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Fuel-Optimal Trajectories for Continuous-Thrust Orbital Rendezvous with Path Constraints

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Aerospace Applications of Control and Optimization,
ENSTA ParisTech, Paris, March 2, 2011



Outline

- 1 Problem statement
 - Dynamical equations
 - Optimal control formulation
- 2 Solving the path-constrained rendezvous problem
 - Smoothing the bang-off-bang control
 - A new approach to deal with the state constraint
- 3 Numerical results - A rendezvous in Highly Elliptical Orbit
 - Statement of the test case
 - Unconstrained rendezvous
 - Rendezvous under collision avoidance constraint
- 4 Conclusion and future prospects

Problem statement

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Problem statement

Dynamical equations

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Tschauner-Hempel equations in Hill's frame

- Keplerian motions - Small intersatellite distance
- (a, e, ν) : semi-major axis, eccentricity and true anomaly of the target satellite

- $X(\nu), Y(\nu), Z(\nu)$: relative coordinates of the chaser

$$\begin{bmatrix} x_1(\nu) \\ x_2(\nu) \\ x_3(\nu) \end{bmatrix} = (1 + e \cos(\nu)) \begin{bmatrix} X(\nu) \\ Y(\nu) \\ Z(\nu) \end{bmatrix}$$

- $x_4(\nu), x_5(\nu), x_6(\nu)$: derivatives of $x_i(\nu)$, ($i = 1, \dots, 3$) w.r.t. ν
- $m(\nu)$: mass of the chaser at true anomaly ν
- $u(\nu)$: normalized thrust vector of the chaser at true anomaly ν

Problem statement

Optimal control formulation

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 - Dynamical equations
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State-constrained minimum-fuel rendezvous (1/2)

The problem to solve

$$(P) \left\{ \begin{array}{l} \text{Find } \bar{u} = \underset{u}{\operatorname{argmin}} J(u) = -m(v_f) \\ \text{s.t.} \\ \dot{x}(v) = A(v)x(v) + B(v) \frac{u(v)}{m(v)} \\ \dot{m}(v) = -c(v) \|u(v)\| \\ \|u(v)\| \leq 1 \quad v \in [v_0, v_f] \\ g(v, x(v)) \leq 0 \quad v \in [v_0, v_f] \\ x(v_0) = x_0 \quad h(x(v_f)) = 0 \\ m(v_0) = m_0 \end{array} \right.$$

State-constrained minimum-fuel rendezvous (2/2)

Collision avoidance constraint

$$g(v, x(v)) = 1 - \frac{\sqrt{x_1(v)^2 + x_2(v)^2 + x_3(v)^2}}{d_{min}(1 + e\cos(v))} \leq 0 \quad v \in [v_0, v_f]$$

Key parameter

d_{min} : minimum safety distance between the chaser and the target

Main issues for shooting methods

- The control is bang-off-bang \implies numerical difficulties
- State constraint \implies the number and location of constrained arcs must be defined beforehand in order to build the MPBVP

Solving the path-constrained rendezvous problem

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- 3 Numerical results - A rendezvous in Highly Elliptical Orbit
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Solving the path-constrained rendezvous problem

Smoothing the bang-off-bang control

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 - Dynamical equations
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 - Unconstrained rendezvous
 - Rendezvous under collision avoidance constraint
- 4 Conclusion and future prospects

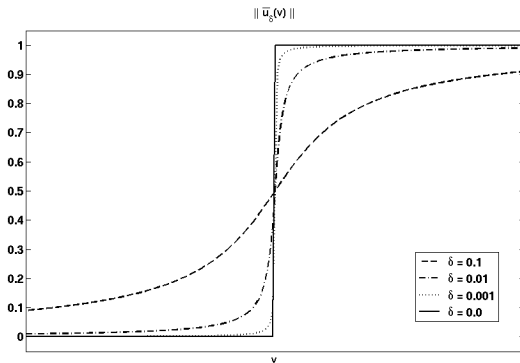
Problem regularization (1/2)

A logarithmic barrier approach

$$(P)_\delta \left\{ \begin{array}{l} \text{Find } \bar{u}_\delta = \underset{u}{\operatorname{argmin}} J_\delta(u) = J(u) - \delta \int_{v_0}^{v_f} F(v, u(v)) dv \\ \text{s.t.} \\ \dot{x}(v) = A(v)x(v) + B(v) \frac{u(v)}{m(v)} \\ \dot{m}(v) = -c(v) \|u(v)\| \\ \|u(v)\| \leq 1 \quad v \in [v_0, v_f] \\ g(v, x(v)) \leq 0 \quad v \in [v_0, v_f] \\ x(v_0) = x_0 \quad h(x(v_f)) = 0 \\ m(v_0) = m_0 \end{array} \right.$$

$$F(v, u(v)) = c(v) (\log(\|u(v)\|) + \log(1 - \|u(v)\|))$$

Problem regularization (2/2)

