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# LOSSY COMPRESSION OF DISTRIBUTED SPARSE SOURCES: A PRACTICAL SCHEME

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#### ABSTRACT

A new lossy compression scheme for distributed and sparse sources under a low complexity encoding constraint is proposed. This architecture is able to exploit both intra- and inter-signal correlations typical of signals monitored, for example, by a wireless sensor network. In order to meet the low complexity constraint, the encoding stage is performed by a lossy distributed compressed sensing (CS) algorithm. The novelty of the scheme consists in the combination of lossy distributed source coding (DSC) and CS. More precisely, we propose a joint CS reconstruction filter, which exploits the knowledge of the side information to improve the quality of both the dequantization and the CS reconstruction steps. The joint use of CS and DSC allows to achieve large bit-rate savings for the same quality with respect to the non-distributed CS scheme, e.g. up to 1.2 bps in the cases considered in this paper. Compared to the DSC scheme (without CS), we observe a gain increasing with the rate for the same mean square error.

#### 1. INTRODUCTION

Lossy compression of sparse but distributed sources consists in finding a cost constrained representation of inherently sparse sources by exploiting their inter- but also intra-correlation, when the communication between the sources is not possible. This problem naturally arises in wireless sensor networks. Indeed, nodes of a sensor network may acquire temperature readings over time. The temperature may vary slowly over time, and hence consecutive readings have similar values. However, they also have inter-sensor correlation, as the sensors may be in the same room, in which the temperature is rather uniform. The question hence arises of how to exploit intra- and inter-sensor correlations without communication between the sensors and with a low complexity acquisition process in order to save energy consumption at the sensor. Therefore, we consider continuous, correlated, distributed and sparse (in some domain) sources and perform lossy universal compression under a low encoding complexity constraint.

Compressed sensing (CS) [1, 2] has recently emerged as an efficient technique for sampling a signal with fewer coefficients than classic Shannon/Nyquist theory. The hypothesis underlying this approach is that the signal to be sampled is sparse or at least "compressible", i.e., it must have a sparse representation in a convenient basis. In CS, sampling is performed by taking a num-

The research leading to these results has received funding from the EC's FP7 under grant agreement n. 216715 (NEWCOM++) and partly from the PRIN project "ARACHNE" funded by the Italian Ministry of Education, University and Research ber of projections of the signal onto pseudorandom sequences. Therefore, the acquisition presents appealing properties such as: low encoding complexity, since the basis in which the signal is sparse does not need to be computed, and universality, since the sensing is blind to the source distribution.

The universality<sup>1</sup> property makes CS well suited for the compression of distributed and correlated sources, since the same measurement matrix can be used for all sources and the inter-correlation is maintained. Therefore, distributed CS (DCS) was proposed in [3]. In that paper, it was shown that a distributed system based on CS could save up to 30% of measurements with respect to separate CS encoding/decoding of each source. However, CS [1, 2] and DCS [3] are mainly concerned with the performance of perfect reconstruction, and do not consider the representation/coding problem in a rate-distortion framework, particularly regarding the rate necessary to encode the measurements.

Recently, [4, 5] considered the cost of encoding the random measurements for single sources. More precisely, rate distortion analysis was performed and it was shown that adaptive encoding (that takes into account the source distribution and is therefore complex) outperforms scalar quantization of random measurements. However, in the distributed context, adaptive encoding may loose the inter-correlation between the sources since it is adapted to the distribution of each single source or even the realization of each source (KLT or Burrows Wheeler Transform). Instead, DCS has the natural property to maintain it.

For this reason, we propose to construct a quantized distributed CS architecture. The design implies to optimize the joint reconstruction filter. The main idea is to exploit the knowledge of Side Information (SI) not only as a way to reduce the encoding rate, but also as an instrument to improve the reconstruction quality. In particular, we exploit knowledge of SI to improve the accuracy of the dequantized signal, improving the method in [6] through clipping of the reconstructed value within the quantization interval. Moreover, we propose two practical algorithms to estimate the common component between the unknown source and the SI, in order to help the CS reconstruction allowing the transmission of fewer measurement samples.

