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# Analysis of flow over cylinders using the AIRONUM software

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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

*Analysis of flow over cylinders using  
the AIRONUM software*

Stephen F. Wornom

N° 7550

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*R*apport  
de recherche



# Analysis of flow over cylinders using the AIRONUM software

Stephen F. Wornom \*

Theme :  
Applied Mathematics, Computation and Simulation  
Équipes-Projets Tropics

Rapport de recherche n° 7550 — — 22 pages

**Abstract:** Flow over cylinders has played an important role in the development of CFD methods and turbulence models in the past 50 years and will continue to play an important role in the future. Due to its geometry facilitating experiments and numerical studies, cylinders flows are among the simplest test case for studying the effect of Reynolds number on bluff body flows involving recirculation regions. For these reasons, the AIRONUM CFD software has added specific features to simplify the analysis for users; automatically computing the following meanflow values: drag and lift coefficients, base pressure coefficient, flow separation angle, L/D of the recirculation zone, the max/min U-velocity, and the surface pressure coefficient, while the simulation is executing. Meanflow ParaView graphics files are also automatically at the completion of the simulation. Also tools to analyse the time series values of drag, lift, separation angles, base pressure coefficients, L/D are included in the software. These analysis features are found in the AIRONUM software version 4.1 and later.

**Key-words:** Flow over cylinders, meanflow characteristics, low Mach, unstructured meshes, VMS-LES-WALE and LES-WALE turbulence models

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## Analysis of flow over cylinders using the AIRONUM software

**Résumé :** L'écoulement autour d'un cylindre a joué un rôle important dans le développement des modèles de la turbulence durant les 50 dernières années et continueront à jouer un rôle important dans l'avenir. En raison de la simplicité de la géométrie le cylindre permet l'étude de l'effet du nombre de Reynolds sur les écoulements autour des corps arrondis avec des zones de recirculation. Pour ces raisons, on a ajouté au logiciel AIRONUM des routines spécifiques pour simplifier l'analyse par l'utilisateur ; on a automatisé le calcul des quantités suivantes tirées de l'écoulement moyen : traînée , portance, coefficient de pression superficiel, angle de décollement de l'écoulement, rapport L/D de la zone de recirculation, rapport vitesse maximum/minimum, et coefficient extérieur de pression. Les structures de donnée de l'écoulement moyen pour ParaView sont également automatiquement créées à la fin de la simulation. Sont également installés des outils d'analyse des séries chronologiques des traînée, portance, angles de séparation, coefficients de pression surface, et rapport L/D.

**Mots-clés :** Ecoulement autour des cylindres, caractéristiques de l'écoulement moyen, petit Mach, les mailles non structurées, model de turbulence VMS-LES, SGS WALE

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## 1 Introduction

Flow over cylinders has played an important role in the development of CFD methods and turbulence models in the past 50 years and will continue to play an important role in the future. Due to its simple geometry facilitating experiments and numerical studies, cylinders flows are among the simplest test case for studying the effect of Reynolds number on buff body flows involving recirculation regions.

To simplify the analysis for users studying flow over cylinders, the AIRONUM CFD software has added specific features that automatically compute the meanflow drag and lift coefficients, base pressure coefficient, flow separation angle, L/D of the recirculation zone, the max/min U-velocity, and the surface pressure coefficient, while the simulation is executing. The meanflow ParaView graphics files are also automatically written at the completion of the simulation.

Specifically for cylinder analysis, two keywords<sup>1</sup> have been added to the list of keywords found in the subroutine Don3D.f. these are: 1) cylinderAnalysis and 2) computeMeanFlow. The next sections explains these keywords and what they do.

## 2 keyword: computeMeanFlow

The computeMeanFlow option was developed at INRIA to compute the meanflow. Note that AIRONUM has a second keyword relating to cylinder flow analysis, "pstp", to computed the meanflow files with the prefix promedia to be analyzed using MATLAB (developed at the University of Pisa, Italy.) The user decides which keyword(s) they want to use (both can be used). Presently,

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<sup>1</sup>Data options in the AIRONUM software are activated with keywords. The DEFAULT is nonactive.



the "cylinderAnalysis" option only works when combined with the "computeMeanFlow" option.

A grep for computeMeanFlow in the subroutine Nsc3Dm.F shows seven options for the keyword "computeMeanFlow":

- 0 inactive (DEFAULT)
- 1 compute MeanFlow solution (initial run)
- 2 compute MeanFlow solution (restart run)

Options 3-6 are discussed in section A.