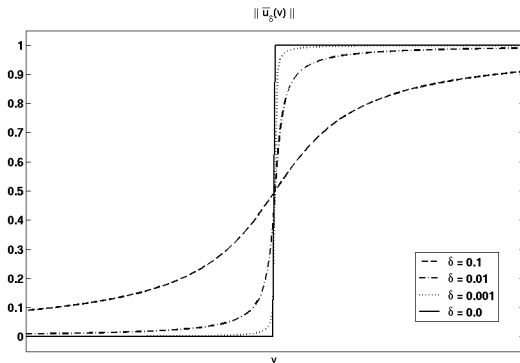


Convergence results - No state constraint

$$J_\delta(\bar{u}_\delta) \rightarrow J(\bar{u}) \quad \text{as } \delta \rightarrow 0$$

$$\bar{u}_\delta \rightarrow \bar{u} \quad \text{for the weak-* topology on } L^\infty([v_0, v_f], \mathbb{R}^3) \quad \text{as } \delta \rightarrow 0$$

Problem regularization (2/2)



Reference

R. Epenoy and R. Bertrand: New Smoothing Techniques for Solving Bang-Bang Optimal Control Problems - Numerical Results and Statistical Interpretation, *Optimal Control Applications and Methods*, Vol. 23, No. 4, 2002, pp. 171-197.

Solving the path-constrained rendezvous problem

A new approach to deal with the state constraint

- 1 Problem statement
 - Dynamical equations
 - Optimal control formulation
- 2 Solving the path-constrained rendezvous problem
 - Smoothing the bang-off-bang control
 - A new approach to deal with the state constraint
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The smoothed exact penalty method

The penalized problem $(P)_{\sigma,\alpha,\epsilon}^2$

$$\left\{ \begin{array}{l} \text{Find } u_{\sigma,\alpha,\epsilon}^2 = \underset{u}{\operatorname{argmin}} J_{\sigma,\alpha,\epsilon}^2(u) = J_{\delta}(u) + \int_{v_0}^{v_f} \psi_{\sigma,\alpha,\epsilon}(g(v, x(v))) dv \\ \text{s.t.} \\ \dot{x}(v) = A(v)x(v) + B(v) \frac{u(v)}{m(v)} \\ \dot{m}(v) = -c(v) \|u(v)\| \\ \|u(v)\| \leq 1 \quad v \in [v_0, v_f] \\ x(v_0) = x_0 \quad h(x(v_f)) = 0 \\ m(v_0) = m_0 \end{array} \right.$$

$$\psi_{\sigma,\alpha,\epsilon}(z) = \sigma \log \left(1 + \exp \left[\frac{z}{\sigma \epsilon} \left(1 + \frac{\epsilon}{\alpha - z} \right) \right] \right) \xrightarrow{\sigma \rightarrow 0} \operatorname{Max} \left\{ 0, \frac{z}{\epsilon} \left(1 + \frac{\epsilon}{\alpha - z} \right) \right\}$$

The smoothed exact penalty method

The penalized problem $(P)_{\sigma,\alpha,\epsilon}^2$

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G. Liuzzi and S. Lucidi: A Derivative-Free Algorithm for Inequality Constrained Nonlinear Programming via Smoothing of an l_{∞} Penalty Function, *SIAM Journal on Optimization*, Vol. 20, No. 1, 2009, pp. 1-29.

Algorithm

Let $0 < q_1 < q_2 < 1$, $0 < \alpha_{lim} < \alpha_0$, $\epsilon_0 > 0$, $\sigma_0 > \alpha_{lim}^{q_1}$, $0 < \theta < 1$, $0 < \tau < 1$

Let $k = 0$, end = false

WHILE (end = false)

Solve problem $(P)_{\sigma_k, \alpha_k, \epsilon_k}^2 \rightarrow (x_{\sigma_k, \alpha_k, \epsilon_k}^2, m_{\sigma_k, \alpha_k, \epsilon_k}^2, u_{\sigma_k, \alpha_k, \epsilon_k}^2)$

IF $(\alpha_k \leq \alpha_{lim})$ THEN

end = true

$(\bar{x}_\delta, \bar{m}_\delta, \bar{u}_\delta) \leftarrow (x_{\sigma_k, \alpha_k, \epsilon_k}^2, m_{\sigma_k, \alpha_k, \epsilon_k}^2, u_{\sigma_k, \alpha_k, \epsilon_k}^2)$

ELSE

IF $\text{Min} \left\{ \epsilon_k, \int_{v_0}^{v_f} \text{Max} \left(0, g(v, x_{\sigma_k, \alpha_k, \epsilon_k}^2(v)) \right) dv \right\} > \frac{\alpha_k^{q_2}}{\sigma_k}$ THEN

$$\epsilon_{k+1} = \tau \frac{\alpha_k^{q_2}}{\sigma_k}$$

ELSE

$$\epsilon_{k+1} = \epsilon_k$$

ENDIF

$$\alpha_{k+1} = \theta \alpha_k$$

$$\sigma_{k+1} = \text{Min} \left\{ \sigma_k, \alpha_{k+1}^{q_1} \right\}$$

$$k = k + 1$$

ENDIF

END WHILE

Convergence results

Lemma

- Let $\alpha > 0$, $z < \alpha$ and $\epsilon > 0$ be given. Then, $(\sigma \rightarrow \psi_{\sigma,\alpha,\epsilon}(z))$ is strictly increasing on $]0, +\infty[$
- Let $z < 0$, $\epsilon > 0$ and $\sigma > 0$ be given. Then, $(\alpha \rightarrow \psi_{\sigma,\alpha,\epsilon}(z))$ is strictly increasing on $]0, +\infty[$