The construction of optimal quantizers for sparse sources has been addressed in [7] and the rate-distortion behaviour of sparse memoryless sources was described in [8]. In both papers, authors give hints about the application of the methods they propose to a DCS scenario,

<sup>&</sup>lt;sup>1</sup>We stress here that with the term *universality* we refer to the fact that the transmitter is blind to the source distribution.

but they consider either the case where the sources are observed in the domain where they are sparse or they consider non sparse sources with sparse difference. Instead we consider here the more general case of a source not observed in its sparsity basis, with non sparse difference.

On the other hand, the authors of [9] propose a method to exploit the knowledge of a SI at the decoder to recover sparse signals. In that case, the SI consists in the (even approximate) knowledge of the position of nonzero components. This information is used to reduce the computational complexity of the Orthogonal Matching Pursuit algorithm for the recovery of the sparse signal. Instead, the SI we use is a sparse signal correlated to the unknown source through a common component. Therefore it contains information about the position of nonzero components but also about the coefficients and is used in the CS reconstruction step (estimation of the common part between the unknown source and the SI) and in the dequantizer.

#### 2. BACKGROUND

#### 2.1 Compressed sensing

In the standard CS framework, introduced in [2], a signal  $\mathbf{x} \in \mathbb{R}^{N \times 1}$  which has a sparse representation in some basis  $\mathbf{\Psi} \in \mathbb{R}^{N \times N}$ , *i.e*:

$$\mathbf{x} = \mathbf{\Psi} \boldsymbol{\theta}, \quad \|\boldsymbol{\theta}\|_{\ell_0} = K, \quad K \ll N$$

can be recovered by a smaller vector  $\mathbf{y} \in \mathbb{R}^{M \times 1}$ , K < M < N, of linear measurements  $\mathbf{y} = \mathbf{\Phi} \mathbf{x}$ , where  $\mathbf{\Phi} \in \mathbb{R}^{M \times N}$  is the *sensing matrix*. The optimum solution, requiring at least M = K + 1 measurements, would be

$$\widehat{oldsymbol{ heta}} = rg \min_{oldsymbol{ heta}} \|oldsymbol{ heta}\|_{\ell_0} \quad ext{ s.t. } \quad oldsymbol{\Phi} oldsymbol{\Psi} oldsymbol{ heta} = \mathbf{y} \ .$$

Since the  $\ell_0$  norm minimization is a NP-hard problem, one can resort to a linear programming reconstruction by minimizing the  $\ell_1$  norm

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left\| \boldsymbol{\theta} \right\|_{\ell_1} \quad \text{ s.t. } \quad \boldsymbol{\Phi} \boldsymbol{\Psi} \boldsymbol{\theta} = \mathbf{y} \ ,$$

provided that M is large enough ( $\sim K \log(N/K)$ ).

When the measurements are noisy, like in the case of quantized data which are subject to quantization noise, the  $\ell_1$  minimization with relaxed constraints is used for reconstruction:

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \|\boldsymbol{\theta}\|_{\ell_1} \quad \text{ s.t. } \|\boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\theta} - \mathbf{y}\|_{\ell_2} < \varepsilon , \quad (1)$$

where  $\varepsilon$  bounds the amount of noise in the data. Extracting the elements of  $\Phi$  at random from a Rademacher distribution (i.e.,  $\pm 1$  with the same probability) allows a correct reconstruction with overwhelming probability.

## 2.2 Distributed source coding

Distributed source coding (DSC) refers to the problem of compressing correlated sources without cooperation at their encoders. In our scheme, DSC applies after the CS module that spreads the intra-correlation of the sources among measurement samples **y**. Therefore, we assume that the sources compressed with DSC are i.i.d.

