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*What Topology tells us about Diagnosability in  
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## What Topology tells us about Diagnosability in Partial Order Semantics

Stefan Haar\*

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**Abstract:** From a partial observation of the behaviour of a labeled Discrete Event System, *fault diagnosis* strives to determine whether or not a given “invisible” fault event has occurred. The *diagnosability problem* can be stated as follows: does the labeling allow for an outside observer to determine the occurrence of the fault, no later than a bounded number of events after that unobservable occurrence? When this problem is investigated in the context of concurrent systems, partial order semantics adds to the difficulty of the problem, but also provides a richer and more complex picture of observation and diagnosis. In particular, it is crucial to clarify the intuitive notion of “*time after fault occurrence*”. To this end, we will use a unifying metric framework for event structures, providing a general topological description of diagnosability in both sequential and nonsequential semantics for Petri nets.

**Key-words:** Discrete event systems, diagnosis, Petri nets, events, observability, partial order semantics, Event structures.

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\* INRIA and LSV (CNRS and ENS Cachan), 61, avenue du Président Wilson, 94235 CACHAN Cedex, France (e-mail:haar@lsv.ens-cachan.fr,stefan.haar@inria.fr).

## What Topology tells us about Diagnosability in Partial Order Semantics

**Résumé :** Description topologique de diagnosticabilité dans des sémantiques séquentielles et non-séquentielles des Réseaux de Petri.

**Mots-clés :** Systèmes à événements discrets, diagnostiques, Réseau de Petri, observabilité, sémantique d'ordre partiel, structures d'événements.

## 1 Introduction

*Diagnosis under partial observation* is a classical problem in automatic control in general, and has received considerable attention in *discret event system (DES)* theory, among other fields. In the DES setting, the approach that we will call “classical” here supposes that the observed system is an automaton with transition set  $T$ , (behavioural) language  $\mathcal{L} \subseteq T^*$ , and a set of *observable transition labels*  $\mathbb{O}$ . The associated labeling map, let us call it  $\eta : T \rightarrow \mathbb{O}$  in line with the formalism used below, may not be required injective, and leaves some transitions from  $T$  unobservable, in particular *fault*  $\phi$ . The observations have the form of words  $w \in \mathbb{O}^*$  obtained by extending  $\eta$  into a homomorphism  $T^* \rightarrow \mathbb{O}^*$ . A classical definition of diagnosability is given in [CL99], following [SSL<sup>+</sup>95]; writing  $s \sim_\eta s'$  iff  $s, s' \in T^*$  are mapped to the same observable word in  $\mathbb{O}^*$ , we can state it as follows:

$\mathcal{L}$  is *non-diagnosable* iff there exist sequences  $s_N, s_Y \in \mathcal{L}$  such that:

1.  $s_Y$  is faulty,  $s_N$  is healthy, and  $s_N \sim_\eta s_Y$ ;
2. moreover,  $s_Y$  with the above is arbitrarily long after the first fault, i. e. for every  $k \in \mathbb{N}$  there exists a choice of  $s_N, s_Y \in \mathcal{L}$  with the above properties and such that the suffix  $s_{Y/\phi}$  of  $s_Y$  after the first occurrence of fault  $\phi$  in  $s_Y$  satisfies  $|s_{Y/\phi}| \geq k$ .

*Concurrent systems* are difficult to supervise using the classical approach because of the state explosion problem. Moreover, consider intrinsically asynchronous distributed systems, such as encountered in telecommunications or more generally in networked systems. Here, the use of models that reflect the local and distributed nature of the observed system, such as Petri nets or graph grammars, is helpful not only in terms of computational efficiency, but also *conceptually*. Putting these ideas together, we were led in [BFHJ03] to carry over diagnosis to asynchronous models *and their non-interleaved semantics*; see also the discussion of the necessity for using partial order methods in [FB07]. This generalized methodology for fault diagnosis is based on the non-sequential executions of labeled Petri nets, that is, the partial order semantics in occurrence nets and event structures. The approach was extended to graph transformation systems for modelling dynamically evolving system topologies in [BCHK10]. We have provided a series of results [HBFJ03, Haa07, Haa09, Haa10] on *partial order diagnosability* for Petri nets, in the spirit of the above definition. While the sequential case is embedded and generalized in these results, new features emerge in partial ordered runs that have no counterpart in sequential behaviour; this led to the distinction between *strong* and *weak* observability and diagnosability properties in [HBFJ03, Haa10].

Bauer and Pinchinat [BP08] have given a topological view on diagnosability in terms of sequential languages. The present work develops a framework that includes both sequential and partial order semantics, retrieving and generalizing as a special case the results of [BP08] and showing connections between weak and strong properties. The key construction is that of suitable metrics on event structures. For this, we generalize a standard construction to be found in [BMP90, Kwi90] and others, in such a way that progress and observation properties can be captured in the resulting topology. Event structures provide a unifying semantical model both for the sequential and non-sequential viewpoints.

That is, both sequential languages as in [CL99, BP08] AND the partial order semantics given in [Eng91, NPW81] and used in [FBHJ05, Haa10], associate event structures to a system; and the metric topology given here coincides, on the sequential semantics, with the Cantor topology used in [BP08]. With these tools, the properties of weak and strong diagnosability from [HBFJ03, Haa10] become different instances of a general property, *eventual diagnosability*, for general labeled event structures. The difference between the weak and strong properties lies thus in the choice of *semantics* that produces the event structure model of behaviour for the system that is investigated.

**Structure of the paper:** We begin in Section 2. with the basic definitions for (labeled) event structures. The following Section 3. investigates partial observation and diagnosability, and develops the main general results of this paper. Section 4 specializes to safe Petri nets, and studies properties characterizing weakly diagnosable nets. We then conclude in Section 5.

## 2 Event Structures

Let  $A$  be a set.  $A^* \triangleq \{a_1 \dots a_n \mid a_i \in A\}$  is the set of all finite words over  $A$ ; the set of *infinite* words over  $A$  is denoted  $A^\omega$ . Let  $\mathbf{1}_A$  be the indicator function of  $A$ , i.e.  $\mathbf{1}_A(x) = 1$  iff  $x \in A$  and  $\mathbf{1}_A(x) = 0$  for  $x \notin A$ . Let  $f : A \rightarrow B$  be a partial function. Write  $f(a) \downarrow$  if  $f$  is defined on  $a \in A$ , and  $f(a) \uparrow$  otherwise. The *domain* of  $f$  is  $\text{dom}(f) \triangleq \{a \in A \mid f(a) \downarrow\}$ , and the *image* of  $f$  is

$$f(A) \triangleq \{b \in B \mid \exists a \in \text{dom}(f) : f(a) \downarrow \wedge f(a) = b\}.$$

We shall be using throughout this paper *prime event structures (PES)* following Winskel et al [NPW81, Win], with particular attention to labeling. Fix some alphabet  $\mathbb{A} \neq \emptyset$ .

**Definition 1**  $A$  (labeled) prime event structure (over alphabet  $\mathbb{A}$ ) is a tuple  $\mathcal{E} = (E, \leq, \#, \lambda)$ , where

1.  $E = \text{supp}(\mathcal{E})$  is the support, or set of events of  $\mathcal{E}$ ,
2.  $\leq \subseteq E \times E$  is a partial order satisfying the property of finite causes, i.e. setting  $[e] \triangleq \{e' \in E \mid e' \leq e\}$ , one has

$$\forall e \in E : |[e]| < \infty, \tag{1}$$

3.  $\# \subseteq E \times E$  an irreflexive symmetric conflict relation satisfying the property of conflict heredity, i.e.

$$\forall e, e', e'' \in E : e \# e' \wedge e' \leq e'' \Rightarrow e \# e'', \tag{2}$$

4.  $\lambda : E \rightarrow \mathbb{A}$  is a total mapping called the labelling. Events  $e, e' \in E$  are concurrent, written  $e \mathbf{co} e'$ , iff neither  $e = e'$  nor  $e \leq e'$  or  $e' \leq e$  nor  $e \# e'$  hold. If  $\mathbf{co} = \perp$ , i.e. if  $\mathbf{co}$  is the empty relation, we call  $\mathcal{E}$  sequential. An  $\mathbb{A}$ -labeled event structure is called *simple*<sup>1</sup> iff no label can occur concurrently on two different events; that is, iff

$$e \mathbf{co} e' \Rightarrow \lambda(e) \neq \lambda(e'). \tag{3}$$

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<sup>1</sup>one might call it *safe* or *auto-concurrency free*

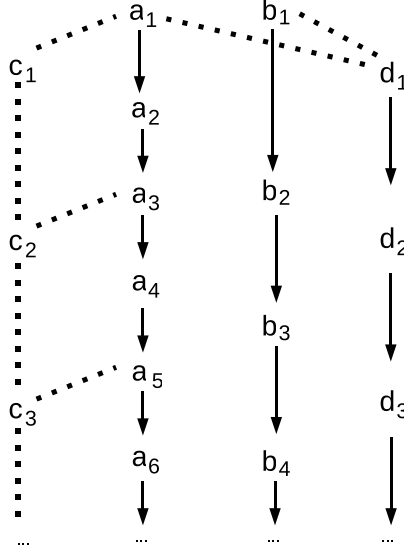


Figure 1: The simple event structure of Example 1. Arrows represent causal precedence  $\leq$ , and dashed lines stand for conflict  $\#$ ; only minimal relations are represented, all others are generated by transitivity and inheritance.

