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► **To cite this version:**

Olivier Bouré, Nazim A. Fatès, Vincent Chevrier. Robustness of Cellular Automata in the Light of Asynchronous Information Transmission. [Research Report] 2011, pp.13. inria-00563904v1

HAL Id: inria-00563904

<https://inria.hal.science/inria-00563904v1>

Submitted on 7 Feb 2011 (v1), last revised 30 Mar 2011 (v2)

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Robustness of Cellular Automata in the Light of Asynchronous Information Transmission

Olivier Bouré, Nazim Fatès, Vincent Chevrier
{olivier.boure, nazim.fates, vincent.chevrier}@loria.fr
Nancy Université – INRIA Nancy–Grand-Est – LORIA

February 7, 2011

Abstract

Cellular automata are classically synchronous: all cells are simultaneously updated. However, it has been proved that perturbations in the updating scheme may induce qualitative changes of behaviours. This paper presents a new type of asynchronism, the β -synchronism, where cells still update at each time step but where the transmission of information between cells is disrupted randomly. We experimentally study the behaviour of β -synchronous models. We observe that, although many effects are similar to the perturbation of the update, novel phenomena occur. We particularly study phase transitions as an illustration of a qualitative variation of behaviour triggered by continuous change of the disruption probability β .

Keywords: asynchronous cellular automata, discrete dynamical systems, robustness, phase transitions, directed percolation

1 Introduction

Cellular automata are a parallel, spatially-extended, model of computation, which has been studied as an alternative to the sequential computing models, for instance Turing machines. By their very structure, they are well-suited for modelling natural phenomena and for the design of massively parallel algorithms. These mathematical objects have been mostly considered in the synchronous case, that is, when all their components are simultaneously updated. However, this hypothesis of perfect synchrony is somehow inadequate when modelling systems that are subject to noise or non-ideal information transmission, as often met in various natural systems or in asynchronous parallel computing devices. This is why authors tackled the question of whether a cellular automaton is robust to non-ideal updating, either without changing its local transition rule (*e.g.* [4]), or by adding adapted constructs (*e.g.* [8]).

The aim of this paper is to study the robustness of cellular automata by taking into account the possibility of disruptions in the transmission of

information between cells. To this end, we describe the updating process of the system in the frame of *cellular cycle*. This cycle consists of two steps: (a) the local computation and (b) the transmission of the updated state of a cell to its neighbours. This dichotomy induces two possible types of asynchronism: the update-wise asynchronism, or α -synchronism, which disrupts step (a) and a novel type of asynchronism, the influence-wise asynchronism or β -synchronism, which disrupts step (b) and perturbs the interaction between a cell and its neighbours.

The α -synchronous updating is now a relatively well-studied perturbation, whose effect can either be “inoffensive” or trigger drastic qualitative changes, such as phase transitions [2, 3, 6]. Our goal is now to examine β -synchronism on simple cellular automata, in particular, comparing its effects to α -synchronism. We aim at extending the range of perturbations, in order to gain insight on how complex collective behaviour emerges from numerous simple local interactions.

On the formal side, this type of asynchronism leads us to extend the cell state space in order to distinguish the *eigenstate*, the actual state of the cell, and the *perceived state*, the state of the cell which is perceived by its neighbours. These new definitions are introduced in Sec. 2 while experimental observations are presented in Sec. 3. We then analyze more particularly the occurrence of phase transitions in Sec. 4 and proceed to bring discussion in Sec. 5.

2 Asynchronous Cellular Automata

2.1 Cellular Automata

A synchronous cellular automaton is a discrete dynamical system defined by $A = \{\mathcal{L}, Q, \mathcal{N}, f\}$ where :

- $\mathcal{L} \subset \mathbb{Z}^d$ the array of the cellular space, where an element of \mathcal{L} represents a cell.
- Q is a nonempty finite set of states.
- $\mathcal{N} \subset \mathcal{L}$ is the neighbourhood, which associates to a cell the set of its neighbouring cells.
- f is the local transition rule, which defines the next state of a cell according to the states of that cell and the ones of its neighbours.

