



HAL
open science

Maximizing the Cohesion is NP-hard

Adrien Friggeri, Eric Fleury

► **To cite this version:**

Adrien Friggeri, Eric Fleury. Maximizing the Cohesion is NP-hard. [Research Report] RR-7734, 2011. inria-00621065v1

HAL Id: inria-00621065

<https://inria.hal.science/inria-00621065v1>

Submitted on 9 Sep 2011 (v1), last revised 9 Oct 2011 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Maximizing the Cohesion is NP-hard

Adrien Friggeri — Eric Fleury

N° 7734

8 September 2011

A large, light blue stylized 'R' logo is positioned to the left of the text. The text 'Rapport de recherche' is written in a light blue serif font, with 'Rapport' on the top line and 'de recherche' on the bottom line. A horizontal line is drawn below the text.

*Rapport
de recherche*

Maximizing the Cohesion is NP-hard

Adrien Friggeri , Eric Fleury

Thème : Réseaux et télécommunications
Équipe-Projet DNET

Rapport de recherche n° 7734 — 8 September 2011 — 10 pages

Abstract: We show that the problem of finding a set with maximum cohesion in an undirected network is **NP**-hard.

Key-words: social networks, complex networks, cohesion, np-complete, complexity

Maximiser la Cohésion est NP-dur

Résumé : Nous montrons que le problème de trouver un ensemble de cohésion maximum dans un graphe non orienté est **NP**-dur.

Mots-clés : réseaux sociaux, réseaux complexes, cohésion, np-complet, complexité

Introduction

In [1], we have introduced a new metric called the *cohesion* which rates the community-ness of a group of people in a social network from a sociological point of view. Through a large scale experiment on Facebook, we have established that the cohesion is highly correlated to the subjective user perception of the communities. In this article, we show that finding a set of vertices with maximum cohesion is **NP**-hard.

Notations

Let $G = (V, E)$ be a graph with vertex set V and edge set E of size $n = |V| \geq 4$. For all vertices $u \in V$, we write $d_G(u)$ the degree of u , or more simply $d(u)$ ¹. A *triangle* in G is a triplet of pairwise connected vertices.

For all sets of vertices $S \subseteq V$, let $G[S] = (S, E_S)$ be the subgraph induced by S on G . We write $m(S) = |E_S|$ the number of edges in $G[S]$, and $i(S) = |\{(u, v, w) \in S^3 : (uv, vw, uw) \in E^3\}|$ the number of triangles in $G[S]$. We define $o(S) = |\{(u, v, w), (u, v) \in S^2, w \in V \setminus S : (uv, vw, uw) \in E^3\}|$, the number of *outbound* triangles of S , that is: triangles in G which have exactly two vertices in S .

Moreover, for all (u, v) in E , let $\Delta(uv) = |\{w \in V : (uw, vw) \in E^2\}|$ be the number of triangles the edge uv belongs to in G .

Finally, we recall the definition of the cohesion of a set S in G :

$$\mathcal{C}(S) = \frac{i(S)^2}{\binom{|S|}{3}(i(S) + o(S))}$$

An example is given on Figure 1. The cohesion of a given set S in G can naively be computed in $\mathcal{O}(n^3)$ by listing all triangles in G and counting those inside and outbound to S .

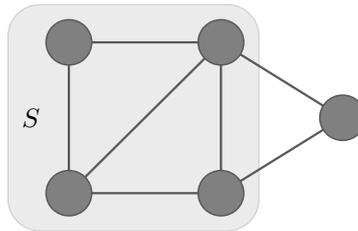


Figure 1: In this example, $i(S) = 2$ and $o(S) = 1$, thus $\mathcal{C}(S) = \frac{1}{6}$

In this article we examine the problem of finding a set of vertices $S \subseteq V$ of maximum cohesion, i.e. for all subset $S' \subseteq V$, $\mathcal{C}(S') \leq \mathcal{C}(S)$.

