- introduces an IIR filter which can lead to potential instability
- leads to closed-form expressions for learning curves that are difficult to exploit [2]
- brings a Hessian matrix that is time-varying even if the input signals are stationary
- renders the independence theory almost void, see the following analysis

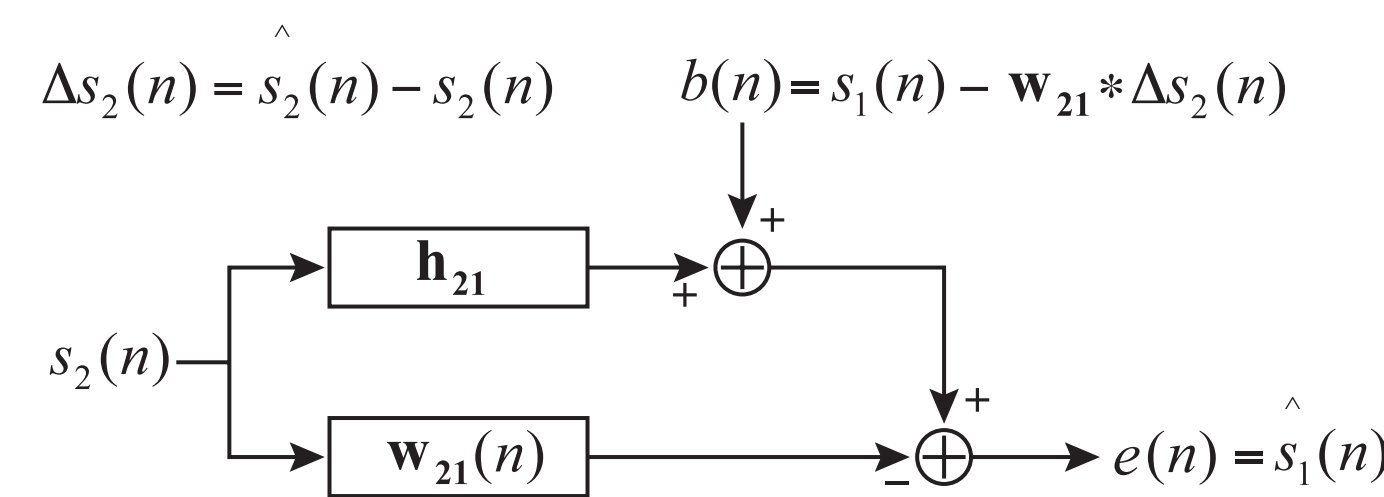


Fig. 2: Equivalent adaptive plant identification of  $h_{21}$

According to the equivalent adaptive plant identification of Fig. 2, we see that in contrast with [3]

- the sequences  $\{s_2(n)\}$  and  $\{b(n)\}$  are mutually dependent
- the noise sequence  $b(n)$  depends on  $w_{21}(n)$  and is not stationary

## ARTIFACT ANALYSIS ON THE OUTPUT SIGNAL

According to the mixing and separation models of Fig. 1, the artifact signals  $\Delta s_i(n)$  on  $s_i(n)$  can be formulated as

$$\Delta s_1(n) = y_1(n) - w_{21}^T(n) \hat{s}_2(n) - s_1(n)$$

$$= h_{21}^T(n) s_2(n) - w_{21}^T(n) y_2(n) + w_{21}^T(n) \text{diag}\{W_{12}^T(n) \hat{S}_1(n-1)\}$$

Where diag operator denotes diagonal elements of matrix and

$$\hat{S}_1(n-1) = [\hat{s}_1(n-1) \hat{s}_1(n-2) \dots \hat{s}_1(n-L)]$$

$$W_{12}(n) = [w_{12}(n) w_{12}(n-1) \dots w_{12}(n-L+1)]$$

### Assumptions:

- $w_{12}(n-j) = w_{12}(n) \forall j = 0, \dots, L-1 \Rightarrow$  slowly time varying
- $h_{12}(0) = 0 \Rightarrow$  causality constraint that is a stability condition