A *configuration* x^t represents the state of the automaton at time t ; it is defined as a function $x^t : \mathcal{L} \rightarrow Q$ which maps each cell to a state. Classically, cellular automata are synchronously updated, meaning that at each time the local transition rule is applied simultaneously of all cells. The *global transition function* is therefore defined as $x^{t+1} = F(x^t)$, so that, for $\mathcal{N} = \{n_1, \dots, n_k\}$:

$$\forall c \in \mathcal{L}, x^{t+1}(c) = f(x^t(c), x^t(c + n_1), \dots, x^t(c + n_k)).$$

Without loss of generality, we assume that the neighbourhood \mathcal{N} does *not* contain the cell itself. This hypothesis is necessary to explicitly represent

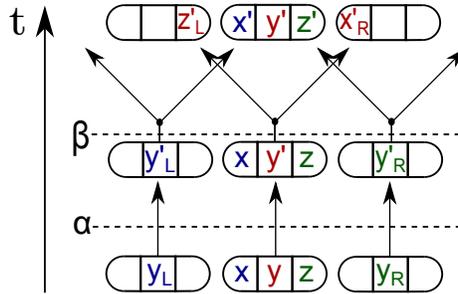


Figure 1: Representation of a cell cycle, for a cell with two neighbours, denoted by indices L and R. A triplet represents a cell with its state (y) and the perceived states of its neighbours (x for left cell and z for right). The prime sign indicates states that are updated. Solid arrows show the updates of states, otherwise states are conserved from the previous step. In α -synchronism, the local transition update applies with probability α and in β -synchronism, the information transmission is applied with probability β . The lower arrow is the local transition update, which performs $y' = f(x, y, z)$ when applied. The forking upper arrow is the information transmission, which performs $z'_L = y'$ and $x'_R = y'$ when applied.

the flow of information between a cell and its neighbours. Note that this does not restrict the expressiveness of f since the current state of a cell $x^t(c)$ is always a parameter of f , possibly not taken into account in the transition calculus.

2.2 Asynchronism as a disruption of cell activity

Cell cycle. The update of a cell can be represented by a *cell cycle*, which we decompose into two steps (see Fig. 1):

- the *state update* step, where a cell changes its states according to the local transition function.
- the *information transmission* step, where the cell transmits the updated state to its neighbours.

We give an example of the cell cycle for the different updating schemes on Fig. 2. The update-wise asynchronous updating in cellular automata, or α -*synchronism* [3], is defined as follows: at each time step, each cell is updated with probability α , or else left unchanged. We introduce a new type of asynchronism, the β -*synchronism*, where each cell is always updated but the transmission of the new state to the neighbourhood is realized with probability β , thus allowing us to model outdated information. As a result, both perturbations consist in applying one of the steps of the cell cycle with a probability defined as the *synchrony rate*.

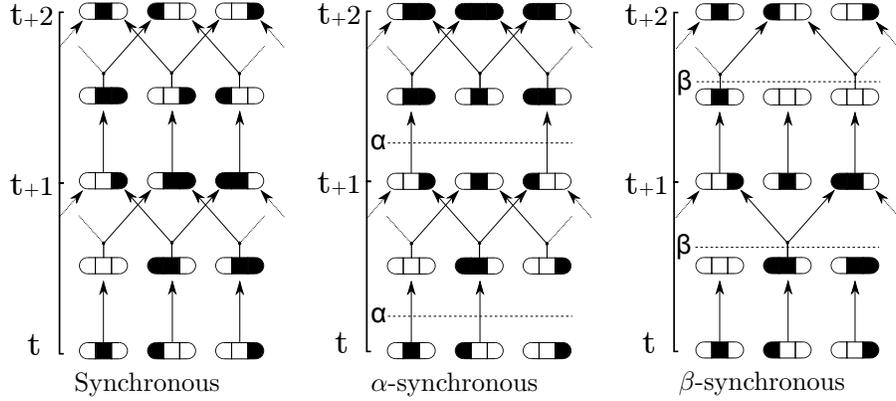


Figure 2: Example of the time cycle for a 3-cell sample for synchronous (left), α -synchronous (middle) and β -synchronous (right) updating schemes (0 is denoted by a white space, 1 by black). The rule is ECA50 (see Sec. 2.3) but is not important here.

