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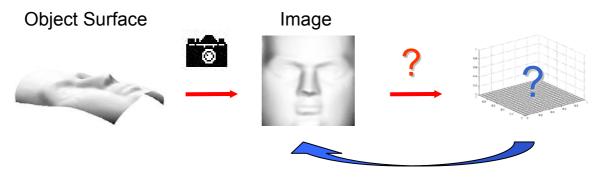
Viscosity solutions For Realistic Shape-From-Shading

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Workshop "Numerical Methods for Viscosity Solutions and Applications" Rome, September 6-8, 2004¹

The Shape From Shading Problem



SFS = Inverse problem

Problem:

- Inverse problem to image synthesis,
- Recover the surface(s) which yield(s) the same image.

Used Information:

Shading.



Shape From Shading Data

One image;

- Intrinsic parameters of the camera:
 - Focal length,
 - Size of the pixels
- Surface parameters:
 - Reflectance of the surface (albedo)
- Lighting parameters:
 - Flash...
- Database of *real* images of faces available online:
 - Experimental setup: scene illuminated by the flash of the camera.
 - Parameters are detailed.
 - WWW link:



http://www-sop.inria.fr/odyssee/team/Emmanuel.Prados/







Report:

- Most of the SFS methods model the problem very basically: Ex.: Camera=Orthographic Projection...
 - \Rightarrow Not Realistic!
- 2) Lots of SFS algorithms require boundary data (generally Dirichlet BC) ⇒ Not Available with Real Images!

The results on real images are generally very disappointing.



Shape From Shading Contributions:

- We propose more realistic modeling: Example: camera = pinhole (perspective projection)
- We reduce as much as possible the requirement of the boundary data
 - (in particular on the boundary of the images).



Modeling / Mathematical formulations

The Basic Assumption: Lambertian Surface

(without specularities or highlights)

- \Rightarrow Involves a connection between:
 - \succ the brightness of the image (intensity) : variable I,

> the normal surface: n, << Depends on the parameterization >>.

> the lighting direction: L << Depends on the lighting>>.

 $I(X) = cos(\mathbf{n}(X), \mathbf{L}(\mathbf{X})) = \frac{\mathbf{n}(X)}{|\mathbf{n}(X)|} \cdot \mathbf{L}(\mathbf{X})$

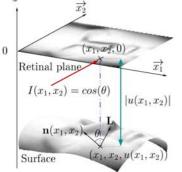
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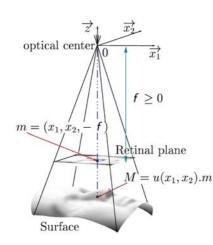
Modeling / Mathematical formulations



- Orthographic projection (classical modeling)
 - Far light source:

 $I(x)\sqrt{1+|\nabla u|^2}+\nabla u\cdot \mathbf{l}-\gamma=0,$

- Frontal far light source: (Eikonal equation) |
abla u|=f(x), $_{f(x)=\sqrt{rac{1}{f(x)^2}-1},}$



- More realistic Modeling: Perspective Projection:
 - Far light source:

$$I(x)\sqrt{f^2|\nabla u|^2 + (x\cdot\nabla u+1)^2} - (f\mathbf{l}+\gamma x)\cdot\nabla u - \gamma = 0,$$

- Light source located at the Optical center:

$$I(x)\sqrt{f^2|\nabla u|^2 + (x \cdot \nabla u)^2 + Q(x)^2} - Q(x) = 0.$$

 $Q(x) = \frac{f}{\sqrt{|x|^2 + f^2}}.$

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A Generic Hamiltonian

All these SFS equations are special cases of the "generic" equation: $\kappa_x \sqrt{|A_x \nabla \mathbf{u} + \mathbf{v_x}|^2 + K_x^2} + \mathbf{w_x} \cdot \nabla \mathbf{u} + c_x = 0$ (1)

> Where $\kappa_x, K_x \ge 0$, $A_x \in \mathcal{M}_{2 \times 2}$, $\mu_x, \nu_x \in \mathbb{R}^*$, $\mathbf{v}_x, \mathbf{w}_x \in \mathbb{R}^2$ and $c_x \in \mathbb{R}$.