### Definitions:

- $\Delta h_{21}(n) = h_{21} - w_{21}(n) \Rightarrow$  weight deviation vector
- $\tilde{X} \Rightarrow$  truncated version of  $X$
- $\Delta S_1(n) = S_1(n) - \hat{S}_1(n) \Rightarrow$  artifacts matrix

Thus, expressions of artifact signals on each estimated signal are:

$$\Delta s_1(n) = \Delta h_{21}^T(n) s_2(n) - w_{21}^T(n) [\tilde{S}_1(n-1) \Delta h_{12}(n-1) + \Delta S_1^T(n-1) w_{12}(n)]$$

and

$$\Delta s_2(n) = \Delta h_{12}^T(n-1) \tilde{s}_1(n-1) - w_{12}^T(n) [\tilde{S}_2^T(n-1) \Delta h_{21}(n-1) - \Delta S_2^T(n-1) w_{21}(n-1)]$$

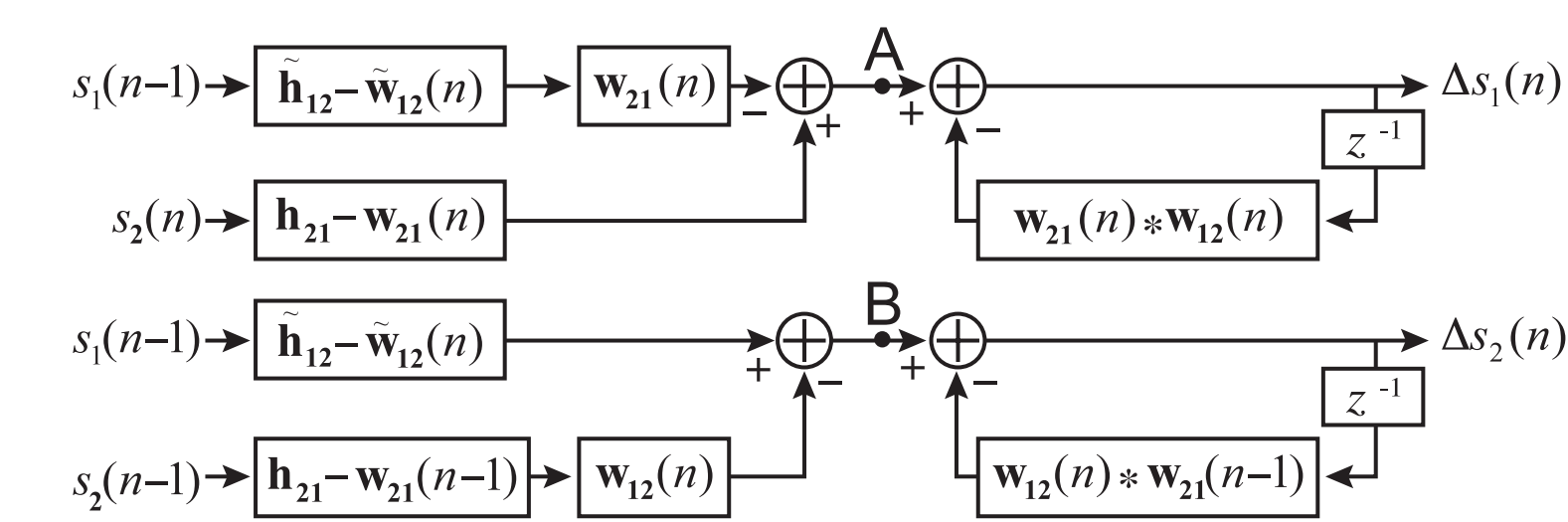


Fig. 3: Global network for getting the artifact signals on each channel

## EQUILIBRIUM STATE

Let us assume that the equilibrium state of the feedback structure given in Fig. 1 is obtained when the energy of the artifacts on the output signal is the same on each channel:

$$E(\Delta s_1^2(n)) = E(\Delta s_2^2(n))$$

By considering that the filters are slowly time varying, i.e.  $w_{21}(n) * w_{12}(n) = w_{12}(n) * w_{21}(n-1)$  equal output artifact energies ensure identical correlation properties (\*\*) of the signals present at points A and B (see Fig. 3).

