



HAL
open science

Model Predictive Control of Nonholonomic Vehicle Formations

Fernando A.C.C. Fontes

► **To cite this version:**

Fernando A.C.C. Fontes. Model Predictive Control of Nonholonomic Vehicle Formations. SADCO Kick off, Mar 2011, Paris, France. inria-00585716

HAL Id: inria-00585716

<https://inria.hal.science/inria-00585716>

Submitted on 13 Apr 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Model Predictive Control of Nonholonomic Vehicle Formations

Fernando A. C. C. Fontes

Universidade do Porto
faf@fe.up.pt

ITN-SADCO 1st Industrial workshop and kick-off meeting
Paris, March 3, 2011

- 1 The Problem
 - Challenges
 - Control of Nonholonomic Systems
- 2 The MPC framework
- 3 Control of vehicle formations
- 4 Path-following
- 5 Mission objective

Outline

- 1 **The Problem**
 - Challenges
 - Control of Nonholonomic Systems
- 2 The MPC framework
- 3 Control of vehicle formations
- 4 Path-following
- 5 Mission objective

The Problem

To control a formation of nonholonomic vehicles

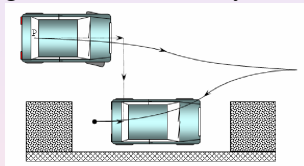
Objective: devise trajectories, and corresponding controls for all vehicles, maintaining some (pre-specified) relative positions between them.



Nonholonomic systems At a given instant cannot move in certain directions (although they can reach any point in the state space).
Example: wheeled vehicles, airplanes, robot manipulators, ...

Which are the main challenges of this problem?

nonholonomy simple controllers (with continuous feedback) cannot, in general, stabilize the system.



constraints input saturation, collision avoidance (between vehicles and with obstacles).

distributed control stability of the formation, communication delays or failure, computational burden in each agent.

Why is MPC a good tool for this problem?

- **MPC can overcome nonholonomy challenges**
 - It involves *planning*, not just reactive control.
 - Can generate required nonlinear, discontinuous feedback.
- **MPC can overcome constraints challenges**
 - It is known to be a (if not the main) technique to deal appropriately with constraints.
 - Simply dealt with within the optimization
- **Appropriate for distributed control**
 - Computational burden can be distributed among agents, while kept at feasible levels
 - Overall stability can be guaranteed
 - In case of communication delays or failures, we have the computed open-loop control as backup plan

Moreover

- It has desirable performance characteristics (optimization-based method)
- It has intrinsic robustness

Nonholonomic Systems

Are completely controllable but instantaneously cannot move in certain directions (Ex: wheeled vehicles, robot manipulators, rolling sphere, ...)

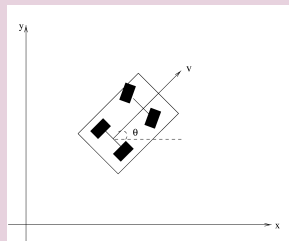
Example: A car-like vehicle

$$\begin{cases} \dot{x} = v \cdot \cos \theta \\ \dot{y} = v \cdot \sin \theta \\ \dot{\theta} = v \cdot c \end{cases}$$

with control inputs v and c satisfying

$$v \in [0, v_{max}] \text{ and } c \in [-c_{max}, c_{max}].$$

(Minimum turning radius $R_{min} = c_{max}^{-1}$)



- Nonholonomic constraint: velocity vector is orthogonal to the wheel axle
 $(\dot{x}(t), \dot{y}(t))^T (\sin \theta(t), -\cos \theta(t)) = 0$
- Linearizing around any point $(x_0, y_0, \theta_0, v = 0)$ results in an uncontrollable system.
- Cannot be stabilized by a continuous (time-invariant) feedback.

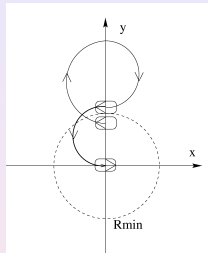
Cannot be stabilized by a continuous feedback.

Suppose $x = 0$ and $\theta = \pi$

If $y \geq 2R_{min} \Rightarrow$ turn left.

If $y < 2R_{min} \Rightarrow$ turn right.

which is a discontinuous feedback.



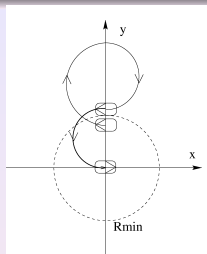
Cannot be stabilized by a continuous feedback.