In the lossless case, Slepian and Wolf [10] showed that the separate encoding of two i.i.d. sources, say  $Y_1$ 

and  $Y_2$ , does not incur any loss relative to joint encoding (in terms of compression sum rate  $R_1 + R_2$ ) provided decoding is performed jointly. In the asymmetric setup, one source  $(Y_2 \text{ for instance})$  is compressed at its entropy  $H(Y_2)$  and is hence available at the decoder; the other source  $(Y_1)$  is compressed at its conditional entropy and can be recovered exploiting  $Y_2$  as a "side-information". This problem is also referred to as source coding with SI at the decoder. It has been shown that this problem can be optimally solved by using channel codes matched to the channel that models the correlation between the source and the SI. This setup has been extended to lossy compression of general sources by Wyner and Ziv (WZ) [11], where it is shown that separate encoding incurs a loss relative to joint encoding except for some distributions (Gaussian sources, or more generally Gaussian correlation noise): practical WZ solutions compress and decompress the data relying on an inner SW codec and an outer quantization plus reconstruction filter.

#### 3. PROPOSED ALGORITHM

The problem we wish to solve is the following. In a WZ setting, we want to encode continuous jointly sparse and correlated sources, achieving the smallest distortion for a given rate. We assume that the encoding stage must have low complexity, hence either CS is used to take advantage of intra-sensor correlations because of its low complexity with respect to transform coding, or the system employs a CS camera that directly outputs the linear measurements. In the latter case, measurements can be quantized in order to meet a rate (or distortion) constraint. However, in the former case, a new degree of freedom is introduced since the quantizer can be placed either before or after the sensing module. This issue is discussed in the following. The main ingredients of the proposed solution are a CS stage to exploit sparsity, reducing the signal length from N to M, coupled with a WZ stage to exploit inter-sensor correlation, i.e. a scalar quantizer with joint reconstruction to set the desired degree of distortion, a SW codebook to achieve minimum rate, and a joint CS reconstruction stage.

## 3.1 Jointly sparse source model

Let source signals  $\mathbf{x}_1$  and  $\mathbf{x}_2$  be acquired by the system. We assume that  $\mathbf{x}_1$  and  $\mathbf{x}_2$  follow the JSM-1 model [3]. According to this model, a set of J sparse correlated signals  $\mathbf{x}_{i} \in \mathbb{R}^{N \times 1}$ ,  $j = 1, \dots, J$  is modeled as the sum  $\mathbf{x}_{i} = \mathbf{x}_{\mathsf{C}} + \mathbf{x}_{\mathsf{I},i}$  of two sparse components: i) a common component  $\mathbf{x}_{\mathsf{C}}$ , which is shared by all signals, and ii) an innovation component  $\mathbf{x}_{l,j}$ , which is unique to each signal  $\mathbf{x}_{j}$ . Both  $\mathbf{x}_{C}$  and  $\mathbf{x}_{I,j}$  are sparse in some domain represented by the orthonormal basis matrix  $\Psi \in \mathbb{R}^{N \times N}$ , namely:  $\mathbf{x}_{\mathsf{C}} = \boldsymbol{\Psi}\boldsymbol{\theta}_{\mathsf{C}}$  and  $\mathbf{x}_{\mathsf{I},j} = \boldsymbol{\Psi}\boldsymbol{\theta}_{\mathsf{I},j}$ , with  $\|\boldsymbol{\theta}_{\mathsf{C}}\|_{\ell_0} = K_{\mathsf{C}}$ ,  $\|\boldsymbol{\theta}_{l,j}\|_{\ell_0} = K_j$  and  $K_{\mathsf{C}}, K_j \ll N$ . This model is a good fit for signals acquired by a group of sensors monitoring the same physical event in different points of space, where local factors can affect the innovation component of a more global behavior taken into account by the common component. With respect to the original JSM-1 scheme described before, we introduce a parameter  $\alpha$  to modulate the correlation between the sources, i.e.  $\mathbf{x}_1 = \mathbf{x}_{\mathsf{C}} + \alpha \mathbf{x}_{\mathsf{I},1} \text{ and } \mathbf{x}_2 = \mathbf{x}_{\mathsf{C}} + \alpha \mathbf{x}_{\mathsf{I},2}.$ 