Outline

We now proceed to prove that finding a set of vertices with maximum cohesion in G is **NP**-hard. We will first show in Section 1 that this problem is equivalent

¹Here, as elsewhere, we drop the index referring to the underlying graph if the reference is clear.

to that of finding a connected set of vertices with maximum cohesion in G . The decision problem associated to the latter is **CONNECTED-COHESIVE**.

In order to prove that **CONNECTED-COHESIVE** is **NP**-complete, we introduce an intermediary problem, **CONNECTED-COHESIVE-DEG-GT-2**. In Section 2 we prove that **CONNECTED-COHESIVE-DEG-GT-2** is **NP**-complete by reducing **CLIQUE** (problem GT19 in [2]). Then, in Section 3 we show that **CONNECTED-COHESIVE-DEG-GT-2** is reducible to **CONNECTED-COHESIVE** and conclude that **CONNECTED-COHESIVE** is **NP**-complete. From there we deduce that the optimization problem of finding a set of vertices with maximum cohesion is **NP**-hard.

Problems

1. **CONNECTED-COHESIVE**:

Input A graph $G = (V, E)$, $k \in \mathbb{N}$, $k \leq |V|$ and $\lambda \in \mathbb{Q}$, $\lambda \in [0, 1]$

Question Is there a subset S of V such that $G[S]$ is connected, $|S| = k$ and $\mathcal{C}(S) = \lambda$?

2. **CONNECTED-COHESIVE-DEG-GT-2**:

Input A graph $G = (V, E)$, $k \in \mathbb{N}$, $k \leq |V|$ and $\lambda \in \mathbb{Q}$, $\lambda \in [0, 1]$

Question Is there a subset S of V such that $G[S]$ is connected, $|S| = k$, $\mathcal{C}(S) = \lambda$ and all vertices in S have a degree strictly greater than 2?

3. **CLIQUE**:

Input A graph $G = (V, E)$, $k \in \mathbb{N}$, $k \leq |V|$

Question Is there a subset S of V such that $|S| = k$ and the subgraph induced by S is a clique?

1 A maximum cohesive group is connected

In order to prove that a set of vertices with maximum cohesion in a given network is connected, we need the following lemma:

Lemma 1.1. *Let $S_1 \subseteq V$ and $S_2 \subseteq V$ be two disconnected sets of vertices ($(S_1 \times S_2) \cap E = \emptyset$). If $\mathcal{C}(S_1) \leq \mathcal{C}(S_1 \cup S_2)$ then $\mathcal{C}(S_2) > \mathcal{C}(S_1 \cup S_2)$.*

Proof. Suppose $\mathcal{C}(S_1) \leq \mathcal{C}(S_1 \cup S_2)$ and $\mathcal{C}(S_2) \leq \mathcal{C}(S_1 \cup S_2)$. Given that S_1 and S_2 are disconnected, $i(S_1 \cup S_2) = i(S_1) + i(S_2)$ and $o(S_1 \cup S_2) = o(S_1) + o(S_2)$. We can then write:

$$\frac{i(S_1)^2}{\binom{|S_1|}{3}} \leq (i(S_1) + o(S_1))\mathcal{C}(S_1 \cup S_2) \quad (1)$$

$$\frac{i(S_2)^2}{\binom{|S_2|}{3}} \leq (i(S_2) + o(S_2))\mathcal{C}(S_1 \cup S_2) \quad (2)$$

By summing (1) and (2), we obtain:

$$\begin{aligned} \frac{i(S_1)^2}{\binom{|S_1|}{3}} + \frac{i(S_2)^2}{\binom{|S_2|}{3}} &\leq (i(S_1) + o(S_1) + i(S_2) + o(S_2))\mathcal{C}(S_1 \cup S_2) \\ &\leq (i(S_1 \cup S_2) + o(S_1 \cup S_2))\mathcal{C}(S_1 \cup S_2) \\ &\leq \frac{(i(S_1) + i(S_2))^2}{\binom{|S_1|+|S_2|}{3}} \end{aligned}$$

