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## A Gaussian Derivative Model of the Complex Cell

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## 1. Introduction

- ▶ Visual filters can be modelled by derivatives  $G_k$  of the Gaussian function.
- ▶ This **Gaussian jet** representation is convenient because it is:
  - ▷ Steerable, hence all orientations can be represented concisely.
  - ▷ Dimensionally separable, hence easily defined in 2D and 3D.
  - ▷ The natural code for typical image features, e.g. edges and blobs.
- ▶ But what about **complex cells**, cf. the Gabor energy model?
- ▶ Can the jet be made insensitive to small **shifts of the image**?

## 2. Subunit Filters

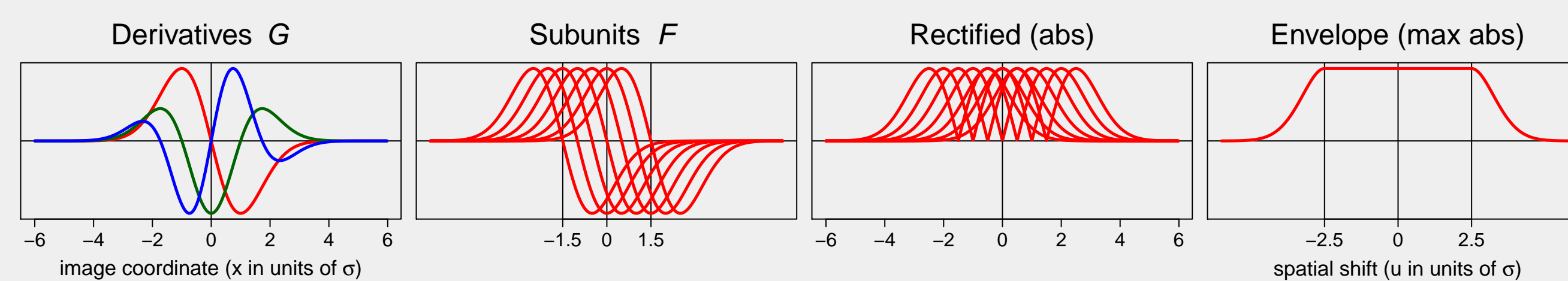
- ▶ Let  $F_*(x, u)$  be a family of **subunit filters**, parameterized by shift  $u$ .
- ▶ These can be Taylor-approximated from  $F_*(x, 0)$  and its derivatives.
- ▶ In particular, choose the edge-filters  $F_*(x, u) = G_1(x - u, \sigma)$ .
- ▶ Approximate filters  $F \approx F_*$  can be obtained from the  $D$ th-order jet:

$$F(x, u) = \sum_{k=0}^{D-1} \frac{(-u)^k}{k!} G_{k+1}(x, \sigma)$$

- ▶ Problems: Unstable, and nature of the approximation is unclear.
- ▶ Solution: Allow **polynomial weights**  $P_k(u)$ , and solve by least-squares.

## 3. Invariant Response

- ▶ The basis  $G$  is used to synthesize the filters  $F_i$ , for each shift  $u_i$ .
- ▶ Each subunit filter is applied to the signal  $s$ , giving  $q_i = F_i \cdot s$ .
- ▶ The response-envelope is computed by the operation  $\max_i |q_i|$ .



- ▶ More derivatives are needed in practice (see box 6).
- ▶ The range of shifts must cover  $\pm\sigma$  for a unimodal impulse-response.
- ▶ Note that the family of subunit filters is continuous (only 7 shown).

## 4. Neural Implementation

- ▶ A neurally plausible 'softmax' is used to compute the response-envelope:

$$\max_i |q_i| \approx \sum_i w_i |q_i|$$

- ▶ The weights  $w_i$  are defined by a **nonlinearity** and **normalization**:

$$w_i = \exp(\mu |q_i|) / \sum_i \exp(\mu |q_i|)$$

## 5. Matrix Formulation

- ▶  $F$  : Subunit filters (rows)       $P$  : Polynomials (columns)
- ▶  $G$  : Gaussian derivatives (rows)       $M$  : Monomials (columns)
- ▶  $s$  : Input signal (column)       $C$  : Estimated coefficients

- ▶ The subunit-filters are polynomially-weighted Gaussian derivatives:

$$F = PG \quad \text{where} \quad P = MC$$

- ▶ The filter-design problem is to estimate  $C$ , given ideal filters  $F_*$ .

