

Factorization Methods for Projective Structure & Motion

Bill Triggs, INRIA Rhône-Alpes

PROJECTIVE DEPTH + FACTORIZATION



PROJECTIVE STRUCTURE + MOTION

NEW “*Projective depth*” & *depth recovery relations*

NEW Factorization-based *projective* reconstruction [4]

~~Affine~~ *Fully Perspective* camera model

NEW *Lines* as well as points

NEW Can also use *trivalent tensor*

⊕ Very *stable* & *accurate* for points

⊕ All images & tokens treated equally

⊕ No initialization required

⊖ Points must be matched across all images

⊖ Requires epipolar (or tri-polar,...) geometry

Projective Depth

POINT PROJECTION (homogeneous coordinates):

$$\lambda \mathbf{x} = \mathbf{P} \mathbf{X}$$

Diagram illustrating the point projection equation $\lambda \mathbf{x} = \mathbf{P} \mathbf{X}$. The term $\lambda \mathbf{x}$ is labeled as 'projective depth' and 'image point'. The term \mathbf{P} is labeled as 'image projection'. The term \mathbf{X} is labeled as 'world point'.

- For m images of n points:

$$\begin{pmatrix} \lambda_{11} \mathbf{x}_{11} & \cdots & \lambda_{1n} \mathbf{x}_{1n} \\ \vdots & \ddots & \vdots \\ \lambda_{m1} \mathbf{x}_{m1} & \cdots & \lambda_{mn} \mathbf{x}_{mn} \end{pmatrix} = \begin{pmatrix} \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_m \end{pmatrix} (\mathbf{X}_1 \cdots \mathbf{X}_n)$$

rank 4: factorize \implies motion + structure

PROJECTIVE DEPTHS

- **Projective scale factors** λ_{ip}
- The projective counterpart of **optical depths**
 - reduce to optical depths in a Euclidean frame
- The same as Sparr's "**depths**" [3]
- Different from Shashua's "**projective depths**" [2]
 - these are cross ratios, not camera-relative depths

Joint Image Approach

PROJECTIVE DEPTHS

contain an

IMPLICIT PROJECTIVE RECONSTRUCTION

JOINT IMAGE

- Rescaled points lie in *Joint Image* [6]
 - a 3D projective subspace of space of combined image coordinates
 - a *faithful projective copy* of 3D world

DOF

- The mn depths λ_{ip} have $(m - 1)(n - 1)$ independent d.o.f.
 - only defined modulo rescalings of projections & world points
i.e. rows and columns of rescaled data matrix
- *Depth cross ratios* $\frac{\lambda_{ip} \lambda_{jq}}{\lambda_{jp} \lambda_{iq}}$ are *invariants of camera geometry*

Individual projective depths are arbitrary

BUT

Together they determine *3D projective structure!*

Depth Recovery Relations

IMAGE POINTS + F-MATRICES + EPIPOLES



PROJECTIVE DEPTHS

$$(\mathbf{F}_{ij} \mathbf{x}_{jp}) \lambda_{jp} = (\mathbf{e}_{ji} \wedge \mathbf{x}_{ip}) \lambda_{ip}$$

DEPTH RECOVERY

OLD Epipolar line passes through corresponding point & epipole

NEW *Relative scale factor* gives projective depths
i.e. $\lambda_{ip}/\lambda_{jp} \approx$ relative distance of points from epipoles

• Solve in least squares:

$$\lambda_{jp} = \frac{(\mathbf{F}_{ij} \mathbf{x}_{jp}) \cdot (\mathbf{e}_{ji} \wedge \mathbf{x}_{ip})}{\|\mathbf{F}_{ij} \mathbf{x}_{jp}\|^2} \lambda_{ip}$$