A simple labeled event structure will be called an SES.

Let  $\mathcal{E}_1 = (E_1, \leq_1, \#_1, \lambda_1)$  and  $\mathcal{E}_2 = (E_2, \leq_2, \#_2, \lambda_2)$  be two  $\mathbb{A}$ -labeled event structures. If (i)  $E_1 \subseteq E_2$  and (ii) for all  $e, e' \in E_1$ ,

$$e \#_1 e' \Leftrightarrow e \#_2 e' \quad \text{and} \quad e \leq_1 e' \Leftrightarrow e \leq_2 e',$$

then  $\mathcal{E}_1$  is a sub-event structure of  $\mathcal{E}_2$ .

**Example 1.** Let

$$\begin{aligned} E &\triangleq \{a_i, b_i, c_i, d_i \mid i \in \mathbb{N}\} \\ \mathbb{A} &\triangleq \{a, a^*, b, b^*, c, c^*, d, d^*\} \end{aligned}$$

and for all  $i \in \mathbb{N}$ ,

$$\begin{aligned} \lambda_p(a, 2i) &= a \quad \wedge \quad \lambda_p(a, 2i+1) = a^* \\ \lambda_p(b, 2i) &= a \quad \wedge \quad \lambda_p(b, 2i+1) = b^* \\ \lambda_p(c, 2i) &= a \quad \wedge \quad \lambda_p(c, 2i+1) = c^* \\ \lambda_p(d, 2i) &= a \quad \wedge \quad \lambda_p(d, 2i+1) = d^*. \end{aligned}$$

Define sets  $A \triangleq \lambda_p^{-1}(\{a\})$ ,  $A^* \triangleq \lambda_p^{-1}(\{a^*\})$ ,  $\overline{A} \triangleq A \cup A^*$  and analogously  $B, B^*, \overline{B}, C, C^*, \overline{C}, D, D^*, \overline{D}$ . Let



1. for  $i < j$ ,  $a_i < a_j$ ,  $b_i < b_j$  and  $d_i < d_j$ , but  $c_i \# c_j$ ,
2.  $a_{2i} \# c_i$ ,  $a_i \# d_j$  and  $b_i \# d_j$  for any  $i, j \in \mathbb{N}$ ;

an illustration is given by Figure 1. One easily checks that  $\mathcal{E} = (E, \leq, \#, \lambda)$  thus defined is an SES.

**Prefixes and Configurations.** The *set of causes* or *prime configuration* of  $e \in E$  is  $[e] \triangleq \{e' \mid e' \leq e\}$ , as defined above. A *prefix* of  $\mathcal{E}$  is any downward closed subset  $D \subseteq E$ , i.e. such that for every  $e \in D$ ,  $[e] \subseteq D$ . Prefixes of  $\mathcal{E}$  induce, in the obvious way, sub-event structures of  $\mathcal{E}$  in the sense of the above definition. Denote the set of  $\mathcal{E}$ 's prefixes as  $\mathcal{D}(\mathcal{E})$ . Prefix  $\mathbf{c} \in \mathcal{D}(\mathcal{E})$  is a *configuration* if and only if it is conflict-free, i.e. if  $e \in \mathbf{c}$  and  $e \# e'$  imply  $e' \notin \mathbf{c}$ . Denote as  $\mathcal{C}(\mathcal{E})$  the set of  $\mathcal{E}$ 's configurations. Call any  $\subseteq$ -maximal element of  $\mathcal{C}(\mathcal{E})$  a *run* of  $\mathcal{E}$ ; denote the set of  $\mathcal{E}$ 's runs as  $\Omega(\mathcal{E})$ , or simply  $\Omega$  if no confusion can arise.

In the context of Example 1, one checks that, e.g.,  $[c_i] \cup [b_j]$  and  $[a_i] \cup [b_j]$  are some of the configurations for all  $i, j \in \mathbb{N}$ ; the runs are  $\omega_{AB} \triangleq A \cup B$ ,  $\omega_{c_i B} \triangleq [c_i] \cup B$  for  $i \in \mathbb{N}$ , and  $\omega_D \triangleq D$ .

## 2.1 Labeled event structure morphisms

The modeling of observation projection leads us to introduce a dedicated class of morphisms for labeled event structures, which specializes Winskel's morphisms for event structures (see [Win, BCM01]):

**Definition 2** Let  $\mathcal{E}_1 = (E_1, \leq_1, \#_1, \lambda_1)$  and  $\mathcal{E}_2 = (E_2, \leq_2, \#_2, \lambda_2)$  be two prime event structures. A partial mapping  $f : E_1 \rightarrow E_2$  is a morphism iff for all  $e_1 \in \text{dom}(f)$ ,

1.  $[f(e_1)] \subseteq f([e_1])$ ,
2. and for all  $e'_1 \in \text{dom}(f)$ ,
  - (a)  $f(e_1) \#_2 f(e'_1)$  implies  $e_1 \#_1 e'_1$ , and
  - (b)  $f(e_1) = f(e'_1)$  and  $e_1 \neq e'_1$  together imply that  $e_1 \#_1 e'_1$ .

A morphism  $f : E_1 \rightarrow E_2$  is called an  $(\mathbb{A}-)$  morphism iff, in addition,

1.  $\text{dom}(\lambda_1) \subseteq \text{dom}(f)$  and  $\text{dom}(f) \subseteq \text{dom}(\lambda_2)$ ,
2.  $\forall e \in E_1 : \lambda_1(e) = \lambda_2(f(e))$ .

$\mathcal{E}_1$  and  $\mathcal{E}_2$  are  $(\mathbb{A}-)$ isomorphic, written  $\mathcal{E}_1 \sim_{\mathbb{A}} \mathcal{E}_2$ , iff there exist morphisms  $f : E_1 \rightarrow E_2$  and  $f^{-1} : E_2 \rightarrow E_1$  such that for all  $e_1 \in \text{dom}(f)$  and all  $e_2 \in \text{dom}(f^{-1})$ ,

$$f^{-1}(f(e_1)) = e_1 \quad \text{and} \quad f(f^{-1}(e_2)) = e_2.$$

Note that Abbes [Abb06] defines a different class of morphisms: *full* mapping  $f : E_1 \rightarrow E_2$  is a morphism iff it is order-preserving between the underlying posets and if moreover  $f$  reflects conflict. This class is less appropriate than the above for our purposes since it does not allow for fusion of observationally equivalent conflicting configurations, nor for unobservable events.

Write  $D_1 \sqsubseteq_{\mathbb{A}} D_2$  iff  $D_1$  is  $\mathbb{A}$ -isomorphic to a prefix of  $D_2$ . For  $\mathbf{c}_1, \mathbf{c}_2 \in \mathcal{C}(\mathcal{E})$ , let

$$[[\mathbf{c}_1]]_{\mathbb{A}} \sqcap [[\mathbf{c}_2]]_{\mathbb{A}} \triangleq [[\mathbf{c}_3]]_{\mathbb{A}},$$

where  $\mathbf{c}_3$  is the unique  $\sqsubseteq$ -maximal prefix of  $\mathbf{c}_1$  such that  $\mathbf{c}_3 \sqsubseteq_{\mathbb{A}} \mathbf{c}_2$ . This symmetric operation can be seen as the *intersection* of two configurations *up to  $\mathbb{A}$ -isomorphism*.

