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► **To cite this version:**

Vinay Kumar, Bhooshan Sunil. Two Dimensional Linear Phase Multiband Chebyshev FIR Filter. Acta Electrotehnica, 2010, 51 (4), pp.248-253. inria-00593395v1

HAL Id: inria-00593395

<https://inria.hal.science/inria-00593395v1>

Submitted on 17 May 2011 (v1), last revised 1 Aug 2011 (v2)

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Two Dimensional Linear Phase Chebyshev FIR Filters

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Abstract

In this paper we discuss a method to design 2 dimensional (2D) FIR filters with the help of Chebyshev polynomials. The polynomial is first converted into a 2D function and later a mapping function is used to design the filters. By the proposed technique one can design filters with various types of passband.

Keywords: Chebyshev Polynomials, FIR Filters.

I. INTRODUCTION

For image processing designing 2D FIR filters is one of the fundamentals tasks. The issue Chebyshev polynomial based filter design is addressed by various authors [1], [2], [3], [4]. Hu and Rabiner [3] introduced a linear programming based technique. Fiasconaro's [5] reduced the amount computations required in the aforesaid method and introduced a better algorithm. An equiripple linear phase FIR filter design technique was discussed by McClellan [6]. Lu [7] and Lu and Hinamoto [8] introduced methods based on semidefinite programming (SDP) and sequential quadratic programming (SQP). These methods work quite well except for the fact that the design complexity becomes rather high even for filters of moderate order.

the design technique we are going to discuss in the present paper is produces linear phase 2D FIR filters. The present method can be used to design filter having a variety of pass bands. The parameters involved in the design of these filters are few, therefore, computation time is also less. Another advantage of the present method is the ease of understanding and application of present method. The transformation involved in the present discussion is much easier to understand and apply than discussed in [9], [10].

II. DESIGN

Design procedure for the FIR filters based on Chebyshev polynomials is discussed in the present section. One dimensional Chebyshev polynomials are given by

$$T_m(x) = \begin{cases} \cos(m \cos^{-1} x) & -1 < |x| < 1 \\ \cosh(m \cosh^{-1} x) & |x| > 1 \end{cases} \quad (1)$$

where, m represents the order of the filter.

To design a 2 dimensional FIR filter we first have to represent the Chebyshev polynomial in 2 dimensional domain. We replace x with a new variable ρ and the corresponding Chebyshev polynomial is represented by

$$T_m(\rho) = \begin{cases} \cos(m \cos^{-1} \rho) & -1 < |\rho| < 1 \\ \cosh(m \cosh^{-1} \rho) & |\rho| > 1 \end{cases} \quad (2)$$

where ρ represents the mapping function; or in other words, ρ is mapped onto two variables by $\rho^2 = x^2 + y^2$, where $x - y$ is the plane of reference.

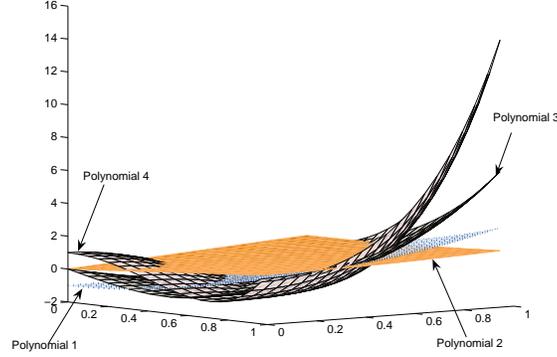


Figure 1. Chebyshev Polynomials in 2 Dimension

Some of these polynomials, Equation 2, are represented in Figure (1).

These polynomials will be used to design the required FIR filters. To convert these polynomials into frequency domain (ω -domain); or to get the corresponding filter we use a special transformation function given by [11], [12], [13]

$$\rho = \rho_0 \cos\left(\frac{\omega}{2}\right) \quad -\pi \leq \omega \leq \pi \quad (3)$$

where ρ_0 is the maximum value of ρ . To understand the transformation of Equation (3) better we discuss it in the following paragraph.