DEPTH CHAINING

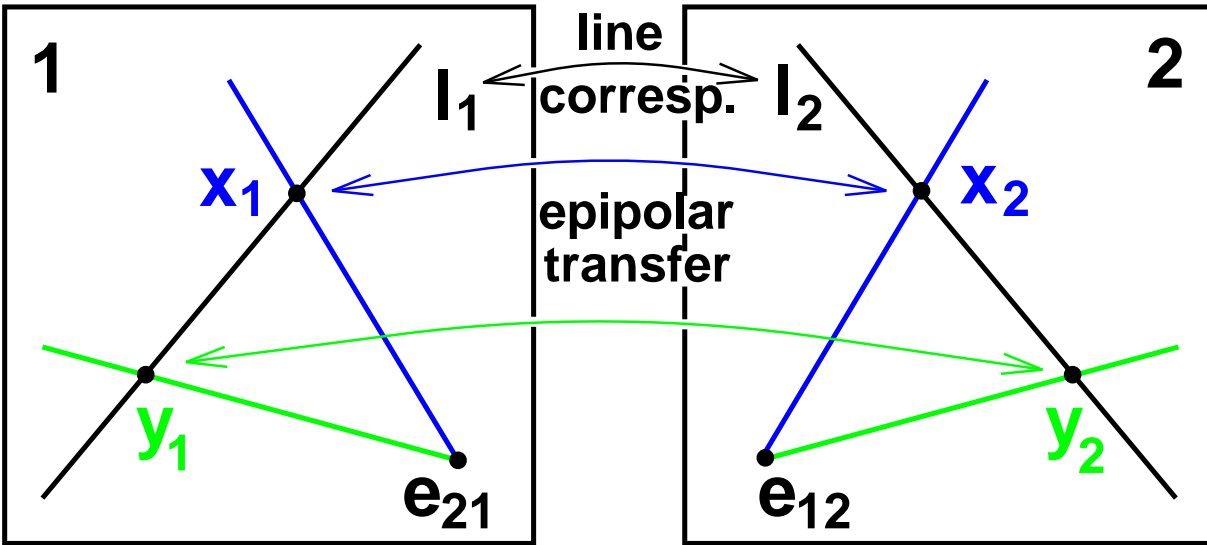
- For each point, propagate solution through all images
 - parallel (key image) & serial (adjacent images) chains
 - redundant chains also possible (consistent scaling needed!)

COMING SOON Analogous relations & reconstruction methods for:

- trivalent & quadrivalent tensors
- projection matrices instead of points [7]

Line Reconstruction

- 1 Represent lines by *pairs of via points*
- 2 Via point correspondence by *epipolar transfer*
- 3 *Reconstruct via points* to find 3D lines



Via point correspondence by epipolar transfer

LINES + DEPTH RECOVERY RELATIONS

ONE \Downarrow STEP

POINT TRANSFER + DEPTH RECOVERY

$$\mathbf{l}_i \wedge (\mathbf{F}_{ij} \mathbf{y}_j) = -(\mathbf{l}_i \cdot \mathbf{e}_{ji}) \mathbf{y}_i$$

VIA POINT TRANSFER

- NB**
- Transfer is *ill-conditioned* for near-epipolar lines
 - For good conditioning use *widely spaced* via points

Factorization

- + Concretizes *reconstruction implicit in depths*
- + *Reduces error* by least-squares averaging/rank truncation

- *Rescaled data matrix* has *rank 4 decomposition*

$$\begin{pmatrix} \lambda_{11} \mathbf{x}_{11} & \cdots & \lambda_{1n} \mathbf{x}_{1n} \\ \vdots & \ddots & \vdots \\ \lambda_{m1} \mathbf{x}_{m1} & \cdots & \lambda_{mn} \mathbf{x}_{mn} \end{pmatrix} = \begin{pmatrix} \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_m \end{pmatrix} (\mathbf{X}_1 \cdots \mathbf{X}_n)$$

- All such decompositions are *equivalent*.
 - freedom is 3D projective transform $\mathbf{X}_p \rightarrow \mathbf{T}\mathbf{X}_p, \mathbf{P}_i \rightarrow \mathbf{P}_i\mathbf{T}^{-1}$

SVD

- Factorize using *Singular Value Decomposition*
 - *c.f.* Tomasi-Kanade [5]
 - very stable, but slow for large problems

FIXED RANK

- *Fixed rank factorizations* are $\mathcal{O}(\min(3m, n))$ faster
 - linear in size of input data
 - still quite stable
 - iterative QR method (*c.f.* [1]) slightly more accurate than 1 sweep Gram-Schmidt method in paper

Algorithm

- 0 Extract & match points & lines across all images.
- 1 Standardize image coordinates to unit box.
- 2 Estimate chain of F-matrices & epipoles linking images.
- 3 **Points:** Estimate projective depths by chaining depth equations.
Lines: Choose via points & propagate through all images.
- 4 Build & balance* rescaled point matrix.
- 5 Factorize using SVD or fixed-rank method.
- 6 Read off 3D projective structure & motion.