For a given configuration  $\mathbf{c} \in \mathcal{C}(\mathcal{E})$ , we denote the set of configurations in  $\mathcal{E}$  that are  $\mathbb{A}$ -isomorphic images of  $\mathbf{c}$  as

$$[[\mathbf{c}]] \triangleq \{\mathbf{c}' \in \mathcal{C} \mid \mathbf{c}' \sim_{\mathbb{A}} \mathbf{c}\}.$$

## 2.2 Metrics.

The sets  $\mathcal{C}(\mathcal{E})$  and  $\Omega(\mathcal{E})$  can be equipped with Lawson or Scott topologies, or with natural *metrics*; we will follow and generalize the latter approach, similar to metrizations of traces as studied in [KK03]. Our pseudometrics allow to capture in particular partial observation and fault equivalence. Our principal tool are  $\mu$ -**Heights**: Let  $\mu : \mathbb{A} \rightarrow \mathbb{R}_0^+$  be any total mapping; we shall refer to  $\mu$  as a *weight* function. As a particular case, consider  $\mu(e) \equiv \mathbf{1}_E$ : we will refer this as the *counting weight*. The following construction yields pseudometrics that are equivalent (in topological terms) to the prefix metric [Kwi90] and the Foata normal form metric [BMP90], see [KK03], when the counting weight is chosen; other choices of weights allow to generalize to observation and fault equivalence.

The  $\mu$ -*induced  $*$ -height*  $\mathcal{H}_{\mu}^*(D)$  of a *prefix* is defined recursively by setting, for  $\emptyset$  representing the empty preset,

$$\mathcal{H}_{\mu}^*(\emptyset) \triangleq 0 \tag{4}$$

$$\mathcal{H}_{\mu}^*([e]) \triangleq \mathcal{H}_{\mu}^*([e] \setminus \{e\}) + \mu(e) \tag{5}$$

$$\mathcal{H}_{\mu}^*(D) \triangleq \sup_{e \in D} (\mathcal{H}_{\mu}^*([e])). \tag{6}$$

Now, for  $\tau \in [0, \infty)$  let  $\mathcal{U}_{\tau}^{\mu}$  be the  $\tau$ -*prefix under  $\mu$* , i.e.

$$\mathcal{U}_{\tau}^{\mu} \triangleq \bigcup \{D \in \mathcal{D}(\mathcal{E}) \mid \mathcal{H}_{\mu}^*(D) \leq \tau\}, \tag{7}$$

and let  $\mathcal{E}_{\tau}^{\mu}$  be the prime event structure that  $\mathcal{E}$  induces on  $\mathcal{U}_{\tau}^{\mu}$ . Then define  $\mathcal{H}_{\mu}(\mathbf{c})$  for all  $\mathbf{c} \in \mathcal{C}(\mathcal{E})$  as

$$\mathcal{H}_{\mu}(\mathbf{c}) \triangleq \sup\{\tau \mid \mathbf{c} \in \Omega(\mathcal{E}_{\tau}^{\mu})\}. \tag{8}$$

Note that in general, for any configuration  $\mathbf{c}$ ,

$$\mathcal{H}_{\mu}(\mathbf{c}) \leq \mathcal{H}_{\mu}^*(D); \tag{9}$$

we will call any configuration such that equality holds in (9) *progressive*.

Note that  $\mathcal{H}_{\mu}(\bullet)$  is invariant under  $\mathbb{A}$ -isomorphism. Thus, let  $\Psi_{\mu}(\bullet) : \mathcal{C}(\mathcal{E}) \rightarrow [0, 1]$  and the  $\mu$ -*pseudometric*  $\mathbf{d}_{\mu}(\bullet, \bullet)$  be given by

$$\Psi_{\mu}(\mathbf{c}) \triangleq 2^{-\mathcal{H}_{\mu}(\mathbf{c})} \tag{10}$$

$$\mathbf{d}_{\mu}(\mathbf{c}_1, \mathbf{c}_2) \triangleq \Psi_{\mu}(\mathbf{c}_1 \sqcap \mathbf{c}_2). \tag{11}$$

Again, consider  $\mu(e) \equiv \mathbf{1}_E$ ; denote as  $\mathcal{H}(\bullet)$ ,  $\Psi(\bullet)$  and  $\mathbf{d}(\bullet, \bullet)$  the associated height, conciseness and pre-distance. We observe for this special case:

**Lemma 1** For all  $\mathbf{c} \in \mathcal{C}$ ,

$$\mathcal{H}(\mathbf{c}) = \infty \quad \Rightarrow \quad \mathbf{c} \in \Omega. \quad (12)$$

**Proof:** Assume  $\mathbf{c} \notin \Omega$ , and let  $e \in E \setminus \mathbf{c}$  such that there is no  $e' \in \mathbf{c}$  such that  $e' \# e$ , and let  $n \triangleq \mathcal{H}([e'])$ . Then  $\mathcal{H}(\mathbf{c}) \leq n < \infty$  by definition of  $\mathcal{H}(\bullet)$ .  $\square$

As noted above,  $\mathcal{H}_\mu(\bullet)$  - and thus all the above functions derived from it - are invariant under isomorphisms.

**Example 1 continued.** In the context of example 1, see Figure 1, observe first that  $\overline{A}$  and  $\overline{B}$  are configurations but not maximal. Consider now the counting height. Here - as in any event structure - all sets of the form  $S_{\mathbf{c}} \triangleq \{\omega \in \Omega \mid \mathbf{c}\}$  for  $\mathbf{c} \in \mathcal{C}(\mathcal{E})$  finite, are open sets; the set  $\{\omega_{AB}\}$  coincides e.g. with  $S_{\mathbf{c}_{31}}$ , where  $\mathbf{c}_{31} \triangleq [(a, 3)] \cup [(b, 1)]$ . One obtains that  $\{\omega_{AB}\}$ ,  $\{\omega_D\}$  and all  $\{\omega_{c_i B}\}$  are open; so are of course their unions and intersections. In particular,  $S_B = \{\omega_{AB}, \omega_{c_1 B}, \omega_{c_2 B}, \dots\}$  is also an open set. However, for the configuration  $A_2 = A \cup \{b_1, b_2\}$ ,  $S_{A_2} = \{\omega_{AB}\}$  not an open set, since any open neighbourhood of  $\omega_{AB}$  must contain some  $\omega_{c_i B}$ . Hence it is not the case in general for infinite configurations  $\mathbf{c}$  that  $S_{\mathbf{c}}$  is open, in contrast with the case where  $\mathbf{c}$  is finite. Further, one checks that configurations  $[a_2] \cup [b_2]$  and  $[c_2] \cup [b_4]$  are progressive, but e.g.  $[a_6] \cup [b_4]$  is not.

Let us now choose a weight  $\mu$  on  $E$  such that for all  $i$ ,  $\mu(a, 2i) = \mu(c, i) = 1$  but  $\mu(a, 2i + 1) = \mu(b, i) = \mu(d, i) = 0$ . Then  $\{\omega_D\}$  is not open in  $\mathfrak{T}^\mu$  since any neighborhood of  $\omega_D$  contains  $\omega_{c_1 B}$ .

### 3 Observability and Diagnosability

Let  $\mathcal{E} = (E, \leq, \#, \lambda)$  with  $\lambda : E \rightarrow \mathbb{A}$ , and  $\eta : \mathbb{A} \rightarrow \mathbb{O}$  a partial *observation mapping* into an *observation alphabet*  $\mathbb{O}$ . For a given labeled prime event structure, let  $E_\eta \triangleq \{e \mid \eta(\lambda(e)) \downarrow\}$  be the set of *visible* events, and  $E_\varepsilon \triangleq \{e \mid \eta(\lambda(e)) \uparrow\}$  the set of *invisible* events. Using the above construction, we obtain the *visible height*  $\mathcal{H}_\eta(\bullet)$ , *observable conciseness*  $\Psi_\eta(\bullet)$  and pre-distance  $\mathbf{d}_\eta(\bullet, \bullet)$ , respectively, by setting  $\mu \equiv \mathbf{1}_{E_\eta}$ . Write  $\mathcal{E}_1 \sim_\eta \mathcal{E}_2$  iff the two structures with  $\lambda$  replaced by  $\eta \circ \lambda$  are  $\mathbb{O}$ -isomorphic.

