HAL open science

Filter Transformations for Shift-Insensitive Feature Detection

Miles Hansard, Radu Horaud

To cite this version:

Miles Hansard, Radu Horaud. Filter Transformations for Shift-Insensitive Feature Detection. Applied Vision Association Christmas meeting, Dec 2009, Bristol, United Kingdom. 2009. inria-00590258

HAL Id: inria-00590258

https://inria.hal.science/inria-00590258v1

Submitted on 3 May 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Filter Transformations for Shift-Insensitive Feature Detection

Miles Hansard and Radu Horaud
INRIA Rhône-Alpes

Introduction

- Visual filters can be modelled by derivatives G_k of the Gaussian function.
- The combined responses characterize the local structure of the image.
- This *Gaussian jet* representation is convenient because it is:
  - Steerable: Get any $G_k(x,\sigma,\theta)$ from $G_k(x,\sigma,\theta_1)\cdots G_k(x,\sigma,\theta_{k+1})$.
  - Dimensionally separable, hence easily defined in 2D and 3D.
  - The natural code for typical image features (ridges, blobs, etc).
- But what about *complex cells*, cf. the Gabor 'energy model'?
- Can the jet be made insensitive to small shifts of the image?

Replica Filters

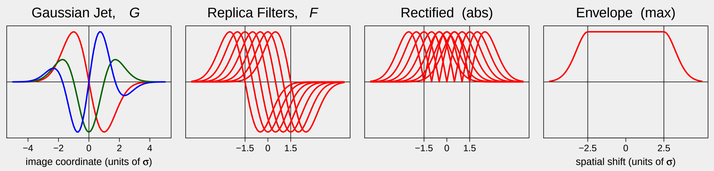
- Let $F_\star(x,u)$ be a family of ideal filters, shifted by u.
- These can be Taylor-approximated from $F_\star(x,0)$ and its derivatives.
- In particular, choose the edge-templates $F_\star(x,u) = G_1(x-u,\sigma)$.
- Now define the replica filters, $F \approx F_\star$, from the Dth-order jet:

$$F(x,u) = \sum_{k=0}^{D-1} \frac{-u^k}{k!}\, G_{k+1}(x,\sigma)$$

- Problems: Unstable, and nature of the approximation is unclear.
- Solution: Allow polynomial weights $P_k(u)$, and solve by least-squares.

Impulse Response

- Schematic representation:



- More derivatives are needed in practice (see 'Filter Construction' box).
- The interval of approximation is $\pm\,\rho\,\sigma$, with $\rho = 1.5$ here.
- Note that the family of replica filters is *continuous* (only 7 shown).

Neural Implementation

- The linear-response vector is $q_j = F_j \cdot s$, one value for each shift.
- A neurally plausible 'soft-max' is used to compute the envelope:

$$q_\star = \sum_j w_j\,|q_j| \approx \max |q_j|$$

- The weights are defined by a nonlinearity and normalization:

$$w_j = \exp\big(\mu|q_j|\big) \Big/ \sum_j \exp\big(\mu|q_j|\big)$$

Matrix Formulation

- F : Replica filters (rows) — P : Polynomials (columns)
  G : Gaussian derivatives (rows) — M : Monomials (columns)
  s : Input signal (column) — C : Estimated coefficients
- The defining equations are:

$$F = PG \quad \text{where} \quad P = MC$$

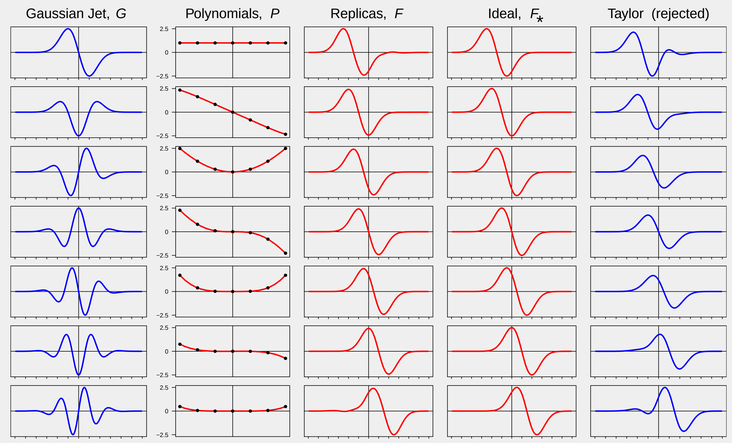
- The 'filter design' problem is to estimate C, given ideal filters $F_\star$.
- Least-squares solution is the pseudo-inverse of a Kronecker product:

$$\mathrm{vec}(C) = \big(G^\top \otimes M\big)^+ \mathrm{vec}(F_\star)$$

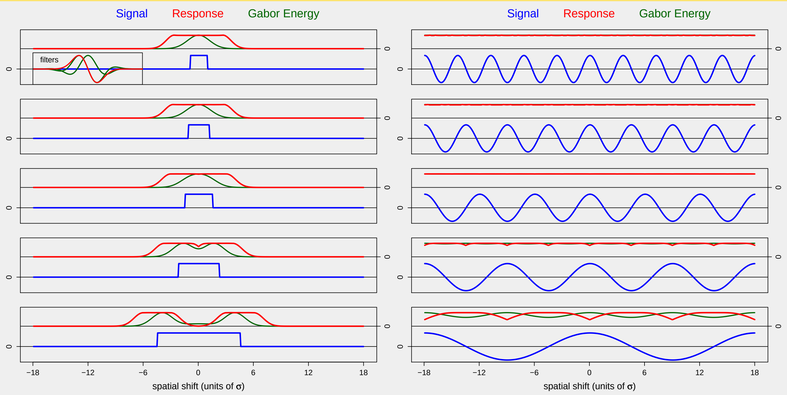
- The replica response q is a linear transformation of the jet response Gs:

$$q = Fs = P(Gs)$$

Complete Example



Bar and Grating Responses



Two-Dimensional Filters

- The image gradient at 2D position x, and scale σ, is estimated as:

$$\nabla S(x) = \big[G_1(x,\sigma,0)\cdot S,\; G_1(x,\sigma,\pi/2)\cdot S\big]$$

- The replica filters can find the local maximum of the gradient:

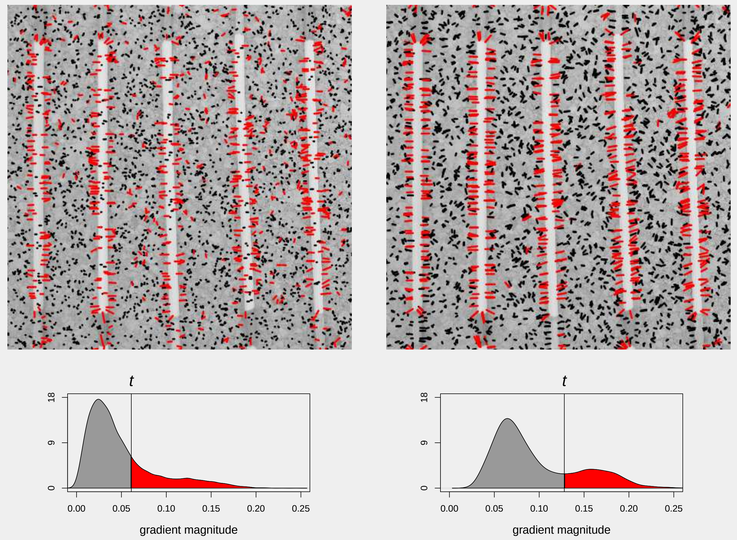
$$u_\star = \arg\max_{u\in\mathcal{F}} \big|\nabla S(x+u)\big| \quad \text{where} \quad \mathcal{F} = \{\, u : |u| < \rho\,\sigma \,\}$$

- The disk $\mathcal{F}$ is the receptive field of the mechanism.

Natural Image Experiments

- Make a coarse (∼1% pixels) random sampling of the gradient.
- Compare $\nabla S(x)$ to $\nabla S(x+u_\star)$, using $\sigma = 3$ pixels and $\rho = 1.5$.
- Split each distribution at $\mathrm{P}\big(|\nabla S| < t\big) = 0.75$; plot strong vectors in red.

Raw Gradient $\nabla S(x)$ — Adjusted Gradient $\nabla S(x+u_\star)$



- The undersampled structure is better represented by $\nabla S(x+u_\star)$.
- The true gradient distribution is bi-modal (boundaries plus texture).
- But the randomly-placed filters are unlikely to fall on the boundaries.
- If x is within $\rho\,\sigma$ of an edge, then $|\nabla S(x+u_\star)| \gg |\nabla S(x)|$.

Conclusions

- A shift-insensitive response can be obtained from the Gaussian jet.
- The signal structure can be represented geometrically.
- The new model is steerable, and works in any number of dimensions.
- High-order filters, as seen in neural data, are needed in the jet basis.

http://perception.inrialpes.fr/~Hansard/
miles.hansard@inrialpes.fr