Furthermore, given that $|S_1|, |S_2| > 1$,

$$\binom{|S_1|}{3} + \binom{|S_2|}{3} < \binom{|S_1| + |S_2|}{3}$$

We then have:

$$\frac{i(S_1)^2}{\binom{|S_1|}{3}} + \frac{i(S_2)^2}{\binom{|S_2|}{3}} < \frac{(i(S_1) + i(S_2))^2}{\binom{|S_1|}{3} + \binom{|S_2|}{3}}$$

Which simplifies to:

$$\left(\binom{|S_2|}{3} i(S_1) - \binom{|S_1|}{3} i(S_2) \right)^2 < 0$$

Hence the contradiction. Therefore, for all $S_1, S_2 \subseteq V$, disconnected:

$$\mathcal{C}(S_1) \leq \mathcal{C}(S_1 \cup S_2) \Rightarrow \mathcal{C}(S_2) > \mathcal{C}(S_1 \cup S_2) \quad \square$$

Theorem 1.2. *Let S be the set of vertices of G with the highest cohesion, S is connected.*

Proof. Suppose S is not connected, then there exist two disconnected subsets $S_1, S_2 \subseteq S$ such that $S = S_1 \cup S_2$. Given that S has maximum cohesion, we have $\mathcal{C}(S) \geq \mathcal{C}(S_1)$. Thus per Lemma 1.1: $\mathcal{C}(S) < \mathcal{C}(S_2)$ and S does not have the highest cohesion, hence the contradiction. \square

Corollary 1.3. *Per Theorem 1.2, the problem of searching for a set of vertices with maximum cohesion is strictly equivalent to that of searching a set of connected vertices with maximum cohesion.*

2 CONNECTED-COHESIVE-DEG-GT-2 is NP-complete

First note that given a set S of vertices of G , it is possible to verify that S is a solution of CONNECTED-COHESIVE-DEG-GT-2 by computing its cohesion, its size, its connectivity and the minimum degree of its vertices, all in polynomial time. Therefore CONNECTED-COHESIVE-DEG-GT-2 is in **NP**.

Let us now reduce CLIQUE to CONNECTED-COHESIVE-DEG-GT-2. Let $(G = (V, E), k \in \mathbb{N})$ be an instance of CLIQUE². We can assume that G is connected (if not, we use the following reasoning separately on each connected component of

²We consider here that $|G| > 2$ and $k > 2$, although this is not exactly CLIQUE, this problem is clearly **NP**-complete, given that the complexity of CLIQUE does not arise from those small values.

Algorithm 1 Transforms an instance of CLIQUE in an instance of CONNECTED-COHESIVE-DEG-GT-2

Require: $G = (V, E), k \in \mathbb{N}$

```

1:  $W := \emptyset$ 
2:  $E' := E$ 
3: for  $uv \in E$  do
4:   for  $i = \Delta(uv) + 1$  to  $2\binom{n}{3}^3$  do
5:     let  $w$  be a new vertex
6:      $W \leftarrow W \cup \{w\}$ 
7:      $E' \leftarrow E' \cup \{uw, vw\}$ 
8:   end for
9: end for
10: return  $G' = (V \cup W, E'), k, \lambda = \frac{\binom{k}{3}^2}{2\binom{k}{3}(\binom{k}{2}\binom{n}{3}^3 - \binom{k}{3})}$ 

```

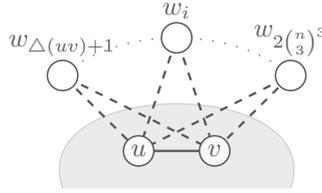


Figure 2: Illustration of Algorithm 1. At this step, we add $2\binom{n}{3}^3 - \Delta(uv)$ neighbors w_i to u and v , forming one new triangle for each added neighbor.