### **3 keywords: computeMeanFlow+cylinderAnalysis**

The option `computeMeanFlow > 0` is very useful when combined with `cylinderAnalysis` as creates the meanflow Paraview files and the following files:

- `cylinder_delta_rad_wall.data`: Gives the radial step size at the cylinder surface.
- `cylinder_Mean_cpbase_using_pdyn_inf_Mesh.data`: Gives the upper/lower and average base pressure based on the mesh dynamic pressure.
- `cylinder_Mean_drag_using_pdyn_inf_Mesh.data`: Gives the mean drag coefficient based on the mesh dynamic pressure.
- `cylinder_Mean_LOD_recirculation_zone.data`: Gives the mean L/D value.
- `cylinder_Mean_separation_angle_using_tau.data`: Gives the separation angle based on the tau method.
- `cylinder_Mean_separation_angle_using_umin.data`: Gives the separation angle based on the umin method.

- `cylinder_pressure_coefficient_lower.data`: Gives the upper surface pressure coefficient based on upstream infinity conditions and the pressure coefficient based upon the mesh upstream dynamic pressure and pressure.
- `cylinder_pressure_coefficient_upper.data`: Gives the lower surface pressure coefficient based on upstream infinity conditions and the pressure coefficient based upon the mesh upstream dynamic pressure and pressure.
- `cylinder_upstream_values.data`: Give the mesh upstream non-dimensional values of density, velocity, and pressure.
- `cylinder_Mean_maxUmin.data`: Gives the minimum/maximum meanflow u-velocity.
- `cylinder_Mesh_pdyn.data`: Gives theoretical and mesh dynamic pressures.

## 4 Notes on Meshes

A mesh with 1.8 million mesh vertices was used. The mesh was created for a Reynolds number 20000 study using integration to the wall. However, examination of the mesh shows that it does not have sufficient refinement near the cylinder for a  $Re=20000$  study but is well suited for  $Re=3900$ .

### 4.1 Meanflow analysis

Using the "pstp" and analysing the results using MATLAB assumes that the cylinder surface vertices lie on  $\Theta = \text{constant}$  lines as MATLAB averages the values along span lines.

The "computeMeanFlow" options are more general in that it does NOT average<sup>2</sup> over  $\Theta = \text{constant}$  span lines. The "com-

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<sup>2</sup>At the present time but this feature could be added in the future.

puteMeanFlow" averages over the surface faces that touch the z=center plane.

## 5 Results using the WALE turbulence models

The keyword "cylinderAnalysis" combined with the keyword "computeMeanFlow" will be explained with several test cases using the LES-WALE model of Nicoud and Ducos [8] and the VMS<sup>3</sup>-LES-WALE version.

### 5.1 Surface pressure coefficient

The surface pressure coefficient is defined as:

$$Cp = (P - P_{pref}) / (\text{Dynamic pressure}) \quad (1)$$

where

$$\text{Dynamic pressure} = (1/2\rho_{ref}U_{ref}^2) \quad (2)$$

and *ref* are the upstream infinity values.

head cylinder\_pressure\_coefficient\_upper.data

| #             | X            | theta | Cp-Pdyn      | Cp-PdynMesh  |
|---------------|--------------|-------|--------------|--------------|
| -0.4999366000 | 0.9124348113 |       | 1.2985039761 | 1.0058836483 |
| -0.4998081000 | 1.5874644435 |       | 1.2960973022 | 1.0034769744 |
| -0.4992902333 | 3.0532461332 |       | 1.2871981327 | 0.9945778050 |

shows four columns: X, theta, the pressure coefficient based on upstream theoretical infinity conditions and the pressure coefficient based upon the mesh upstream dynamic pressure(PdynMesh) and pressure.

<sup>3</sup>Variational-Multi-Scale version of the LES-WALE model.

Shown in Figures 1-2 are the surface pressure coefficients using the undisturbed dynamic pressure<sup>4</sup> and the dynamic pressure at the mesh upstream inflow boundary (5 Diameters upstream from the cylinder center). As can be seen in Figure 1, the surface pressure coefficient based on the undisturbed dynamic pressure gives extremely poor agreement with the experimental data of Norberg (2002) for both the LES-WALE and VMS-LES-WALE models whereas using the dynamic pressure at the upstream mesh inflow boundary gives good agreement with the experimental data. A probable reason why using the the undisturbed dynamic pressure to compute the pressure coefficient is due to the fact that characteristic boundary conditions were used at the mesh exterior boundaries located at a finite distance from the cylinder and not at infinity, thus the infinity dynamic pressure is not applied. Future meshes should be constructed so the that the upstream mesh boundary is greater than five diameters upstream from the cylinder center.

Note that any small differences in estimating the dynamic pressure at the upstream mesh inflow boundary greatly affects the agreement with the experimental data. This is illustrated in Figure 3 where the mesh dynamic pressure coefficient has been increased by five percent (assumes it was underestimated by five percent). The point of this figure is to show the sensitivity of the pressure coefficient to the dynamic pressure used to compute it<sup>5</sup>.