**Observability.** To avoid tedious case distinctions, we assume henceforth that all runs of  $\mathcal{E}$  are of infinite height; if necessary, consider any finite-height run extended by an infinite chain of dummy events.

**Definition 3** A labeled ES  $\mathcal{E}$  is observable w.r.t.  $\eta$  iff

$$\mathcal{H}(\mathbf{c}) = \infty \quad \Rightarrow \quad \mathcal{H}_\eta(\mathbf{c}) = \infty. \quad (13)$$

For an illustration, let  $\mathbb{O} = \{a\}$  and define - in the context of Example 1 - the partial mapping  $\eta : \mathbb{A} \rightarrow \mathbb{O}$  such that  $\eta$  maps  $a$  to  $a$  and is undefined otherwise. Then  $\mathcal{E}$  is not observable w.r.t.  $\eta$  since one has, for every  $i \in \mathbb{N}$ ,

$$\mathcal{H}(\omega_{c_i B}) = \infty \quad \wedge \quad \mathcal{H}_\eta(\omega_{c_i B}) = i - 1.$$

**Topologies.** Clearly, any choice of  $\mu : \mathbb{A} \rightarrow \mathbb{R}_0^+$  and hence of  $\mathbf{d}_\mu(\bullet, \bullet)$  defines a topology  $\mathfrak{T}^\mu$ , called the  $\mu$ -topology, on  $\Omega$ . Note that for  $\mu \equiv \mathbf{1}_E$ , we obtain the restriction - to  $\Omega$  - of the Scott topology on  $\mathcal{C}$ ; call this topology  $\mathfrak{T}$ . Further, denote as

$$\begin{aligned}\mathcal{C}_{/\mu}(\mathcal{E}) &\triangleq \{[[\mathbf{c}]]_\eta \mid \mathbf{c} \in \mathcal{C}(\mathcal{E})\} \\ \Omega_{/\mu}(\mathcal{E}) &\triangleq \{[[\mathbf{c}]]_\eta \mid \mathbf{c} \in \mathcal{C}(\mathcal{E})\}\end{aligned}$$

the quotient spaces of configurations and runs, respectively, under  $\mu \circ \lambda$ -preserving isomorphism, with associated quotient topology  $\mathfrak{T}_\mu$  on  $\Omega_{/\mu} = \Omega_{/\mu}(\mathcal{E})$ . In particular, set  $\mathfrak{D} \triangleq \mathfrak{T}_\eta$ .

**Defining diagnosability.** Let  $\Phi \subseteq E$  be a set of *invisible fault events*; in particular, no event in  $\Phi$  is observable, i.e.  $\lambda(\Phi) \cup \text{dom}(\eta) = \emptyset$ . A configuration  $\mathbf{c} \in \mathcal{C}(\mathcal{E})$  is called *faulty* iff  $\mathbf{c} \cap \Phi \neq \emptyset$ , and *healthy* otherwise. Denote as  $\Omega_F$  ( $\mathcal{C}_F$ ) the set of faulty runs (configurations), and  $\Omega_{NF}$  the set of healthy runs. We observe that if  $\mathbf{c}$  is faulty, so is every extension of  $\mathbf{c}$ , i.e. every  $\mathbf{c}' \in \mathcal{C}(\mathcal{E})$  such that  $\mathbf{c} \subseteq \mathbf{c}'$  is faulty. As a consequence, we have:

**Lemma 2**  $\Omega_F$  is open in  $\mathfrak{T}$ .

Note, however, that  $\Omega_F$  is in general neither open nor closed in  $\mathfrak{D}$ .

We can distinguish three *diagnosis states*, given by sets of runs:

$$\begin{aligned}\text{Fault} - \text{definite} : FD &\triangleq \{\omega \in \Omega \mid [[\omega]]_\eta \subseteq \Omega_F\} \\ \text{NF} - \text{definite} : ND &\triangleq \{\omega \in \Omega \mid [[\omega]]_\eta \subseteq \Omega_{NF}\} \\ \text{Indefinite} : ID &\triangleq \Omega \setminus (FD \cup ND).\end{aligned}$$

If the system is in state  $FD$  (or  $ND$  or  $ID$ ), this means that its current configuration  $\mathbf{c}$  is such that

$$\Omega_{\mathbf{c}} \triangleq \{\omega \in \Omega \mid \mathbf{c} \subseteq \omega\} \subseteq FD(ND, ID)$$

It is of course not feasible to verify directly the *infinite* runs. In [CL99], a diagnoser system is built over *diagnoser states* that correspond to finite observation sequences : a diagnoser state represents the knowledge that can be derived about the eventual diagnosis, from a given finite observation. We shall not proceed here by constructing a diagnoser, since it is not feasible in general event structures; its state space would be infinite in general<sup>2</sup>. Rather, we give directly a definition of *eventual diagnosability* notions:

**Definition 4**  $\Phi$  is eventually F-diagnosable for  $(\mathcal{E}, \eta)$  iff  $\Omega_F$  is open in  $\mathfrak{D}$ . Dually,  $\Phi$  is eventually N-diagnosable for  $(\mathcal{E}, \eta)$  iff  $\Omega_{NF}$  is open in  $\mathfrak{D}$ .

This is a notion that does not at all take the time after fault occurrence into account, contrary to e.g. [SSL<sup>+</sup>95, GL]. It generalizes the traditional definition from [CL99] given in the introduction, and the ones we presented for Petri nets in [HBFJ03, Haa07, Haa09].

<sup>2</sup>Note that, for the case of Petri nets with sequential semantics (see below), the diagnoser construction is carried out in [MND10]

**Metric characterization.** Exploring the topology  $\mathfrak{D}$  to characterize F- and NF-diagnosability shows us that both are equivalent, confirming corresponding results (see [WLY05]) in the sequential case:

**Theorem 1** *If  $(\mathcal{E}, \eta)$  is observable, then  $\Phi$  is eventually F-diagnosable for  $(\mathcal{E}, \eta)$  iff for every faulty  $\omega_\Phi \in \Omega_F$ , there exists a finite-height prefix  $\mathbf{c}_\Phi$  of  $\omega_\Phi$  such that  $\Omega_{\mathbf{c}_\Phi} \subseteq \Omega_F$ . Dually, if  $(\mathcal{E}, \eta)$  is observable, then  $\Phi$  is eventually NF-diagnosable for  $(\mathcal{E}, \eta)$  iff for every healthy  $\omega_0 \in \Omega_{NF}$ , there exists a finite prefix  $\mathbf{c}_0$  of  $\omega_0$  such that  $\Omega_{\mathbf{c}_0} \subseteq \Omega_{NF}$ .*

**Proof:** Fix  $\omega_\Phi$  and assume  $\Phi$  is eventually F-diagnosable; then there exists  $\delta = \delta(\omega_\Phi)$  such that

$$\forall \omega \in \Omega_{NF} : \mathbf{d}_\eta(\omega_\Phi, \omega) > \delta. \quad (14)$$

Let  $k$  be any integer such that  $k > \log_2(\delta)$ ; then let  $\mathbf{c}_\Phi$  be the smallest prefix of  $\omega_\Phi$  such that  $\mathcal{H}_\eta(\mathbf{c}_\Phi) = k$ . By observability,  $\mathcal{H}(\mathbf{c}) < +\infty$ , and (14) implies that  $\Omega_{\mathbf{c}_\Phi} \subseteq \Omega_F$ . The reverse implication is obvious. Finally, the proof for the characterization of NF-diagnosability is exactly analogous.  $\square$

We obtain the following additional result:

**Theorem 2** *If  $(\mathcal{E}, \eta)$  is observable, then:  $\Phi$  is eventually NF-diagnosable for  $(\mathcal{E}, \eta)$  iff it is eventually F-diagnosable for  $(\mathcal{E}, \eta)$ .*

**Proof:** Follows from the symmetry of  $\mathbf{d}_\eta(\bullet, \bullet)$  in the proof of Theorem 1.  $\square$

The astute reader will notice that a system may be diagnosable even without being observable as defined in Def. 3. In the case of non-observability, all runs  $\omega, \omega'$  for which  $\mathcal{H}_\lambda(\mathbf{c})$  is finite, satisfy  $\mathbf{d}_\eta(\omega, \omega') = 0$ . For  $\Phi$  to be F- or NF-diagnosable in  $(\mathcal{E}, \eta)$ , the runs of finite observable height must either all be faulty or all be healthy. In our view, this fact illustrates that all *interesting* diagnosis problems concern *observable* systems.