### Assumptions:

- (\*\*) is equivalent to the equality of the power spectral densities (not universally correct):  
 $\Theta_{AA}(e^{j\omega}) = \Theta_{BB}(e^{j\omega}) \Rightarrow$  Equality principle
- $s_1(n)$  and  $s_2(n)$  are independent and stationary random variables with zero mean and variance  $\sigma_{s_1}^2$  and  $\sigma_{s_2}^2$  respectively

Applying the usual spectral analysis of time-invariant system for some fixed  $W_{12}(z)$  and  $W_{21}(z)$  on the artifact signals leads to the following equilibrium relationship:

$$\frac{|\Delta \tilde{H}_{12}(e^{j\omega})|^2}{|\Delta \tilde{H}_{21}(e^{j\omega})|^2} = \frac{\Theta_{S_2 S_2}(e^{j\omega}) |W_{12}(e^{j\omega})|^2 - 1}{\Theta_{S_1 S_1}(e^{j\omega}) |W_{21}(e^{j\omega})|^2 - 1}$$

Let us define the signal to artifact ratio (SAR) as:

$$\frac{\Theta_{S_1 S_1}(e^{j\omega})}{\Theta_{\Delta S_1 \Delta S_1}(e^{j\omega})} = \frac{1 - |W_{12}(e^{j\omega}) W_{21}(e^{j\omega}) e^{-j\omega}|^2}{\Theta_{AA}(e^{j\omega})}$$

$$SAR_1(e^{j\omega}) = \Theta_{S_1 S_1}(e^{j\omega}) \cdot \frac{1 - |W_{12}(e^{j\omega}) W_{21}(e^{j\omega}) e^{-j\omega}|^2}{\Theta_{AA}(e^{j\omega})}$$

$$\frac{\Theta_{S_2 S_2}(e^{j\omega})}{\Theta_{\Delta S_2 \Delta S_2}(e^{j\omega})} = \frac{1 - |W_{12}(e^{j\omega}) W_{21}(e^{j\omega}) e^{-j\omega}|^2}{\Theta_{BB}(e^{j\omega})}$$

$$SAR_2(e^{j\omega}) = \Theta_{S_2 S_2}(e^{j\omega}) \cdot \frac{1 - |W_{12}(e^{j\omega}) W_{21}(e^{j\omega}) e^{-j\omega}|^2}{\Theta_{BB}(e^{j\omega})}$$

According to the stated equality principle, we get:

$$\frac{SAR_1(e^{j\omega})}{SAR_2(e^{j\omega})} = \frac{\Theta_{S_1 S_1}(e^{j\omega})}{\Theta_{S_2 S_2}(e^{j\omega})}$$

This important relation describes the behavior of the feedback structure and defines its equilibrium point.

## RESULTS WITH SYNTHETIC SIGNALS

Let us define the frequency average SAR noted  $SAR^{f.b.}$  and given by

$$SAR_i^{f.b.} = \int_{-\pi}^{\pi} SAR_i(e^{j\omega}) d\omega$$

The full-band SAR values obtained during the steady-state convergence phase for different configurations of the input sources power ratio  $\sigma_{s_1}^2 / \sigma_{s_2}^2$  are plotted in Fig. 4 and Fig. 5.