### 5.1.1 Surface pressure coefficient: median vs Barth cells

Figure 4 compares the surface pressure coefficient using median and Barth cells [11]. The cells near the cylinder surface are highly stretch to capture the surface boundary layer. For the median

---

<sup>4</sup>Theoretically at upstream infinity.

<sup>5</sup>The mesh upstream dynamic pressure is computed in the subroutine GetUpStream.f. Here it was taken at the average values over the most upstream face nearest the centerline. This is an approximation and further study is needed to resolve how PdynMesh should be computed.

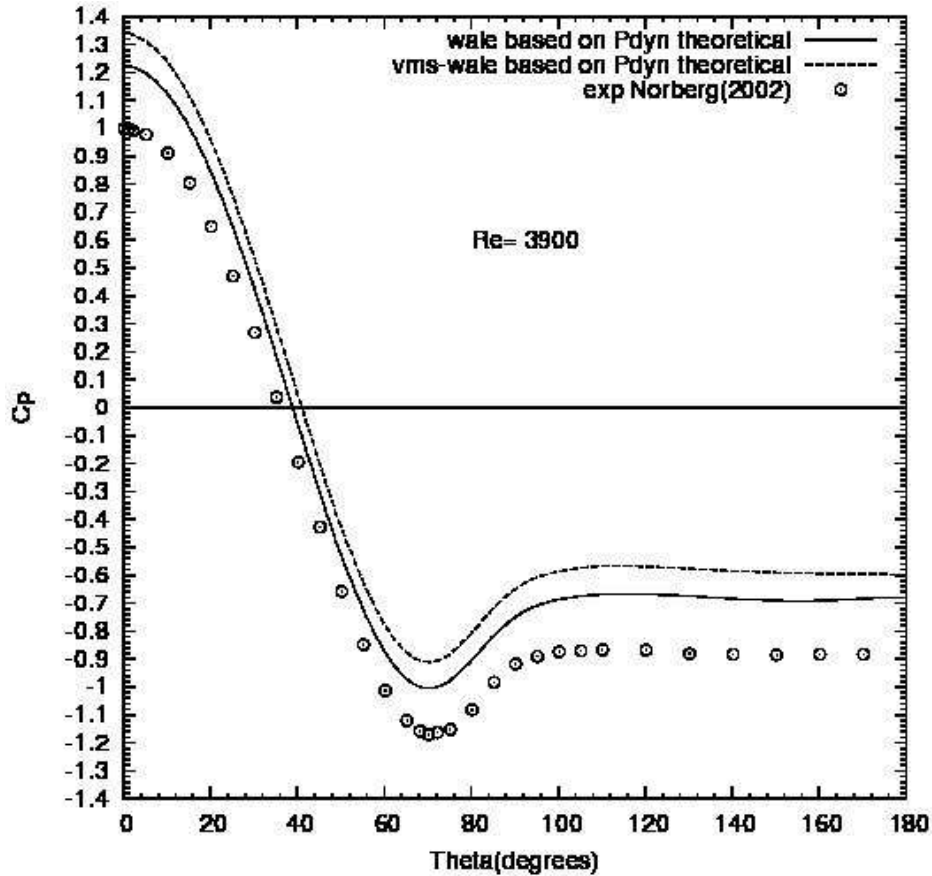


Figure 1: Pressure coefficient based on Pdyn theoretical

method, the control volume is limited by part of the medians of surrounding triangles (2D). With the Barth method[11], the control volume is built joining the center of edges with the center of the smallest circle containing the considered triangle. For highly stretched cells, the method of Barth produces a more accurate solution as can be seen in the pressure coefficient.

