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## Interval numerical observer: Application to a discrete time nonlinear fish model

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**RÉSUMÉ.** Le but de ce travail est de reconstruire les états d'un système discret non linéaire représentant la dynamique d'une population de poisson soumise à l'action de la pêche. Nous adaptons à un modèle de population de poisson un algorithme appelé "Interval Moving Horizon State Estimation" (IMHSE) qui permet d'estimer les états d'un système par des intervalles. Cette algorithme est développé dans [6] pour une classe générale de systèmes discrets.

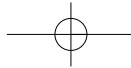
**ABSTRACT.** The aim of this work is to reconstitute the state of a discrete time nonlinear system representing a dynamical model of a harvested fish population. We adapt to this model an algorithm called "Interval Moving Horizon State Estimation" (IMHSE) which gives an estimated interval of the system states. This algorithm is carried out in [6] for some general class of discrete time systems.

**MOTS-CLÉS :** Systèmes discrets non linéaires; Observateurs intervalles; Dynamique des populations.

**KEYWORDS :** Nonlinear discrete time systems; Interval observers; Populations dynamics .

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## 1. Introduction

Nowadays the management of renewable resource has become one of the great priorities of political and economic decision-makers. Mathematical models combined with simulations on computer enable to observe, understand and regulate very complex phenomena of nature. Basing oneself on mathematical models and using some tools of control theory, one can reconstruct the states of a dynamical system. Our aim in this paper is to evaluate the stock of an exploited fish population whose dynamics are represented by the following discrete time nonlinear model (see for instance [5] page 76 ) :

$$\begin{cases} x_1(k+1) = f(\sum_{i=1}^n b_i x_i(k)) \\ x_2(k+1) = x_1(k) \exp(-M_1 - \tau q_1 E(k)) \\ \vdots \\ x_n(k+1) = x_{n-1}(k) \exp(-M_{n-1} - \tau q_{n-1} E(k)) \end{cases} \quad (1)$$

Where

- $n$  is the number of age group
- $x_i(k)$  is the number of individual in age group  $i$  at time  $k$
- $b_i$  is the fecundity rate of individuals in age group  $i$
- $M_i$  and  $q_i$  are respectively death and capturability rate of individuals in age group  $i$
- $E(k)$  is the fishing effort at time  $k$  and  $\tau$  is the annual fishing time.

$f$  is a continuous recruitment function. Many recruitment functions exist in the literature, the widely used are :

Beverton-Holt : $f(x) = \alpha x / (1 + \beta x)$	Ricker : $f(x) = \alpha x e^{-\beta x}$	
Power function : $f(x) = \alpha x^{1-\beta}$	Shepherd : $f(x) = \alpha x / (1 + \beta x^c)$ ( $c > 0$ )	
Saila-Lorda : $f(x) = \alpha x^\gamma e^{-\beta x}$	Deriso-Schute : $f(x) = \alpha x (1 - \beta x)^{1/\gamma}$	

where  $\alpha, \beta$  and  $\gamma$  are positive real parameters.

We assume that the fish population is submitted to fishing action and that the harvested number fish during each period  $[k, k+1)$  is available for measurement. The catch in the time interval period  $[k, k+1)$  can be expressed as follows (see [5] page 146 for instance) :

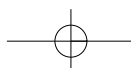
$$y(k) = \sum_{i=1}^n \int_k^{k+1} q_i E(k) \tau x_i(t) dt \quad (2)$$

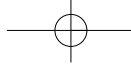
After some computations we can write :

$$y(k) = \sum_{i=1}^n \frac{q_i E(k) \tau}{q_i E(k) \tau + M_i} \left( 1 - \exp(-M_i - q_i E(k) \tau) \right) x_i(k) \quad (3)$$