When we consider the value of $\omega = 0$, it gives $\rho = \rho_0$; that is, maximum value of ρ . As we increase the value of ω from 0 to $\pi/2$ the value of ρ comes out to be 0 and when the value of ω is increased further to π , the value of ρ becomes $-\rho_0$. In other words, as the value of ω increases from 0 to π the value of ρ decreases from ρ_0 to $-\rho_0$. Similarly, when ω varies from $-\pi$ to 0, ρ will vary from $-\rho_0$ to ρ_0 . Therefore, we conclude that this transform converts the polynomial to low pass filter; that is, lower values of the polynomial, variable ρ , will be converted to higher values in filter characteristics, variable ω , and vice versa.

After transformation the Chebyshev polynomial becomes

$$T_m(\omega) = \begin{cases} \cos [m \cos^{-1} \{\rho_0 \cos(\omega/2)\}] & -1 < |\rho| < 1 \\ \cosh [m \cosh^{-1} \{\rho_0 \cos(\omega/2)\}] & |\rho| > 1 \end{cases} \quad (4)$$

ω represents the 2 dimensional frequency domain, or $\omega^2 = u^2 + v^2$, where u and v represent axis in frequency domain. Therefore, Equation (4) becomes

$$T_m(u, v) = \begin{cases} \cos \left[m \cos^{-1} \left\{ \rho_0 \cos \left(\frac{\sqrt{u^2 + v^2}}{2} \right) \right\} \right] & -1 < |\rho| < 1 \\ \cosh \left[m \cosh^{-1} \left\{ \rho_0 \cos \left(\frac{\sqrt{u^2 + v^2}}{2} \right) \right\} \right] & |\rho| > 1 \end{cases} \quad (5)$$

Equation (5) represents the transfer function of the 2D linear phase FIR filter.

The value of ρ depends on order of the filter and the attenuation required by the user for the side bands. Following the procedure given in [14] we can state that the value of ρ_0 is given by

$$\rho_0 = \cosh(\cosh^{-1} b/m) \quad (6)$$

where, m is the order of the filter, and b is the absolute value of the attenuation and is given by

$$b = 10^{(\text{attenuation in dB}/20)} \quad (7)$$

Following the procedure outlined in [14] we can find out the values of stop band, ω_s , and pass band, ω_p , frequencies are given by

$$\omega_s = 2 \cos^{-1} \left[1 / \left\{ \cosh(\overline{1/m} \cosh^{-1} b) \right\} \right] \quad (8)$$

$$\omega_p = 2 \cos^{-1} \left[\frac{\cosh \left\{ (1/m) \cosh^{-1} (b/\sqrt{2}) \right\}}{\cosh(\overline{1/m} \cosh^{-1} b)} \right] \quad (9)$$

The design procedure discussed above produces a filter with pass band centered at $(0, 0)$. To make sure that the user has control over the placement of the pass band we change the Chebyshev polynomial of Equation(5) to

$$T_m(u, v) = \begin{cases} \cos \left[m \cos^{-1} \left\{ \rho_0 \cos \left(\frac{\sqrt{(u-u_0)^2 + (v-v_0)^2}}{2} \right) \right\} \right] & -1 < |\rho| < 1 \\ \cosh \left[m \cosh^{-1} \left\{ \rho_0 \cos \left(\frac{\sqrt{(u-u_0)^2 + (v-v_0)^2}}{2} \right) \right\} \right] & |\rho| > 1 \end{cases} \quad (10)$$

where, u_0 and v_0 represents the location of the pass band. By recursively applying the above formula we can design a multiband filter also.

Another constraint related with the filter design discussed above is the size of pass band. The size of pass band is not user dependent to make it so we introduce another variable α . This variable will be multiplied with ρ . Therefore, the updated Chebyshev polynomial will be

$$T_m(\alpha\rho) = \begin{cases} \cos(m \cos^{-1} \alpha\rho) & -1 < |\rho| < 1 \\ \cosh(m \cosh^{-1} \alpha\rho) & |\rho| > 1 \end{cases} \quad (11)$$

New value of ω_s can easily be calculated and it comes out to be

$$\omega_s = 2 \cos^{-1} \left[1/\alpha \left\{ \cosh(\overline{1/m} \cosh^{-1} b) \right\} \right] \quad (12)$$

value of ω_p remains the same.

Few examples discussed in the next section will clarify the procedure discussed in the present section.

III. EXAMPLES

To design a filter user have to provide order of the filter, m , value of α , value of attenuation of side bands and the coordinates of the centre of the pass band.