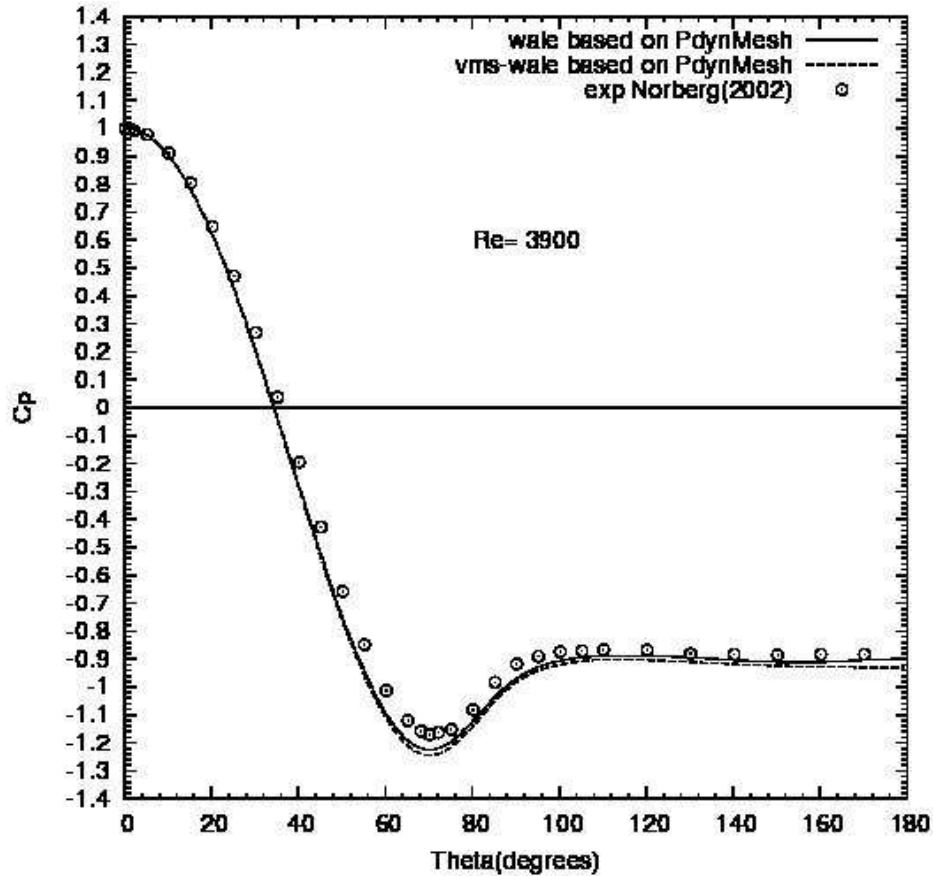


Figure 2: Pressure coefficient based on PdynMesh

## 6 Meanflow separation angle

The meanflow separation angle is calculated by two methods. The tau-method is a shear-stress method. Using integration to the wall, we create a 2D stress vector from the stress tensor components  $\vec{S}$  and a 2D vector normal to the cylinder surface,  $\vec{N}$ . Flow separation occurs when the vector product  $\vec{S} \cdot \vec{N}$  changes sign. Figure 6 shows the case when the method is applied to the meanflow solution for The VMS-LES-Smagorinsky model, the VMS-LES-WALE and LES-WALE models are similar. The scale of the vertical axis is not important as we are only interested in when the vector product changes sign.

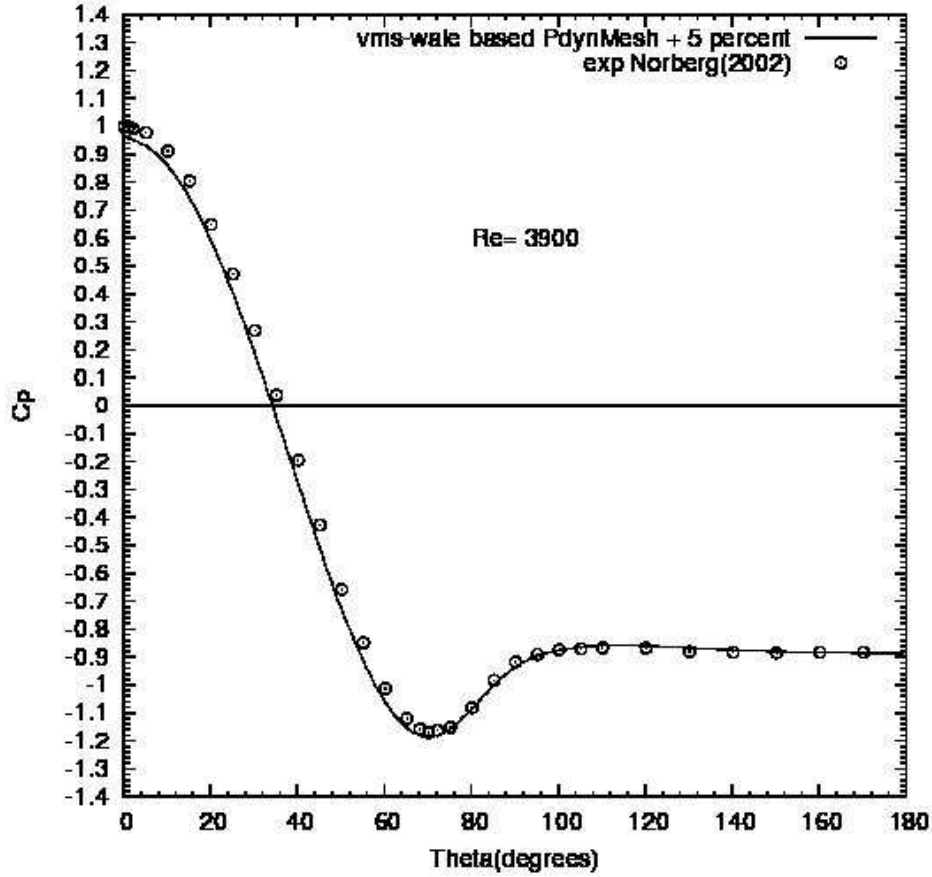


Figure 3: Pressure coefficient based on PdynMesh with a 5% correction

The umin-method, shown in Figure 7, locates the upstream minimum x-location for negative u-velocity. This occurs near the cylinder surface in the recirculation region. The umin-method is valid for integration to the wall as well as the wall law whereas the tau-method has been validated only for the case of integration to the wall. The separation angle is given by:

$$\Theta_{sep} = \cos^{-1}(-X_{min}U_{min1}/\text{cyl radius}) \times 180/\pi \quad (3)$$