Remark that in order to implement the dynamical system (1) we must know the initial condition  $x(k_0)$  at a given time  $k_0$ , that is we have to know the number of fish in each class at time  $k_0$ . But it is a difficult task to count the number of a fish population in the sea, all that one can measure is the value of the catch  $y(k)$  for a given time  $k$ . The motivation of this paper is to assess the state  $x_i(k)$  for a given fish population whose dynamics are described by the model (1) using the capture data. To solve this problem we use a classical method which consists in building an observer for the system (1). We briefly recall the meaning of an observer in control theory. Let us consider a discrete time system of the form :

$$x(k+1) = G(x(k), u(k)) \quad (4)$$





with the measurable output :

$$y(k) = O(x(k), u(k)) \quad (5)$$

Where  $x(k) \in \mathbb{R}^n$  is the system state at time  $k$  and  $u(k)$  is a control term at time  $k$ . As we mentioned it above, in most cases we do not always have access to the state  $x(k)$  but we can measure only a function  $y(k)$  of the state. This function is called the measurable output. An observer of the system (4) is an auxiliary dynamical system whose input is the input and the output of system (4), which produce an estimator  $\hat{x}(k)$  of the state  $x(k)$  such that the estimation error  $x(k) - \hat{x}(k)$  tends towards zero as time  $k$  goes to infinity and must remain small if it starts small. For example, for linear system of the form :

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \quad (6)$$

Where  $x(k) \in \Omega \subset \mathbb{R}^n$ ,  $u(k) \in \mathbb{R}^m$ ,  $A \in \mathbb{R}^n \times \mathbb{R}^n$ ,  $B \in \mathbb{R}^n \times \mathbb{R}^m$ ,  $y(k) \in \mathbb{R}^q$  and  $C \in \mathbb{R}^q \times \mathbb{R}^n$ , the synthesis of the observer is entirely determined by the Luemberger method [10] if the pair  $(A, C)$  is uniformly observable i.e if  $\text{rank}(C^t \ A^t C^t \ \dots \ A^{n-1} C^t) = n$ . In fact, the candidate observer is put as being

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K(y(k) - C\hat{x}(k)) \quad (7)$$

It is shown that if the pair  $(A, C)$  is observable one can choose  $K$  such that the estimation error  $x(k) - \hat{x}(k)$  tends towards zero when  $k$  tends towards infinity. For discrete time nonlinear systems, the synthesis of an observer is a difficult task, only few results exist in the literature ([3],[4],[8],[11],[13],[14]) and most of these results give local convergence of the observer. While it is difficult to build an observer for the model (1)(3) due to the high non linearity of the first equation one can use interval estimator theory to estimate the system states ([3],[6],[7]). In [6] an algorithm called "Interval Moving Horizon state Estimator" (IMHSE) which builds an interval estimator state of a nonlinear dynamical system is developed. The algorithm IMHSE has the advantage of being applicable to a large class of dynamical system with global convergence. We would like to notice that an observer has been build for system (1)(3) in [12] but this observer uses the output at time  $k+1$  instead of  $k$  to estimate the system state at time  $k+1$ . Our paper is organized as follows : first Section is devoted to the presentation of the model fish for which we aim to build an interval observer and to the motivation problem. In Section two we briefly recall the IMHSE algorithm and adapt the algorithm to the considered harvested model fish (1)(3). A numerical example is developed in the last Section.

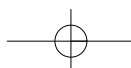
To simplify the notation let us denote equations (1) and (3) respectively by

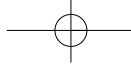
$$x(k+1) = F(x(k), E(k)) \quad (8)$$

$$y(k) = H(x(k), E(k)) \quad (9)$$

where

$$\begin{aligned} F : \mathbb{R}^n \times \mathbb{R} &\mapsto \mathbb{R}^n \\ (x, E(k)) &\mapsto (f(\sum_{i=1}^n b_i x_i), v_1(k)x_1, \dots, v_{n-1}(k)x_{n-1})^t \\ H : \mathbb{R}^n \times \mathbb{R} &\mapsto \mathbb{R} \\ (x, E(k)) &\mapsto \sum_{i=1}^n C_i(k)x_i \end{aligned}$$





$$x = (x_1, x_2, \dots, x_n)^t, v_i(k) = \text{Exp}(-M_i - q_i E(k)\tau), i = 1, \dots, n-1 \text{ and}$$

$$C_i(k) = \frac{q_i E(k)\tau}{q_i E(k)\tau + M_i} \left(1 - \text{Exp}(-M_i - q_i E(k)\tau)\right), i = 1, \dots, n.$$