Suppose $x = 0$ and $\theta = \pi$

If $y \geq 2R_{min} \Rightarrow$ turn left.

If $y < 2R_{min} \Rightarrow$ turn right.

which is a discontinuous feedback.



More precisely ...

Theorem (Brockett 83)

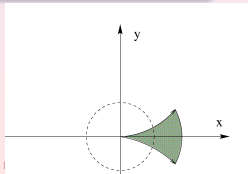
If the system $\dot{x} = f(x, u)$, with $u \in U$, admits a continuous stabilizing feedback, then for any neighbourhood \mathcal{X} of zero, the set $f(\mathcal{X}, U)$ contains a neighbourhood of zero.

In the car example, if

$\mathcal{X} := \{(x, y, \theta) : |x| < 1, |y| < 1, |\theta| < \pi/6\}$

then

$f(\mathcal{X}, U)$ has no points of the form $(\dot{x}, \dot{y}, \dot{\theta})$ with $\dot{x} < 0$.



Discontinuous Feedback: Difficulties

- The trajectory x , solution to

$$\dot{x}(t) = f(x(t), k(x(t))), \quad x(0) = x_0$$

is only defined, in classical (Caratheodory) sense, if (among other requirements) if the right-hand side is a locally Lipschitz continuous function of x .

- Using Filippov solution-concept some controllable systems cannot be stabilized. (Ryan 94, Coron&Rosier 94).
- Problem solved if we use “sampling-feedback” solution concept (Clarke,Ledyaev,Sontag,Subbotin 97)

Discontinuous Feedback: “sampling-feedback” solution

(Clarke, Ledyev, Sontag, Subbotin IEEE TAC97)

- Take a sequence of sampling instants $\pi := \{t_i\}_{i \geq 0}$ in $[0, +\infty)$ with $t_0 < t_1 < t_2 < \dots$

$$\dot{x}(t) = f(x(t), k(x(\lfloor t \rfloor_\pi))), \quad x(0) = x_0$$

where $\lfloor t \rfloor_\pi := \max_i \{t_i \in \pi : t_i \leq t\}$

i.e. The feedback is not a function of the state at every instant of time, rather it is a function of the state at the last sampling instant.

This solution concept combines naturally with our MPC sampled-data framework.

Outline

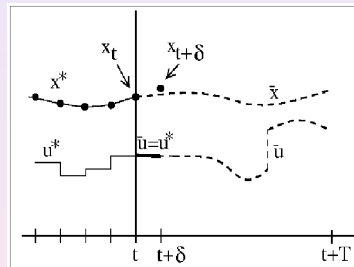
- 1 The Problem
 - Challenges
 - Control of Nonholonomic Systems
- 2 The MPC framework
- 3 Control of vehicle formations
- 4 Path-following
- 5 Mission objective

Our MPC framework

Consider a sequence $\{t_i\}_{i \geq 0}$ s.t. $t_{i+1} = t_i + \delta$, $\delta > 0$:

1. Measure state of the plant x_{t_i}
2. Get $\bar{u} : [t_i, t_i + T] \mapsto \mathbb{R}^m$ solution to the OCP:

$$\begin{aligned} &\text{Minimize} && \int_{t_i}^{t_i+T} L(t, x(t), u(t)) dt + W(x(t_i + T)) \\ &\text{subject to} && \dot{x}(t) = f(t, x(t), u(t)) \quad \text{a.e. } t \in [t_i, t_i + T] \\ & && x(t_i) = x_{t_i} \\ & && u(t) \in U(t) \quad \text{a.e. } t \in [t_i, t_i + T] \\ & && x(t) \in X \quad \text{a.e. } t \in [t_i, t_i + T] \\ & && x(t_i + T) \in S \end{aligned}$$



$\{t_i\}$ Sequence of sampling instants

δ Sampling interval

x_{t_i} Plant state measured at time t_i .

(\bar{x}, \bar{u}) OL state/control solving the OCP

(x^*, u^*) CL state/control of the MPC

T, L, W, S Design parameters

3. Apply to the plant the control $u^*(t) := \bar{u}(t)$ in the interval $[t_i, t_i + \delta]$. (the remaining control $\bar{u}(t)$, $t > t_i + \delta$ is discarded)
4. Repeat for the next sampling time $t_i = t_i + \delta$

A Sufficient Condition for Stability

Choose the design parameters: time horizon T , objective functions L and W , and terminal constraint set S , to satisfy:

SC1–SC4 ... S is closed, contains the origin is reachable in time T ... W is locally Lipschitz continuous and positive definite ...