Convergence theorem - See Epenoy (2011) in JGCD

- $J_{\sigma_k, \alpha_k, \epsilon_k}^2(u_{\sigma_k, \alpha_k, \epsilon_k}^2) \rightarrow J_\delta(\bar{u}_\delta)$ as $k \rightarrow \infty$
- $u_{\sigma_k, \alpha_k, \epsilon_k}^2 \rightarrow \bar{u}_\delta$ according to the weak-* topology on $L^\infty([v_0, v_f], \mathbb{R}^3)$ as $k \rightarrow \infty$
- $x_{\sigma_k, \alpha_k, \epsilon_k}^2 \rightarrow \bar{x}_\delta$ uniformly on $[v_0, v_f]$ as $k \rightarrow \infty$
- $m_{\sigma_k, \alpha_k, \epsilon_k}^2 \rightarrow \bar{m}_\delta$ uniformly on $[v_0, v_f]$ as $k \rightarrow \infty$

Numerical results - A rendezvous in Highly Elliptical Orbit

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 - Dynamical equations
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- 3 Numerical results - A rendezvous in Highly Elliptical Orbit
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 - Unconstrained rendezvous
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Numerical results - A rendezvous in Highly Elliptical Orbit

Statement of the test case

- 1 Problem statement
 - Dynamical equations
 - Optimal control formulation
- 2 Solving the path-constrained rendezvous problem
 - Smoothing the bang-off-bang control
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- 3 Numerical results - A rendezvous in Highly Elliptical Orbit
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Numerical data - SIMBOL-X project

$$a = 106246.9753 \text{ km}$$

$$F_{max} = 0.1 \text{ N}$$

$$m_0 = 960.0 \text{ kg}$$

$$e = 0.798788$$

$$I_{sp} = 220 \text{ s}$$

$$v_0 = 3.317940017547 \text{ rad}$$

$$t_0 = 0.0 \text{ s}$$

$$v_f = 3.349161118514 \text{ rad}$$

$$t_f = 8000.0 \text{ s}$$

$$\begin{bmatrix} X(t_0) \\ Y(t_0) \\ Z(t_0) \end{bmatrix} = \begin{bmatrix} -100 \text{ m} \\ -100 \text{ m} \\ -100 \text{ m} \end{bmatrix}$$

$$\begin{bmatrix} X(t_f) \\ Y(t_f) \\ Z(t_f) \end{bmatrix} = \begin{bmatrix} 500 \text{ m} \\ 500 \text{ m} \\ 500 \text{ m} \end{bmatrix}$$

$$\begin{bmatrix} \frac{dX}{dt}(t_0) \\ \frac{dY}{dt}(t_0) \\ \frac{dZ}{dt}(t_0) \end{bmatrix} = \begin{bmatrix} 0.0 \text{ m/s} \\ 0.0 \text{ m/s} \\ 0.0 \text{ m/s} \end{bmatrix}$$

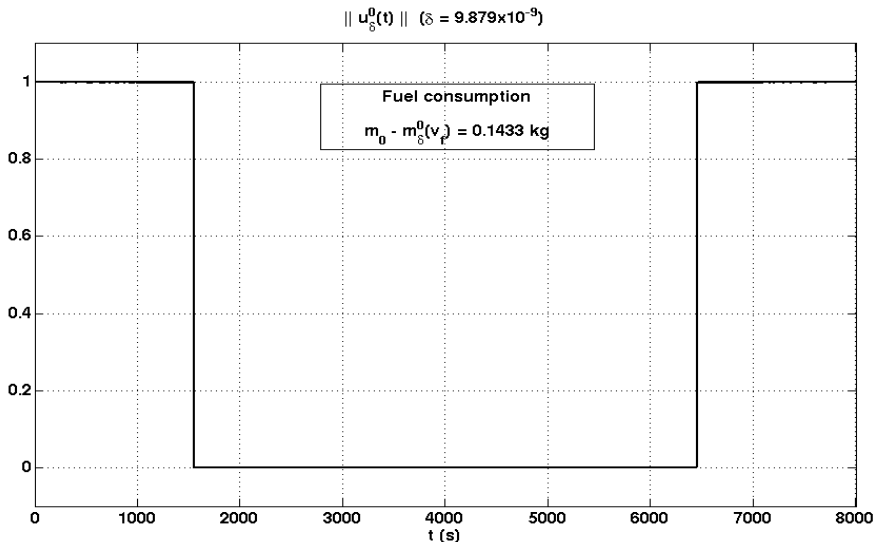
$$\begin{bmatrix} \frac{dX}{dt}(t_f) \\ \frac{dY}{dt}(t_f) \\ \frac{dZ}{dt}(t_f) \end{bmatrix} = \begin{bmatrix} 0.0 \text{ m/s} \\ 0.0 \text{ m/s} \\ 0.0 \text{ m/s} \end{bmatrix}$$