α -synchronism. Formally, we introduce a *selection function* $\Delta : \mathbb{N} \rightarrow \mathcal{P}(\mathcal{L})$ which returns for time t the subset of cells to be updated. When $\alpha = 1$ the updating is fully synchronous and the system is deterministic. As α decreases, the updating becomes more stochastic. The global transition function becomes $\forall t \in \mathbb{N}, \forall c \in \mathcal{L}, \mathcal{N} = \{n_1, \dots, n_k\}$:

$$F_{\Delta}(x^t(c)) = \begin{cases} f(x^t(c), x^t(c+n_1), \dots, x^t(c+n_k)) & \text{if } c \in \Delta(t) \\ x^t(c) & \text{otherwise.} \end{cases}$$

β -synchronism. To define this new asynchronism, we need to extend the classical definition of cellular automata by taking into account the difference between the *eigenstate* of a cell, and the *perceived state*.

Let us consider a cellular automaton $A = \{\mathcal{L}, Q, \mathcal{N}, f\}$. From A we derive a new cellular automaton $A' = \{\mathcal{L}, Q', \mathcal{N}, f'\}$ where :

- $Q' = Q^2$ is the new set of states.

For a given configuration x^t , a cell state is denoted by $x^t = \begin{pmatrix} x_e^t \\ x_p^t \end{pmatrix}$

with $x_e^t : \mathcal{L} \rightarrow Q$ the eigenstate of the cell, and $x_p^t : \mathcal{L} \rightarrow Q$ the state of the cell perceived by its neighbourhood.

- the local transition function f' is splitted into two parts to decompose its action: state update and information transmission.

Therefore, we write $f' = f_t \circ f_u$, so that:

- $f_u : Q'^{k+1} \rightarrow Q'$ is the *update function*, which computes the new state of the cell based on its eigenstate and the perceived state of the neighbours:

$$f_u \left(\begin{pmatrix} e \\ p \end{pmatrix}, \begin{pmatrix} \cdot \\ p_1 \end{pmatrix}, \dots, \begin{pmatrix} \cdot \\ p_k \end{pmatrix} \right) = \begin{pmatrix} f(e, p_1, \dots, p_k) \\ p \end{pmatrix}$$

- $f_t : Q' \rightarrow Q'$ is the *transmission function*, the effect of which is the transmission of the eigenstate by replacing the perceived state:

$$f_t \left(\begin{pmatrix} e \\ p \end{pmatrix} \right) = \begin{pmatrix} e \\ e \end{pmatrix}.$$

We use the same function Δ introduced for α -synchronism and define the global transition function as, $\forall t \in \mathbb{N}, \forall c \in \mathcal{L}, \mathcal{N} = \{n_1, \dots, n_k\}$:

$$F_\Delta(x^t(c)) = \begin{cases} f_t \circ f_u(x^t(c), x^t(c+n_1), \dots, x^t(c+n_k)) & \text{if } c \in \Delta(t) \\ f_u(x^t(c), x^t(c+n_1), \dots, x^t(c+n_k)) & \text{otherwise.} \end{cases}$$

2.3 Models studied

Now that we defined our different updating schemes for cellular automata, we choose to study their effects on two well-studied models.