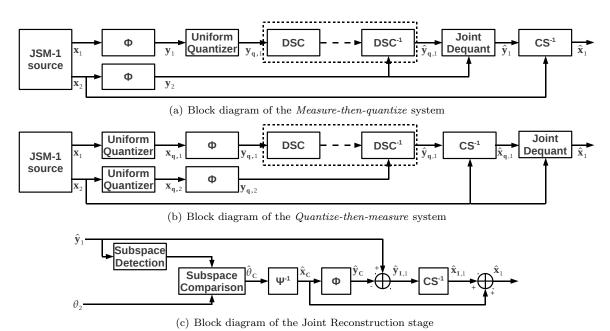


Figure 1: Block diagrams of M+Q and Q+M systems and of the Joint Reconstruction stage

### 3.2 Quantization and coding

When the data are first acquired through a conventional process and then sensed through linear random measurements, the quantizer can either be placed before or after the sensing module. In the single source case, it is known that CS is robust to quantization noise [4]. Here we raise the problem of the optimal position of the quantizer vs the CS module in the context of separate encoding of distributed sources. To answer this question, we define and compare two different schemes for encoding jointly sparse and correlated sources. They are depicted in fig. 1. In the first scheme (fig. 1(a)), denoted M+Q, first linear measurements of the sources are taken with a Gaussian  $\Phi$  matrix such that  $(\Phi)_{mn} \sim \mathcal{N}(0, \frac{1}{M})$  to obtain  $y_1$  and  $y_2$ . Then, the measurements are quantized with a scalar uniform quantizer and coded using a SW coder. In the second scheme (fig. 1(b)), denoted Q+M, the sources are first quantized with a scalar uniform quantizer having step size  $\Delta$ . After quantization, we take the linear measurements with a random i.i.d. integer  $\Phi$  matrix with Rademacher distributed entries  $(\Pr((\mathbf{\Phi})_{mn} = \pm 1) = 0.5)$ , yielding a measurements vectors  $\mathbf{y}_{q,1}, \mathbf{y}_{q,2}$  composed by integer elements, which are encoded in a lossless way by the SW encoder. It should be noted that, since  $\mathbf{x}_1$  and  $\mathbf{x}_2$  are correlated sample by sample, their linear measurements are still correlated if we use the same sensing matrix  $\Phi$  for either sources, justifying the use of a SW coder.

#### 3.3 Slepian-Wolf coding

In both systems, after the quantization/measurement stage, a SW zero-error co/decoding stage follows. Let  $\mathbf{y}_{b,1}$  and  $\mathbf{y}_{b,2}$  denote the  $N_b$ -long binary versions of  $\mathbf{y}_{q,1}$  and  $\mathbf{y}_{q,2}$  respectively. Let  $\mathbf{H}$  be the parity-check matrix representing a  $K_b/N_b$  rate channel code, of size  $(N_b - K_b) \times N_b$ .  $\mathbf{y}_{b,1}$  is compressed by sending its syndrome  $\mathbf{s}_1 = \mathbf{H} \cdot \mathbf{y}_{b,1}$ , of size  $(N_b - K_b)$ , to the decoder; the side-information  $\mathbf{y}_{b,2}$  is exploited at the decoder using

the modified Belief Propagation algorithm presented in [12] to find the best estimate  $\hat{\mathbf{y}}_{b,1}$ , as the closest sequence to  $\mathbf{y}_{b,2}$  with syndrome  $\mathbf{s}_1$ . Here, we use rate-adaptive accumulated LDPC codes that have been proved to perform close to the SW bound [13], and allow to achieve zero-error coding through rate adaptation.