SC5 $\forall x_t \in S, \exists \tilde{u} : [t, t + \delta] \rightarrow U$ [or $\tilde{u}_t \in U$] such that

$$W(x(t + \delta; t, x_t, \tilde{u})) - W(x_t) \leq - \int_t^{t+\delta} L(x(s), \tilde{u}(s)) ds, \quad (\text{SC5a})$$

$$\text{[or } \langle \xi, f(x_t, \tilde{u}_t) \rangle \leq -L(x_t, \tilde{u}_t), \quad \forall \xi \in \partial_P W(x_t)]$$

and

$$x(s; t, x_t, \tilde{u}) \in S, \quad s \in [t, t + \delta]. \quad (\text{SC5b})$$

$$\text{[or } \langle \xi, f(x_t, \tilde{u}_t) \rangle \leq 0, \quad \forall \xi \in N_S^P(x_t)]$$

Then, the MPC strategy is stabilizing (i.e. for a sufficiently small inter-sample time δ we have $\|x^*(t)\| \rightarrow 0$ as $t \rightarrow +\infty$).

Consider the system

$$\begin{cases} \dot{x}(t) = (u_1(t) + u_2(t)) \cdot \cos \theta(t) \\ \dot{y}(t) = (u_1(t) + u_2(t)) \cdot \sin \theta(t) \\ \dot{\theta} = (u_1(t) - u_2(t)). \end{cases}$$

with $\theta(t) \in [-\pi, \pi]$, and the controls
 $u_1, u_2(t) \in [-1, 1]$.

Stability achieved choosing:

S to be the set of points s.t. the heading angle θ
 is pointing towards the origin of the plane $(x, y) = (0, 0)$.

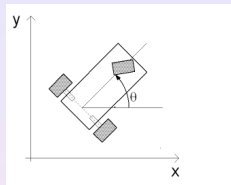
$$T = \pi/2.$$

$$L(x, y, \theta) := x^2 + y^2 + \theta^2,$$

$$W(x_0, y_0, \theta_0) := \int_0^{\bar{t}} L(x(t), y(t), \theta(t)) dt$$

where \bar{t} is the time to reach the origin.

SC is satisfied. MPC is stable.



Example: A car-like vehicle

Consider the system

$$\begin{cases} \dot{x} = v \cdot \cos \theta \\ \dot{y} = v \cdot \sin \theta \\ \dot{\theta} = v \cdot c \end{cases}$$

with control inputs v and c satisfying

$$v \in [0, v_{max}] \text{ and } c \in [-c_{max}, c_{max}].$$

Stability achieved choosing

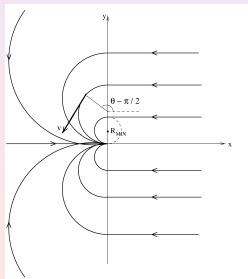
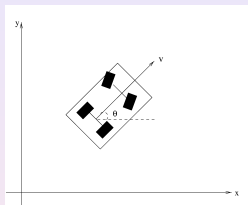
S as in picture

$$T = 2\pi R_{min} / v_{max}.$$

$$L(x, y, \theta) := x^2 + y^2 + \theta^2,$$

$$W(x_0, y_0, \theta_0) := \int_0^{\bar{t}} L(x(t), y(t), \theta(t)) dt$$

where \bar{t} is the time to reach the origin.

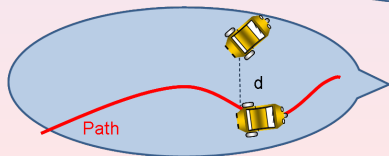
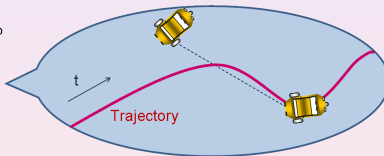


Problem Objective

What do we mean by “to control”?

- point-to-point motion (what we've seen so far)
- trajectory tracking (much easier)
- path-following (one more degree of freedom)
- accomplish mission objective (why not explore further degrees of freedom?)