Numerical results - A rendezvous in Highly Elliptical Orbit

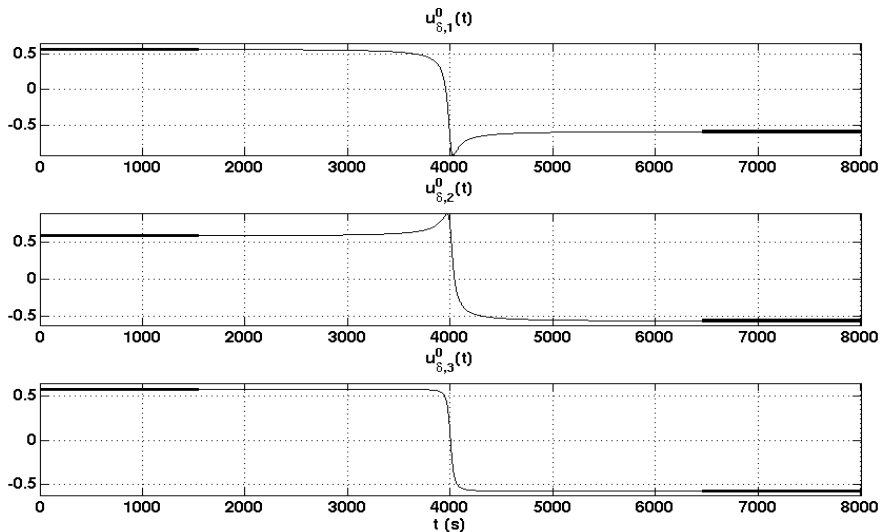
Unconstrained rendezvous

- 1 Problem statement
 - Dynamical equations
 - Optimal control formulation
- 2 Solving the path-constrained rendezvous problem
 - Smoothing the bang-off-bang control
 - A new approach to deal with the state constraint
- 3 Numerical results - A rendezvous in Highly Elliptical Orbit
 - Statement of the test case
 - **Unconstrained rendezvous**
 - Rendezvous under collision avoidance constraint
- 4 Conclusion and future prospects

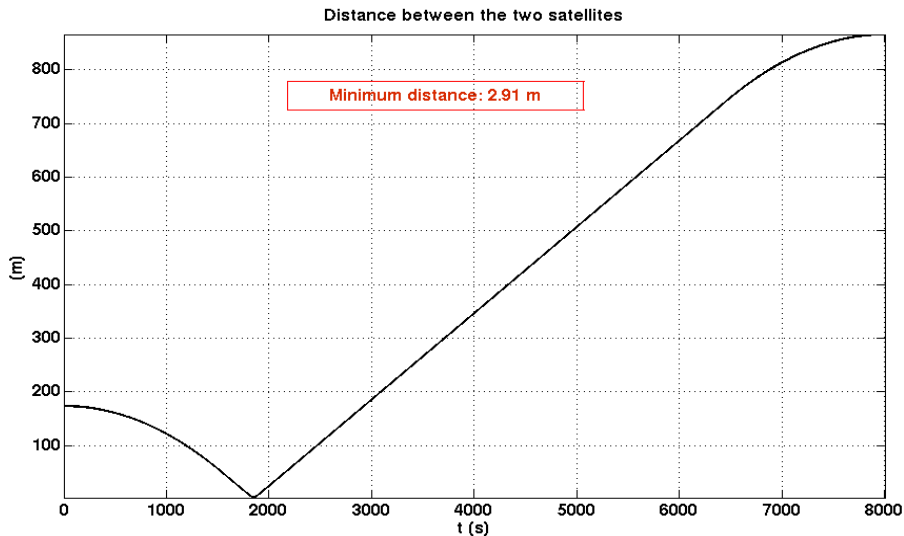
Norm of the normalized thrust vector



Normalized thrust vector



Intersatellite distance



Numerical results - A rendezvous in Highly Elliptical Orbit

Rendezvous under collision avoidance constraint

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 - Dynamical equations
 - Optimal control formulation
- 2 Solving the path-constrained rendezvous problem
 - Smoothing the bang-off-bang control
 - A new approach to deal with the state constraint
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Parameters of the algorithm