## 3.4 Reconstruction

Finally, reconstruction is performed in order to obtain an estimate of the unknown source  $\hat{\mathbf{x}}_1$ . The performance of the reconstruction stage can be improved exploiting knowledge of the SI at the decoder, for both dequantization and CS reconstruction. In this section, for simplicity we will refer to the scenario of "coding with SI", or "asymmetric" coding, in which  $\mathbf{x}_1$  has to be communicated, and  $\mathbf{x}_2$  is known at the receiver. We describe the implementation for the M+Q scheme, but the extension to Q+M is straightforward.

### 3.4.1 Joint Dequantization

The first improvement is given by joint signal dequantization. Under the classical Gaussian additive correlation noise model, i.e.  $\mathbf{y}_1$  and  $\mathbf{y}_2$  are Gaussian, and their difference is independent from  $\mathbf{y}_2$ , we derive the optimal joint dequantizer

friequantizer
$$\hat{y}_1 = y_2 + \sqrt{\frac{2}{\pi}} \sigma \frac{e^{\frac{(a-y_2)^2}{2\sigma^2}} - e^{\frac{(b-y_2)^2}{2\sigma^2}}}{\operatorname{erf}\left(\frac{b-y_2}{\sqrt{2}\sigma}\right) - \operatorname{erf}\left(\frac{a-y_2}{\sqrt{2}\sigma}\right)}, \quad (2)$$

where  $y_1$  is an element of  $\mathbf{y}_1$  and  $y_2$  is the corresponding element in  $\mathbf{y}_2$ .  $\sigma$  is the variance of  $\mathbf{y}_1 - \mathbf{y}_2$ ,  $a = \widehat{y}_{q,1} - \Delta/2$  and  $b = \widehat{y}_{q,1} + \Delta/2$ .

However, in the JSM-1 model  $\mathbf{y}_1 - \mathbf{y}_2$  is not independent from either  $\mathbf{y}_1$  or  $\mathbf{y}_2$ . Since (2) is not optimal under JSM-1, we resort to a suboptimal more robust estimator. In [6], the output of the inverse quantizer is obtained as  $\hat{y}_1 = \frac{\sigma_q^2}{\sigma_q^2 + \sigma^2} y_2 + \frac{\sigma^2}{\sigma_q^2 + \sigma^2} \hat{y}_{q,1}$ . The scheme we use is an improved version in which  $\hat{y}_1$  is clipped within the interval [a, b] when it happens to lie outside of it.

#### 3.4.2 Joint Reconstruction

The second improvement regards CS reconstruction. In particular, the idea is to estimate the sparsity support of the common component  $\theta_{\mathsf{C}}$ , i.e. the set  $\widehat{\Theta}_{\mathsf{C}}$  of the positions of its nonzero coefficients. This estimation allows us to improve CS reconstruction, since we further sparsify the vector to be reconstructed, and hence require less measurements to successfully reconstruct the signal (or, equivalently, improve the quality of the reconstruction given the same number of measurements). In particular, we subtract from  $\mathbf{y}_{q,1}$  the "measurements common component"  $\mathbf{y}_{\mathsf{C}} = \mathbf{\Phi} \mathbf{x}_{\mathsf{C}}$ ; the resulting measurements vector only contains the innovation component of the source  $\mathbf{x}_{\mathsf{I},1}$ , which is sparser than  $\mathbf{x}_1$  and requires fewer measurements to be recovered. After recovering the unique component, we re-add the estimated common component to obtain the final estimate of  $\mathbf{x}_1$ . The entire process is depicted in the block diagram of fig. 1(c).

We propose two algorithms to estimate  $\widehat{\Theta}_{\mathsf{C}}$ . Both take as inputs the set of nonzero elements of  $\theta_2 = \Psi^\mathsf{T} \mathbf{x}_2$ , and initially perform non-joint reconstruction to obtain an initial estimate  $\widehat{\mathbf{x}}_1$  using (1). This is used to compute  $\widehat{\theta}_1 = \Psi^\mathsf{T} \widehat{\mathbf{x}}_1$ . The output is the set  $\widehat{\Theta}_{\mathsf{C}}$  of positions of the nonzero elements of  $\widehat{\theta}_{\mathsf{C}}$ . In short, Algorithm 1 estimates the intersection of the positions of the significant components of both sources; algorithm 2 sorts by decreasing magnitude the nonzero elements of the estimated source, and then compares them to the SI coefficients.