A. Example I

Let us suppose that user needs to design a filter with its order 10, side band 40dB down, α 1 (no change in the size of passband) and centre of the passband to be at $(0,0)$. The values of ω_s and ω_p comes out to be 1.0133 and 0.3607, respectively. The lowpass and highpass filters for these values are shown in Figures (2) and (3), respectively.

If we shift the centre of the passband to $(\pi/2, \pi/2)$ the above filters with all other values unchanged become as shown in Figures (4) and (5)

Let us extend the exapmle with a value of α other than 1. Suppose we take the value of α equal to 1.3 and we shift the centre of the passband to $(\pi/2, \pi/2)$ the above filters with all other values unchanged become as shown in Figures (6) and (7)

Various figures in the present example show how various parameters are interdependent and they change the characteristics of the filter. In the next example we design a filter with noncircular passband.

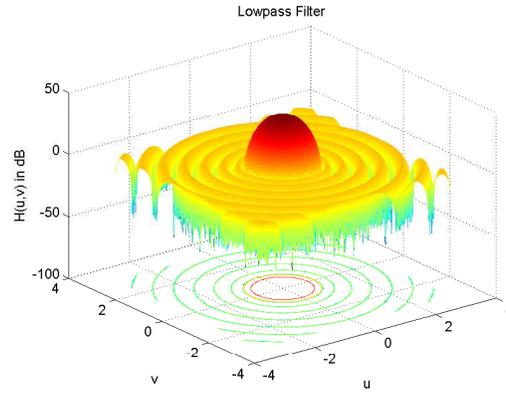


Figure 2. 10^{th} order lowpass filter with value of $\alpha = 1$, sidebands $40dB$ down and centered at $(0,0)$.

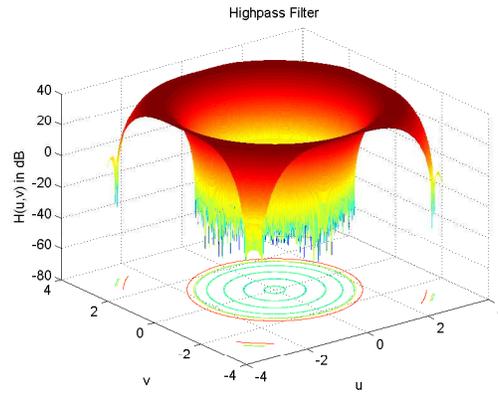


Figure 3. 10^{th} order highpass filter with value of $\alpha = 1$, sidebands $40dB$ down and centered at $(0,0)$.

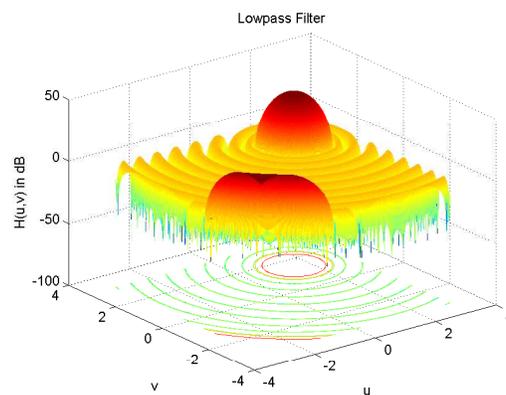


Figure 4. 10^{th} order lowpass filter with value of $\alpha = 1$, sidebands $40dB$ down and centered at $(\pi/2, \pi/2)$.

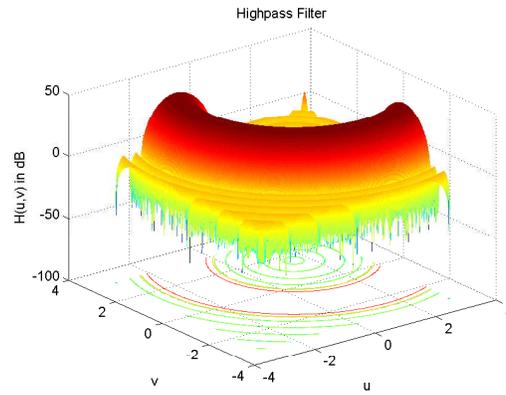


Figure 5. 10^{th} order highpass filter with value of $\alpha = 1$, sidebands $40dB$ down and centered at $(\pi/2, \pi/2)$.