Smoothing parameter: The same as for the unconstrained problem

$$\delta = 9.879 \times 10^{-9}$$

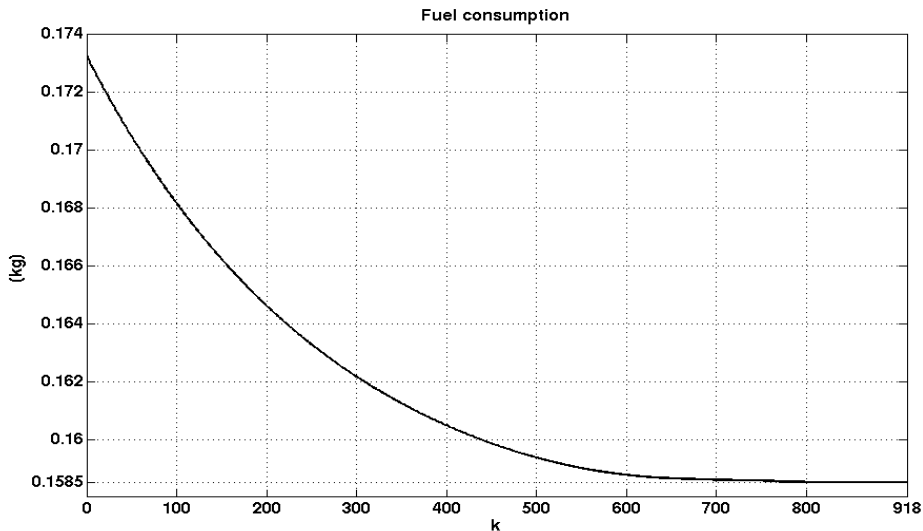
Case 1: $d_{min} = 50.0$ m

$q_1 = 0.3$	$q_2 = 0.6$	$\alpha_{lim} = 4.73 \times 10^{-4}$	$\alpha_0 = 0.98$
$\epsilon_0 = 0.0999$	$\sigma_0 = 1.0$	$\theta = 0.99$	$\tau = 0.9$

Case 2: $d_{min} = 140.0$ m

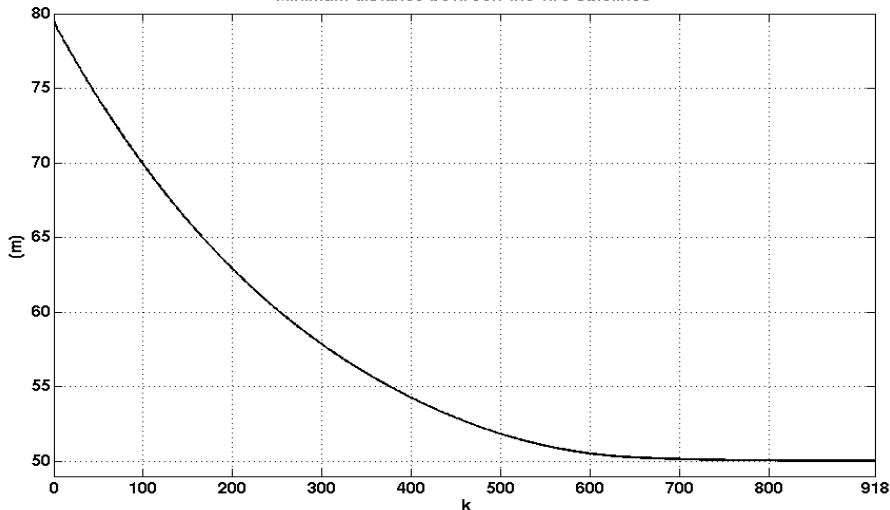
$q_1 = 0.3$	$q_2 = 0.6$	$\alpha_{lim} = 1.37 \times 10^{-4}$	$\alpha_0 = 0.98$
$\epsilon_0 = 0.0157$	$\sigma_0 = 1.0$	$\theta = 0.99$	$\tau = 0.9$