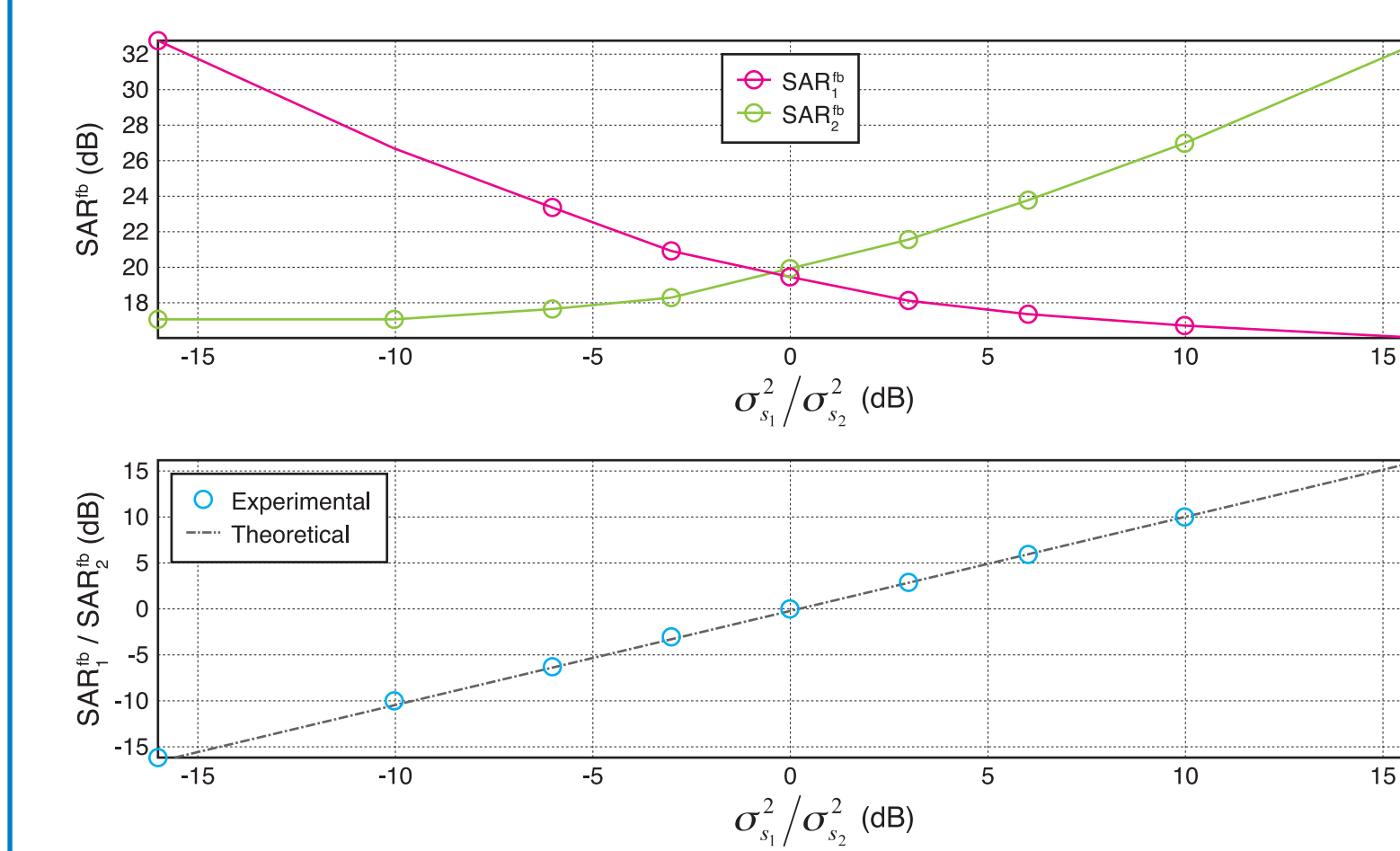


Fig. 4: Frequency average SAR obtained for two white Gaussian noises

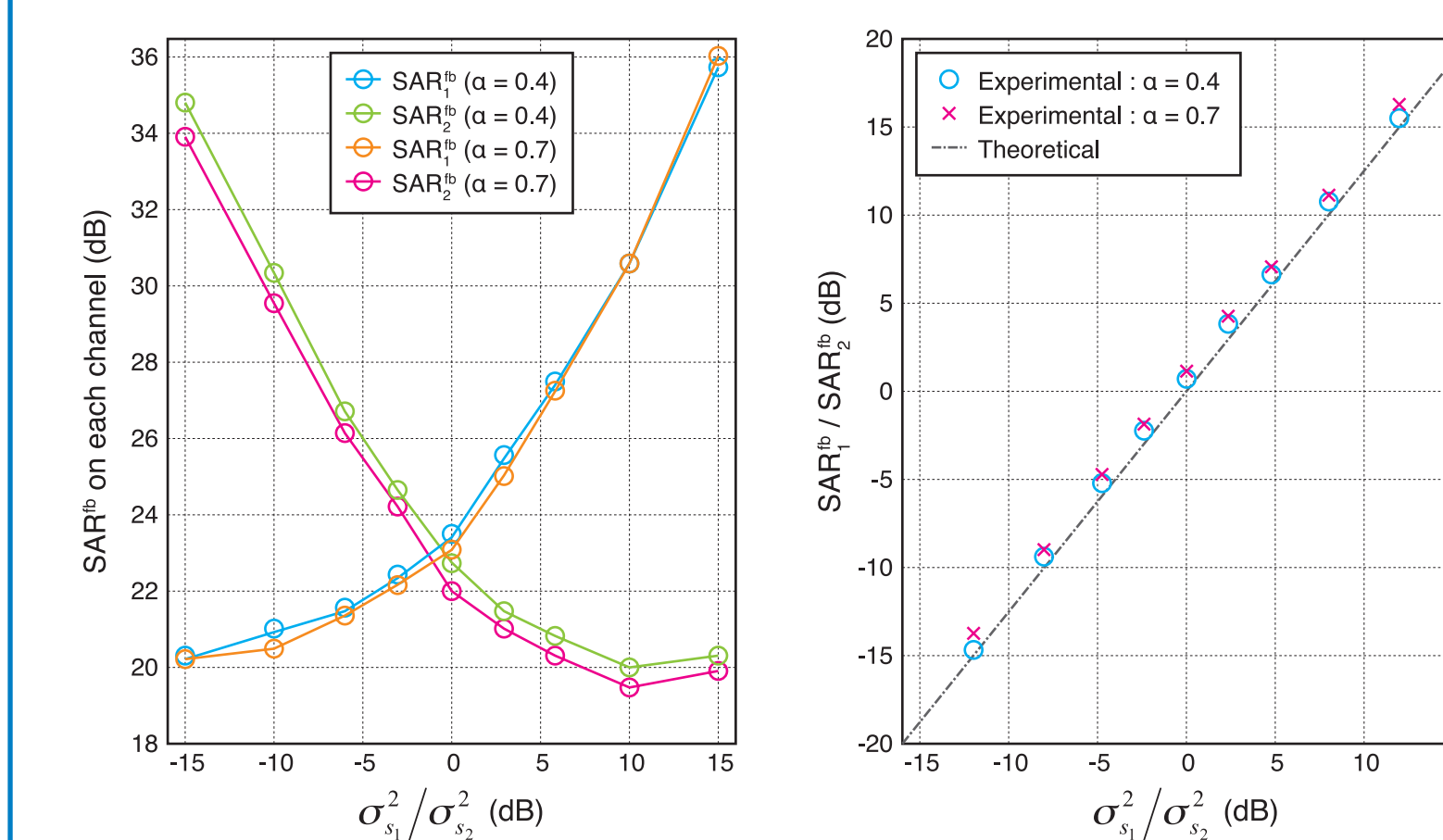


Fig. 5: Frequency average SAR obtained for AR(1) signals with pole  $\alpha = 0.4$  and  $\alpha = 0.7$

- In all considered cases, the ratio between the experimental  $SAR^{f.b.}$  of each channel is equal to  $\sigma_{s_1}^2 / \sigma_{s_2}^2$

The two adaptive filters of the feedback structure adjust their time-varying impulse responses according to the following rule: the energies of the artifact signals on each output are equal.

## RESULTS WITH SPEECH SIGNALS

Non compliant with the stationary signals assumption !