G). We construct an instance $(G' = (V', E'), k, \lambda)$ of CONNECTED-COHESIVE-DEG-GT-2 by adding new neighbors to all vertices in G such that in the resulting graph, all edges in E belong to $2\binom{n}{3}^3$ triangles as described in Algorithm 1 and illustrated by Figure 2. Given that the outer for loop (line 3) is executed $\mathcal{O}(n^2)$ times – once for each edge in E – and that the inner for loop (line 4) runs in $\mathcal{O}(n^9)$ – at most $2\binom{n}{3}^3$ time, Algorithm 1 has a complexity of $\mathcal{O}(n^{11})$, thus runs in polynomial time. G' has the following properties:

Properties of G'

Property 1. G is an induced subgraph of G' .

Proof. No vertices or edges are deleted when constructing G' and moreover no edge is created between vertices in V . \square

Property 2. By construction, for all edge $uv \in E$, $\Delta_{G'}(uv) = 2\binom{n}{3}^3$.

Property 3. The set of vertices with degree > 2 in G' is exactly V .

Proof. Each vertex $w \in W = V' \setminus V$ has a degree $d_{G'}(w) = 2$. Moreover, G being connected, each vertex $u \in V$ belongs to at least one edge $uv \in E$ and thus $d_{G'}(u) \geq \Delta_{G'}(uv) = 2\binom{n}{3}^3 > 2$ (Property 2). \square

Lemma 2.1. *Let $S \subseteq V$ be a set of vertices of G' , then*

$$\mathcal{C}_{G'}(S) = \frac{i_{G'}(S)^2}{2 \binom{|S|}{3} (m_{G'}(S) \binom{n}{3}^3 - i_{G'}(S))}$$

Proof. By summing the number of triangles belonging to each edge in $E \cap S^2$, each inner triangle is counted three times – once for each edge – and each outbound triangle once:

$$\sum_{uv \in E \cap S^2} \Delta_{G'}(uv) = 3i_{G'}(S) + o_{G'}(S)$$

Therefore, using Property 2:

$$\begin{aligned} i_{G'}(S) + o_{G'}(S) &= \sum_{uv \in E \cap S^2} \Delta_{G'}(uv) - 2i_{G'}(S) \\ &= 2m_{G'}(S) \binom{n}{3}^3 - 2i_{G'}(S) \end{aligned}$$

Replacing in the definition of the cohesion leads to the desired result:

$$\begin{aligned} \mathcal{C}_{G'}(S) &= \frac{i_{G'}(S)^2}{\binom{|S|}{3} (i_{G'}(S) + o_{G'}(S))} \\ &= \frac{i_{G'}(S)^2}{2 \binom{|S|}{3} (m_{G'}(S) \binom{n}{3}^3 - i_{G'}(S))} \end{aligned}$$

□

Theorem 2.2. *There exist a clique of size k in G iff there exist a group of vertices of G' of size k , cohesion $\lambda = \frac{\binom{k}{3}}{2 \binom{k}{2} \binom{n}{3}^3 - \binom{k}{3}}$ and in which all vertices have a degree strictly greater than 2.*

Proof. Let $K \subseteq V$, be a clique of size $|K| = k$ in G , then K forms a clique in G' too – as G is an induced subgraph of G' by Property 1. From here it comes that:

$$\mathcal{C}_{G'}(K) = \frac{\binom{k}{3}^2}{2 \binom{k}{2} \left(\binom{k}{2} \binom{n}{3}^3 - \binom{k}{3} \right)}$$

Moreover, all vertices $u \in K$ have a degree $d_{G'} > 2$.

Conversely, let $S \subseteq V'$ such that $|S| = k$, $\mathcal{C}_{G'}(S) = \frac{\binom{k}{3}^2}{2 \binom{k}{2} \left(\binom{k}{2} \binom{n}{3}^3 - \binom{k}{3} \right)}$ and for all $u \in S$, $d_{G'}(u) > 2$. Suppose that S is not a clique.