Trajectory tracking: The aim is to follow, as close as possible, a trajectory in state space (i.e., a geometric path in the cartesian space together with an associated timing law) starting from a given initial configuration .

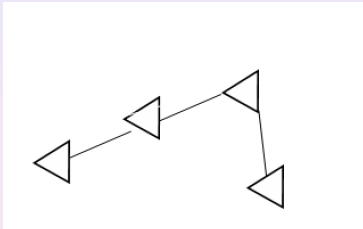


Path following: The robot must reach and follow a geometric path in the cartesian space starting from a given initial configuration, but no associated timing law is specified

Outline

- 1 The Problem
 - Challenges
 - Control of Nonholonomic Systems
- 2 The MPC framework
- 3 Control of vehicle formations
- 4 Path-following
- 5 Mission objective

Control of formations



Method: A two-layer model predictive control (MPC) scheme

- The top layer controls the trajectory of the whole group of vehicles.
Based on Nonlinear MPC allowing discontinuous feedbacks.
- The bottom layer controls the relative position between vehicles.
Based on MPC with a linearized model (solves constrained LQR).

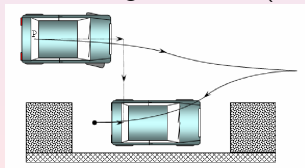
Why two layers?

Because there are two intrinsically different control problems

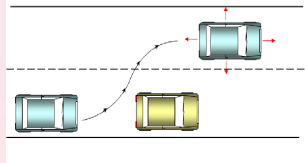
- The nonholonomic vehicles require a nonlinear controller allowing discontinuous feedbacks (point-to-point motion)
- However, while in motion, the relative position between the vehicles in a formation can be changed in all directions (as if they were holonomic). Can be controlled by a linear controller. (trajectory tracking)

In some robotics literature called leader-follower formation control.

Example: Consider a car in a parking manoeuvre (nonholonomic), or in a overtaking manoeuvre (can move in all directions relative to the other car)



Parking manoeuvre



Overtaking manoeuvre

The two layer MPC scheme

- The top layer, **the trajectory controller**, applied to the group of vehicles as a whole.
 - Is a nonlinear MPC controller allowing discontinuous feedbacks. The control is computed centrally and a unique control is broadcasted to all vehicles.
- The bottom layer, **the formation controller**, applied to each vehicle individually.
 - Aims to compensate for small changes around a nominal trajectory maintaining the relative positions between vehicles
 - can be adequately carried out by a linear controller accommodating input and state constraints
 - This has the advantage that the control laws for each vehicle are much simpler to compute (are piecewise affine feedback laws and can even be pre-computed off-line [Bemporad, Morari, Dua, Pistikopoulos 2002]) They are computed and implemented in a distributed way in each vehicle.

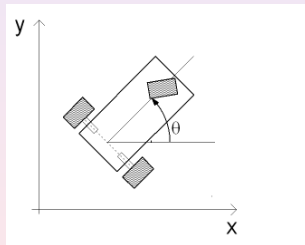
In the end the two controls are added and applied to the actuators in each vehicle.

Nonholonomic vehicle model for the Trajectory controller

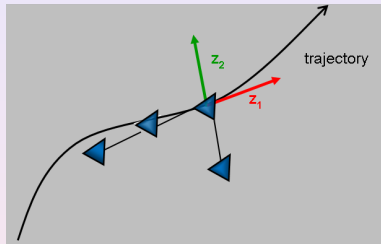
Our vehicle: A differential-drive mobile robot

$$\begin{cases} \dot{x}(t) = (u_1(t) + u_2(t)) \cdot \cos \theta(t) \\ \dot{y}(t) = (u_1(t) + u_2(t)) \cdot \sin \theta(t) \\ \dot{\theta} = (u_1(t) - u_2(t)). \end{cases}$$

with constrained controls $u_1, u_2(t) \in [-1, 1]$.



Linearized vehicle model for the Formation Controller



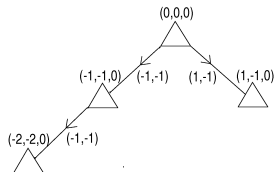
We consider a linearized model for each of the differential drive mobile robots:

- with the z_1 axis aligned with the velocity of the *reference vehicle*
- for the nominal velocity $v_n = (u_{1n} + u_{2n})/2$, with $u_{1n} = u_{2n}$ being the nominal velocities of the wheels.

