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A Gaussian Derivative Model of the Complex Cell

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► **To cite this version:**

Miles Hansard, Radu Horaud. A Gaussian Derivative Model of the Complex Cell. ECVP 2010 - 33rd European Conference on Visual Perception, Aug 2010, Lausanne, Switzerland. 2010. inria-00590263

HAL Id: inria-00590263

<https://inria.hal.science/inria-00590263>

Submitted on 3 May 2011

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1. Introduction

- ▶ Visual filters can be modelled by derivatives G_k of the Gaussian function.
- ▶ This **Gaussian jet** representation is convenient because it is:
 - ▷ Steerable, hence all orientations can be represented concisely.
 - ▷ Dimensionally separable, hence easily defined in 2D and 3D.
 - ▷ The natural code for typical image features, e.g. edges and blobs.
- ▶ But what about **complex cells**, cf. the Gabor energy model?
- ▶ Can the jet be made insensitive to small **shifts of the image**?

2. Subunit Filters

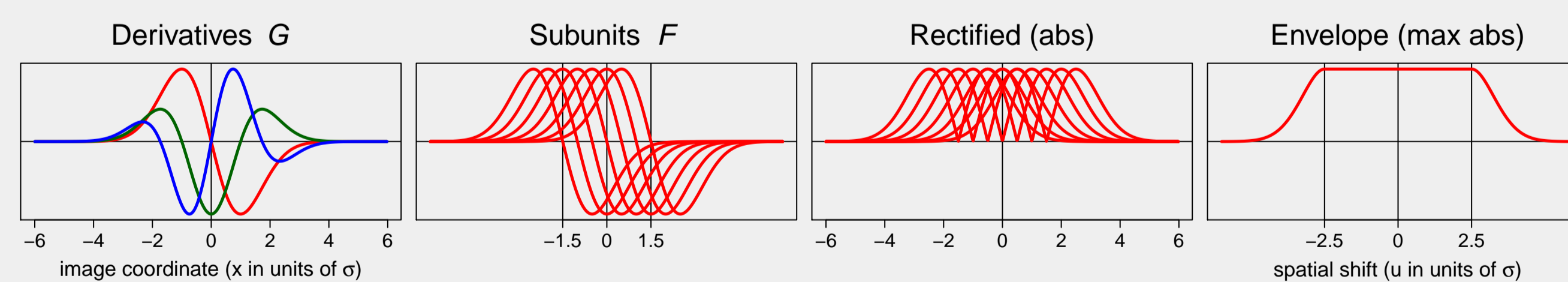
- ▶ Let $F_*(x, u)$ be a family of **subunit filters**, parameterized by shift u .
- ▶ These can be Taylor-approximated from $F_*(x, 0)$ and its derivatives.
- ▶ In particular, choose the edge-filters $F_*(x, u) = G_1(x - u, \sigma)$.
- ▶ Approximate filters $F \approx F_*$ can be obtained from the D th-order jet:

$$F(x, u) = \sum_{k=0}^{D-1} \frac{(-u)^k}{k!} G_{k+1}(x, \sigma)$$

- ▶ Problems: Unstable, and nature of the approximation is unclear.
- ▶ Solution: Allow **polynomial weights** $P_k(u)$, and solve by least-squares.

3. Invariant Response

- ▶ The basis G is used to synthesize the filters F_i , for each shift u_i .
- ▶ Each subunit filter is applied to the signal s , giving $q_i = F_i \cdot s$.
- ▶ The response-envelope is computed by the operation $\max_i |q_i|$.



- ▶ More derivatives are needed in practice (see box 6).
- ▶ The range of shifts must cover $\pm\sigma$ for a unimodal impulse-response.
- ▶ Note that the family of subunit filters is continuous (only 7 shown).