Note that equivalence of F-diagnosability and NF-diagnosability had been shown in [WLY05] for the classical approach, using an enumeration argument that requires *sequential* semantics; the above generalization shows that it is an intrinsic, semantics-independent feature of diagnosis.

In the light of Theorem 2, we will henceforth drop the reference to F and NF as well as the qualifier "eventually", and speak simply of *diagnosable* labeled event structures.

**Example.** In the context of the event structure in Example 1, let us now choose  $\mathbb{O} = \{b, d\}$  with  $\text{dom}(\eta) = \{b, b^*, d, d^*\}$ , where  $\eta(b) = \eta(b^*) = b$  and  $\eta(d) = \eta(d^*) = d$ . If  $\Phi \subseteq \{c_2, c_3, c_4, \dots\}$ , then the net is not diagnosable since  $\Omega_F = \bigcup_{i \in \mathbb{N}} \{\omega_{c_i B}, \omega_{c_i D}\}$  is not an open set in  $\mathfrak{D}$ ; any neighborhood of  $\Omega_F$  in  $\mathfrak{D}$  contains  $\omega_{AB} \in \Omega_{NF}$ .

If one has, on the other hand,  $\Phi \subseteq \overline{B}$ ,  $\mathbb{O} = \{a, d\}$  and  $\text{dom}(\eta) = \{a, a^*, d, d^*\}$ , where  $\eta(a) = \eta(a^*) = a$  and  $\eta(d) = \eta(d^*) = d$ , then  $\mathcal{E}$  is diagnosable with respect to  $\eta$  and  $\Phi$ , since  $\Omega_F = \{\omega_{c_i B} \mid i \in \mathbb{N}\} \cup \{\omega_{AB}\}$  is open in  $\mathfrak{D}$ .

**Suffixes.** Note that all prefixes of  $\mathcal{E}$ , and in particular all its configurations, constitute sub-event-structures of  $\mathcal{E}$ ; we will denote these structures with the

same symbols as the corresponding sets. We have the following *suffix* objects :  
For  $\mathbf{c} \in \mathcal{C}$  and  $S \subseteq \mathcal{C}$ , let

$$\begin{aligned} \mathcal{C}_{\mathbf{c}} &\triangleq \{\bar{\mathbf{c}} \in \mathcal{C} \mid \mathbf{c} \subseteq \bar{\mathbf{c}}\}, \quad \Omega_{\mathbf{c}} \triangleq \{\omega \in \Omega \mid \mathbf{c} \subseteq \omega\} \\ \text{and} \quad \Omega_S &\triangleq \bigcup_{\mathbf{c} \in S} \Omega_{\mathbf{c}}. \end{aligned}$$

Further, for any  $\mathbf{c} \in \mathcal{C}(\mathcal{E})$ , denote as

$$\begin{aligned} \mathcal{E}_{\mathbf{c}} &= (E_{\mathbf{c}}, \leq_{|E_{\mathbf{c}}}, \#_{|E_{\mathbf{c}}}, \lambda_{|E_{\mathbf{c}}}), \\ \text{where } E_{\mathbf{c}} &\triangleq \{e \in E \setminus \mathbf{c} \mid \forall e' \in \mathbf{c} : \neg(e \# e')\}, \end{aligned}$$

the *shift* of  $\mathcal{E}$  by  $\mathbf{c}$ . If  $\mathbf{c}' \in \mathcal{C}(\mathcal{E}_{\mathbf{c}})$ , then  $\mathbf{c} \circ \mathbf{c}'$  is the unique configuration of  $\mathcal{E}$  such that (i)  $\mathbf{c}$  is a prefix of  $\mathbf{c} \circ \mathbf{c}'$ , and (ii)  $\mathbf{c} \circ \mathbf{c}' \cap E_{\mathbf{c}} = \mathbf{c}'$ . For every  $\mathbf{c}' \in \mathcal{C}(\mathcal{E}_{\mathbf{c}})$ , we observe that  $\mathbf{c}'' \triangleq \mathbf{c} \cup \mathbf{c}' \in \mathcal{C}(\mathcal{E})$ ; write in this case  $\mathbf{c}'' = \mathbf{c} \circ \mathbf{c}'$ , and say that  $\mathbf{c}''$  is obtained by *appending*  $\mathbf{c}'$  to  $\mathbf{c}$ .

**Structural Characterization.** The following characterization result lifts the analogous one unfoldings of safe Petri nets presented in [HBFJ03, Haa10] to regular event structures. For any two finite configurations  $\mathbf{c}_1, \mathbf{c}_2 \in \mathcal{C}(\mathcal{E})$ , say that  $\mathbf{c}_2$  *corresponds to*  $\mathbf{c}_1$ , written  $\mathbf{c}_1 \sim_{\mathcal{E}} \mathbf{c}_2$ , iff  $\mathcal{E}_{\mathbf{c}_1} \sim_{\mathbb{A}} \mathcal{E}_{\mathbf{c}_2}$ . Clearly,  $\sim_{\mathcal{E}}$  is an equivalence; event structure  $\mathcal{E}$  is *regular* iff it has a finite number of distinct  $\sim_{\mathcal{E}}$ -classes. In particular, all unfoldings of 1-safe Petri nets are regular. In fact, all infinite runs of these unfoldings must pass through an infinite number of finite configurations corresponding to the behaviour after the same net marking, since the number of reachable markings is finite. Any pair  $(\mathbf{c}_1, \mathbf{c}_2)$  of such configurations with  $\mathbf{c}_1 \subseteq \mathbf{c}_2$  satisfies  $\mathbf{c}_1 \sim_{\mathcal{E}} \mathbf{c}_2$  by construction of the unfolding. The converse - can all regular event structures be constructed as unfoldings of 1-safe nets ? - is known as Thiagarajan's conjecture [Thi02].

To complete our preparations for Theorem 3, let  $\mathbf{c} \sim_{\eta} \mathbf{c}'$  iff there is an  $\eta$ -isomorphism between  $\mathbf{c}$  and  $\mathbf{c}'$ , and  $\mathbf{c} \sim_{\Phi} \mathbf{c}'$  iff  $\mathbf{c}$  and  $\mathbf{c}'$  are either both healthy or both faulty.

**Theorem 3** *If  $(\mathcal{E}, \eta)$  is observable and regular,  $\Phi$  is eventually F-diagnosable for  $(\mathcal{E}, \eta)$  iff for all configurations  $\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}'_1, \mathbf{c}'_2 \in \mathcal{C}(\mathcal{E})$  of finite height such that*

$$\begin{aligned} \mathbf{c}_1 \subseteq \mathbf{c}'_1 \quad \wedge \quad \mathbf{c}_1 \sim_{\mathcal{E}} \mathbf{c}'_1 \\ \mathbf{c}_2 \subseteq \mathbf{c}'_2 \quad \wedge \quad \mathbf{c}_2 \sim_{\mathcal{E}} \mathbf{c}'_2, \end{aligned}$$

*the following holds:*

$$\left. \begin{aligned} &\mathbf{c}_1 \sim_{\eta} \mathbf{c}_2 \\ \wedge &\mathbf{c}'_1 \sim_{\eta} \mathbf{c}'_2 \\ \wedge &\mathcal{H}(\mathbf{c}_1) < \mathcal{H}(\mathbf{c}'_1) \end{aligned} \right\} \Rightarrow \mathbf{c}'_1 \sim_{\Phi} \mathbf{c}'_2. \quad (15)$$

**Proof:** To show the "if" part, assume  $\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}'_1, \mathbf{c}'_2$  violate (15), i.e. without loss of generality

1.  $\mathbf{c}'_2$  is faulty, but neither  $\mathbf{c}'_1$  nor  $\mathbf{c}_1$  are,
2. for  $i \in \{1, 2\}$ ,  $\mathbf{c}'_i = \mathbf{c}_i \circ \mathbf{d}_i$ , where  $\mathbf{d}_i \in \mathcal{C}(\mathcal{E}_{\mathbf{c}_i})$  and  $\mathbf{d}_1^{\dagger} \neq \emptyset$  ( $\mathbf{d}_2$  may be empty), and

3. for  $i \in \{1, 2\}$ ,  $\mathbf{c}'_i \sim_\eta \mathbf{c}_i$  and  $\mathbf{c}'_i \sim_{\mathcal{E}} \mathbf{c}_i$ .