## 2. Interval Moving Horizon State Estimation (IMHSE)

The algorithm IMHSE is well developed in [6], we briefly recall the algorithm description for system (8) with measurable output (9). Let  $sh$  be the start horizon time and  $lh$  the length horizon time observation. It is assumed that the measurable output  $y(k)$  is available for  $k = sh, \dots, sh + lh - 1$ . The algorithm IMHSE uses the available output  $y(k)$  to compute an interval estimator state of system (8). For a given  $sh \in \mathbb{N}$  and  $lh \in \mathbb{N}$ , let  $x(sh), x(sh + 1), \dots, x(sh + lh - 1)$  the values of the unknown real states with the measurable outputs  $y(sh), y(sh + 1), \dots, y(sh + lh - 1)$ .

For any  $z \in \Omega$ , we define :

$$j_{sh}(z) = \frac{1}{2} \sum_{k=sh}^{sh+lh-1} (H(z_k, E(k)) - y(k))^2 \quad (10)$$

Where  $z_{sh} = z$  and  $z_k = F(z_{k-1}, E(k-1))$  for  $k = sh + 1, \dots, sh + lh - 1$ .

Let us consider the following inclusion functions :

$$\begin{aligned} \mathcal{F} : \mathcal{B}(\mathbb{R}^n) \times \mathbb{R} &\mapsto \mathcal{B}(\mathbb{R}^n) \\ (I, E) &\mapsto \mathcal{F}(I, E) = \{F(x, E)/x \in I\} \end{aligned}$$

$$\begin{aligned} \mathcal{J}_{sh} : \mathcal{B}(\mathbb{R}^n) &\mapsto \mathcal{B}(\mathbb{R}) \\ I &\mapsto \mathcal{J}_{sh}(I) = \{j_{sh}(x)/x \in I\} \end{aligned}$$

We wish to compute at time  $sh + lh$  an interval with wide less than  $\epsilon$  (arbitrary chosen) that contains the solution  $x(sh + lh)$  of system (1) at time  $sh + lh$ . Consider the problem :

$$\min_{z \in \Omega} j_{sh}(z) \quad (11)$$

First of all what we have to do is to find in an invariant domain  $\Omega$  for (8) all the intervals with width less than  $\eta$  (arbitrary chosen) that contain a solution of problem (11) by the use of global optimization by interval analysis [9].

### 2.1. Global optimization algorithm

A simple algorithm that globally minimizes the nonlinear function (10) can be formulated [6] :

let  $\eta > 0$  arbitrary chosen and  $L$  a box list that contained solution of problem (11).

A) Consider an invariant domain  $\Omega$  for the model (1). We initialize  $L$  to be  $\Omega$

B) Compute  $d$  the wide of  $\Omega$

C) Compute  $\mathcal{J}_{sh}(\cdot)$  over  $\Omega : [L_\Omega, U_\Omega] = \mathcal{J}_{sh}(\Omega)$  and put  $U_j := U_\Omega$

D) While  $d > \eta$  do :

D<sub>1</sub>) Remove the first box  $X$  of the list  $L$ .

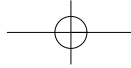
D<sub>2</sub>) Bisect  $X$  by its widest such that  $X = X_1 \cup X_2$ .

D<sub>3</sub>) Compute  $\mathcal{J}_{sh}(\cdot)$  over  $X_1$  to obtain  $[L_{X_1}, U_{X_1}] := \mathcal{J}_{sh}(X_1)$

if  $L_{X_1} > U_j$  then delete  $X_1$ , else add  $X_1$  at the end of the list  $L$

if  $X_1$  is listed, compute the midpoint  $\beta_{X_1}$  of  $X_1$  in  $\mathcal{J}_{sh}(\cdot) : U_{J_{X_1}} := \mathcal{J}_{sh}(\beta_{X_1})$





