



# Numerical experiments on a Factored Approximate Inverse Preconditioner

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# Numerical experiments on a Factored Approximate Inverse Preconditioner

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We consider an algebraic approach to preconditioning linear systems by computing approximate factors. Let  $\mathcal{W}$  and  $\mathcal{V}_1$  be sparse matrix subspaces of  $\mathbb{C}^{n \times n}$  over  $\mathbb{C}$  containing invertible elements and assume that nonsingular elements of  $\mathcal{V}_1$  are readily invertible. In addition, let the sparsity structures of  $\mathcal{W}$  and  $\mathcal{V}_1$  determine their dimension.

To approximately factor the inverse of a large and sparse nonsingular matrix  $A \in \mathbb{C}^{n \times n}$  into the product  $WV_1^{-1}$ , we consider the problem  $AW \approx V_1$  with non-zero matrices  $W \in \mathcal{W}$  and  $V_1 \in \mathcal{V}_1$  regarded as variables *both*. Denote by  $P_1$  an orthogonal projection onto  $\mathcal{V}_1$ . We then have the optimality criterion

$$\min_{W \in \mathcal{W}, \|W\|_F=1} \|(I - P_1)AW\|_F \quad (1)$$

for generating factors  $W$  and  $V_1 = P_1AW$ . For an algorithm for solving the minimization problem (1) and computing the approximate factors of the inverse, we refer to [1].

In this talk we focus on the choice of subspaces  $\mathcal{W}$  and  $\mathcal{V}_1$ . We also discuss how the quality of the computed preconditioner relates to the conditioning of the matrix  $V_1$  and consider the parallel scalability of the algorithm for computing the approximate factors in practice.

As a preprocessing step, we first use **MC64** to compute diagonal scaling matrices  $D_1$  and  $D_2$  and a permutation  $Q$  such that the matrix  $A_Q = D_1AD_2Q$  has nonzero diagonal entries [4]. We then apply a symmetric permutation  $P$  such that the entries which are large in magnitude are located either in the diagonal blocks or in the block upper triangular part of  $PA_QP^T$ . In our experiments, the permutation  $P$  is computed with **XPABLO** [5] or with a recent technique for finding strongly connected components of a matrix [3].

The sparsity structure of the subspace  $\mathcal{W}$  is determined by the sparsity structure of the powers of the sparsified matrix  $PA_QP^T$ , i.e., we use a technique similar to [2]. The sparsity structure of the subspace  $\mathcal{V}_1$  is chosen as the sparsity structure of the block diagonal or the block upper triangular part of the permuted matrix  $PA_QP^T$ . With these, the algorithm for generating the factors  $W$  and  $V_1$  can be interpreted as a process for improving a preconditioner  $V_1 = P_1A$ .

## References

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