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & w_n & 0 \\ -w_n & 0 & v_n \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix}$$

Formation Model

- Consider M vehicles and a *reference vehicle, leader* (that might not exist) that follows exactly the trajectory predicted by the trajectory controller.
- Consider that the $M + 1$ vehicles are nodes of a directed graph $G = (V, E)$, that is a directed tree rooted at the reference vehicle. That is, all vertices are connected, there are no cycles, and all edges are directed in a path away from the root.
- To each vertex i associate a triple $z^i = (z_1^i, z_2^i, z_3^i)$ with the relative position w.r.t. the reference vehicle.
- To each edge $(i, j) \in E$ associate the pair $\tilde{z}^{ij} = (\tilde{z}_1^{ij}, \tilde{z}_2^{ij})$ with the desired relative position in the plane of vehicle i with respect to its **parent** node j .



The performance index for each vehicle i with parent j is

$$J^i(z) = \int_{t=0}^T \left(\|(\tilde{z}_1^{ij}, \tilde{z}_2^{ij}, 0) - (z^j(t) - z^i(t))\|_Q^2 + \|(v(t), w(t))\|_R^2 \right) dt + \|(\tilde{z}_1^{ij}, \tilde{z}_2^{ij}, 0) - (z^j(N) - z^i(N))\|_P^2$$

Formation Controller (cont.)

Stability of the formation

- If we choose the design parameter L, W, T , and S for the trajectory controller as seen previously, and
- if we choose the matrices Q and R as p.d. and P solving the Riccati equation $A'P + PA - PBR^{-1}B'P + Q = 0$, in the objective function of the formation controller.

Then,

- The trajectory of the reference vehicle is stable.
- The trajectory of each vehicle is stable with respect to the desired relative position to its parent vehicle.
- Since there is exactly one path from each vehicle to the reference vehicle, stability of any vehicle follows recursively

Outline

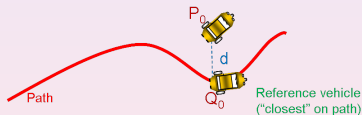
- 1 The Problem
 - Challenges
 - Control of Nonholonomic Systems
- 2 The MPC framework
- 3 Control of vehicle formations
- 4 Path-following
- 5 Mission objective

Path-following

Given a path parameterized by $\tau \mapsto (x_d(\tau), y_d(\tau))$ for $\tau \in [0, T_d]$:

- 1 Find an initial point Q_0 in the path to start the trajectory of the virtual reference vehicle. Given the initial (current) position of our vehicle (x_0, y_0) , find the path parameter value that corresponds to a point in the path that is the closest to (x_0, y_0) .

$$\tau_0 = \operatorname{argmin}_{\tau \in [0, T_d]} \|(x_d(\tau), y_d(\tau)) - (x_0, y_0)\|$$



- 2 Select a speed profile at which the path is to be followed

$$\nu : [0, T] \mapsto \mathbb{R}_+$$

- 3 Find a feedback control to track the trajectory of the virtual reference vehicle $t \mapsto (x_r(t), y_r(t))$

Path-following with MPC

Minimize over $\nu \in \mathbb{R}_+$ and u

$$\int_0^{T_d/\nu} (\|q(t) - q_d(\tau(t))\|_Q^2 + \|u(t)\|_R^2) dt + \|q(T_d/\nu)\|_P^2, \quad (1)$$

subject to:

$$\begin{aligned} \dot{q}(t) &= f(t, q(t), u(t)) && a.e. t \in [0, T_d/\nu] \\ q(0) &= q_{t_i} \\ \tau_0 &= \operatorname{argmin}_{\tau \in [0, T_d]} \|q_d(\tau) - q_{t_i}\| && (2) \\ \tau(t) &= \tau_0 + \nu t \\ u(t) &\in U && \forall t \in [0, T_d/\nu]. \end{aligned}$$

where the path is parameterized by

$$t \mapsto q_d(\tau), \quad \tau \in [0, T_d]$$

Path-following example

Consider a differential-drive robot to follow a eight-shaped path

$$\begin{aligned}x_d(\tau) &= \sin(\tau/10), & \tau \in [0, 38\pi] \\y_d(\tau) &= \sin(\tau/20), & \tau \in [0, 38\pi]\end{aligned}$$