It follows that there is a configuration  $\mathbf{d}_i^2 \in \mathcal{C}(\mathcal{E}_{\mathbf{c}'_i})$  that is an isomorphic copy of  $\mathbf{d}_i$ . Iterating this argument, let  $\mathbf{c}_i^1 \triangleq \mathbf{c}'_i = \mathbf{c}_1 \circ \mathbf{d}_i^1$  and  $\mathbf{c}_i^{n+1} \triangleq \mathbf{c}_i^n \circ \mathbf{d}_i^{n+1}$  for  $n \in \mathbb{N}$ . Then by assumption,  $\mathcal{H}(\mathbf{c}_1^n) \rightarrow_{n \rightarrow \infty} \infty$  (the same need not be true for the sequence of  $\mathbf{c}_2^n$ ). We have  $\mathbf{c}_i^n \sim_\eta \mathbf{c}_i$  for all  $n$ ; by construction, all  $\mathbf{c}_2^n$  are healthy, so  $\Phi$  can not be F-diagnosable for  $(\mathcal{E}, \eta)$ .

For “only if”, suppose  $\Phi$  is not F-diagnosable for  $(\mathcal{E}, \eta)$ . Then there exists  $\omega \in \Omega_F$  such that for any finite-height prefix  $\mathbf{c}$  of  $\omega$ , there is  $\mathbf{c}' \in \mathcal{C}(\mathcal{E})$  that satisfies  $\mathbf{c}' \sim_\eta \mathbf{c}$  and  $\Omega_{\mathbf{c}'} \cap \Omega_{NF} \neq \emptyset$ . But then one obtains a violation of (15) from the assumption that  $\mathcal{E}$  is regular.  $\square$

## 4 Application to Petri Nets

**Petri Nets.** We will turn now to an important instance of event structures, those linked to Petri net models.

**Definition 5** A *net* is a tuple  $N = (P, T, F)$  where

- $P \neq \emptyset$  is a set of **places**,
- $T \neq \emptyset$  is a set of **transitions** such that  $P \cap T = \emptyset$ ,
- $F \subseteq (P \times T) \cup (T \times P)$  is a set of **flow arcs**.

A **marking** is a multiset  $m$  of places, i.e. a map from  $P$  to  $\mathbb{N}$ . A **Petri net** is a tuple  $\mathcal{N} = (P, T, F, m)$ , where

- $(P, T, F)$  is a finite net, and
- $m : P \rightarrow \mathbb{N}$  is an **initial marking**.

Elements of  $P \cup T$  are called the *nodes* of  $\mathcal{N}$ . For a transition  $t \in T$ , we call  $\bullet t = \{p \mid (p, t) \in F\}$  the *preset* of  $t$ ,  $t^\bullet = \{p \mid (t, p) \in F\}$  the *postset* of  $t$ . In Figure 2, we represent as usual places by empty circles, transitions by squares,  $F$  by arrows, and the marking of a place  $p$  by putting the corresponding number of *black tokens* into  $p$ . A transition  $t$  is *enabled* in marking  $m$  if  $\forall p \in \bullet t, m(p) > 0$ . This enabled transition can *fire*, resulting in a new marking  $m' = m - \bullet t + t^\bullet$ ; this firing relation is denoted by  $m[t]m'$ . A marking  $m$  is *reachable* if there exists a *firing sequence*, i.e. transitions  $t_0 \dots t_n$  such that  $m_0[t_0]m_1[t_1] \dots [t_n]m$ . A net is *safe* if for all reachable markings  $m$ ,  $m(p) \subseteq \{0, 1\}$  for all  $p \in P$ .

**Sequential semantics.** The language  $\mathcal{L}$  of  $\mathcal{N}$  is the set of words  $e_0 \dots e_n$  over a set  $E$  with a mapping  $\lambda : E \rightarrow T$  such that  $\lambda(e_0) \dots \lambda(e_n)$  is a firing sequence. Assume now that  $\mathcal{L}$  is *trim*: any two words  $w, w'$  in  $\mathcal{L}$  share their common prefix, i.e. if there are  $u \in E^*, x, x' \in E^\infty$  and  $e, e' \in E$  such that  $w = uex$  and  $w' = ue'x'$ , then  $\lambda(e) = \lambda(e')$  implies  $e = e'$ . The *sequential semantics* of  $\mathcal{N}$  is given by event structure  $\mathcal{E}_{seq} = (E, \leq_{seq}, \#_{seq}, \lambda)$ , obtained from  $\mathcal{L}$  by setting

1.  $e \leq_{seq} e'$  iff there exist  $u, v \in E^*$  and  $w \in E^\infty$  such that  $ueve'w \in \mathcal{L}$ , and
2.  $e \#_{seq} e'$  iff there exist  $\bar{e}, \bar{e}' \in E$  and  $u, v \in E^*$  such that  $u\bar{e}, u\bar{e}' \in \mathcal{L}$  with  $\lambda(\bar{e}) \neq \lambda(\bar{e}')$ .

**Partial order unfolding semantics.** In a net  $N = (P, T, F)$ , let  $<_N$  the transitive closure of  $F$ , and  $\leq_N$  the reflexive closure of  $<_N$ . Further, set  $t_1 \#_{im} t_2$  for transitions  $t_1$  and  $t_2$  if and only if  $t_1 \neq t_2$  and  $\bullet t_1 \cap \bullet t_2 \neq \emptyset$ , and define  $\# = \#_N$  by

$$a \# b \Leftrightarrow \exists t_a, t_b \in T : \begin{cases} t_a \#_{im} t_b \\ \wedge t_a \leq_N a \\ \wedge t_b \leq_N b. \end{cases}$$

Finally, define  $\mathbf{co} = \mathbf{co}_N$  by setting, for any nodes  $a, b \in P \cup T$ ,

$$a \mathbf{co} b \iff \neg(a \leq b) \wedge \neg(a \# b) \wedge \neg(b < a).$$

**Definition 6** A net  $ON = (B, E, G)$  is an *occurrence net* if and only if it satisfies

1.  $\leq_{ON}$  is a partial order;
2. for all  $b \in B$ ,  $|\bullet b| \in \{0, 1\}$ ;
3. for all  $x \in B \cup E$ , the set  $[x] = \{y \in B \cup E \mid y \leq_{ON} x\}$  is finite;
4. no self-conflict, i.e. there is no  $x \in B \cup E$  such that  $x \#_{ON} x$ ;
5. the set  $cut_0$  of  $\leq_{ON}$ -minimal nodes is contained in  $B$  and finite.

The nodes of  $E$  are the *events*, those of  $B$  *conditions*. One notices quickly that complete occurrence nets form particular cases of event structures. The canonical association of an event structure to an occurrence net  $ON$  is by restricting  $\leq$  and  $\#$  to the event set  $E$ , "forgetting" conditions. In particular, configurations of occurrence nets are defined as sets of events, i.e. configurations defined as above for the "stripped" event structure.

Occurrence nets are the mathematical form of the *partial order unfolding semantics* for Petri nets [JEV02]; although more general applications are possible, we will focus here on unfoldings of *safe* Petri nets only.

If  $N_1 = (P_1, T_1, F_1)$  and  $N_2 = (P_2, T_2, F_2)$  are nets, a *homomorphism* is a mapping  $h : P_1 \cup T_1 \rightarrow P_2 \cup T_2$  such that

- $h(P_1) \subseteq P_2$  and
- for every  $t_1 \in T_1$ , the restriction to  $\bullet t_1$  is a bijection between the set  $\bullet t_1$  in  $N_1$  and the  $\bullet h(t_1)$  in  $N_2$ , and similarly for  $t_1 \bullet$  and  $(h(t_1)) \bullet$ .