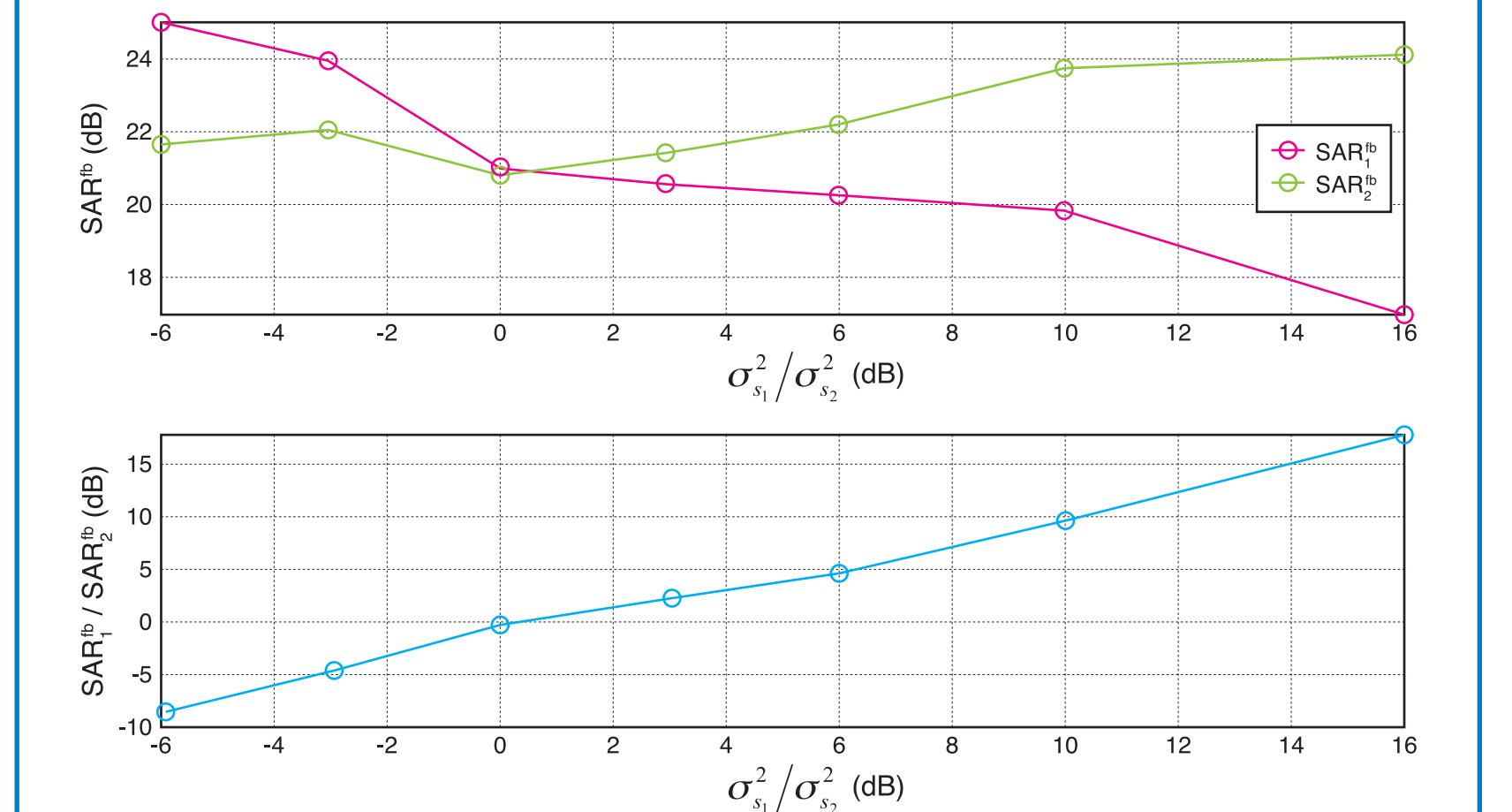


Fig. 6: Frequency average SAR obtained for two speech sources

- The proposed equilibrium state seems to be consistent even with speech signals.

## CONCLUSIONS

Based on the analysis of the relationship between output signals and their artifacts, we bring up the following contributions:

- new relations for further analyzing the convergence and steady-state properties of the feedback structure
- equilibrium state of the adaptive feedback structure obtained when the energy of the artifacts on the output is the same on each channel

More accurate theoretical description is needed to discuss uniqueness of this equilibrium point

## REFERENCES

- [1] N.R. Yousef and A. H. Sayed. A unified approach to the steady-state and tracking analyses of adaptive filter. IEEE Trans. on Signal Processing, 49(2):341-324, February 2001
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- [3] Eweda Eweda. A new approach for analyzing the limiting behavior of the normalized LMS algorithm under weak assumptions. Signal Processing, 89(11):2143-2151, November 2009