The Game of Life. This 2-dimensional cellular automaton is expressed in our formalism as $A_{GL} = \{\mathcal{L}, \mathcal{N}, \{0, 1\}, f\}$ where:

- $\mathcal{L} = \{\mathbb{Z}/L\mathbb{Z}\} \times \{\mathbb{Z}/L\mathbb{Z}\}$ is a square grid of size L with periodic boundary conditions.
- $\mathcal{N} = \{c \in \mathcal{L}, ||c|| = 1\}$ represents the 8-cell neighbourhood.
- the local transition function $f : Q^9 \rightarrow Q$ is outer-totalistic, that is, it can be written $f(x^t(c), x^t(c+n_1), \dots, x^t(c+n_8)) = \delta(x^t(c), s)$ where $s = \sum_{c' \in \mathcal{N}} x^t(c')$.

$$\text{If } x^t(c) = 0, \text{ then } \delta(x^t(c), s) = \begin{cases} 1 & \text{if } s = 3 \\ 0 & \text{otherwise.} \end{cases} \quad (\textit{Birth rule})$$

$$\text{If } x^t(c) = 1, \text{ then } \delta(x^t(c), s) = \begin{cases} 1 & \text{if } s \in \{2, 3\} \\ 0 & \text{otherwise.} \end{cases} \quad (\textit{Survival rule})$$

Elementary Cellular Automata (ECA). An *ECA* is a 1-D binary cellular automaton with nearest-neighbourhood, whose transition function is determined according to Wolfram's notation W . In our formalism, ECAs are denoted by $A_E = \{\mathcal{L}, \mathcal{N}, \{0, 1\}, f\}$ where:

- $\mathcal{L} = \{\mathbb{Z}/L\mathbb{Z}\}$ is a 1-dimensional ring.
- $\mathcal{N} = \{-1, +1\}$, i.e. the 2 nearest neighbours.
- the local transition function δ is determined by its code $W = f(0, 0, 0).2^0 + f(0, 0, 1).2^1 + \dots + f(1, 1, 1).2^7$.

3 Qualitative observations

We now observe qualitatively the effect of β -synchronism. We are in particular interested in knowing whether the effects will differ from α -synchronism. In the rest of this paper, random initial conditions are constructed, for each cell, with a perceived state equal to the eigenstate, and chosen uniformly in $\{0, 1\}$.

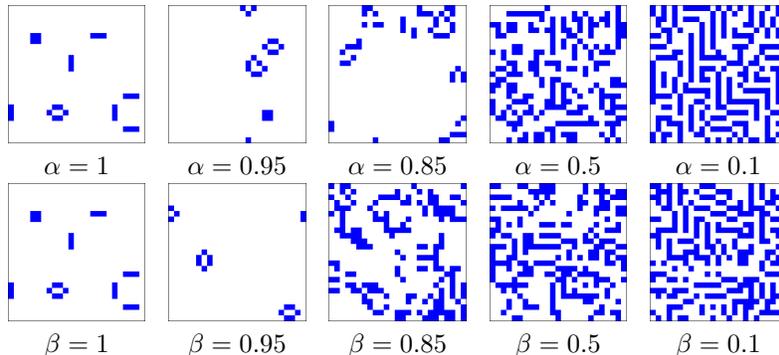


Figure 3: Game of Life configurations for different values of α and β (0 is denoted by a white space, 1 by blue). These are obtained with the software FiatLux for identical initial states, for a transient time $t \approx 1000$. The first row shows the different behaviors obtained with different values of α -synchronism. The second row displays them for the same values of β -synchronism.

3.1 The Game of Life

Figure 3 shows sample configurations appearing for random initial conditions after a transient time of 1000, which was observed sufficient for reaching a steady state. We observe that the behaviour separates into two phases for both types of asynchronism: the system converges to a fixed point for values of the synchrony rate α or β above 0.9, whereas a *labyrinth-like pattern* appears for values below 0.9.