Case 1 - Fuel consumption vs. iteration index k

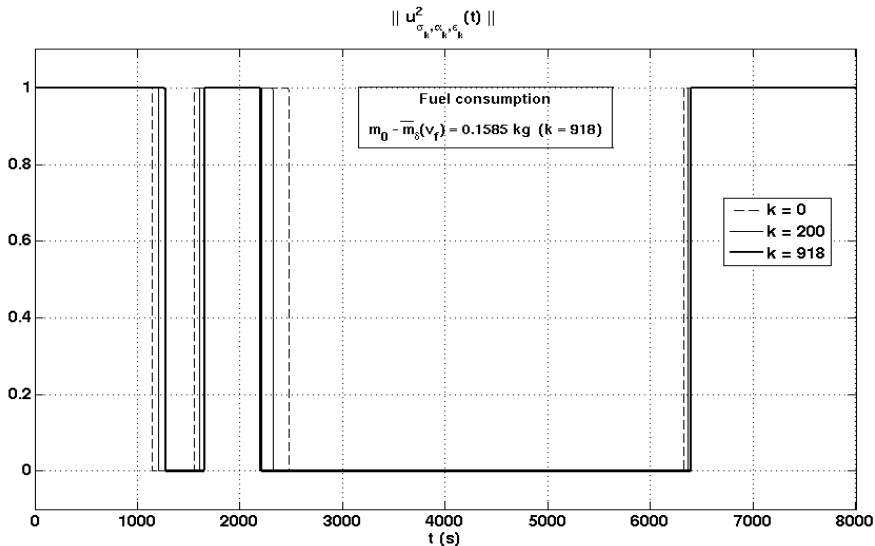


Case 1 - Minimum distance vs. iteration index k

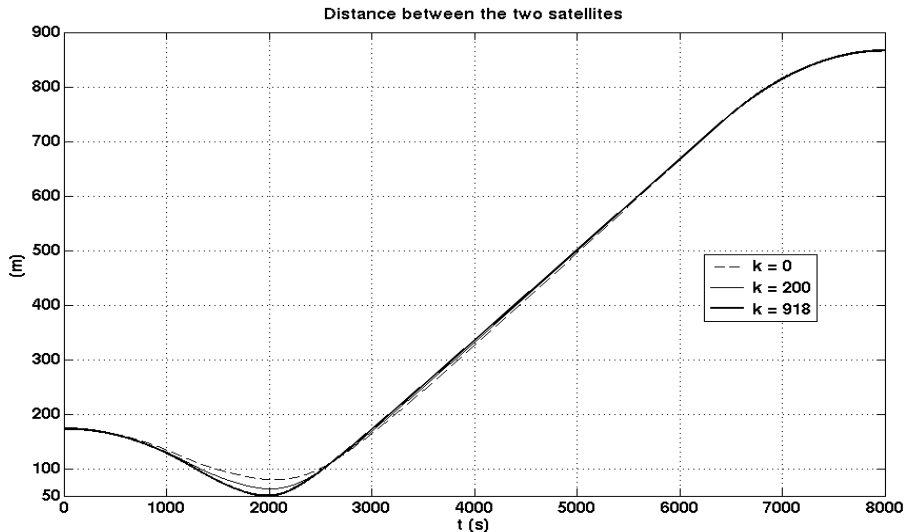
Minimum distance between the two satellites



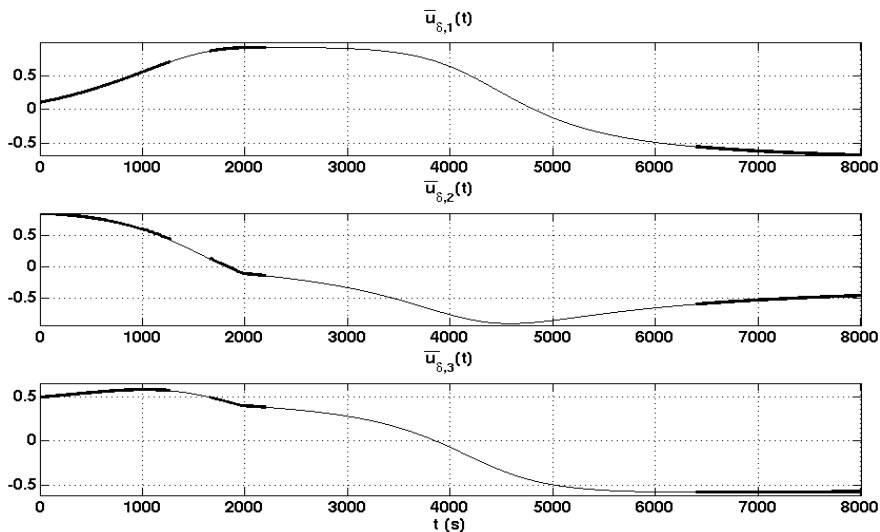
Case 1 - Norm of the normalized thrust vector