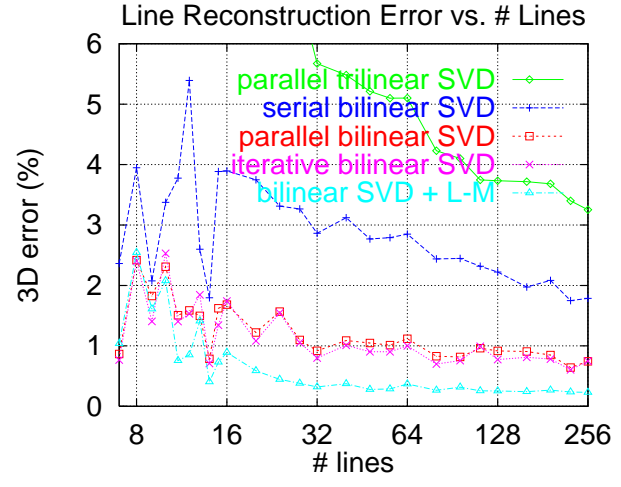
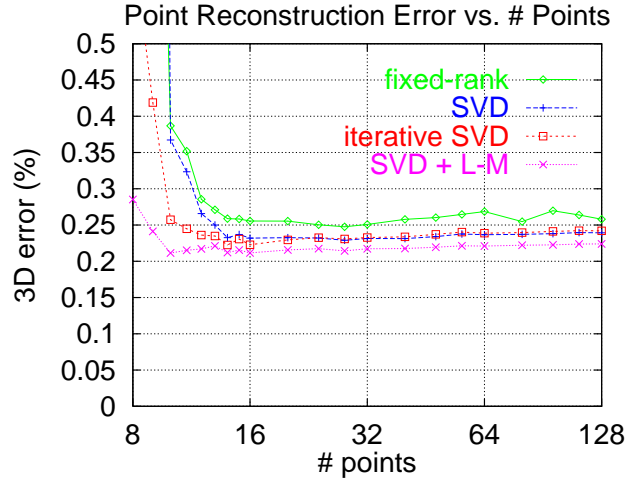
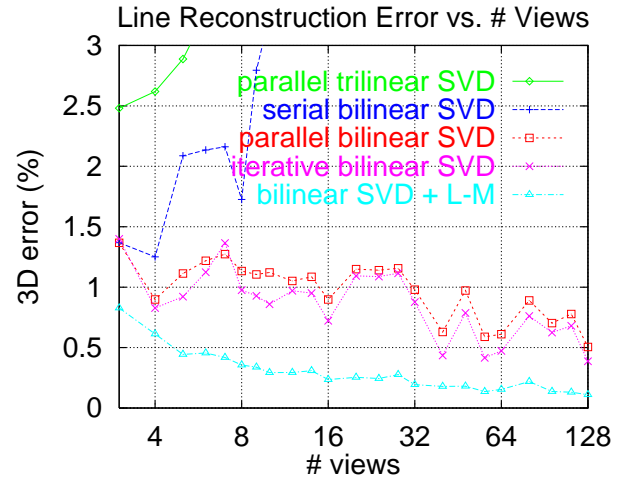
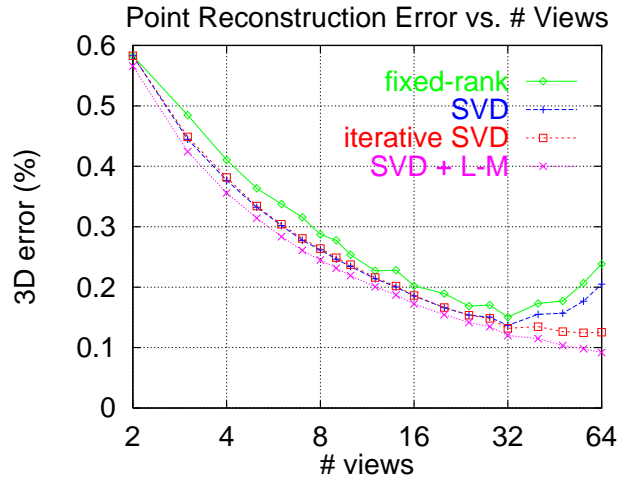
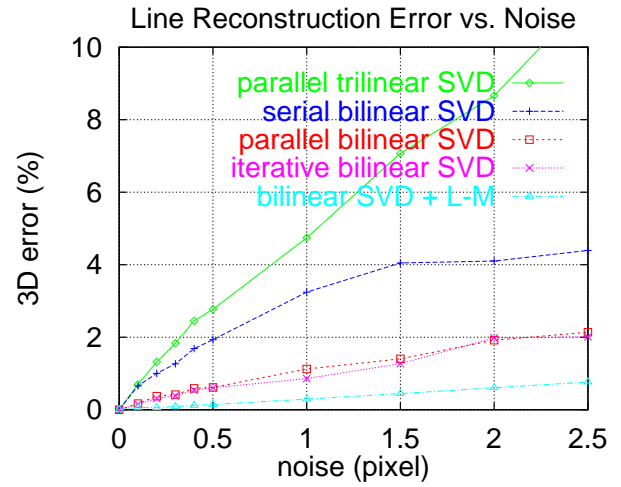
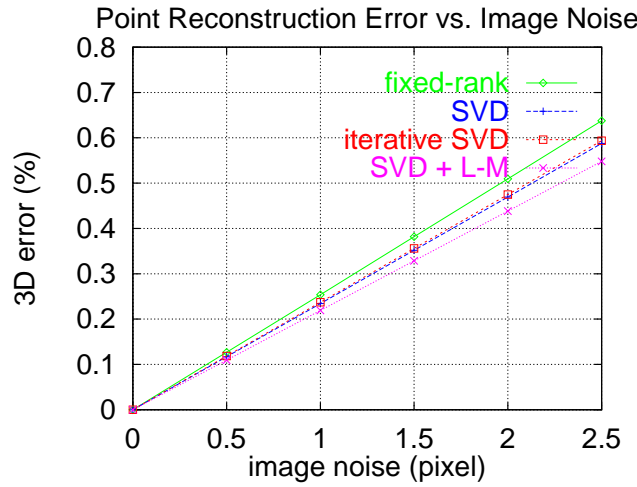
*Rescale rows & columns for good numerical conditioning.