Our experimentations for the Game of Life rely on several macroscopic parameters that aim at quantifying the behavioral changes:

- the *density* d is the ratio of cells with state (or eigenstate) 1.
- the *activity* a is the ratio of instable cells. A cell is said *instable* at time t if a synchronous update would modify its state (or eigenstate). This definition cannot be readily transposed for β -synchronism: what does a synchronous update mean for a cell whose eigenstate and perceived state are desynchronized? As neighbourhood knowledge is always absolute in synchronous cellular automata, we reckon that it should also be the cases for the estimation of activity. Therefore, using the notations defined in Fig. 1, we define a cell as instable if $y \neq f(y_L, y, y_R)$.

Figure 4 compares the two types of asynchronism through the activity parameter at steady state as a function of the synchrony rate. At first sight, the β -synchronous Game of Life seems to react in a similar way to its α -synchronous counterpart.

1. A singularity occurs for activity in the synchronous case ($\alpha, \beta = 1$). This phenomenon has been explained for α -synchronism as the loss of stable periodic patterns when a noise is introduced no matter

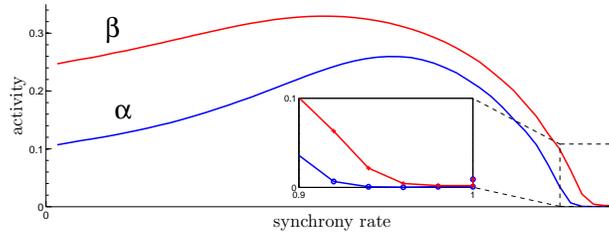


Figure 4: Activity a_∞ for Game of Life in the continuous state (here $t \in [10000, 11000]$) with synchrony rates $\tilde{\alpha}$ and β , averaged over sample of 50 automata of size $L=100$. Inset: a closeup for values $\alpha, \beta \in [0.9, 1]$.

how small (*i.e.* $\alpha < 1$) [1]. The same explanation stands for β -synchronism. This means that the traditional construct to make the Game of Life Turing-universal no longer holds. Nevertheless, the question of the Turing-universality of the asynchronous Game of Life still stands.

2. The macroscopic behaviour confirms the phase separation for a critical value of the synchrony rate α_c (resp. β_c). For values $\alpha > \alpha_c$ (resp. $\beta > \beta_c$), the system converges to a stable fixed point of low density, which constitutes a passive phase. However, for values $\alpha < \alpha_c$ (resp. $\beta < \beta_c$), the system enters an active phase, characterized by the labyrinth pattern (though less regular in β - than in α -synchronism).

The comparison of both types of asynchronism reveals a similar reaction of the Game of Life, but this calls for a close observation of the characteristics of the phase transition.

3.2 Elementary Cellular Automata

Through the search of reflexive and complementary symmetries, it is possible to reduce the number of ECA to study from the 256 possible models to 88 inequivalent ECA.

Our experimentations for these 88 ECA rely on several macroscopic parameters that aim at capturing the behavioral changes between different systems:

- the *density* d , as defined above.
- the *01 block density* (00 , respectively) is the ratio of successive cells with states $0 - 1$ (resp., $0 - 0$).

A great range of results were obtained from the comparison of both types of asynchronism applied to the 88 minimal ECA¹. Although these perturbations have similar effects on most models, some surprising divergences of behaviour appeared, for example rules ECA6, 22, 50, 72 and 200. For