Given that all vertices in S have a degree strictly greater than 2, it comes from Property 3 that $S \subseteq V$ and thus we can write, using Lemma 2.1:

$$\frac{i_{G'}(S)^2}{2 \binom{k}{3} (m_{G'}(S) \binom{n}{3}^3 - i_{G'}(S))} = \frac{\binom{k}{3}^2}{2 \binom{k}{2} \left(\binom{k}{2} \binom{n}{3}^3 - \binom{k}{3} \right)}$$

From there, it comes:

$$\frac{i_{G'}(S)^2}{m_{G'}(S) \binom{n}{3}^3 - i_{G'}(S)} = \frac{\binom{k}{3}^2}{\binom{k}{2} \binom{n}{3}^3 - \binom{k}{3}}$$

And thus:

$$\binom{n}{3}^3 \left(m_{G'}(S) \binom{k}{3}^2 - i_{G'}(S)^2 \binom{k}{2} \right) = i_{G'}(S) \binom{k}{3} \left(\binom{k}{3} - i_{G'}(S) \right)$$

Moreover, $i_{G'}(S) < \binom{k}{3} \leq \binom{n}{3}$, therefore $\binom{k}{3} - i_{G'}(S) > 0$ which leads to $m_{G'}(S) \binom{k}{3}^2 - i_{G'}(S)^2 \binom{k}{2} > 0$. Given that $m_{G'}(S) \binom{k}{3}^2 - i_{G'}(S)^2 \binom{k}{2} \in \mathbb{N}$, $m_{G'}(S) \binom{k}{3}^2 - i_{G'}(S)^2 \binom{k}{2} \geq 1$. Finally it comes that:

$$\binom{n}{3}^3 \leq i_{G'}(S) \binom{k}{3} \left(\binom{k}{3} - i_{G'}(S) \right) < \binom{n}{3}^3$$

Hence the contradiction. Therefore, S is a clique. \square

Theorem 2.3. CONNECTED-COHESIVE-DEG-GT-2 is **NP**-complete.

Proof. Per Theorem 2.2, there exist a clique of size k in G iff there exist a subset of vertices of G' of size k , cohesion $\lambda = \frac{\binom{k}{3}}{2(\binom{k}{2}\binom{n}{3}^3 - \binom{k}{3})}$ and in which all vertices have a degree strictly greater than 2, and the transformation from G, k to G', k, λ runs in polynomial time. Thus CLIQUE is reducible to CONNECTED-COHESIVE-DEG-GT-2 and CONNECTED-COHESIVE-DEG-GT-2 is **NP**-hard.

Given that CONNECTED-COHESIVE-DEG-GT-2 is in **NP**, the problem is thus **NP**-complete. \square

3 CONNECTED-COHESIVE is NP-complete

Let S be a set of vertices of G , it is possible to verify in polynomial time that S is a solution of CONNECTED-COHESIVE by computing its cohesion, its size and its connectivity. Therefore CONNECTED-COHESIVE is in **NP**.

We will need the following theorem:

Theorem 3.1. Let $G = (V, E)$ a graph and $E_\Delta = \{uv \in E : \Delta(uv) > 0\}$, we call $G_\Delta = (V, E_\Delta)$ the graph obtained by removing all edges of G which belong to no triangles. Then for all $S \subseteq V$, $\mathcal{C}_G(S) = \mathcal{C}_{G_\Delta}(S)$.