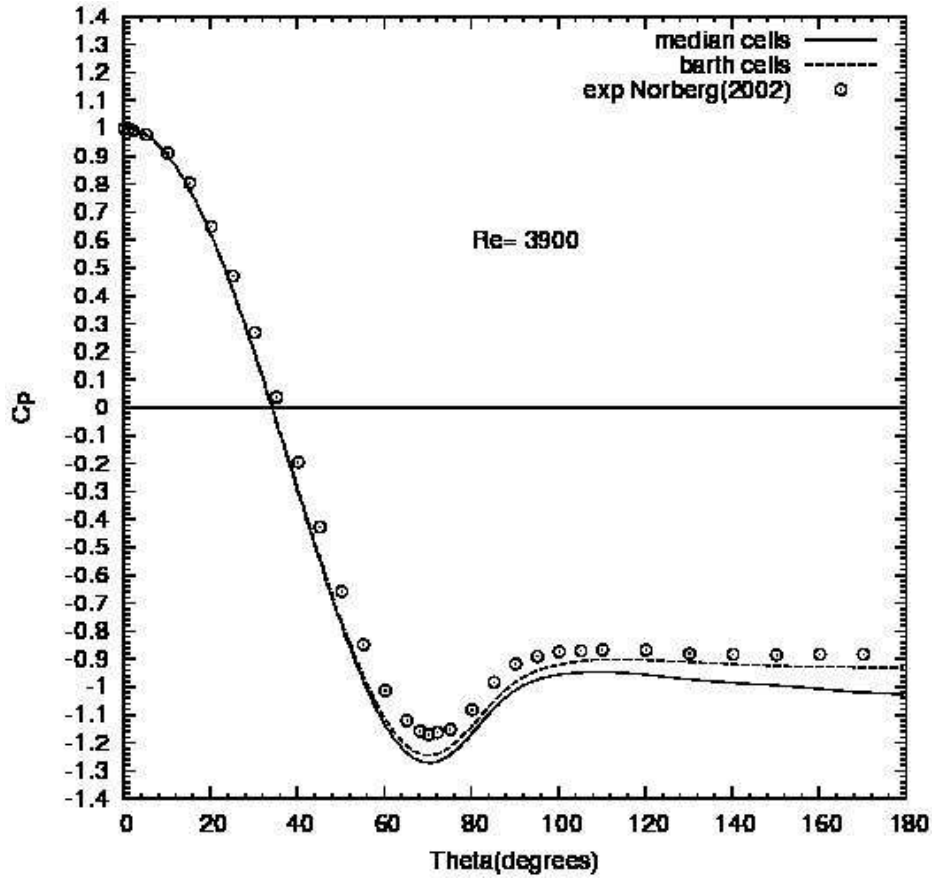


Figure 4: Surface pressure coefficient: median cells vs Barth cells

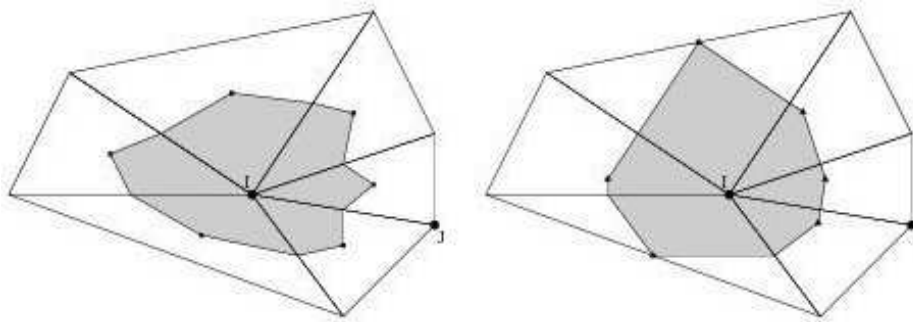


Figure 5: 2D Schematic of median and Barth cells construction: left) Median cells, right) Barth cells

These separation angles are written in the two files:  
 cylinder\_separation\_angle\_using\_umin.data