- if  $X_1$  is listed and  $UJ_{X_1} < U_j$  do  $U_j := UJ_{X_1}$   
**D<sub>4</sub>**) Compute  $\mathcal{J}_{sh}(\cdot)$  over  $X_2$  to obtain  $[L_{X_2}, U_{X_2}] := \mathcal{J}_{sh}(X_2)$   
 if  $L_{X_2} > U_j$  then delete  $X_2$ , else add  $X_2$  at the end of the list  $L$   
 if  $X_2$  is listed, compute the midpoint  $\beta_{X_2}$  of  $X_2$  in  $\mathcal{J}_{sh}(\cdot) : UJ_{X_2} := \mathcal{J}_{sh}(\beta_{X_2})$   
 if  $X_2$  is listed and  $UJ_{X_2} < U_j$  do  $U_j := UJ_{X_2}$   
**D<sub>5</sub>**) Compute the widest side of the next box  $X$  in the list :  $d := width(X)$

## 2.2. IMHSE Algorithm

The IMHSE algorithm computes an enclosure of the state estimation at time  $sh + lh$  as follows :

- a) Define the invariant domain  $\Omega$  for the system (8)  
 b) Solve the minimization problem (11) using the global optimization algorithm.  $X_{sh}$  is initialized to each box with width less than the prescribe tolerance  $\eta$  that contained solution of problem (11).  
 c) Compute the interval estimator vector  $X_{sh+lh}$  at the end of the time horizon :  

$$X_{sh+lh} = \underbrace{\mathcal{F} \circ \mathcal{F} \dots \mathcal{F}}_{lh-1}(X_{sh})$$
  
 d) Go to step b) to calculate the next interval estimator :  $sh := sh + 1$   
 See [6] for more detail on this algorithm.

**Definition 1**  $(X_k)_{k \in \mathbb{N}} \subset \mathcal{B}(\mathbb{R}^n)$  is an interval observer of system (1) if there exists  $k_0 \in \mathbb{N}$  such that  $\forall k \geq k_0$   $x(k) \in X_k$ , where  $x(k)$   $k \geq k_0$  are the solutions of (1) for a given solution initial state  $x(k_0)$ .

Consider the following hypothesis :

$\mathcal{H}_1$  : the model  $(F, H)$  in (8),(9) is perfect.

$\mathcal{H}_2$  : the model  $(F, H)$  is free noise.

$\mathcal{H}_1$  and  $\mathcal{H}_2$  have no biological sens, we give these hypothesis to well pose the mathematical problem.

**Proposition 1** The sequence box  $(X_k)_{k \in \mathbb{N}}$  generated by the IMHSE algorithm is an interval observer of (8).

As we mentioned it above, the result in **Proposition 1** is given in [6], we adapt the proof to the harvested fish model (1),(3). To this end, let us consider the following lemma :

**Lemma 1** Assume that  $\mathcal{H}_1, \mathcal{H}_2$  are satisfied and that the system (8) with measurable output (9) is locally observable then the problem (11) has a unique optimal solution which corresponds to the solution of (8) at the beginning of the time horizon  $sh$ .

The proof of this lemma can be seen in [2]

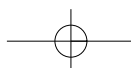
**Proof** (Proposition 1) We restrict the proof in the case where  $n = 3$  and the fishing effort  $E(k) = E$  is constant. Where  $n > 3$  and considering a time varying fishing effort the proof is similar but the computation are too long.

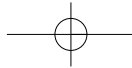
Let  $x(sh)$  the solution at time  $sh$  of system (8)

$\mathcal{H}_1$  and  $\mathcal{H}_2$  imply that  $j_{sh}(x(sh)) = 0$  and thus  $x(sh)$  is a global minimum of the positive function  $j_{sh}$ .