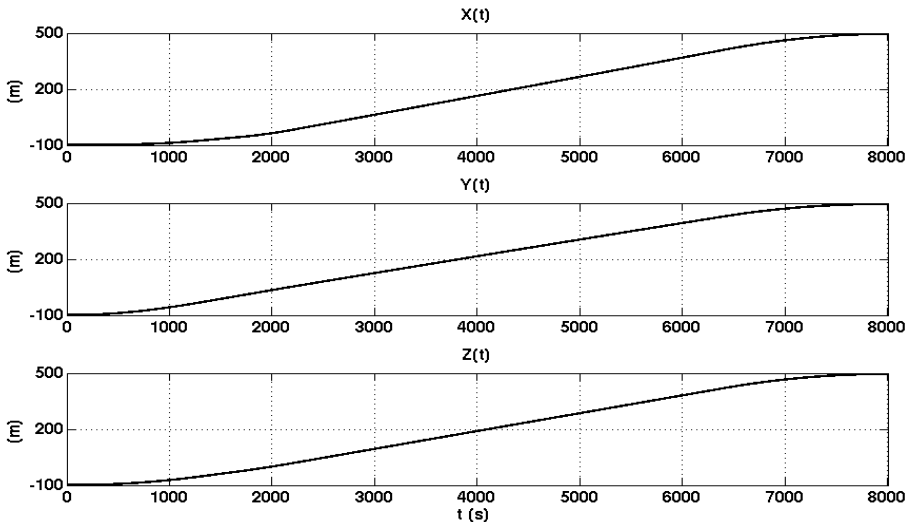
Case 1 - Intersatellite distance



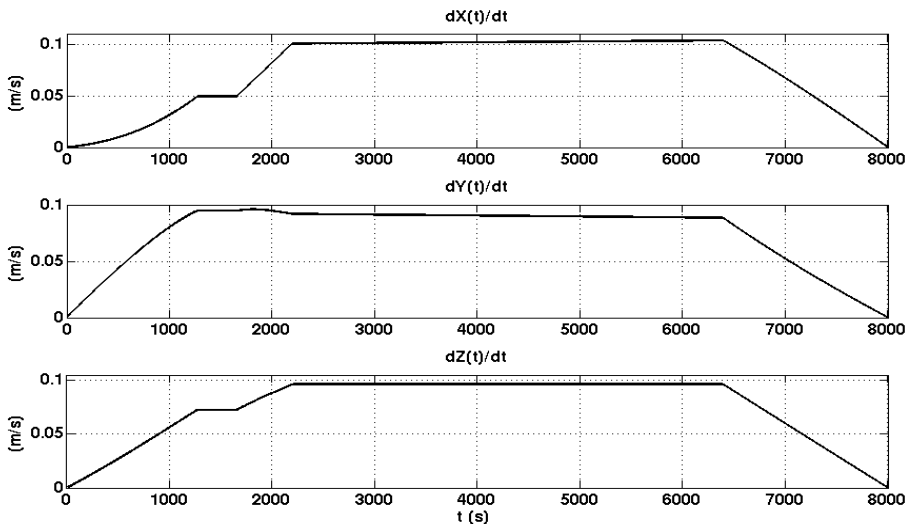
Case 1 - Normalized thrust vector



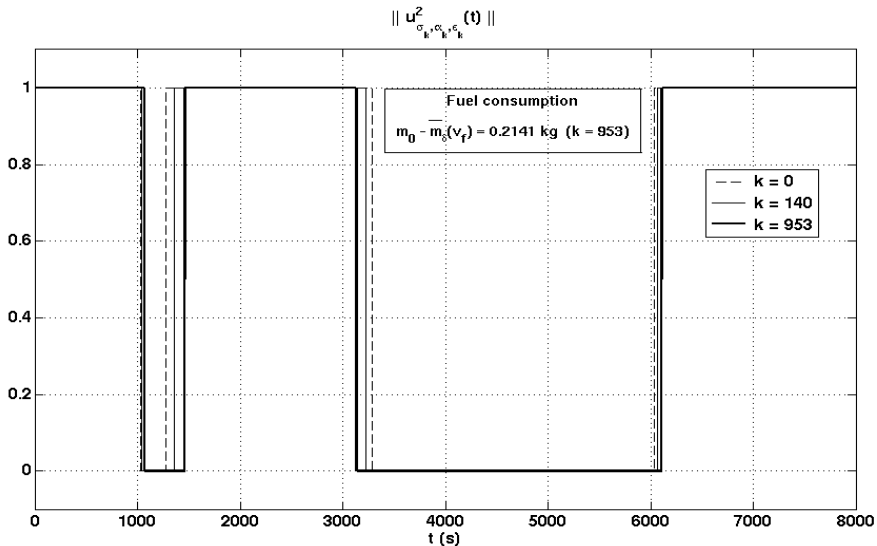
Case 1 - Relative position vector



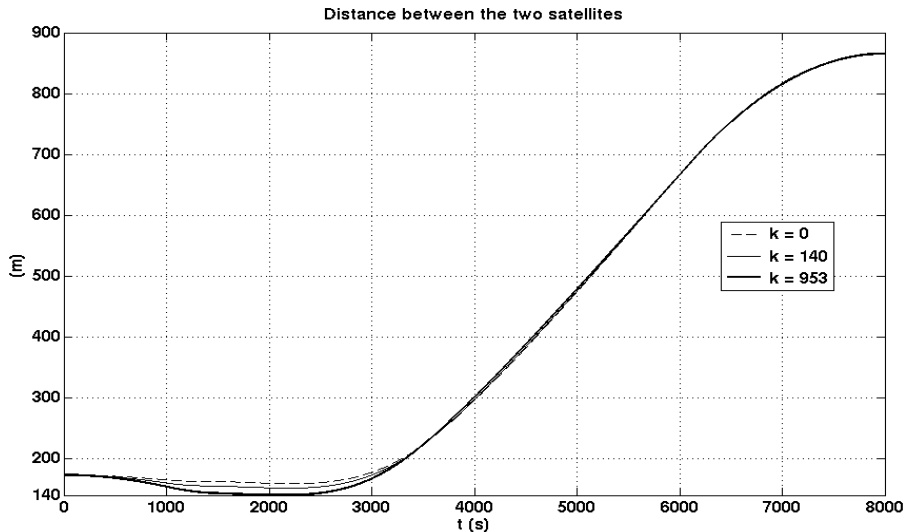
Case 1 - Relative velocity vector



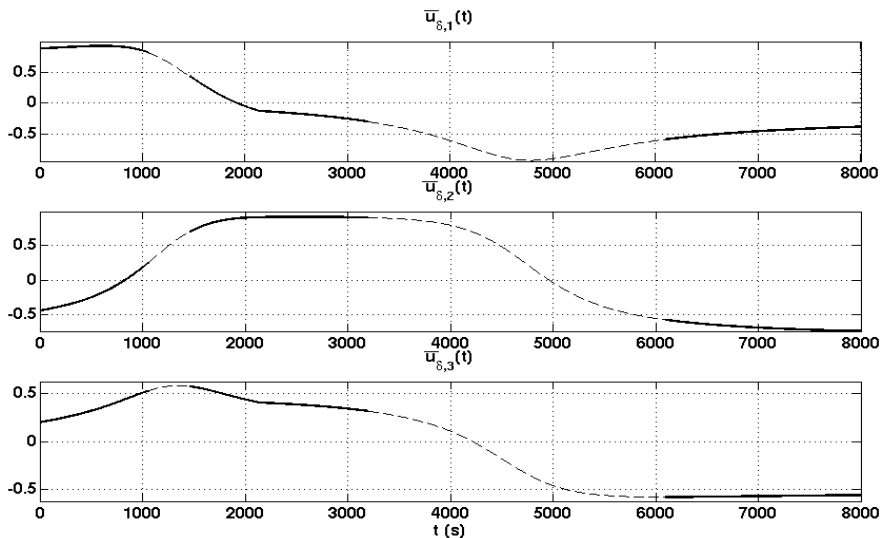
Case 2 - Norm of the normalized thrust vector