By selecting ν and an initial point in the path we define the trajectory for a virtual reference vehicle

$$\begin{aligned}x_r(t) &= \sin(\tau(t)/10) \\y_r(t) &= \sin(\tau(t)/20) \\ \tau(t) &= \tau_0 + \int_0^t \nu(s) ds.\end{aligned}$$

and by inverse kinematics we obtain the control of the reference vehicle

$$\theta_r(t) = \text{ATAN2}(\dot{y}_r, \dot{x}_r)$$

$$v_r(t) = \pm \sqrt{\dot{x}_d^2(t) + \dot{y}_d^2(t)}$$

$$w_r(t) = \frac{\ddot{y}_d(t)\dot{x}_d(t) - \ddot{x}_d(t)\dot{y}_d(t)}{\dot{y}_d^2(t) + \dot{x}_d^2(t)}, \quad \text{a.e.}$$

Path-following example

Apply the MPC with a linearized model:

Minimize

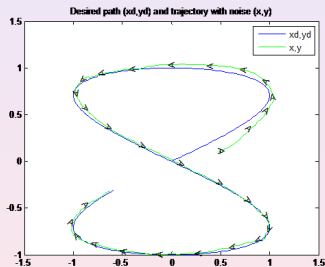
$$\int_0^{T_d/\nu} (\|q(t) - q_d(\tau(t))\|_Q^2 + \|u(t)\|_R^2) dt + \|q(T_d/\nu)\|_P^2, \quad (3)$$

subject to:

$$\begin{aligned} \dot{e}(t) &= Ae(t) + Bu(t) & a.e. t \in [0, T_d/\nu] \\ e(0) &= \mathcal{R}_\theta(q_d(\tau_0) - q(t_i)) \\ \tau_0 &= \operatorname{argmin}_{\tau \in [0, T_d]} \|q_d(\tau) - q_{t_i}\| \\ \tau(t) &= \tau_0 + \nu t & \forall t \in [0, T_d/\nu] \\ |u_1(t)| &\leq U_{max} & \forall t \in [0, T_d/\nu] \\ |u_2(t)| &\leq U_{max} & \forall t \in [0, T_d/\nu]. \end{aligned} \quad (4)$$

where Q and R are chosen to be p.d., and P solves the Riccati equation

$A'P + PA - PBR^{-1}B'P + Q = 0$, guaranteeing stability.

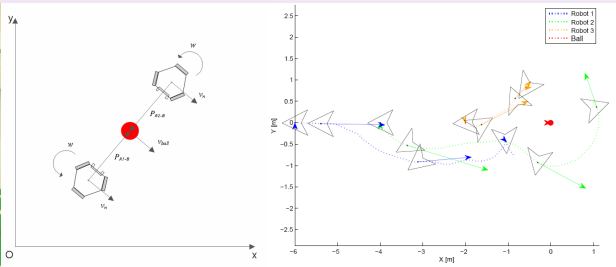


Outline

- 1 The Problem
 - Challenges
 - Control of Nonholonomic Systems
- 2 The MPC framework
- 3 Control of vehicle formations
- 4 Path-following
- 5 Mission objective

Current and Future Research

Mission Objective: Minimize the estimation error of a target (ball) position and velocity.





Fontes, F.A.C.C.:

A general framework to design stabilizing nonlinear model predictive controllers.
Systems & Control Letters **42** (2001) 127–143



Fontes, F.A.C.C.:

Discontinuous feedbacks, discontinuous optimal controls, and continuous-time model predictive control.

International Journal of Robust and Nonlinear Control **13**(3–4) (2003) 191–209



F. A. C. C. Fontes and L. Magni.

Min-max model predictive control of nonlinear systems using discontinuous feedbacks.

IEEE Transactions on Automatic Control, 48:1750–1755, 2003.



Fontes, F.A.C.C., Magni, L., Gyurkovics, E.:

Sampled-data model predictive control for nonlinear time-varying systems: Stability and robustness.

In *Assessment and Future Directions of NMPC*. LNCIS 358, Springer 2007



F. A. C. C. Fontes, D. B. M. M. Fontes and A. C. D. Caldeira.

Model Predictive Control of Vehicle Formations

In *Optimization and Cooperative Control Strategies*, LNCIS 381, Springer 2009.



A. C. D. Caldeira, F. A. C. C. Fontes.

Model Predictive Control for Path-following of Nonholonomic Systems

In *Proceedings of Control 2010*, 2010.