Proof. Let uvw be a triangle in G , each edge uv, vw, uw , belongs to at least one triangle and all three edges are in E_Δ . Therefore, uvw is a triangle in G_Δ . Conversely, let uvw be a triangle in G_Δ , given that $E_\Delta \subseteq E$, uvw is a triangle in G . Hence, for all $S \subseteq V$: $i_G(S) = i_{G_\Delta}(S)$ and $o_G(S) = o_{G_\Delta}(S)$. Therefore, $\mathcal{C}_G(S) = \mathcal{C}_{G_\Delta}(S)$. \square

Theorem 3.1 implies that finding a group with maximum cohesion in G is equivalent to finding a group with maximum cohesion in G_Δ , which is G restricted to vertices belonging to triangles. In particular, all vertices in G_Δ have a degree greater or equal than 2.

Let $G = (V, E)$, k, λ an instance of CONNECTED-COHESIVE-DEG-GT-2 and $n = |V|$. We can safely assume that in all vertices $u \in V$ have a degree $d_G(u) \geq 2$: if not, per Theorem 3.1, we can remove all edges of G which do not belong to a triangle, without changing the answer to the problem and ensure that all vertices have degree at least 2 in G_Δ .

Let $V_2 = \{u \in V : d_G(u) = 2\}$. We construct a graph G' by adding, for each edge uv such that $u \in V_2$ or $v \in V_2$, a clique K_{uv} of size $\binom{n^4}{3}$ to the graph. We then join u and v to each vertex in K_{uv} , as described in Algorithm 2 and illustrated in Figure 3.

Algorithm 2 Transforms an instance of CONNECTED-COHESIVE-DEG-GT-2 in an instance of CONNECTED-COHESIVE

Require: $G = (V, E), k \in \mathbb{N}, \lambda \in \mathbb{Q}$

```

1:  $V' := V$ 
2:  $E' := E$ 
3: for  $uv \in E$  do
4:   if  $d_G(u) = 2$  or  $d_G(v) = 2$  then
5:     let  $K$  be a clique of size  $\binom{n^4}{3}$ 
6:      $V' \leftarrow V' \cup K$ 
7:      $E' \leftarrow E' \cup (\{u, v\} \times K)$ 
8:   end if
9: end for
10: return  $G' = (V', E'), k, \lambda$ 

```

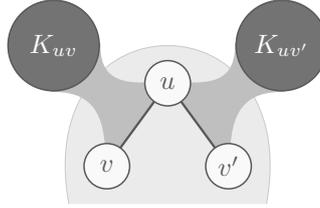


Figure 3: Illustration of Algorithm 2. To all edges uv having an extremity of degree 2 we add a clique K_{uv} of size $\binom{n^4}{3}$ and we join u and v to each vertex in K_{uv} .

Note that this transformation does not affect edges which have both extremities of degree strictly greater than 2: for all $(u, v) \in (V \setminus V_2)^2$, $\Delta_G(uv) = \Delta_{G'}(uv)$. For this reason, for all $S \subseteq (V \setminus V_2)$, $\mathcal{C}_G(S) = \mathcal{C}_{G'}(S)$.

Moreover, we have the following minoration of $\mathcal{C}_{G'}(S)$:

$$\mathcal{C}_{G'}(S) = \frac{i_{G'}(s)^2}{\binom{|S|}{3}(i_{G'}(S) + o_{G'}(s))} \geq \frac{1}{\binom{n^4}{3}^2}$$

Now, let $S' \subseteq V'$ such that $G'[S']$ is connected and there exist $u \in S'$ such that $d_G(u) = 2$. Note how $i_{G'}(S') + o_{G'}(S')$ is the number of triangles having at least one edge in S' . Given that $G'[S']$ is connected, S' contains at least a vertex u of degree 2 in G and one of its neighbors v , and uv belongs to at least $\binom{n^4}{3}$ triangles – one for each vertex in K_{uv} . Therefore:

$$i_{G'}(S') + o_{G'}(S') \geq \Delta_{G'}(u, v) \geq \binom{n^4}{3}$$

Moreover, given that $|S'| = k \leq n$, $i_{G'}(S') \leq \binom{n}{3}$. Which leads to the following majoration of $\mathcal{C}_{G'}(S')$:

$$\mathcal{C}_{G'}(S') = \frac{i_{G'}(S')^2}{\binom{|S'|}{3}(i_{G'}(S') + o_{G'}(S'))} \leq \frac{\binom{n}{3}^2}{\binom{n^4}{3}}$$