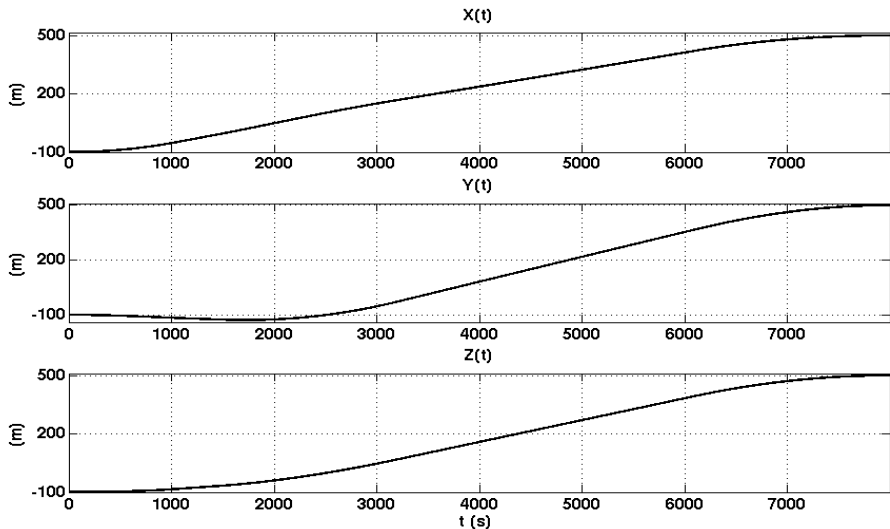
Case 2 - Intersatellite distance



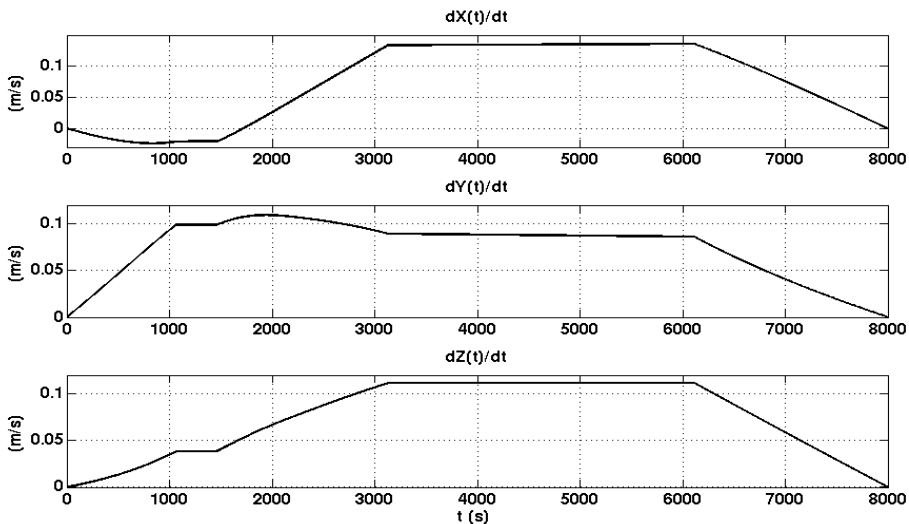
Case 2 - Normalized thrust vector



Case 2 - Relative position vector



Case 2 - Relative velocity vector



Conclusion and future prospects

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 - Dynamical equations
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Conclusion

The smoothed exact penalty approach

- Applied for the first time to an optimal control problem
- Efficient to deal with the collision avoidance constraint
- Implies solving a sequence of state constraint-free problems
- More theoretically based than other penalization techniques
- Can be used to solve a large class of problems

A just published paper

R. Epenoy: Fuel Optimization for Continuous-Thrust Orbital Rendezvous with Collision Avoidance Constraint, *Journal of Guidance, Control and Dynamics*, Vol. 34, No. 2, March-April 2011.

Future prospects

Minimum-fuel rendezvous in perturbed environment

Necessity to modify the dynamical equations

Low Earth Orbit applications $\implies J_2$ term of the Earth's potential

Application to other state-constrained problems

Reentry trajectories under thermal flux constraint

Interplanetary trajectories with minimum flyby altitude constraint

Toward closing the loop

Model Predictive Control methodology

Neighboring extremal paths in the presence of state constraints

Hamilton-Jacobi-Bellman equation for state-constrained problems

Thank you for your attention