¹for the complete results: <http://www.loria.fr/~boure/results/ecaparambeta/>

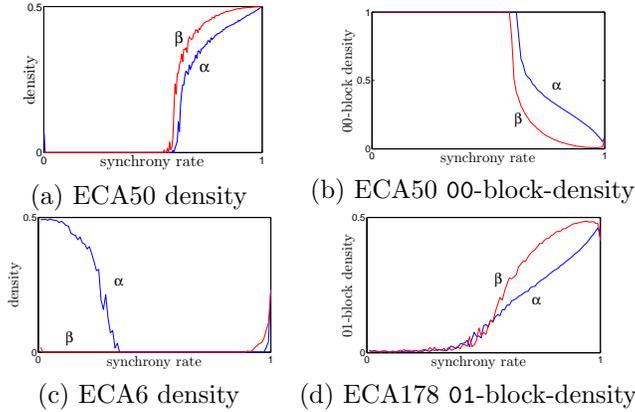


Figure 5: Plots of asynchronous behaviour in continuous state for different ECA and parameters. Values are taken for a grid of size 1000, after a transient time of $T_t = \max(10000, 1000/\alpha)$ (β respectively) and averaged over a sampling time $T_s = \max(1000, 100/\alpha)$ (resp. β).

the sake of conciseness, we choose to focus on a single rule, namely ECA50. An exhaustive study of the phenomena is planned for future work.

The case of ECA50. A notable phenomenon appears for β -synchronous ECA50. The study of density parameter (see Fig. 5a) shows a similar plot profile for both α - and β -synchronism, including a phase transition, which suggests that ECA50 reacts in a similar way to these perturbations.

However, a difference of behaviour is observed with the visual inspection of the evolution of the automaton over a few time steps (see Fig. 6). For the same synchrony rate at 0.75, β -synchronism patterns appear much closer to the synchronous automaton than α -synchronism. In particular, in the long run clusters of 0-states are smaller and less frequent, and clusters of 1-states inexistent.

This can be explained for values of β close to 1 with the following observations:

- the *checkerboard regions* (*i.e.* alternated 0s and 1s) are robust to β -synchronism, that is, the few anomalies (regions of 0s) that disturb the regularity of the region are quickly restored to the original pattern.
- the *pairs* (*i.e.* 00 and 11 pairs) follow a non-biased random walk, and annihilate when they meet.

This example illustrates the qualitative differences that may occur between α - and β -synchronous cellular automata. Although this specific case calls for further analysis, we note that ECA50 is more regular for β -synchronism than for α -synchronism. This regularity can be “seen” as the proximity between rule ECA50 and the inversion rule ECA51, which differ only by one bit in their transition table. As the rule ECA51 is insensitive to β -synchronism (the inversion rule is independent from the

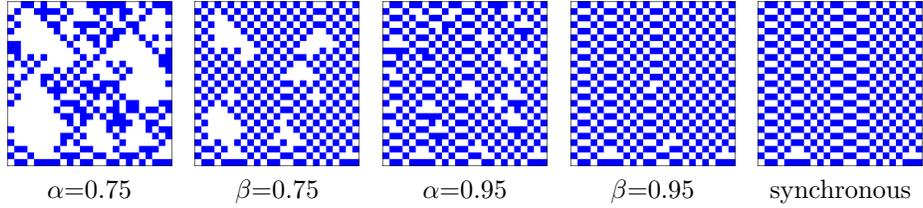


Figure 6: Configurations for ECA50 under different updating schemes (0 is denoted by a white space, 1 by blue).

Table 1: Table of probabilities for updated states to become 1 after an β -synchronous transition of ECA51 (inversion rule). Each column represents a possible state for a cell and its neighbourhood (upper box) and the associated probability to become a 1 for the output of the transition function (lower box).

Synchronized states	. 0 .	. 0 .	. 1 .	. 1 .	. 0 .	. 0 .	. 1 .	. 1 .
	0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1
	1	1	0	0	1	1	0	0
	β	β	$1-\beta$	$1-\beta$	β	β	$1-\beta$	$1-\beta$
Desynchronized states	. 0 .	. 0 .	. 1 .	. 1 .	. 0 .	. 0 .	. 1 .	. 1 .
	0 1 0	0 1 1	0 0 0	0 0 1	1 1 1	1 1 0	1 0 0	1 0 1
	1	1	0	0	1	1	0	0
	1	1	0	0	1	1	0	0

neighbours state), there also seem to exist a proximity in terms of global behaviour between ECA50 and ECA51 for the β -synchronism. One may thus wonder what is the origin of such radical difference between the two updating schemes for such models, difference that we try to explicit through stochastic cellular automata.