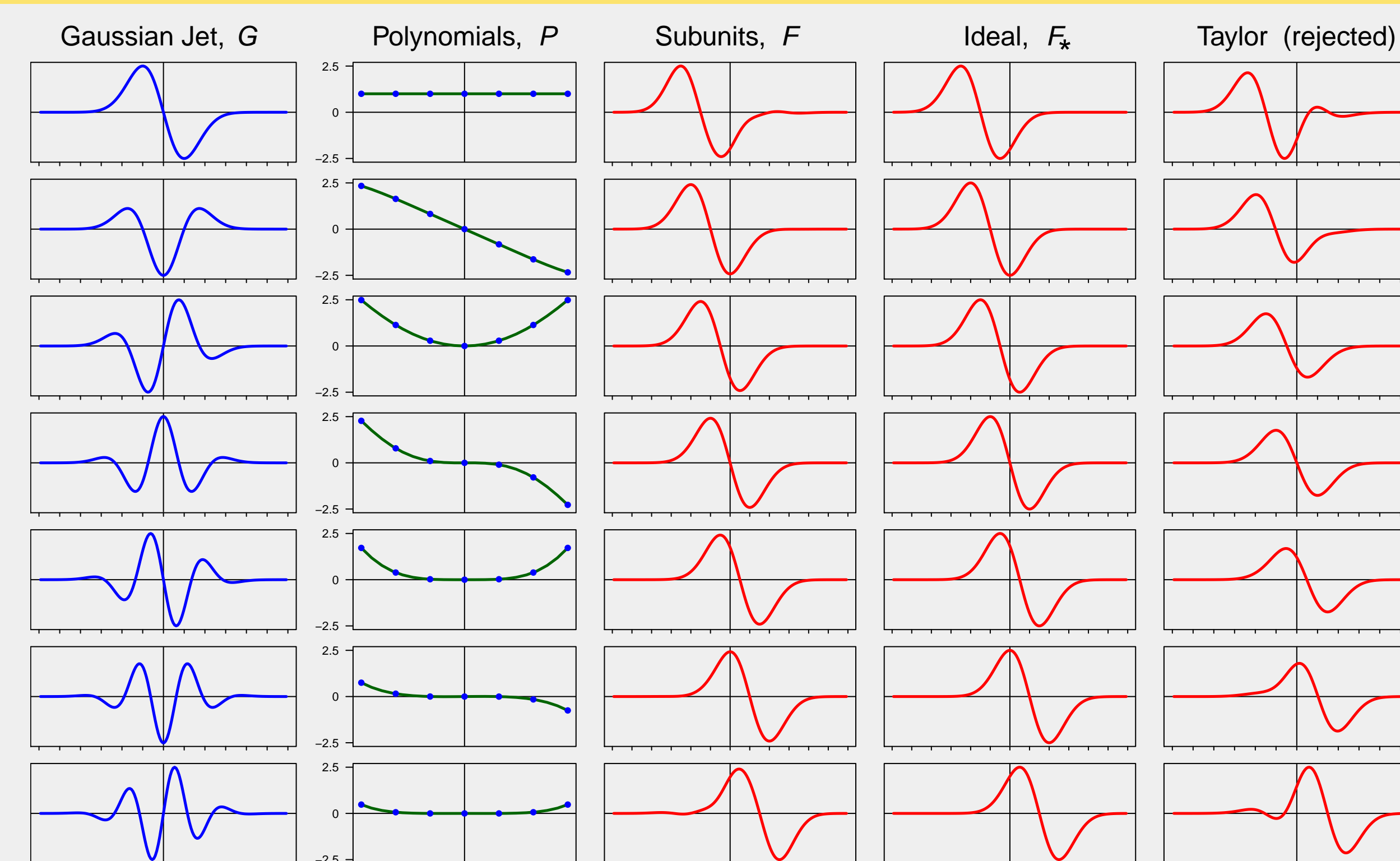
- ▶ The **least-squares solution** involves two pseudo-inverses:

$$F = MCG \quad \text{where} \quad C = M^+ F_* G^+$$

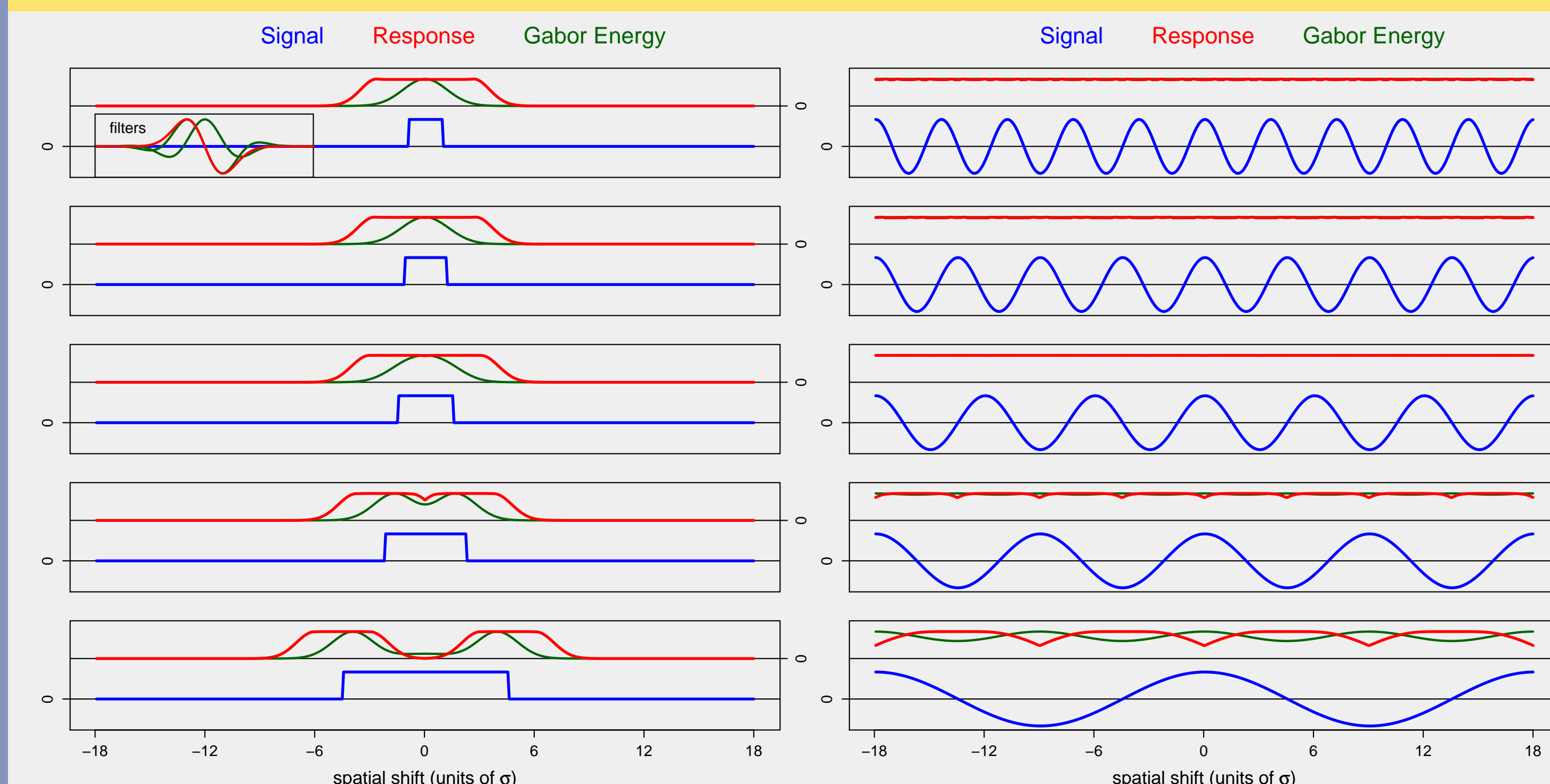
- ▶ The subunit response  $q$  is a **linear transformation** of the jet response:

$$q = Fs = P(Gs)$$

## 6. One-Dimensional Example



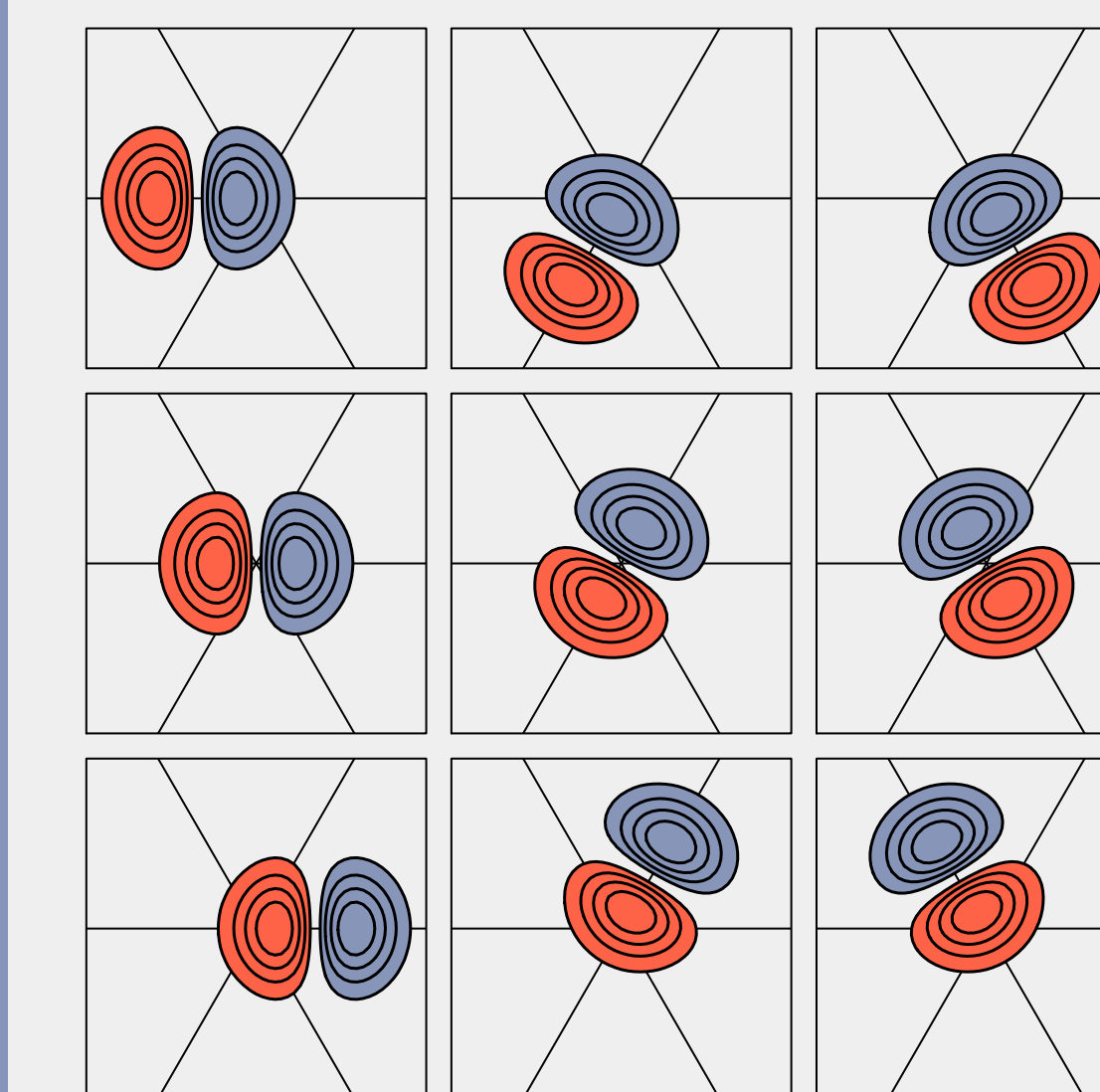
## 7. Bar and Grating Responses



## 8. Steerability

- ▶ Any Gaussian derivative  $G_k(x, \sigma, \theta)$  can be **exactly** synthesized from the basis  $G_k(x, \sigma, \theta_j)$ , where  $j = 1 \dots k + 1$ .
- ▶ A complete basis of order  $D$  requires  $\sum_{k=1}^D (k + 1) = \frac{1}{2}D(D + 3)$  filters.
- ▶ The basis matrix  $G$  must be extended, but there is otherwise no change to the filter-synthesis algorithm (box 5).

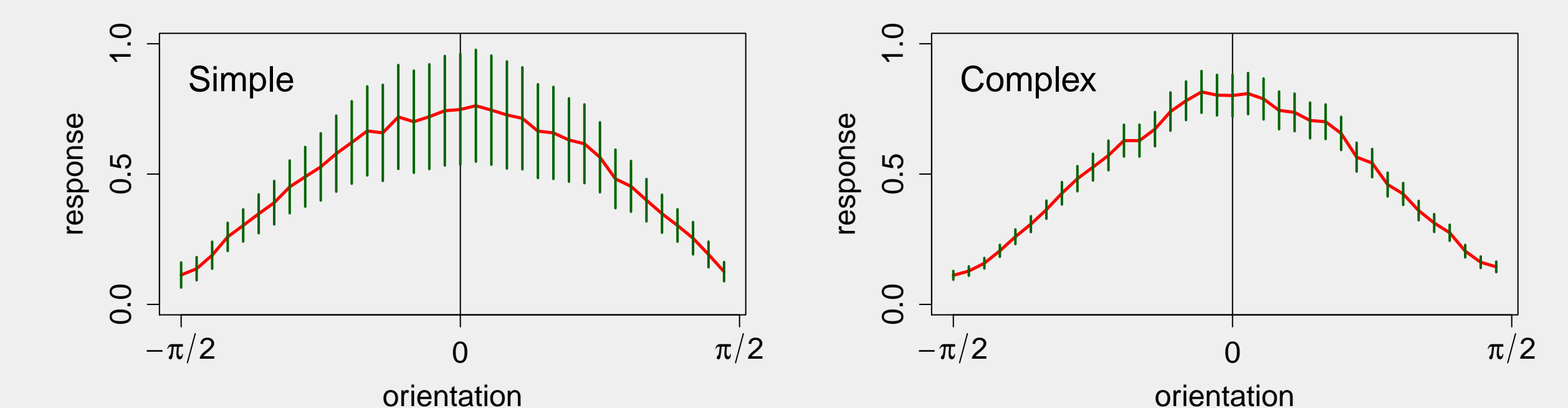
## 9. Two-Dimensional Example



- ▶ The complete basis of order  $D = 8$  contains 44 oriented filters, all centred at the **same location**.
- ▶ This permits accurate synthesis of **any** filter  $G_1(x - u, \sigma, \theta)$ , where  $u \in \pm 1.5\sigma$  and  $\theta \in [0, 2\pi)$ .
- ▶ Each column corresponds to a complex cell RF,  $\theta = 0^\circ, 60^\circ, 120^\circ$ .
- ▶ Each box shows a subunit.

## 10. Natural Image Response

- ▶ The model is evaluated in the framework of 'slow feature analysis'.
- ▶ Pick 100 points at random, from an image with a **dominant orientation**.
- ▶ Choose straight 'tracks' at 36 orientations through each point.
- ▶ Apply simple/complex cell model at 100 points along each track.
- ▶ Compute **mean response** and **SD** in each orientation, over all points.



- ▶ The simple response is orientation tuned (red curve), but highly variable.
- ▶ The complex response is also orientation tuned, but much less variable.

## 11. Conclusions

- ▶ A shift-insensitive response can be obtained from the Gaussian jet.
- ▶ The signal structure can be represented **geometrically**.
- ▶ The new model is steerable, and works in any number of dimensions.
- ▶ High-order filters, as seen in neural data, are needed in the jet basis.