OPTIONAL Refine depths & via points by reprojecting estimated structure: iterate over steps 3–5.

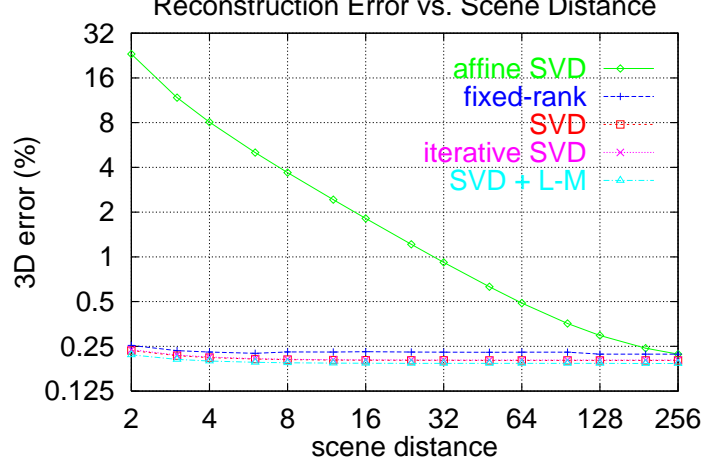
Experiments

Points

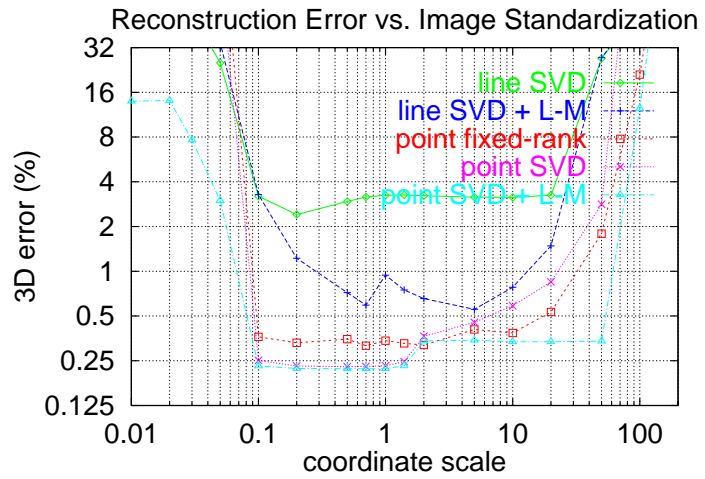
Lines



Synthetic data, wide angle lens, wide viewing angle



Affine vs. projective reconstruction error



Standardize *your* image coordinates!!

Conclusions & Future Work

- ⊕ Depth recovery for points ***extremely stable***
 - even for chains of up to 50 images
- ⊕ Factorization-based point reconstruction very accurate & stable
- ⊖ Line reconstruction works, but much less stable
 - intrinsically ill-conditioned problem, outliers common

**COMING
SOON!**

- Real images!
- Redundant depth recovery equations
- Factorization with incomplete matches
- Trivalent tensor based methods

- 1 T. Morita & T. Kanade. A sequential factorization method for recovering shape & motion from image streams. *IJCV*'94.
- 2 A. Shashua. Projective structure from uncalibrated images: Structure from motion & recognition. *PAMI* 16(8), 1994.
- 3 G. Sparr. Depth computations from polyhedral images. *ECCV*'92.
- 4 P. Sturm & B. Triggs A Factorization Based Algorithm for Multi-Image Projective Structure & Motion. *ECCV*'96.
- 5 C. Tomasi & T. Kanade. Shape & motion from image streams under orthography: a factorization method. *IJCV*, 9(2), 1992.
- 6 B. Triggs. Matching constraints & the joint image. *ICCV*'95.
- 7 B. Triggs. Linear projective reconstruction from matching tensors. *BMVC*'96.

<http://www.inrialpes.fr/movi/Triggs>