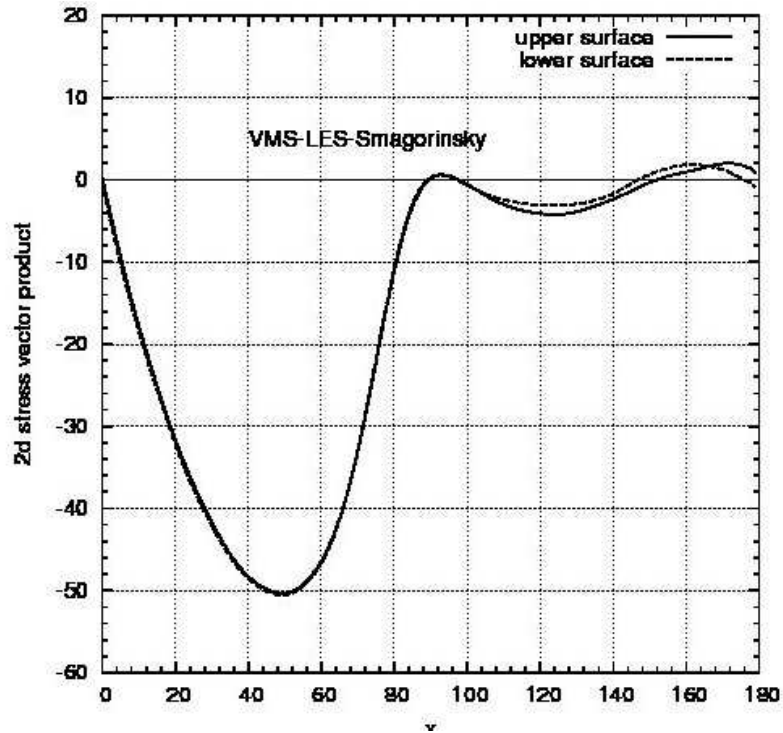


Figure 6: Flow separation location using tau-stress method

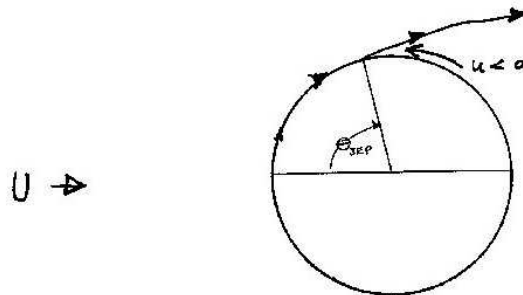


Figure 7: Flow separation location using umin method

cylinder\_separation\_angle\_using\_tau.data

Table 1 gives the angle at which the flow separates from the cylinder surface measured from  $x = -\text{radius}$ .

| method      | Separation angle in degrees |       |         |
|-------------|-----------------------------|-------|---------|
|             | upper                       | lower | Average |
| umin-method | 88.6                        | 88.6  | 88.6    |
| tau-method  | 88.1                        | 88.1  | 88.1    |

Table 1: Flow separation angles in degrees

The differences between the upper and lower surfaces separation angles indicate the degree to which the meanflow is established. Both methods show the separation angle at approximately 89 degrees.

## 7 Meanflow L/D computation

The L/D values are computed using the umin-method by locating the maximum positive x-coordinate for  $u < 0$  in a control volume (assumes the center of the cylinder is at zero) and written in the file:

```
cylinder_Mean_LOD_recirculation_zone.data
```

```
20000 9.43820 1.50 1.50 1.50 (kt, time, upper, lower, average)
```

Using ParaView to view the LES-WALE files

```
paraview_meanflow_results.pvd
```

```
sofmean.020000.pvtu
```

```
sofmean.020000_1.vtu
```

```
sofmean.020000_2.vtu
```

```
...
```

```
sofmean.020000_63.vtu
```

solfmean.020000\_64.vtu

and the ruler tool gives a value of approximately 1.48 as opposed to 1.50 calculated with the `umin` method. The L/D computation was made with ParaView using the meanflow solutions profiles created with `computeMeanFlow 1` vis the following steps:

- 1) locate  $-0.0001 < u < 0.0001$  contour,
- 2) apply the ruler in ParaView

L/D = measurement - Radius.

Figure 8 shows the LES-WALE case. Note 1) The irregular shape of the  $u = 0$  contour is due to the lack of mesh refinement in the cylinder wake.

Note 2) Caution must be used if the initial solutions were obtained with a different turbulence model as the meanflow solutions contain previous histories and the solution must be advanced sufficiently to remove the previous history from the meanflow solution. In the future, the mesh should be define so as to capture the meanflow recirculation region! This will permit a more accurate measure and more accurate numerical results.

Shown in Figure 9 streamline traces for the LES-WALE case.

## 8 Comparison with other results

Table 2 shows the results with the 1.8 million mesh based on the dynamic pressure of the mesh compared with other numerical and experimental results. The present VMS-LES-WALE and the VMS-LES-WALE [1] are the same computation but the latter is based on Figure 3 whereas the present VMS-LES-WALE uses the `Pdyn` of the mesh.

The Strouhal numbers and the  $C_{Lrms}$  values were obtained using tools found in the `AIROToolBox` directory for analysing time series data.