Finally:

$$\mathcal{C}_{G'}(S') \leq \frac{\binom{n}{3}^2}{\binom{n^4}{3}} < \frac{1}{\binom{n}{3}^2} \leq \mathcal{C}_{G'}(S)$$

All subsets of $V \setminus V_2$ have a cohesion in G' greater than $\frac{1}{\binom{n}{3}^2}$, which is equal to their cohesion in G , and all subsets containing vertices which were added in G' or vertices which degree 2 in G have a cohesion in G' strictly smaller than $\frac{1}{\binom{n}{3}^2}$. Therefore, there exist in G a connected group of size k , cohesion λ , and which vertices have a degree strictly greater than 2 iff there exist in G' a connected group of size k and cohesion λ and that $\lambda \geq \frac{1}{\binom{n}{3}^2}$. Thus, CONNECTED-COHESIVE-DEG-GT-2 is polynomial-time Turing-reducible to CONNECTED-COHESIVE as exhibited in Algorithm 3 and therefore **NP**-hard. Finally, CONNECTED-COHESIVE is **NP**-complete. \square

Algorithm 3 Solving CONNECTED-COHESIVE-DEG-GT-2 using CONNECTED-COHESIVE

Require: $G = (V, E)$, $k \in \mathbb{N}$, $\lambda \in \mathbb{Q}$

if $\lambda \geq \frac{1}{\binom{n}{3}^2}$ **then**

$G' \leftarrow$ apply Algorithm 2 to G

return G' contains a connected group of size k and cohesion λ

else

return false

end if

4 Conclusion

The associated decision problem being **NP**-complete, the problem of finding a set of vertices with maximum cohesion is **NP**-hard³.

References

- [1] Adrien Friggeri, Guillaume Chelius, and Eric Fleury. Triangles to Capture Social Cohesion. In *Third IEEE International Conference on Social Computing*, Cambridge, United States, September 2011.
- [2] M.R. Garey and D.S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W.H. Freeman, San Francisco, 1979.

³Note that the problem of finding a set of vertices of maximum cohesion containing a set of predefined vertices is also **NP**-hard, by an immediate reduction



Centre de recherche INRIA Grenoble – Rhône-Alpes
655, avenue de l'Europe - 38334 Montbonnot Saint-Ismier (France)

Centre de recherche INRIA Bordeaux – Sud Ouest : Domaine Universitaire - 351, cours de la Libération - 33405 Talence Cedex
Centre de recherche INRIA Lille – Nord Europe : Parc Scientifique de la Haute Borne - 40, avenue Halley - 59650 Villeneuve d'Ascq
Centre de recherche INRIA Nancy – Grand Est : LORIA, Technopôle de Nancy-Brabois - Campus scientifique
615, rue du Jardin Botanique - BP 101 - 54602 Villers-lès-Nancy Cedex
Centre de recherche INRIA Paris – Rocquencourt : Domaine de Voluceau - Rocquencourt - BP 105 - 78153 Le Chesnay Cedex
Centre de recherche INRIA Rennes – Bretagne Atlantique : IRISA, Campus universitaire de Beaulieu - 35042 Rennes Cedex
Centre de recherche INRIA Saclay – Île-de-France : Parc Orsay Université - ZAC des Vignes : 4, rue Jacques Monod - 91893 Orsay Cedex
Centre de recherche INRIA Sophia Antipolis – Méditerranée : 2004, route des Lucioles - BP 93 - 06902 Sophia Antipolis Cedex

Éditeur
INRIA - Domaine de Voluceau - Rocquencourt, BP 105 - 78153 Le Chesnay Cedex (France)
<http://www.inria.fr>
ISSN 0249-6399