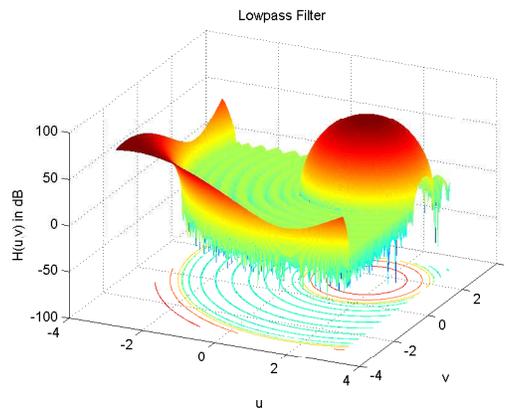


Figure 6. 10^{th} order lowpass filter with value of $\alpha = 1.3$, sidebands $40dB$ down and centered at $(\pi/2, \pi/2)$.

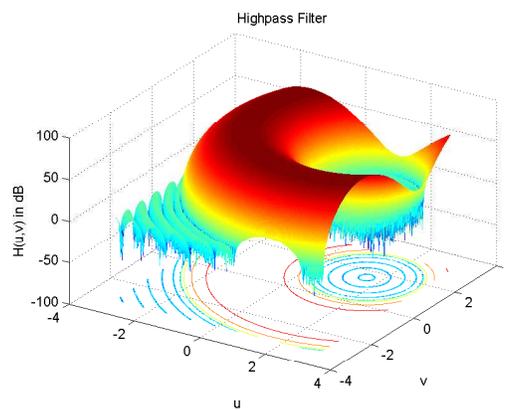


Figure 7. 10^{th} order highpass filter with value of $\alpha = 1.3$, sidebands $40dB$ down and centered at $(\pi/2, \pi/2)$.

B. Example II

In this example we consider that a bank of 7 filter is constituted with locations at $(0,0)(0.1,0.1)(0.2,0.2)(0.3,0.3)(-0.1,-0.1)(-0.2,-0.2)(-0.3,-0.3)$, respectively, other values remains same as in the previous case. The filter characteristics of lowpass and highpass are shown in Figures (8) and (9). To get the detailed view of the characteristics we show them on a frequency scale varying between -2π to 2π and they are shown in Figures (10) and (11), respectively.

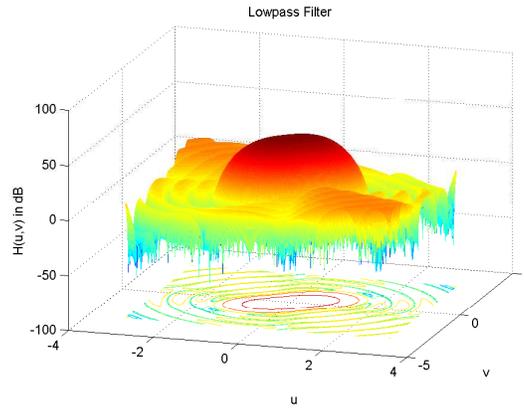


Figure 8. 10th order lowpass filter with value of $\alpha = 1$, sidebands 40dB down.

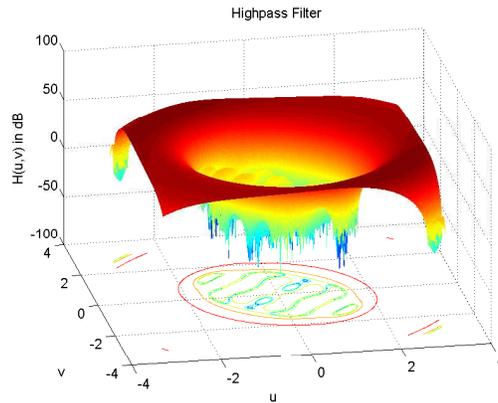


Figure 9. 10th order highpass filter with value of $\alpha = 1$, sidebands 40dB down.

From Figures (10) and (11) it is clear that the shape of the pass band is noncircular.