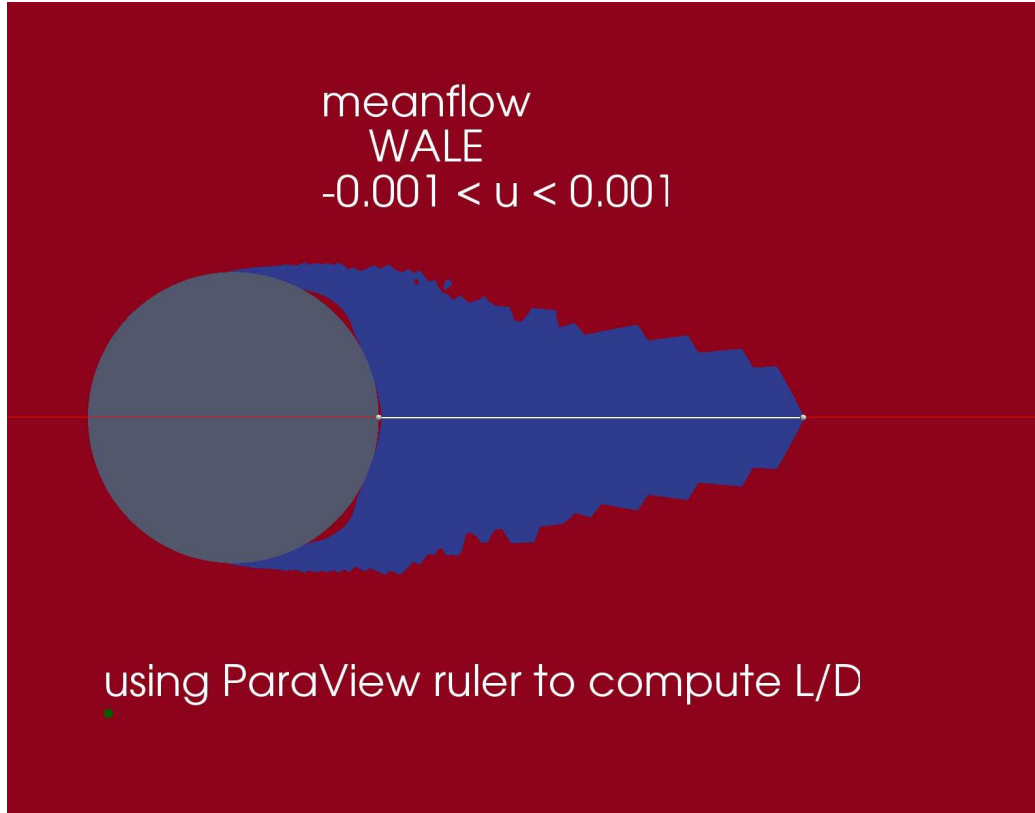


Figure 8: L/D measurement for the LES-WALE case

Table 2: Bulk flow parameters at  $Re=3900$ .  $\overline{C_d}$  is the mean drag coefficient,  $C_{Lrms}$  is the r.m.s. of the time variation of the lift coefficient,  $l_r$  is the mean bubble recirculation length,  $\overline{C_{p_b}}$  is the value of the mean pressure coefficient in the rear part of the cylinder,  $\theta_{sep}$  the mean separation angle, and  $St$  the Strouhal number.

| Simulation       | $\overline{C_d}$ | $C_{Lrms}$ | $l_r$     | $\overline{C_{p_b}}$ | $\theta_{sep}$ | $St$      |
|------------------|------------------|------------|-----------|----------------------|----------------|-----------|
| LES-WALE         | 1.01             | 0.127      | 1.50      | 0.90                 | 89             | .21       |
| VMS-LES-WALE     | 1.02             | 0.108      | 1.44      | 0.93                 | 89             | .21       |
| [1] VMS-LES-WALE | 0.99             | 0.108      | 1.45      | 0.88                 | 89             | .21       |
| LES [5, 9, 10]   | 0.99-1.38        | –          | 1.0-1.56  | 0.89-1.23            | –              | 0.19-0.21 |
| Exp [3, 4, 5, 9] | 0.94-1.04        | –          | 1.47-1.51 | 0.82-0.93            | –              | 0.20-0.22 |

