## 3D Inverse Dynamic Modeling of Strands Alexandre Derouet-Jourdan, Florence Bertails-Descoubes, Joëlle Thollot

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**Goal:** Convert 3D geometric curves into dynamic rods while precisely preserving the rest shape under gravity

## **Motivations**

- > Physics-based modeling of strand-like objects such as hair, rope, tails, plant stems.
- > Bridging the gap between
  - the *modeling* that relies on a pure geometric process

## **Contributions**

- 1. Geometric fitting: A new algorithm for fitting a piecewise helix to a 3D spline curve.
- 2. Static fitting: A method to identify the physical parameters of a super-helix at rest under gravity.

and the *animation* that uses physics-based simulation.

3. Stability analysis: A sufficient condition for guaranteeing the stability of the equilibrium state.

## The super-helix model [1]

- $G^1$  piecewise helical curve
- One clamped end and one free end
- Material frame at the origin  $(t_0, n_{10}, n_{20})$
- Stiffness E and volumetric mass  $\rho$
- For each helical element  $h_i$ :
- -Material curvatures  $\kappa_{1i}$ ,  $\kappa_{2i}$  and twist  $\tau_i$
- -Natural curvatures  $\kappa_{1i}^0$ ,  $\kappa_{2i}^0$  and twist  $\tau_i^0$
- -Length  $l_i$

# **1. Geometric fitting**

 $\rightarrow$  We fit a piecewise helix to a 3D spline with direct control over



- $\rightarrow$  We identify the physical parameters of the super-helix model such that the previous piecewise helix matches the rest shape of the rod under gravity.
  - *Method* (similar to the 2D case [2]):
  - -Formally compute the gradient of the potential energy  $E_p$  of the super-helix model.
  - -Cancel the gradient  $\nabla E_p$ : we get a set of 3N linear equations that have a unique solution  $(\kappa_{1i}^0, \kappa_{2i}^0, \tau_i^0)$  for each helical element  $h_i$  (once  $\rho$  and E are fixed).





Piecewise helix Super-helix at rest under gravity

the number N of helical elements.

#### Algorithm:

- -Approximate the geometric curvature-torsion profile with a piecewise constant function.
- -From this approximation, build a  $G^1$  piecewise helical curve with the origin given by the first point of the spline.
- -Fit this  $G^1$  piecewise helical curve to the original spline using a Levenberg-Marquardt least-squares optimization algorithm.



## **3. Stability analysis**

As in 2D, the study of the Hessian of the potential energy  $E_p$  provides a sufficient condition for stability:



with E the stiffness of the rod,  $\rho$  its volumetric mass and C a constant that only depends on the geometry of the rod.



The tail goes back to its rest shape after deformations.

## Conclusions

• A new method to match an input curve to a physical model at rest under gravity.

• A stability criterion to guarantee that the model goes back to the user-defined rest shape after deformations.

[1] F. Bertails, B. Audoly, M.-P. Cani, B. Querleux, F. Leroy, and J.-L. Lévêque. Super-helices for predicting the dynamics of natural hair. In ACM Transactions on Graphics (Proceedings of the ACM SIGGRAPH'06 conference), 2006.

[2] A. Derouet-Jourdan, F. Bertails-Descoubes, and J. Thollot. Stable inverse dynamic curves. In ACM Transactions on Graphics (Proceedings of the ACM SIGGRAPH Asia'10 conference), 2010.



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