A *branching process* of safe Petri net  $\mathcal{N} = (N, m_0)$  is a pair  $\beta = (ON, \pi)$ , where  $ON = (B, E, G)$  is an occurrence net, and  $\pi$  is a homomorphism from  $ON$  to  $N$  such that:

1. The restriction of  $\pi$  to  $cut_0$  is a bijection from  $cut_0$  to  $m_0$ , and
2. for every  $e_1, e_2 \in E$ , if  $\bullet e_1 = \bullet e_2$  and  $h(e_1) = h(e_2)$  then  $e_1 = e_2$ .



Branching processes  $\beta_1 = (ON_1, \pi_1)$  and  $\beta_2 = (ON_2, \pi_2)$  for  $\mathcal{N}$  are isomorphic iff there exists a bijective homomorphism  $h : ON_1 \rightarrow ON_2$  such that  $\pi_1 = \pi_2 \circ h$ . The unique (up to isomorphism) maximal branching process  $\beta_{\mathcal{U}} = (ON_{\mathcal{U}}, \pi_{\mathcal{U}})$  of  $\mathcal{N}$  is called the *unfolding* of  $\mathcal{N}$ ; see [JEV02] for a canonical algorithm to compute the unfolding of  $\mathcal{N}$ . We will assume that all transitions  $t \in T$  have at least one output place, i.e.  $t^\bullet$  is not empty. In this case, every finite configuration  $\mathbf{c}$  of  $ON_{\mathcal{U}}$  spans a conflict free subnet  $\mathbf{c}_{\mathcal{U}} = (E_{\mathbf{c}}, B_{\mathbf{c}}, G_{|(E_{\mathbf{c}} \times B_{\mathbf{c}}) \cup (B_{\mathbf{c}} \times E_{\mathbf{c}})})$  of  $ON_{\mathcal{U}}$  by setting

$$B_{\mathbf{c}} \triangleq \bigcup_{e \in E} (\bullet t \cup t^\bullet).$$

The following results (see e.g. [JEV02]) justify the use of unfoldings: The set  $cut(\mathbf{c})$  of  $\leq$ -maximal nodes of  $\mathbf{c}_{\mathcal{U}}$  is contained in  $B_{\mathbf{c}}$ . Moreover,  $cut(\mathbf{c})$  is a *co-set*, that is, for all distinct conditions  $b, b' \in cut(\mathbf{c})$ ,  $b \mathbf{co} b'$  holds; and  $cut(\mathbf{c})$  is  $\subseteq$ -maximal with this property, and such sets in occurrence nets are called *cuts*. By setting, for any cut  $s$ ,

$$m(s) \triangleq \pi(s),$$

we obtain a marking of  $\mathcal{N}$ . Now, for  $cut(\mathbf{c})$  as above,  $m(\mathbf{c}) \triangleq m(cut(\mathbf{c}))$  is a reachable marking of  $\mathcal{N}$ , more precisely the marking that  $\mathcal{N}$  is in after executing firable transitions in a sequence compatible with  $\mathbf{c}$ . Conversely, every reachable marking  $m$  of  $\mathcal{N}$  is reflected in this way by at least one configuration  $\mathbf{c}$  in  $ON_{\mathcal{U}}$  such that  $m(\mathbf{c}) = m$ .

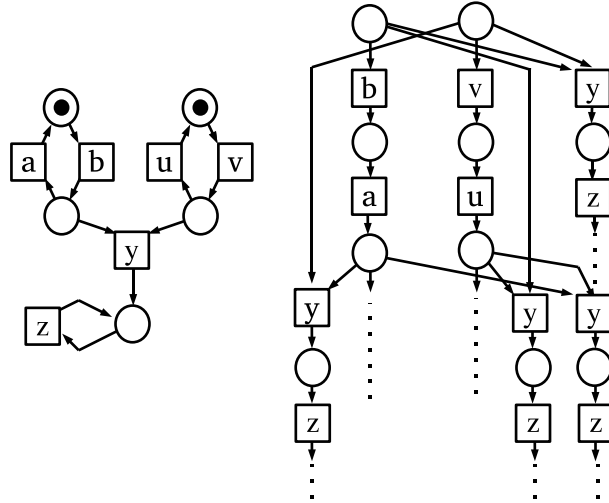


Figure 2: Left: a Petri Net ; right: a prefix of its unfolding, with events bearing the name of their  $\pi$ -image

The **partial order semantics** for  $\mathcal{N}$  is given by the event structure

$$\mathcal{E}_{\mathcal{U}} = (E_{\mathcal{U}}, \leq_{\mathcal{U}}, \#_{\mathcal{U}}, \pi_{\mathcal{U}}^E)$$

where  $E_{\mathcal{U}}$  is the set of events in  $\mathcal{N}$ 's unfolding  $\beta_{\mathcal{U}}$ , and  $\leq_{\mathcal{U}}$ ,  $\#_{\mathcal{U}}$ , and  $\pi_{\mathcal{U}}^E$  are the restrictions to  $E_{\mathcal{U}}$  of the corresponding elements of  $\beta_{\mathcal{U}}$ . By construction, the labeling  $\pi_{\mathcal{U}}^E$  for  $\mathcal{E}_{\mathcal{U}}$  is simple in the above sense: this property simply reflects the fact that no transition can have more than one concurrent occurrence if the net is safe.

**Connecting the diagnosability notions.** The notion of *F-diagnosability* given in Sampath, Lafortune et al [SSL<sup>+</sup>95] involves existence of a uniform bound on the “time” after occurrence of the fault before diagnosis. It can be adapted to our framework - using a sequential event structure  $\mathcal{E}$  obtained from a finite automaton - as follows: let

$$\mathcal{C}_{\Phi}^* \triangleq \{ \mathbf{c} \in \mathcal{C}_F \mid \forall \mathbf{c}' \in \mathcal{C} : \mathbf{c}' \subseteq \mathbf{c} \Rightarrow \mathbf{c}' \notin \mathcal{C}_F \}$$

be the set of *minimal faulty configurations*.  $\Phi$  is *F-diagnosable* for  $(\mathcal{E}, \eta)$  iff for every  $\mathbf{c}_{\Phi} \in \mathcal{C}_{\Phi}^*$ , there exists  $K = K(\mathbf{c}) > 0$  such that the following holds: If  $\mathbf{c} \in \mathcal{C}(\mathcal{E})$  is such that  $\mathbf{c}_{\Phi}$  is  $\eta$ -isomorphic to a prefix of  $\mathbf{c}$ , and the **1**-height of  $\mathbf{c}$  is bounded by  $K$  plus the **1**-height of  $\mathbf{c}_{\Phi}$ , then  $\mathbf{c}$  is also faulty:

$$\mathcal{H}_1(\mathbf{c}_{\Phi}) + K \leq \mathcal{H}_1(\mathbf{c}) \Rightarrow \mathbf{c} \in \mathcal{C}_F. \quad (16)$$

then  $\mathbf{c}$  is also faulty. Note that this definition uses the **1**-height, not observable height; we will see below that, under observability, both are equivalent.

**Characterizing diagnosable Petri nets.** This definition had inspired the analogous one we have given in [HBFJ03, Haa10] for safe Petri nets.

