Cosparse Analysis Modeling
Sangnam Nam, Mike E. Davies, Michael Elad, Rémi Gribonval

▶ To cite this version:
Sangnam Nam, Mike E. Davies, Michael Elad, Rémi Gribonval. Cosparse Analysis Modeling. Workshop on Signal Processing with Adaptive Sparse Structured Representations, Jun 2011, Edinburgh, United Kingdom. inria-00587943

HAL Id: inria-00587943
https://inria.hal.science/inria-00587943
Submitted on 16 Sep 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Cosparse Analysis Modeling

Sangnam Nam*, Michael E. Davies†, Michael Elad‡ and Rémi Gribonval*

*INRIA Rennes – Bretagne Atlantique, France
†IDCOM & Joint Research Institute for Signal and Image Processing, Edinburgh University, UK
‡Department of Computer Science, The Technion – Israel Institute of Technology, Israel

Abstract—In the past decade there has been a great interest in a synthesis-based model for signals, based on sparse and redundant representations. This work considers an alternative analysis-based model, where an analysis operator multiplies the signal, leading to a cosparse outcome. We consider this analysis model, in the context of a generic missing data problem. Our work proposes a uniqueness result for the solution of this problem, based on properties of the analysis operator and the measurement matrix. A new greedy algorithm for solving the missing data problem is proposed along with theoretical study of the success of the algorithm and experimental results.

I. INTRODUCTION

Given a set of incomplete linear observation $y = Mx_0 \in \mathbb{R}^m$ of a signal $x_0 \in \mathbb{R}^d$, $m < d$, the assumption that $x_0$ admits a sparse representation $\tilde{x}_0$ in some synthesis dictionary $\mathbf{D}$ is known to be of significant help in recovering the original signal $x_0$. Indeed, it is now well understood that under incoherence assumptions on the matrix $\mathbf{MD}$, one can recover vectors $x_0$ with sufficiently sparse representations by solving the optimization problem:

$$\hat{x}_S := \mathbf{Dz}; \quad \hat{z} := \arg\min_z \|z\|_p \text{ subject to } y = \mathbf{MDz} \quad (1)$$

for $0 \leq \tau \leq 1$.

An alternative to (1) which has also been used successfully in practice is to consider the analysis $\ell_1$-optimization [2], [6], [7]:

$$\hat{x}_A := \arg\min_x \|\Omega x\|_p \text{ subject to } y = \mathbf{Mx}, \quad (2)$$

where $\Omega : \mathbb{R}^d \to \mathbb{R}^p$ is an analysis operator. Typically the dimensions are $m < d \leq p, n$.

The fact that $\tilde{x}_0$ contains few zeros, i.e., is sparse may be thought of as the principal reason why one can recover the so-called sparse signals via (1). We show that while the optimization (2) has similar look to (1), a different model, which we name the cosparse analysis model, is more closely linked to (2) than the sparse synthesis model. In particular, contrary to the sparse model, we are more interested in the signals $x_0$ whose analysis representation $\hat{\Omega}x_0$ contains many zeros. We call such signals cosparse and the quantity $\ell := p - \|\hat{\Omega}x_0\|_0$ the cosparsity.

II. UNIQUENESS

Based on the existing work [1], [3], we establish [4] the uniqueness of cosparse signals in the context of linear inverse problems above. The result we have derived has simple forms for two particular classes of analysis operators: $\Omega$ that is in general position, which means that the rows of $\Omega$ have no non-trivial linear dependencies, and the popular 2D TV analysis operators $\Omega$ that consists of all of the vertical and horizontal one-step differences in a 2D image. For these two types of $\Omega$, we have:

The authors acknowledge the support by the European Community’s FP7-FET program, SMALL project, under grant agreement no. 225913.

REFERENCES