### Algorithm 1 Intersect algorithm

```
Require: \Theta_{NZ,2} = \{i | (\boldsymbol{\theta}_2)_i \neq 0\}, \ \widehat{\boldsymbol{\theta}}_1, t

Ensure: \widehat{\Theta}_{C} = \{i | (\boldsymbol{\theta}_{C})_i \neq 0\}

1: \widehat{\Theta}_{NZ,1} \leftarrow \{i | |(\widehat{\boldsymbol{\theta}}_1)_i| > t\}

2: \widehat{\Theta}_{C} \leftarrow \widehat{\Theta}_{NZ,1} \cap \Theta_{NZ,2}

3: return \widehat{\Theta}_{C}
```

#### Algorithm 2 Sorting algorithm

```
Require: \Theta_{NZ,2} = \{i | (\boldsymbol{\theta}_2)_i \neq 0\}, \ \widehat{\boldsymbol{\theta}}_1, K_{\mathsf{C}}

Ensure: \widehat{\Theta}_{\mathsf{C}} = \{i | (\boldsymbol{\theta}_{\mathsf{C}})_i \neq 0\}

1: \widehat{\Theta}_{\mathsf{C}} \leftarrow \emptyset

2: for i = \max_i \left| (\widehat{\boldsymbol{\theta}}_1)_i \right| to \min_i \left| (\widehat{\boldsymbol{\theta}}_1)_i \right| do

3: if i \in \Theta_{NZ,2} then

4: \widehat{\Theta}_{\mathsf{C}} \leftarrow \widehat{\Theta}_{\mathsf{C}} \cup i

5: if \left| \widehat{\Theta}_{\mathsf{C}} \right| = K_{\mathsf{C}} then

6: break

7: end if

8: end if

9: end for

10: return \widehat{\Theta}_{\mathsf{C}}
```

#### 4. NUMERICAL RESULTS

In this section, we assess the rate-distortion performance of the proposed scheme. The rate is measured in bits per original source symbol, as output by the zero-error SW encoder. Sources  $\mathbf{x}_1$  and  $\mathbf{x}_2$  are generated according to JSM-1 model described in section 3.1. They have N=512 samples, and we take

 $\alpha = \{10^0, 10^{-1}, 10^{-2}, 10^{-3}\}$  and  $K_{\rm C}, K_1, K_2 = 8$ .  $\Psi$  is the DCT matrix of length N. The quantization step for Q+M is  $\Delta = \{0.1, 0.01, 0.001\}$ , and for M+Q it is  $\Delta = \sqrt{\frac{N}{M}}\{0.1, 0.01, 0.001\}$ , in order to obtain comparable rates. The measurement vector has M = 64, 128, 256 entries. For the SW code,  $L = 66, N_b = 1584$ , and  $K_b$  varies from 24 to 1584.

### 4.1 Comparison between Q+M and M+Q

Fig. 2 shows the rate-distortion performance of the Q+M and M+Q systems, for  $\alpha=10^{-2}$ . It can be noticed that the best performance is achieved by the M+Q system with M=128, while for M=64 the decoder has not enough measurements to properly reconstruct the signal. The Q+M system with M=128 shows a penalty of 0.85 bit per source symbol. This can be interpreted by the fact that: i) in the scheme Q+M the quantized signal may not be sparse (not sparse at all or not sparse in the same basis as the original signal). Therefore, this scheme performs poorly. ii) the scheme M+Q performs good which shows that even in the distributed context, CS is robust to quantization noise.

#### 4.2 Penalty of practical vs. optimal Slepian-Wolf encoder

Moreover, fig. 2 shows the penalty of the Slepian-Wolf encoder previously described with respect to the conditional entropy  $H(\mathbf{y}_{q,1}|\mathbf{y}_{q,2})$ , which bounds its performance. The penalty is about 0.15 bit per source symbol, which is pretty close to the performance of the code shown in [14], although this latter is not a zero-error coder.