Now let  $x(k)$  and  $\bar{x}(k)$ ,  $k \geq 0$  be two solutions of (1) such that  $H(x(k), E) = H(\bar{x}(k), E)$   $k \geq 0$ . Denoting  $z(k) = x(k) - \bar{x}(k)$ , we obtain :

$$C_1 z_1(k) + C_2 z_2(k) + C_3 z_3(k) = 0. \quad (12)$$





$$C_1 z_1(k + 1) + C_2 z_2(k + 1) + C_3 z_3(k + 1) = 0 \tag{13}$$

$$C_1 z_1(k + 2) + C_2 z_2(k + 2) + C_3 z_3(k + 2) = 0 \tag{14}$$

Let  $\alpha_1 = \frac{C_2 v_1}{C_1}$  and  $\alpha_2 = \frac{C_3 v_2 v_1}{C_1}$ , then (14) imply that

$$z_1(k + 2) + \alpha_1 z_1(k + 1) + \alpha_2 z_1(k) = 0 \tag{15}$$

Hence letting  $k = 0$ ,  $z_1(2) = -\alpha_1 z_1(1) - \alpha_2 z_1(0)$

$$z_1(3) = -\alpha_1 z_1(2) - \alpha_2 z_1(1) = -\alpha_1(-\alpha_1 z_1(1) - \alpha_2 z_1(0)) - \alpha_2 z_1(1)$$

$$z_1(3) = (\alpha_1^2 - \alpha_2) z_1(1) + \alpha_1 \alpha_2 z_1(0)$$

$$z_1(3) = f_0(x(2)) - f_0(\bar{x}(2)) \text{ where } f_0(x) = f(\sum_{i=1}^n b_i x_i)$$

$$z_1(3) = f_0(x_1(2), v_1 x_1(1), v_1 v_2 x_1(0)) - f_0(\bar{x}_1(2), v_1 \bar{x}_1(1), v_1 v_2 \bar{x}_1(0))$$

$$z_1(3) = (x_1(2) - \bar{x}_1(2))G_1 + v_1(x_1(2) - \bar{x}_1(2))G_2 + v_1 v_2(x_1(0) - \bar{x}_1(0))G_3 \text{ where}$$

$$G_i = \int_0^1 \frac{\partial f_0}{\partial x_i} (tz_1(2) + \bar{x}_1(2), tv_1 z_1(1) + v_1 \bar{x}_1(1), tv_1 v_2 z_1(0) + v_1 v_2 \bar{x}_1(0)) dt \quad i = 1, 2, 3$$

$$z_1(3) = z_1(2)G_1 + v_1 z_1(1)G_2 + v_1 v_2 z_1(0)G_3$$

$$z_1(3) = G_1(-\alpha_1 z_1(1) - \alpha_2 z_1(0)) + v_1 z_1(1)G_2 + v_1 v_2 z_1(0)G_3$$

$$z_1(3) = (-\alpha_1 G_1 + v_1 G_2) z_1(1) + (-\alpha_2 G_1 + v_1 v_2 G_3) z_1(0)$$

Noting that  $G_i = \frac{b_i}{b_1} G_1$  we finally obtain

$$(v_1 v_2 G_3 - \alpha_2 G_1 - \alpha_1 \alpha_2) z_1(0) + (v_1 G_2 - \alpha_1 G_1 - \alpha_1^2 + \alpha_2) z_1(1) = 0 \tag{16}$$

Let us consider the expression of  $z(4)$ , using the same argument that above we prove that :

$$\begin{aligned} & \left( (\alpha_1 \alpha_2 - \alpha_2 v_1 \frac{b_2}{b_1}) \bar{G}_1 - \alpha_2 (\alpha_2 - \alpha_1^2) \right) z_1(0) \\ & + \left( \alpha_1 (\alpha_1^2 - 2\alpha_2) + ((\alpha_1^2 - \alpha_2) - \alpha_1 v_1 \frac{b_2}{b_1} + v_1 v_2 \frac{b_3}{b_1}) \bar{G}_1 \right) z_1(1) \end{aligned} \tag{17}$$

$$\text{Where } \bar{G}_1 = \int_0^1 \frac{\partial f_0}{\partial x_1} (tz_1(3) + \bar{x}_1(3), tv_1 z_1(2) + v_1 \bar{x}_1(2), tv_1 v_2 z_1(1) + v_1 v_2 \bar{x}_1(1)) dt$$