4. Neural Implementation

- ▶ A neurally plausible 'softmax' is used to compute the response-envelope:

$$\max_i |q_i| \approx \sum_i w_i |q_i|$$

- ▶ The weights w_i are defined by a **nonlinearity** and **normalization**:

$$w_i = \exp(\mu |q_i|) / \sum_i \exp(\mu |q_i|)$$

5. Matrix Formulation

- ▶ F : Subunit filters (rows) P : Polynomials (columns)
- ▶ G : Gaussian derivatives (rows) M : Monomials (columns)
- ▶ s : Input signal (column) C : Estimated coefficients

- ▶ The subunit-filters are polynomially-weighted Gaussian derivatives:

$$F = PG \quad \text{where} \quad P = MC$$

- ▶ The filter-design problem is to estimate C , given ideal filters F_* .

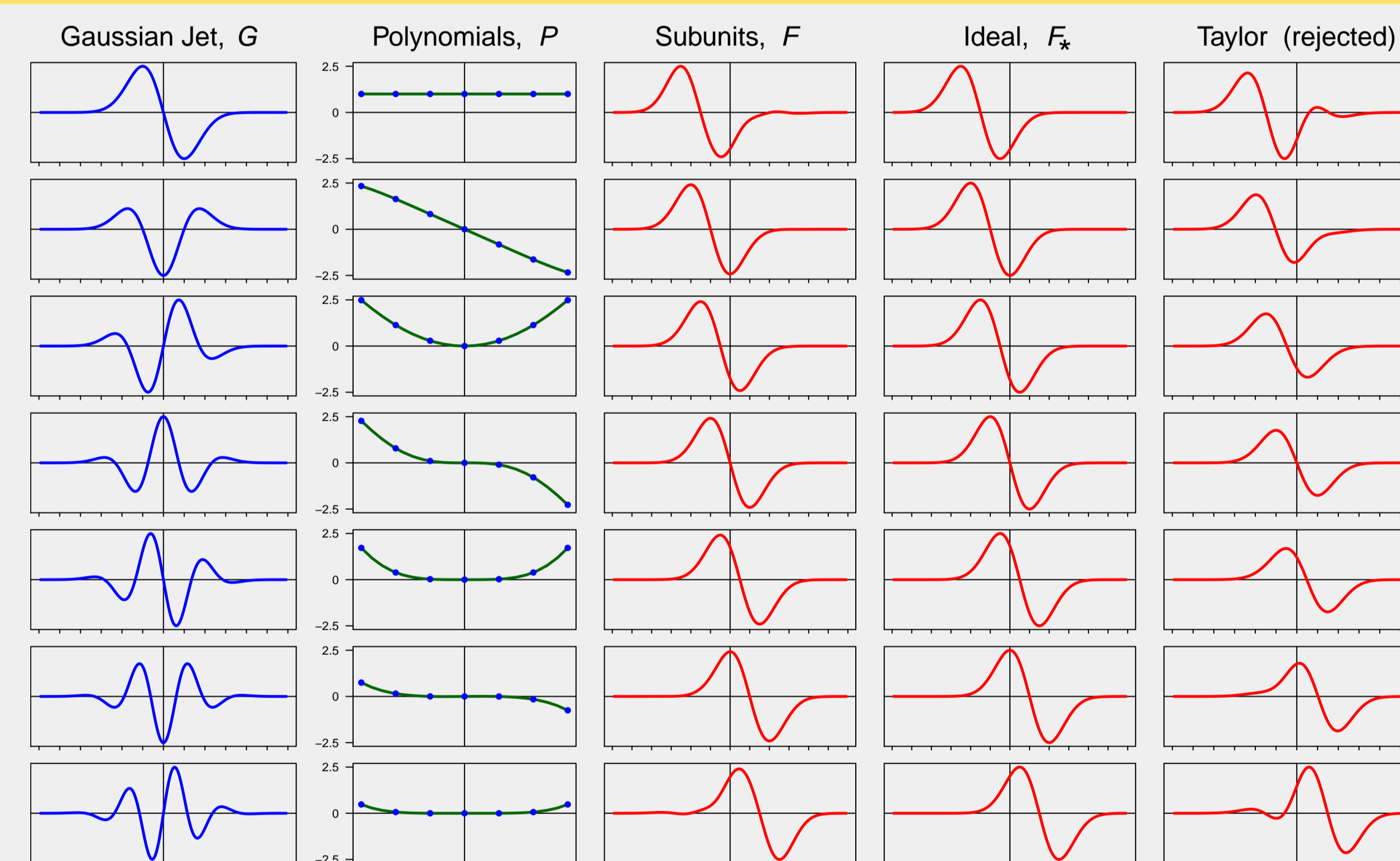
- ▶ The **least-squares solution** involves two pseudo-inverses:

$$F = MCG \quad \text{where} \quad C = M^+ F_* G^+$$

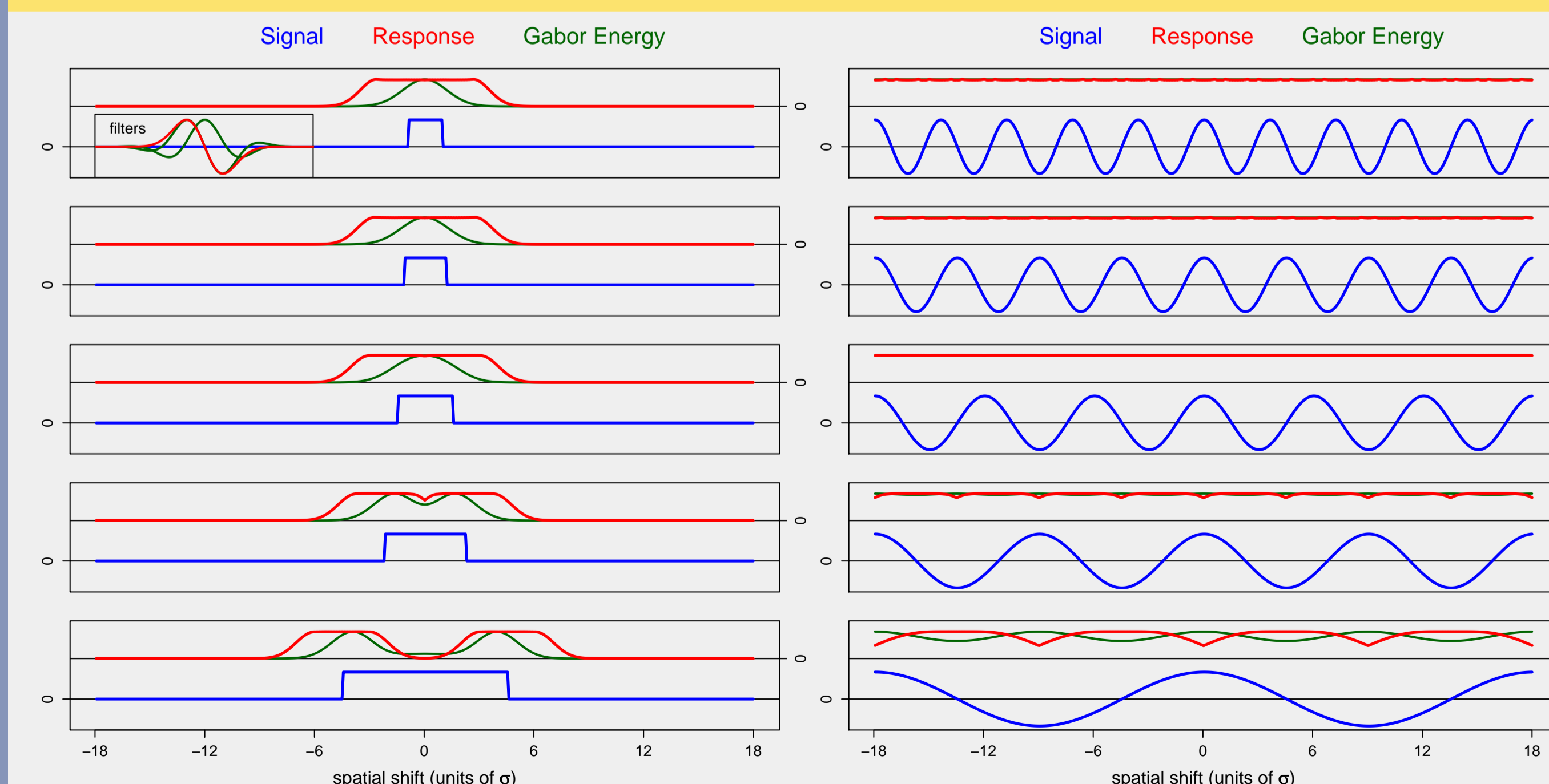
- ▶ The subunit response q is a **linear transformation** of the jet response:

$$q = Fs = P(Gs)$$

6. One-Dimensional Example



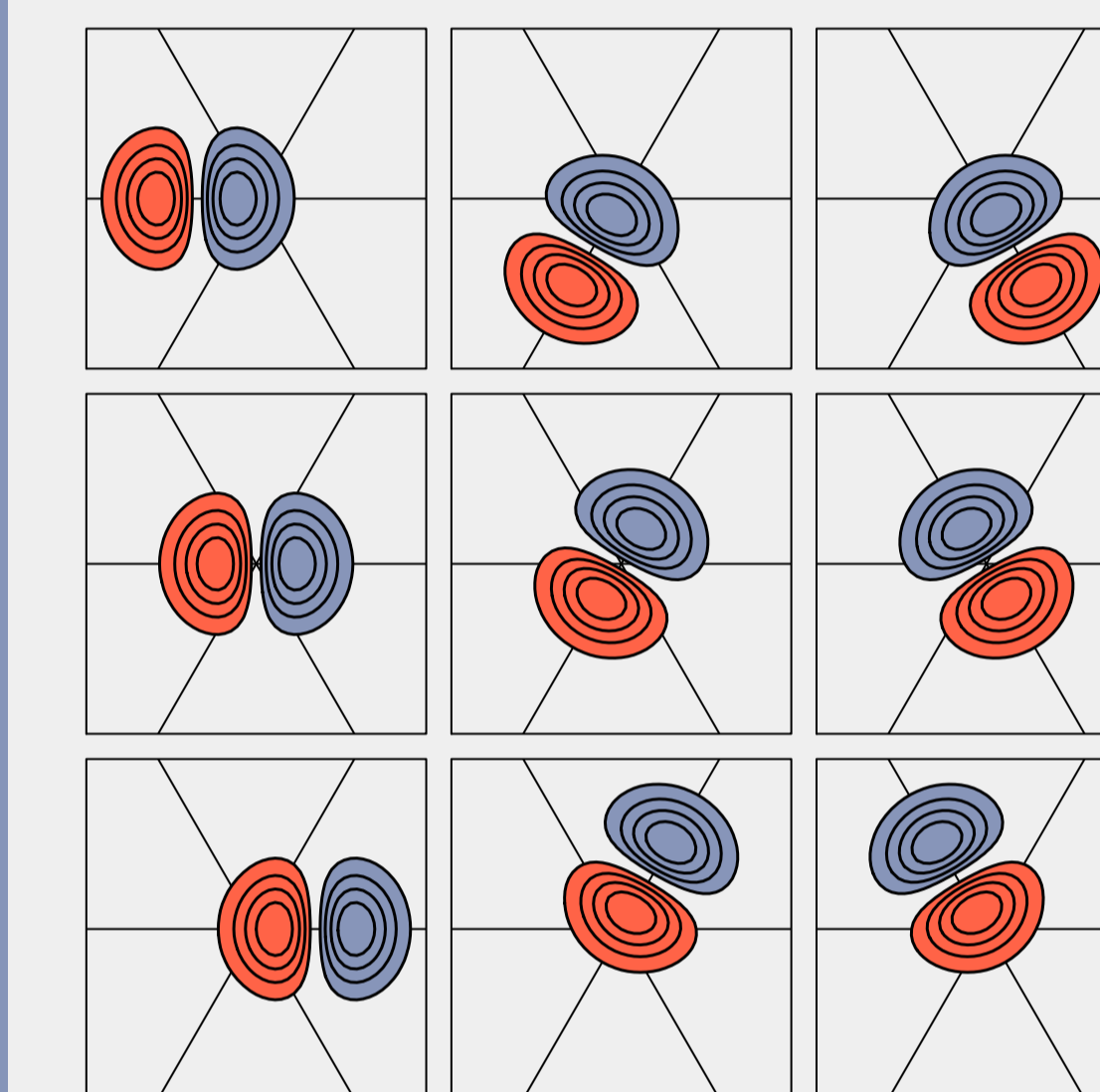
7. Bar and Grating Responses



8. Steerability

- ▶ Any Gaussian derivative $G_k(x, \sigma, \theta)$ can be **exactly** synthesized from the basis $G_k(x, \sigma, \theta_j)$, where $j = 1 \dots k + 1$.
- ▶ A complete basis of order D requires $\sum_{k=1}^D (k + 1) = \frac{1}{2}D(D + 3)$ filters.
- ▶ The basis matrix G must be extended, but there is otherwise no change to the filter-synthesis algorithm (box 5).

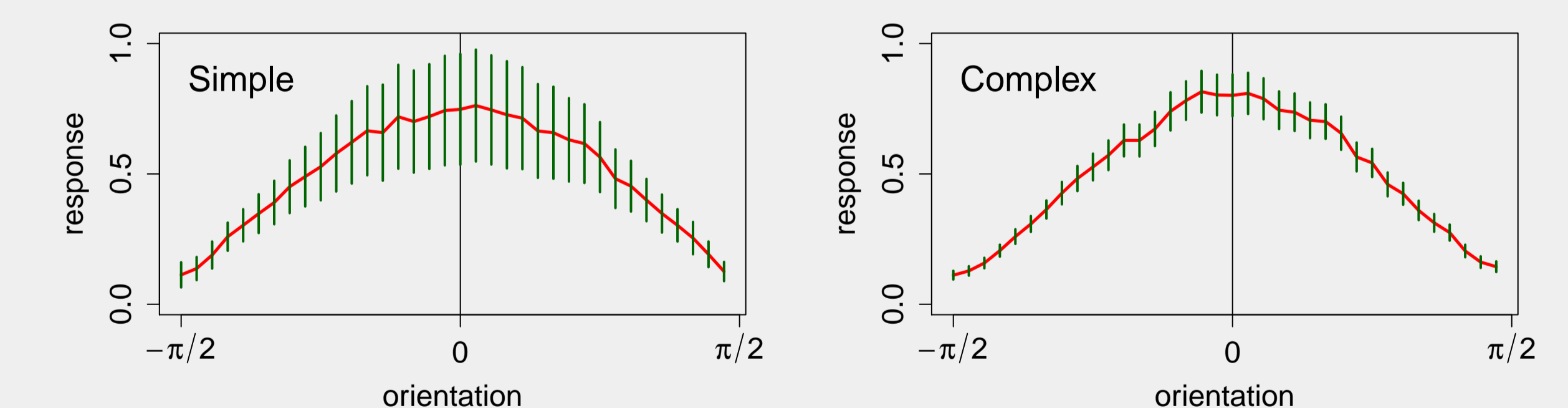
9. Two-Dimensional Example



- ▶ The complete basis of order $D = 8$ contains 44 oriented filters, all centred at the **same location**.
- ▶ This permits accurate synthesis of **any** filter $G_1(x - u, \sigma, \theta)$, where $u \in \pm 1.5\sigma$ and $\theta \in [0, 2\pi)$.
- ▶ Each column corresponds to a complex cell RF, $\theta = 0^\circ, 60^\circ, 120^\circ$.
- ▶ Each box shows a subunit.

10. Natural Image Response

- ▶ The model is evaluated in the framework of 'slow feature analysis'.
- ▶ Pick 100 points at random, from an image with a **dominant orientation**.
- ▶ Choose straight 'tracks' at 36 orientations through each point.
- ▶ Apply simple/complex cell model at 100 points along each track.
- ▶ Compute **mean response** and **SD** in each orientation, over all points.



- ▶ The simple response is orientation tuned (red curve), but highly variable.
- ▶ The complex response is also orientation tuned, but much less variable.

11. Conclusions

- ▶ A shift-insensitive response can be obtained from the Gaussian jet.
- ▶ The signal structure can be represented **geometrically**.
- ▶ The new model is steerable, and works in any number of dimensions.
- ▶ High-order filters, as seen in neural data, are needed in the jet basis.