Next we create a multiband filter with passbands centered at $(0,0)$, $(\pi/2,\pi/2)$, $(-\pi/2,\pi/2)$, $(\pi/2,-\pi/2)$, $(-\pi/2,-\pi/2)$ while other values the same. The results are shown in Figures (12) and (13)

IV. APPLICATION

If we pass image shown in Figure (14) through a high pass filter of order 10 with values of $\alpha=1$, Figure (15), and 1.3 , Figure(16), and passband centered at the origin. It is clear from the figures that when the value of alpha is increased the passband band becomes wider and more low frequency details are visible in the resulting image.

V. CONCLUSION

A design procedure for 2 dimensional FIR filters is presented. The procedure uses 2 dimensional Chebyshev polynomials for creating image filters. The advantage of this type of filter is its ease of design and application. Although only few designs are presented in the paper, one can design a range of different types of filters. As we increase the order of the filter the passband of the resulting filter becomes narrower.

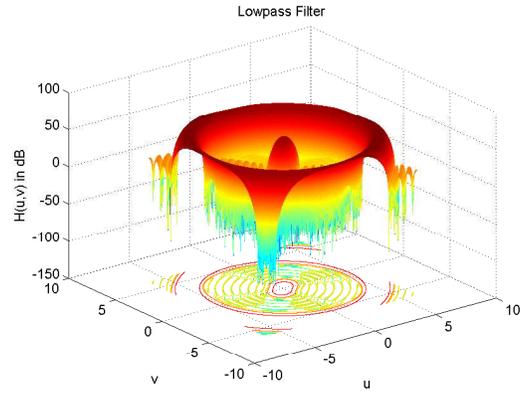


Figure 10. 10^{th} order lowpass filter with value of $\alpha = 1$, sidebands $40dB$ down.

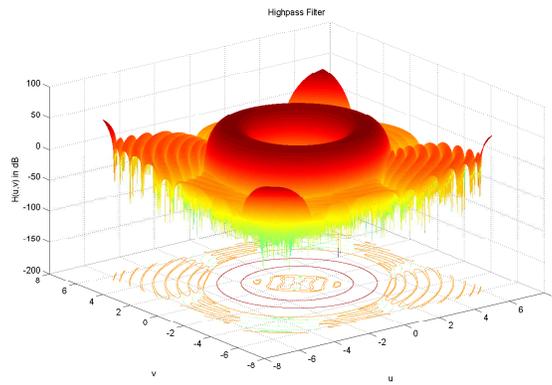


Figure 11. 10^{th} order highpass filter with value of $\alpha = 1$, sidebands $40dB$ down.

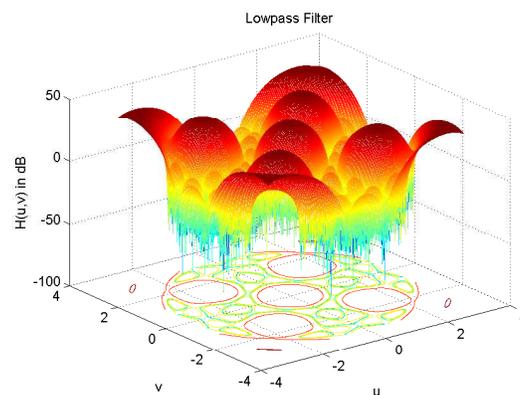


Figure 12. 10^{th} order lowpass filter with value of $\alpha = 1$, sidebands $40dB$ down.

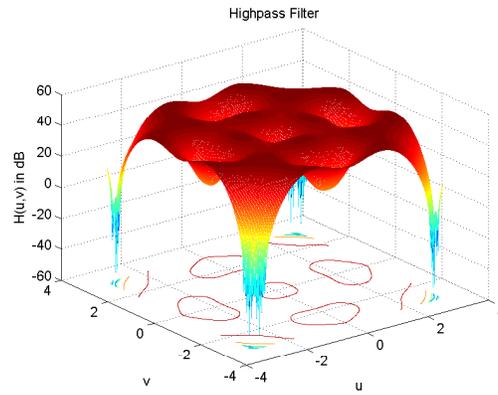


Figure 13. 10^{th} order highpass filter with value of $\alpha = 1$, sidebands $40dB$ down.



Figure 14. Original Image.



Figure 15. Image passed through a 10^{th} order highpass filter with value of $\alpha = 1$, sidebands $40dB$ down.



Figure 16. Image passed through a 10^{th} order highpass filter with value of $\alpha = 1.3$, sidebands $40dB$ down.

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