Assume that :

$$\begin{aligned} & ((\alpha_1 \alpha_2 - \alpha_2 v_1 \frac{b_2}{b_1}) \bar{G}_1 - \alpha_2 (\alpha_2 - \alpha_1^2)) ((\alpha_1 - v_1 \frac{b_2}{b_1}) G_1 + \alpha_1^2 - \alpha_2) + \\ & ((v_1 v_2 \frac{b_3}{b_1} - \alpha_2) G_1 \alpha_1 \alpha_2) (\alpha_1 (\alpha_1^2 - 2\alpha_2) + ((\alpha_1^2 - \alpha_2) - \alpha_1 v_1 \frac{b_2}{b_1} + v_1 v_2 \frac{b_3}{b_1}) \bar{G}_1) \neq 0 \end{aligned} \tag{18}$$

then coupling (16) and (17) we show that  $z_1(0) = z_1(1) = 0$ . Using equation (15), we obtain  $z_1(k) = 0 \forall k \geq 0$ . Remark that  $z_2(2) = v_1 z_1(1)$ , hence  $z_2(2) = 0$

By equation (12) we obtain  $z_3(2) = 0$  and consequently  $z_2(1) = 0$  and  $z_3(1) = 0$

$z_3(1) = 0$  imply  $z_2(0) = 0$ . Hence by equation (12)  $z_3(0) = 0$ . Finally  $z_1(0) = z_2(0) = z_3(0) = 0$ . Hence if (18) hold the model (1)(3) is observable. By lemma 1, the solution of problem (11) is unique. Hence the Global Optimization Algorithm computes a unique interval  $X_{sh}$  that contains  $x(sh)$ . The map  $F^{lh-1}$  is continuous. Hence

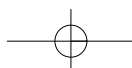
$$\forall \epsilon > 0, \exists \eta > 0 / \|x - x_s h\| < \eta \Rightarrow \|F^{lh-1}(x) - F^{lh-1}(x_s h)\| < \frac{1}{\epsilon}$$

Let  $X_{sh}$  such that its width is less than  $\eta$  and consider the natural inclusion function  $\mathcal{F}$

