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Filter Transformations for Shift-Insensitive Feature Detection

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Introduction

- ▶ Visual filters can be modelled by derivatives G_k of the Gaussian function.
- ▶ The combined responses characterize the local structure of the image.
- ▶ This *Gaussian jet* representation is convenient because it is:
 - ▷ Steerable: Get any $G_k(x, \sigma, \theta)$ from $G_k(x, \sigma, \theta_1) \cdots G_k(x, \sigma, \theta_{k+1})$.
 - ▷ Dimensionally separable, hence easily defined in 2D and 3D.
 - ▷ The natural code for typical image features (ridges, blobs, etc).
- ▶ But what about *complex cells*, cf. the Gabor 'energy model'?
- ▶ **Can the jet be made insensitive to small shifts of the image?**

Replica Filters

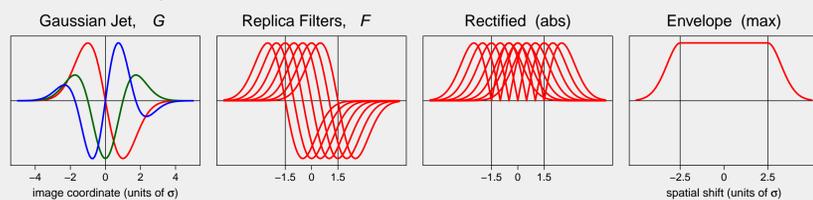
- ▶ Let $F_\star(x, u)$ be a family of ideal filters, shifted by u .
- ▶ These can be Taylor-approximated from $F_\star(x, 0)$ and its derivatives.
- ▶ In particular, choose the edge-templates $F_\star(x, u) = G_1(x - u, \sigma)$.
- ▶ Now define the **replica filters**, $F \approx F_\star$, from the D th-order jet:

$$F(x, u) = \sum_{k=0}^{D-1} \frac{-u^k}{k!} G_{k+1}(x, \sigma)$$

- ▶ Problems: Unstable, and nature of the approximation is unclear.
- ▶ Solution: Allow **polynomial** weights $P_k(u)$, and solve by least-squares.

Impulse Response

- ▶ Schematic representation:



- ▶ More derivatives are needed in practice (see 'Filter Construction' box).
- ▶ The interval of approximation is $\pm \rho \sigma$, with $\rho = 1.5$ here.
- ▶ Note that the family of replica filters is *continuous* (only 7 shown).

Neural Implementation

- ▶ The linear-response vector is $q_j = F_j \cdot s$, one value for each shift.
- ▶ A neurally plausible 'soft-max' is used to compute the envelope:

$$q_\star = \sum_j w_j |q_j| \approx \max |q_j|$$

- ▶ The weights are defined by a **nonlinearity** and **normalization**:

$$w_j = \exp(\mu |q_j|) / \sum_j \exp(\mu |q_j|)$$

Matrix Formulation

- ▶ F : Replica filters (rows) P : Polynomials (columns)
- ▶ G : Gaussian derivatives (rows) M : Monomials (columns)
- ▶ s : Input signal (column) C : Estimated coefficients

- ▶ The defining equations are:

$$F = PG \quad \text{where} \quad P = MC$$

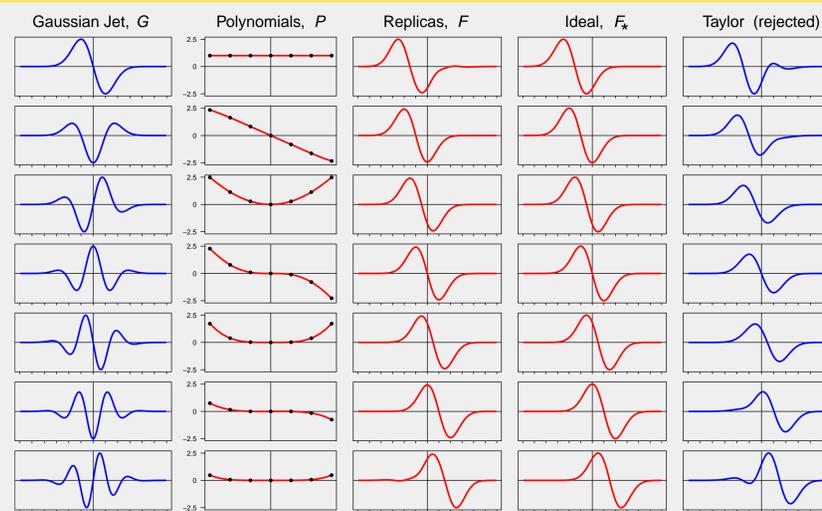
- ▶ The 'filter design' problem is to estimate C , given ideal filters F_\star .
- ▶ Least-squares solution is the pseudo-inverse of a Kronecker product:

$$\text{vec}(C) = (G^T \otimes M)^+ \text{vec}(F_\star)$$

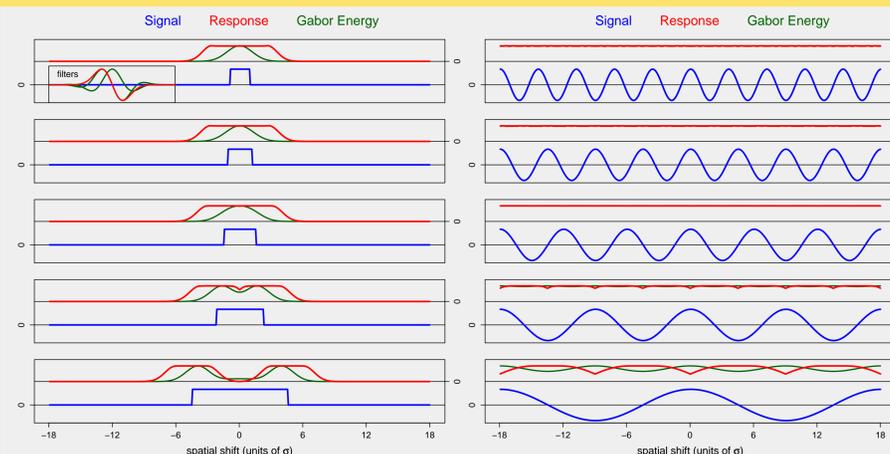
- ▶ The replica response q is a **linear transformation** of the jet response Gs :

$$q = Fs = P(Gs)$$

Complete Example



Bar and Grating Responses



Two-Dimensional Filters

- ▶ The image gradient at 2D position x , and scale σ , is estimated as:

$$\nabla S(x) = [G_1(x, \sigma, 0) \cdot S, G_1(x, \sigma, \pi/2) \cdot S]$$
- ▶ The replica filters can find the local maximum of the gradient:

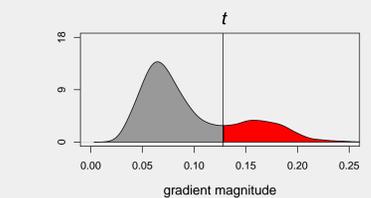
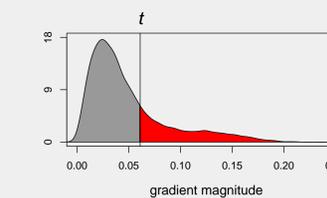
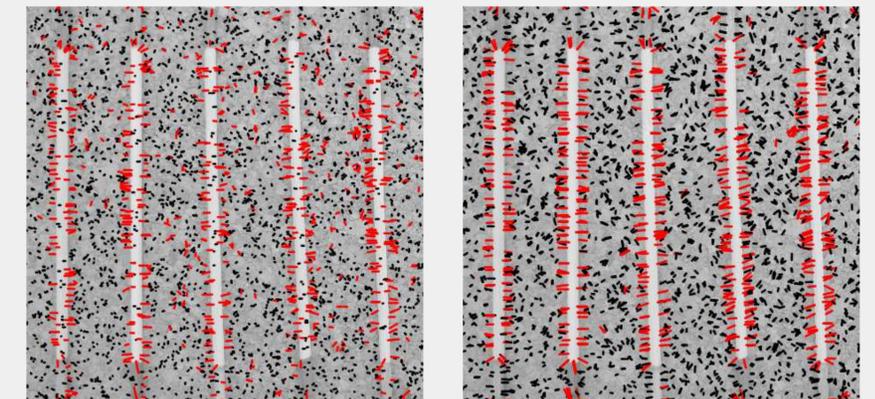
$$u_\star = \arg \max_{u \in \mathcal{F}} |\nabla S(x + u)| \quad \text{where} \quad \mathcal{F} = \{u : |u| < \rho \sigma\}$$
- ▶ The disk \mathcal{F} is the **receptive field** of the mechanism.

Natural Image Experiments

- ▶ Make a coarse ($\sim 1\%$ pixels) random sampling of the gradient.
- ▶ Compare $\nabla S(x)$ to $\nabla S(x + u_\star)$, using $\sigma = 3$ pixels and $\rho = 1.5$.
- ▶ Split each distribution at $P(|\nabla S| < t) = 0.75$; plot strong vectors in red.

Raw Gradient $\nabla S(x)$

Adjusted Gradient $\nabla S(x + u_\star)$



- ▶ The undersampled structure is better represented by $\nabla S(x + u_\star)$.
- ▶ The true gradient distribution is bi-modal (boundaries plus texture).
- ▶ But the randomly-placed filters are unlikely to fall on the boundaries.
- ▶ If x is within $\rho \sigma$ of an edge, then $|\nabla S(x + u_\star)| \gg |\nabla S(x)|$.

Conclusions

- ▶ A shift-insensitive response can be obtained from the Gaussian jet.
- ▶ The signal structure can be represented **geometrically**.
- ▶ The new model is steerable, and works in any number of dimensions.
- ▶ High-order filters, as seen in neural data, are needed in the jet basis.