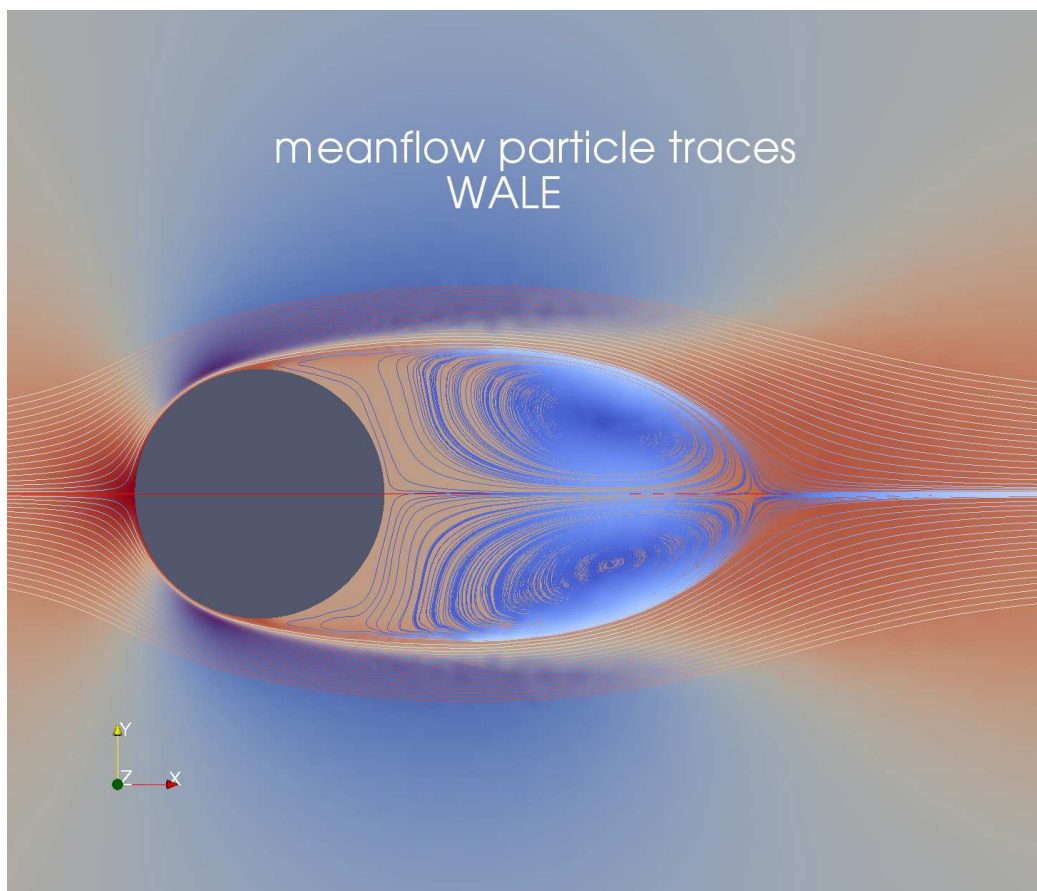


Figure 9: Center plane meanflow stream traces for the LES-WALE case

## 9 Summary

Flow over cylinders has played an important role in the development of CFD methods and turbulence models in the past 50 years and will continue to play an important role in the future. Due to its simple geometry facilitating experiments and numerical studies, cylinders flows are among the simplest test case for studying the effect of Reynolds number on bluff body flows involving recirculation regions.

To simplify the analysis for users studying flow over cylinders, the AIRONUM CFD software has added specific features that automatically compute the meanflow drag and lift coefficients, base pressure coefficient, flow separation angle, L/D of the recirculation zone, the max/min U-velocity, and the surface pressure coefficient, while the simulation is executing. The meanflow ParaView graphics files are also automatically written at the completion of the simulation. These analysis features are found in the AIRONUM software version 4.1 and later.

## 10 Conclusions

- Using the mesh upstream dynamic pressure to compute the surface pressure coefficient agrees well with the experimental data.
- Using the theoretical upstream dynamic pressure to compute the surface pressure coefficient gives poor agreement with the experimental data. This is due to two reasons. First, characteristic boundary condition were used. Thus, the theoretical upstream conditions are not applied in the computation. Secondly, the upstream mesh boundary is too close to the cylinder. The upstream boundary was five diameters from the cylinder center for the mesh used in this study.

- Future meshes should be constructed so that the upstream mesh boundary is greater than five diameters from the cylinder center. This should eliminate the problem when computing the pressure coefficient using the theoretical dynamic pressure as observed in Figure 1.
- In the future, the meshes should be constructed so as to capture the meanflow recirculation region accurately (sufficiently refined mesh). This should lead to more accurate solution.
- A note of caution: if the initial profiles for a new turbulence model computations were obtained with a different turbulence model, the new computation must be run sufficiently long to remove the previous history effects of the other turbulence model.

## A Additional computeMeanFlow options

computeMeanFlow options 3-6 are:

3 compute MeanFlow solution from existing solf.data files

4 create MeanFlow paraview files

5 write promedia files from solfmoy files

6 create lift.data and fort.75 from solf files

Sometimes one forgets to activate the computeMeanFlow keyword but needs the meanflow profiles. The computeMeanFlow = 3 option computes the meanflow from existing solf and startf files. This is a post processor, no time stepping is done. The 'mean\_flow\_solf.inp' and 'mean\_flow\_startf.inp' files contain the names of the files to be processed.

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