Stochastic cellular automata. By its very nature, it is possible to describe an α -synchronous ECA in terms of an *elementary stochastic cellular automaton*, that is, to define it with a function $f_\alpha : \{0, 1\}^3 \rightarrow [0, 1]$ which associates to each neighbourhood state (x, y, z) the probability to update to 1. Indeed, the local rule f_α is simply obtained as the barycentre of the transition function f and the identity Id with weights α and $1 - \alpha$, respectively.

By contrast, β -synchronism requires us to extend the state space in order to differentiate eigenstate and perceived state. As a result, the number of inputs of the transition table is doubled to take into account whether the two states are *synchronized*, that is, if the eigenstate and the perceived state are identical.

As said before, ECA51 is insensitive to β -synchronism. However, this par-

ticular property of ECA51 does not appear readily on its transition table (see Table 1). This makes the stochastic table all the more “cryptic”, as the divergence from synchronism cannot be deduced from the reading of the table, unlike α -synchronism. This shows that the difference between the two types of asynchronism is non-trivial, and justifies the experimental approach as well as calls for further in-depth analysis.

4 Study of phase transitions

The occurrence of phase transitions is probably one of the most remarkable phenomena that arises in asynchronous cellular automata: there exists a non-trivial value of the synchrony rate, the critical threshold, which separates two distinct qualitative behaviours of the system. We are now interested in measuring quantitatively this phenomenon.

4.1 The Game of Life

How to quantify the modification undergone by some cellular automata for a critical synchrony rate? As pointed out in Sec. 3.1, the activity parameter reveals the existence of two distinct phases in the α - and β -synchronous Game of Life. For the α -synchronism, the phase transition has been proved to be second-order [1], that is, if the macroscopic measures that describe the behaviour are continuous, their derivative curve is discontinuous for a critical value of the synchrony rate.

How to determine the critical synchrony rate α_c or β_c ? A simple method to determine its value would consist in estimating the singularity point where the slope jumps from null to infinity, but this technique is rather imprecise as it introduces systematic biases. In order to reduce the biases, we follow a different protocol (see *e.g.* [5]):

1. We fix β , start from a random initial condition and let it evolve for a fix number of steps.
2. We monitor the evolution of the order parameter for a long simulation until we observe a sub-critical or super-critical behavior; In a log-log plot, a concave curve occurs for the passive phase (activity converges to zero) and a convex curve for the active phase (activity converges to a non-zero value). As we expect the order parameter to follow a power law $K.t^{-\delta}$ near criticality, its evolution for the critical synchrony rate should appear as a straight line of slope $-\delta$.
3. We repeat the experiment with a value closer to the critical point until a satisfactory precision is reached (here 10^{-3}).

For the α -synchronous Game of Life, it has been measured that $\alpha_c = 0.9083$ and that the evolution of the order parameter of this value followed a power law in the form $K.t^{-\delta}$ with $\delta = 0.451$ [3], which is the theoretical critical exponent for the directed percolation universality-class in 2+1 dimensions [7]. Figure 7 shows that the measures for β -synchronous updating. They also confirm the directed percolation hypothesis, with $\beta_c = 0.945$.