(1) is a HJ PDE of the form $H(x, \nabla u)$ with: $H(x, p) = \kappa_x \sqrt{|A_x p + \mathbf{v}_x|^2 + K_x^2} + \mathbf{w}_x \cdot p + c_x.$ H(x,p) convex in p.

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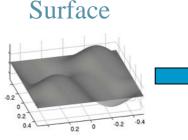
Ill-posedness of the SFS problem

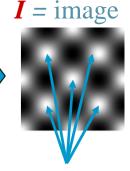
SFS equations are of the form:

 $H(x,\nabla u)=0$, x in Ω (a bounded set).

⇒ They require Boundary Conditions (BC) on $\partial \Omega$ ⇒ But the BC on $\partial \Omega$ are not sufficient! SFS equations can have singularities:

For all the previous SFS equations, the set of **singular points** is





Singular points

{ x | I(x) is maximal (i.e 1) }

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Recall:

In Shape from Shading

Data = one image.

 \Rightarrow We do not have boundary data;

 \Rightarrow i.e. We do not know what are the adequate boundary conditions.



Solution proposed in the SFS literature

Notion	SFS Biblio.	Data (DBC)	Limitations
Continuous Viscosity solutions (Crandall-Lions)	Lions/Rouy/ Tourin (92)	S_{ing} U $\partial \Omega$	Compatibility conditions. Dirich. data on S_{ing} U $\partial \Omega$
C ¹ Functions	Dupuis/ Oliensis (94)	Set of local <i>Min</i> . of <i>u</i>	Value function ? Few intuitive But: Interest : Minimal Data are required !
Discontinuous Viscosity solutions (Ishii)	Prados/ Faugeras (03)	$N(S_{ing})$ U $\partial \Omega$	Dirichlet data on $N(S)_{ing} \cup \partial \Omega$
Singular Viscosity solutions (Camilli-Siconolfi:99)	Camilli/ Falcone (96)	$\partial \Omega$	 not always adapted to SFS. still requires data on ∂Ω





Contributions:

- We have slightly extended and modified the notion of singular viscosity solutions (Camilli/Siconolfi:99,02) in order to:
 - to extend the Dupuis/Oliensis' characterization of the C^1 solutions to the viscosity solutions \Rightarrow data are minimal.
 - such that compatibility conditions are not required.





Notion of SDVS

Definition: We define a notion of solution – which extends the classical notions of

- of the discontinuous viscosity solutions (Ishii...)
- of the singular viscosity solutions (Camilli-Siconolfi:99,02).
- Which allows to fix the values of the solutions at the singular points or on the boundary ∂Ω when we know them, and to send them at the infinity (state constraints), when we do not know them.



Properties of the SDVS:

Existence of the SDVS

- Dynamical programming principle,
- value function;
- Strong uniqueness;
- Stability results;
- Application to the Shape From Shading problem...





Original theorems:

Theorem:

(roughly)

« A discontinuous viscosity solution without "local minima" is the SDVS».

Corollary:

Characterization of the discontinuous viscosity solutions by their « minima ».





Numerical approximations:

4 stages:

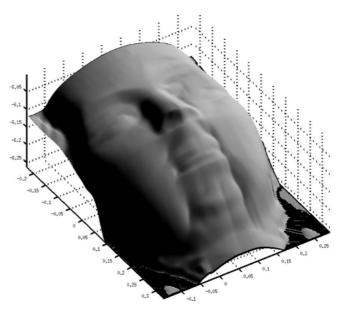
- i) Management of the state constraints;
- ii) Regularization of the equations In SFS (we truncate the image, $I_{\varepsilon}=min(I(x), 1 - \varepsilon)$);
- iii) Designing of the approximation scheme;
- iv) Deduction and implementation of the associated algorithm.



