3D Inverse Dynamic Modeling of Strands

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Goal: Convert 3D geometric curves into dynamic rods while precisely preserving the rest shape under gravity

Motivations

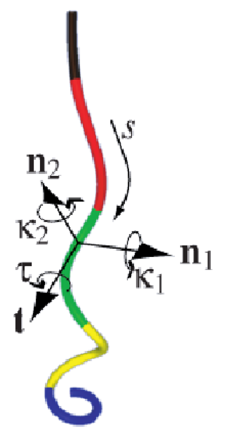
- Physics-based modeling of strand-like objects
  such as hair, rope, tails, plant stems.
- Bridging the gap between:
  the modeling that relies on a pure geometric process and the animation that uses physics-based simulation.

Contributions

1. Geometric fitting: A new algorithm for fitting a 3D spline curve to a piecewise helix.
2. Static fitting: A method to identify the physical parameters of a super-helix model at rest under gravity.
3. Stability analysis: A sufficient condition for guaranteeing the stability of the equilibrium state.

The super-helix model [1]

- G^1 piecewise helical curve
- One clamped end and one free end
- Material frame at the origin (t_0, n_{10}, n_{20})
- Stiffness E and volumetric mass ρ
- For each helical segment h_i:
  - Material curvatures κ_{1i}, κ_{2i} and twist τ_i
  - Natural curvatures κ^0_{1i}, κ^0_{2i} and twist τ^0_i
  - Length l_i

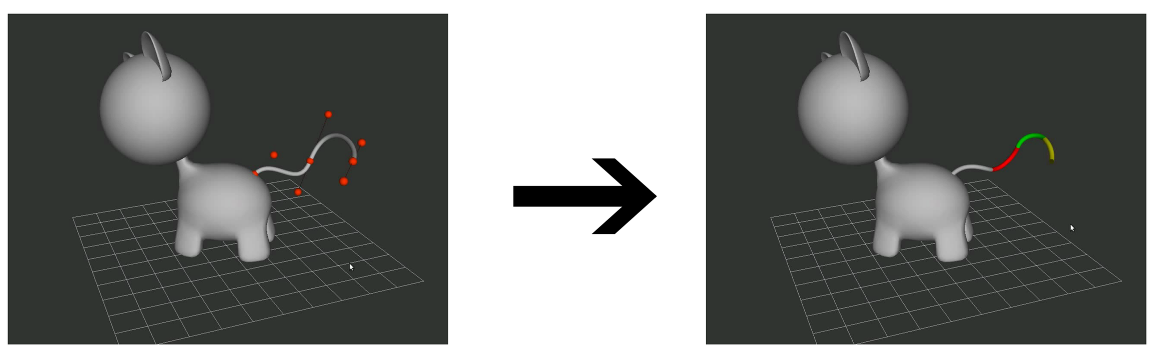


1. Geometric fitting

→ We fit a piecewise helix to a 3D spline with direct control over the number N of helical elements.

Algorithm:

- Approximate the geometric curvature-torsion profile with a piecewise constant function.
- From this approximation, build a G^1 piecewise helical curve with the origin given by the first point of the spline.
- Fit this G^1 piecewise helical curve to the original spline using a Levenberg-Marquardt least-squares optimization algorithm.



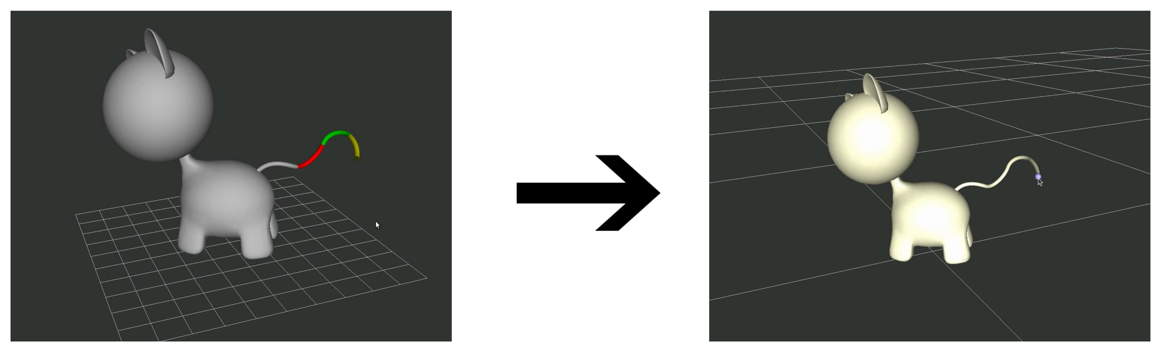
spline → piecewise-helix

2. Static fitting

→ We identify the physical parameters of the super-helix model such that the previous piecewise helix matches the rest shape of the rod under gravity.

Method (similar to the 2D case [2]):

- Formally compute the gradient of the potential energy E_p of the super-helix model.
- Cancel the gradient: we get a set of $3N$ linear equations that have a unique solution $(\kappa^0_{1i}, \kappa^0_{2i}, \tau^0_i)$ for each helical segment h_i (once ρ and E are fixed).



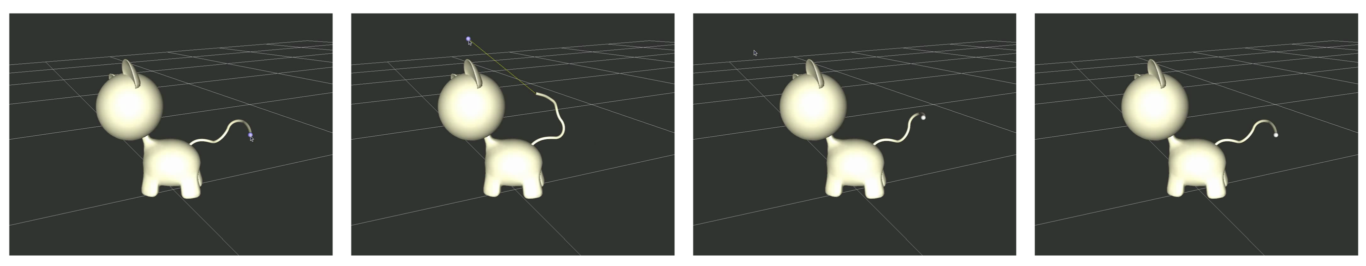
piecewise helix → super-helix at rest

3. Stability analysis

As in 2D, the study of the Hessian of the potential energy E_p provides a sufficient condition for stability:

$$\frac{E}{\rho} > C$$

with E the stiffness of the rod, ρ its volumetric mass and C a constant that only depends on the geometry of the rod.



The tail goes back to its rest shape after deformations.

Conclusions

- A new method to match an input curve to a physical model at rest under gravity.
- A stability criterion to guarantee that the model goes back to the user-defined rest shape after deformations.

[1] F. Bertails, B. Audoly, M.-P. Cani, B. Querleux, F. Leroy, and J.-L. Lévêque. Super-helices for predicting the dynamics of natural hair. In *ACM Transactions on Graphics (Proceedings of the ACM SIGGRAPH'06 conference)*, 2006.

[2] A. Derouet-Jourdan, F. Bertails-Descoubes, and J. Thollot. Stable inverse dynamic curves. *ACM Transactions on Graphics (Proceedings of the ACM SIGGRAPH Asia'10 Conference)*, 2010.

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