# 4.3 Gain of proposed DCS scheme vs. DSC (no CS) and non-distributed CS

In addition, we can read from fig. 2 the gain we obtain singularly by the CS and DSC blocks. Without CS, the rate-distortion curve would be the one depicted in the curve labeled as "No CS", with a penalty increasing with the rate. On the other hand, to evaluate the advantage obtained using DSC, we compare the entropy  $H(\mathbf{y}_{q,1})$  with the conditional entropy  $H(\mathbf{y}_{q,1}|\mathbf{y}_{q,2})$  and see that joint encoding yields a gain of 1.18 bit per source sample.

# 4.4 Conjecture about the optimal number of measurements

Finally, we conjecture that the best RD performance is independent of the number of measurement. The optimal number of measurements would be the one guaranteeing (close-to-)perfect reconstruction. More than this, additional measurements would not yield any improvement. Less than this, the reconstruction distortion would likely dominate over the quantizer distortion. Hence, all the distortion can be optimally tuned at the quantizer only and this should yield a predictable degree of reconstruction distortion.

#### 4.5 Joint Reconstruction

Fig. 3 shows the rate-distortion performance of the reconstruction algorithms proposed in section 3.4 for M = 64,128 for the M+Q scheme. Curves labeled

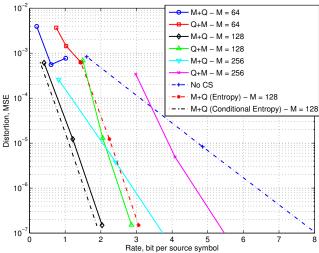


Figure 2: Rate Distortion performance, Q+M vs. M+Q systems.  $N=512,~K_{\rm C}=K_j=8,\alpha=10^{-2}$ 

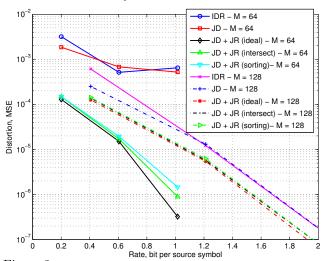


Figure 3: Rate-distortion performance, Joint dequantization and Joint reconstruction.  $N=512, K_{\mathsf{C}}=K_{j}=8, \alpha=10^{-2}$ 

with IDR refer to independent dequantization and reconstruction, i.e., the curves in Fig. 2 . JD labels curves referring to joint dequantization. JD+JR refers to joint dequantization and joint reconstruction. Both the proposed Joint Reconstruction algorithms are tested. Ideal refers to the performance of a joint reconstruction knowing in advance the positions of the nonzero components of  $\theta_C$ .

It can be noticed that the joint reconstruction provides the most significant gain. With M=64, it is now possible to reconstruct the source, significantly outperforming M=128. Moreover, the proposed algorithms for the estimation of the common component perform very close to each other and show a slight penalty with respect to the ideal case.

### 5. DISCUSSION AND CONCLUSIONS

In this paper, we have proposed a system for lossy compression of intra- and inter-correlated sources in application scenarios such as sensor networks. It is based on the joint use of CS to capture memory of a signal, and DSC to take advantage of inter-sensor correlations. The proposed system has satisfactory rate-distortion performance in comparison with a system that does not em-

ploy CS but only DSC, showing that linear measurements represent a viable, universal<sup>2</sup> and low-complexity signal representation.

First, we showed that the resilience of CS to quantization error also holds in the distributed setup. Moreover, the optimal number of measurements can be chosen as the one guaranteeing (close-to-)perfect reconstruction. In addition, using joint decoding, dequantization and reconstruction techniques allows to boost performance even further, making the proposed scheme an attractive choice for environments such as sensor networks, in which the devices performing acquisition and processing are severely constrained in terms of energy and computations.

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 $<sup>^2{\</sup>rm Meaning}$  that the transmitter is blind to the source distribution