$$X_{lh} = \mathcal{F}^{lh-1}(X_{sh}) \text{ and } wide(X_{lh}) < \epsilon$$

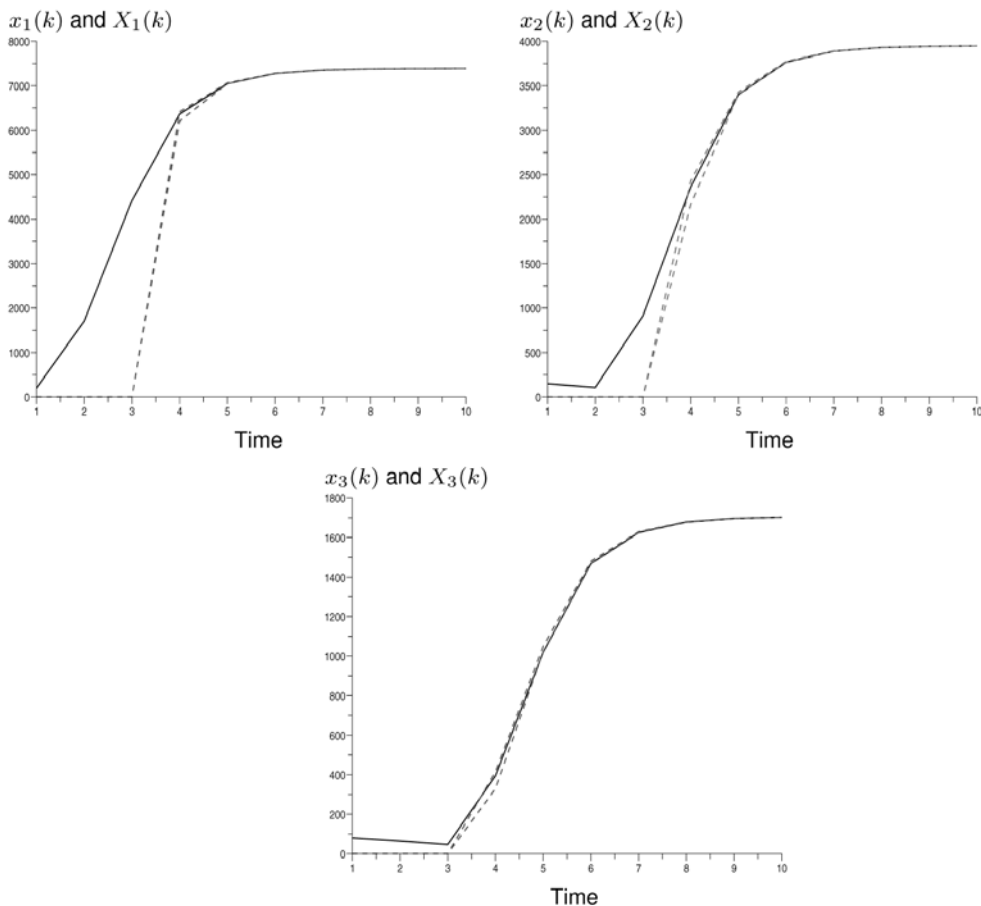
### 2.3. Numerical example

We consider the Beverton-Holt recruitment function  $f(x) = \alpha x / (1 + \beta x)$ . Assume that the length of the time horizon  $lh$  is equal to 3 i.e the past measurable output  $y(k - 3)$ ,  $y(k - 2)$ ,  $y(k - 1)$  are needed to calculate the interval estimator  $X_k$  of the state  $x(k)$  at a given time  $k$ . Let us consider the following parameter (value are not from real data) :



$$\begin{aligned} \alpha &= 1, \quad \beta = 0.0002, \quad b = [5 \ 5 \ 5], \\ q &= [0.24 \ 0.36 \ 0.42], \quad M = [0.2 \ 0.2 \ 0.2], \\ \tau &= 1, \quad E(k) = E = 8/3, \end{aligned}$$

We verify that the parameters relation (18) is true, that is  $0.00734219 + 0.0055823G_1 + 0.00140772\bar{G}_1 + 0.0025079G_1\bar{G}_1 \neq 0$ , which is true because  $G_1 > 0$  and  $\bar{G}_1 > 0$  when  $f$  is the Beverthon-Holt recruitment function. The system is then observable and the IMHSE algorithm computes an interval observer for the model. With the Beverthon-Holt recruitment function  $f$  one can remark that  $\frac{1}{\beta}$  is a global maximum of the map  $x \mapsto f(x)$  on  $\mathbb{R}$ . Let  $m = \min_i M_i$  and  $q = \min_i q_i$ . We have for all  $k > 0$   $x_1(k) = f(\sum_{i=1}^3 b_i x_i(k-1)) \leq \frac{1}{\beta}$ ,  $x_2(k) = e^{-M_1 - q_1 E \tau} x_1(k-1) \leq e^{-m - q E \tau} \cdot \frac{1}{\beta}$ , and  $x_3(k) = e^{-M_2 - q_2 E \tau} x_2(k-1) \leq e^{-2m - 2q E \tau} \cdot \frac{1}{\beta}$ . This shows that  $\Omega = [0, \frac{1}{\beta}] \times [0, \frac{e^{-m - q E \tau}}{\beta}] \times [0, \frac{e^{-2m - 2q E \tau}}{\beta}]$  is an invariant domain for (1) when  $n = 3$ . Simulations are done with Scilab. The real states are represented by solid line and the bound observer are represented by the dashed one.



### 3. Conclusion

We have applied the IMHSE algorithm by considering a discrete time nonlinear model for a harvested fish population. We note that the algorithm works for any  $n$ -age class and



any recruitment function when it is possible to find an invariant domain for the model (1). The IMHSE algorithm works well due to the observability of the model (1)(3). Some numerical examples are given and one can see on simulation that the convergence of the interval observer is very fast. The importance of this algorithm application is that it can help to predicte stock for a given fish population whose dynamic is described by the model (1)(3) using only the measurable output. Hence the numerical interval estimator can be helpful in management policies. This work is an example showing how control theory combined with computer science can be used as a tool for management of renewable resources.

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