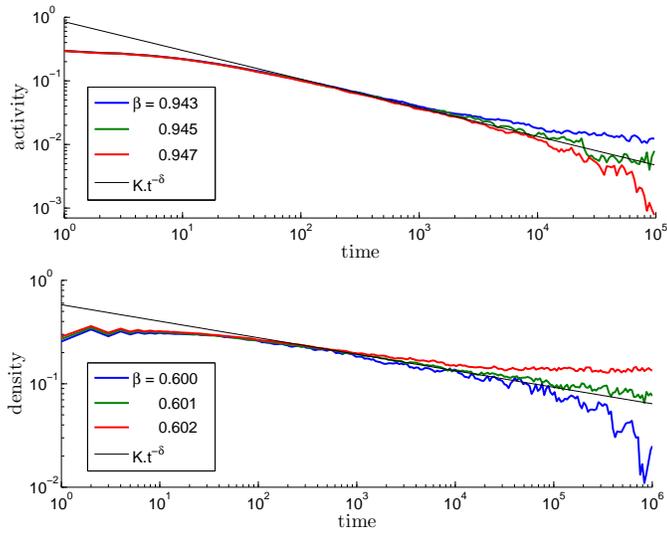


Figure 7: Transition phases analysis for the Game of Life (top) and ECA50 (bottom). The straight lines follow a power law $f(t) = K \times t^{-\delta}$. For the Game of Life, measures are averaged over a sample of 25 automata of size 800×800 . The straight line gives the critical exponent $\delta = 0.451$. For the ECA50, measures are averaged over a sample of 10 automata of size 20000. The straight line gives the critical exponent $\delta = 0.1595$.

4.2 Elementary Cellular Automata

Among the 88 minimal ECAs, it has been observed that several ECA display second-order phase transition in their α -synchronous version. These rules were proved to belong to the directed percolation class [2], and have been divided into 3 distinct subclasses:

- rules 18, 26, 50, 58, 106 and 146 are the DP_{hi} class, for which the active phase of density (respectively the passive phase) occurs for $\alpha > \alpha_c$ (resp. $\alpha < \alpha_c$).
- rules 6, 38 and 134 are the DP_{low} class, where active and passive phases are inverted.
- rule 178 is the sole element of the DP_2 class, where the density is stable but a phase transition appears for the 01-block-density.

As seen on Fig. 5, these subclasses react differently to β -synchronism:

1. The rules of the DP_{hi} class have shown little behavioural change between the two types of asynchronism: the phase transition appearing for the density parameter in Fig. 5 is conserved.
For α -synchronism, the same protocol than for the Game of Life was previously applied [2] to ECA50, and estimated the critical synchrony rate at $\alpha_c = 0.6282$, showing good evidence that the phase transition belongs to the directed percolation universality class.
For β -synchronism (see Fig. 7), we measure $\beta_c = 0.601$, and estimate that the behaviour at critical synchrony rate is in good agreement with a power law of critical exponent $\delta = 0.1595$, which is the theoretical value for the directed percolation in one dimension.
2. Surprisingly enough, no phase transition was observed for the rules of the DP_{low} class in β -synchronism (see ECA6 in Fig. 5), leaving a constant null-density convergent phase for any value of β .
3. Finally, the ECA178 (DP_2 class) reproduced a similar plot profile for the 01-block-density (see ECA178 in Fig. 5).

These first results show how rich the study of β -synchronism can be, and that asynchronism in cellular automata cannot be trivially reduced to the simple perturbation of the local update.

5 Discussion

This paper presented a formalism for a new type of asynchronous updating in cellular automata, the β -synchronism, based on the disruption of information transmission between cells. We compared this perturbation to α -synchronism and observed from a macroscopic point of view that α - and β -synchronism had similar effects. It was observed that β -synchronous updating also produces phase transitions, but for a smaller set of rules than α -synchronism. We remarked that there was no phase transition for the ECA of the DP_{low} class but we have no explanation for this phenomenon so far.

By studying ECA50 more closely, we could exhibit an example for which the macroscopic behaviour was similar for both types of asynchronism but for which additional properties were observed.

This extension is a first step towards a unified view of asynchronism, based on the idea of cell cycles. There exist plenty of other ways the cellular activity can be perturbed. This raises the question of how the space of perturbations can be described.

An interesting lead also lies in the application of these types of asynchronism to collective systems, so that their robustness to different perturbations are studied with regard to expected properties. This may help us characterize the robustness of natural systems and lead to a better understanding of spatially-extended computing models, such as cellular automata.

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