A Numerical Result on a Synthetic Image (Mozart)



a) Synthetic image of the classical Mozart face

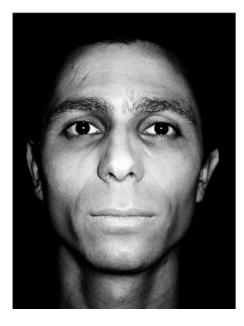


b) Reconstruction

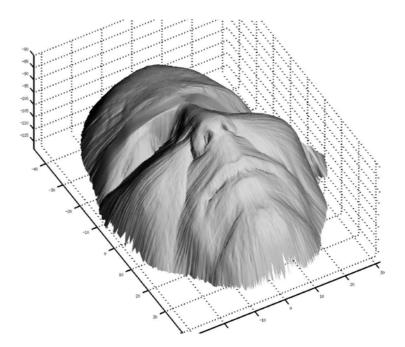




Numerical results on Real images (Faces)...



a) Real image of a face



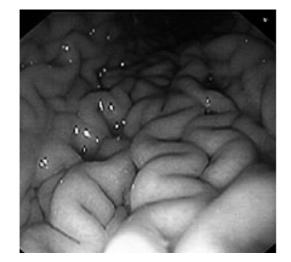
b) Reconstruction

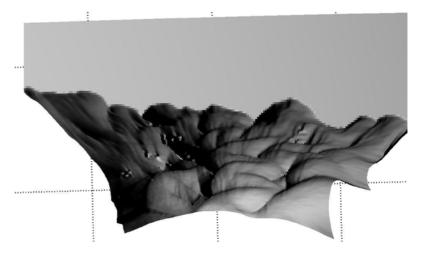






Numerical results on Real images (Stomach)...





a) Real image of a stomach

b) Reconstruction



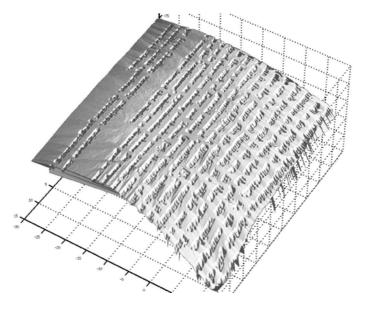


Numerical results on Real images (Pages)

fullen. Rouyeac-lyon.f

estract. This article presents an approach to the shape from shad problem which is based upon the notion of viscosity solutions to the fing partial differential equation, in effect a Hamilton-Jacobi equa The power of this approach is twofolds: 1) it allows nonsmooth, i.e ifferentiable, solutions which allows to recover objects with sharp ths and creases and 2) it provides a framework for deriving a nu al scheme for computing approximations on a discrete grid of thes ons as well as for proving its correctness, i.e. the convergence of approximations to the solution when the grid size vanishes. ork extends previous work in the area in three aspects. First, i with the case of a general illumination in a simpler and a mor I way (since they assume that the solutions are continuously dif able) than in the work of Dupuis and Oliensis [9]. Second, it allow rove the existence and uniqueness of "continuous" solutions to the equation in a more general setting (general direction of illumi than in the work of Rouy and Tourin [24], thereby extending licability of shape from shading methods to more realistic scenes. it allows us to produce an approximation scheme for computing mations of the "continuous" solution on a discrete grid as well as of their convergence toward that solution.

s: Shape from Shading, viscosity solutions, existence and u tion, Hamilton-Jacobi equations, dynamic programming prin a and numerical schemes.



a) Real image of a page

b) Reconstruction

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Conclusion:

SFS problem \neq Eikonal equation.

Realistic SFS consists in solving more complex equations: for ex. the "generic SFS" equation.

In SFS, we do not have at our disposal boundary data.

The notion of SDVS proposes some more adapted solutions than the other classical solutions.



For more details:

Theoretical part:

See the INRIA research report:

"A viscosity method for Shape From Shading without boundary data" (E. Prados, F. Camilli and O. Faugeras), August 2004.

Numerical part: See

- the INRIA research report No RR-5005 :

"A mathematical and algorithmic study of the Lambertian SFS problem for orthographic and pinhole cameras"

(E. Prados and O. Faugeras), September 2003.

- E. Prados' PhD thesis, October 2004.

Web page:

http://www-sop.inria.fr/odyssee/team/Emmanuel.Prados/

Thanks!