**Definition 7** Let  $\mathcal{N} = (P, T, F, m_0)$  a safe Petri net,  $\eta : T \rightarrow \mathbb{O}$  a partial mapping,  $\mathcal{U}_{\mathcal{N}} = (B, E, G, cut_0)$  its unfolding net, with labeling morphism  $\lambda : E \rightarrow T$  given by the unfolding morphism. Let  $\phi \in T \setminus \text{dom}(\eta)$  be a fault transition, and let  $E_{\phi} \triangleq \lambda^{-1}(\phi)$ . Denote by  $\mathcal{C}_{\text{prog}}(\mathcal{N})$  the set of  $\mathcal{N}$ 's progressive configurations (compare (9)):

$$\mathcal{C}_{\text{prog}}(\mathcal{N}) \triangleq \{ \mathbf{c} \in \mathcal{C}(\mathcal{N}) \mid \mathcal{H}(\mathbf{c}) \leq \mathcal{H}_{\mu}^*(D) \}$$

We say that  $\mathcal{N}$  is **weakly observable w.r.t.**  $\eta$  iff its unfolding event structure  $\mathcal{E}_{\mathcal{U}}$  is observable w.r.t.  $\eta$ . A weakly observable (w.r.t.  $\eta$ )  $\mathcal{N}$  is **weakly diagnosable w.r.t.**  $\eta$  and  $\phi$  iff there exists  $n = n_{\mathcal{N}} \in \mathbb{N}$  such that for all configurations  $\mathbf{c}_{\phi} \triangleq [e_{\phi}]$  with  $e_{\phi} \in E_{\phi}$ , every  $\mathbf{c} \in \mathcal{C}_{\text{prog}}(\mathcal{N})$  such that

- (a)  $\mathbf{c}_{\phi} \sqsubseteq \mathbf{c}$ ,
- (b)  $\mathbf{c}$  is not dead, and
- (c)  $H(\mathbf{c}) \geq H(\mathbf{c}_{\phi}) + n$ ,

satisfies:

$$\forall \mathbf{c}' \in \mathcal{L} : \mathbf{c} \sqsubseteq_{\mathbb{O}} \mathbf{c}' \Rightarrow E_{\phi} \cap \mathbf{c}' \neq \emptyset. \quad (17)$$

Notice that the role of the set  $\Phi \subseteq E$ , which was arbitrary in the above study of diagnosability in *event structures*, is played here by the set  $E_{\phi}$  of occurrences of the same transition  $\phi$ . The definition implies that  $\mathcal{N}$  is weakly diagnosable w.r.t.  $\phi$  and  $\eta$  iff  $\mathcal{E}_{\mathcal{U}}(\mathcal{N})$  is diagnosable w.r.t.  $E_{\phi}$  and  $\eta$ .

Let us first show the following auxiliary result:

**Lemma 3** *If  $\mathcal{N}$  is observable, then there exists  $n_{\circ} \in \mathbb{N}$  such that for any two configurations  $\mathbf{c}_1, \mathbf{c}_2 \in \mathcal{C}(\mathcal{N})$  such that  $\mathbf{c}_1 \sqsubseteq \mathbf{c}_2$  and  $\mathbf{c}_1 \sim_{\circ} \mathbf{c}_2$ ,  $\mathcal{H}(\mathbf{c}_2) \leq \mathcal{H}(\mathbf{c}_1)$ .*

**Proof:** Suppose for every  $n \in \mathbb{N}$  there exist  $\mathbf{c}_1, \mathbf{c}_2$  such that  $\mathcal{H}(\mathbf{c}_2) > \mathcal{H}(\mathbf{c}_1)$  while  $\mathbf{c}_1 \sqsubseteq \mathbf{c}_2$  and  $\mathbf{c}_1 \sim_{\circ} \mathbf{c}_2$ . Then the pigeonhole principle implies, since the number of reachable markings of  $\mathcal{N}$  is bounded above by  $2^{|P|}$ , that for any  $n > 2^{|P|}$ , there exist  $\mathbf{c}, \mathbf{c}' \in \mathcal{C}(\mathcal{N})$  such that

1.  $m(\mathbf{c}) = m(\mathbf{c}')$
2.  $\mathbf{c}_1 \sqsubseteq \mathbf{c} \sqsubseteq \mathbf{c}' \sqsubseteq \mathbf{c}_2$ ,
3.  $\mathcal{H}(\mathbf{c}') \geq \mathcal{H}(\mathbf{c}) + 1$ .

It follows that  $\mathbf{c} \sim_{\circ} \mathbf{c}'$ . Moreover, since  $m(\mathbf{c}) = m(\mathbf{c}')$ , any firing sequence leading from  $\mathbf{c}$  to  $\mathbf{c}'$  is again enabled in  $m(\mathbf{c}')$ , hence  $\mathcal{N}$  allows configurations  $\mathbf{c}(n)$ ,  $n \in \mathbb{N}$ , such that  $\mathbf{c} \sqsubseteq \mathbf{c}(1) \sqsubseteq \mathbf{c}(2) \sqsubseteq \dots$  and  $\mathcal{H}(\mathbf{c}(n)) \geq \mathcal{H}(\mathbf{c}) + n$ . This leads to a contradiction with weak observability as  $n \rightarrow \infty$ .  $\square$

We then have:

**Theorem 4** *Use the notations of Definition 7 and assume  $\mathcal{N}$  is weakly observable. Then  $\mathcal{N}$  is weakly diagnosable iff there exists  $n \in \mathbb{N}$  such that for all  $\mathbf{c}_{\phi} \in \mathcal{C}_{\Phi}(\mathcal{N})$  and  $\mathbf{c} \in \mathcal{C}(\mathcal{N})$ ,*

$$\left. \begin{array}{l} \mathbf{c}_{\phi} \sqsubseteq \mathbf{c} \\ \mathbf{c} \text{ not dead} \\ \mathcal{H}_{\circ}(\mathbf{c}) \geq \mathcal{H}_{\circ}(\mathbf{c}_{\phi}) + n \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \forall \omega \in \Omega(\mathcal{N}) : \\ (\mathbf{c} \sqsubseteq_{\circ} \omega) \Rightarrow \omega \in \Omega_F \end{array} \right. \quad (18)$$

**Proof:** Suppose first that  $\mathcal{N}$  is weakly diagnosable, i.e.  $n_{\mathcal{N}}$  as in Definition 7 exists; then  $n \triangleq \max(n_{\mathcal{N}}, n_{\circ})$  with  $n_{\circ}$  from Lemma 3 has the above properties. Similarly, the existence of  $n$  as in the statement of the theorem implies that  $n_{\mathcal{N}} \triangleq \max(n, n_{\circ})$  satisfies the properties required in (18).  $\square$

**Example 2: What Interleavings do and don't see.** Figure 2 illustrates that choosing a partial order vs an interleaving semantics has important consequences. To see this, note that if the net behaviour is recorded in sequential form, we still have an event structure semantics; yet the resulting event structure is degenerate in the sense that  $\mathbf{co}$  is empty. Defining metric topology etc. as above, let  $\Phi = \pi^{-1}(\{v\})$ , and assume the observation labellings for  $\mathcal{E}_{seq}$  and  $\mathcal{E}_{\mathcal{U}}$  both satisfy  $dom(\eta) = \pi^{-1}(\{a\})$ . Then:

a) In sequential semantics, the net is not observable: the run  $\omega_s \in \Omega(\mathcal{E}_{seq})$  which consists only of occurrences of  $u$  and  $v$  satisfies  $\mathcal{H}_{\eta}(\omega_s) = 0$  and  $\mathcal{H}_{\lambda}(\omega_s) = \infty$ . Further,  $(\mathcal{E}_{seq}, \eta)$  is neither F-diagnosable nor NF-diagnosable, since all runs without an occurrence  $y$  are observationally indiscernable from the run  $\omega'$  formed only by occurrences of  $a$  and  $b$ ; this  $\sim_{\eta}$  class therefore contains both faulty and healthy runs.

b) However, with the same assumptions,  $(\mathcal{E}_{\mathcal{U}}, \eta)$  is both observable and diagnosable; in fact, all runs  $\omega \in \Omega(\mathcal{E}_{\mathcal{U}})$  are F-definite.

This example shows that the choice of semantics may decide whether or not a given Petri net is diagnosable. The distinctions in the terminology - weak vs strong diagnosability - are in fact properties of execution semantics.

## 5 Conclusion

We have cast the dynamics of discrete event systems in a general framework that allows to compare properties of the non-sequential and the sequential behaviour. On the level of abstraction granted by event structures, observability and diagnosability become general topological properties that specialize to existing concrete notions once the semantics (sequential or non-sequential) has been chosen. The verification of diagnosability has been shown to **PSPACE**-complete for the sequential case in [BP08]. This theoretical bound is a fortiori true for the non-sequential case. It is important now to develop efficient algorithms for verification of *weak* diagnosability; strong diagnosability has received treated in the existing literature, see e.g. [MC09b, MC09a]). Current work is addressing these issues, based in particular on the results and an investigation of cutoff criteria for constructing suitable finite prefixes of unfoldings.

**Outlook:** The topological framework presented here has the advantage of allowing for unified proofs, based on the properties of event structures regardless of the semantics that generates them. It is applicable to any kind of system model that has an event structure semantics, and potentially useful for capturing extensions such as incomplete models, or loss of alarm. Future work will address such extensions.

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