

Angles between 3D planes — i.e. metric structure up to scale

$$\cos \theta_{12} = \mathbf{n}_1 \cdot \mathbf{n}_2 = (\mathbf{n}_1^\top \ d_1) \Omega \begin{pmatrix} \mathbf{n}_2 \\ d_2 \end{pmatrix}$$

Plane at infinity \mathbf{p}_∞ — i.e. affine structure

$$\mathbf{p}_\infty \sim (\mathbf{0} \ 1) \iff \mathbf{p}_\infty \Omega = 0$$

Image Projection ω is *dual of image of absolute conic*

$$\omega \equiv \mathbf{K} \mathbf{K}^\top \sim \mathbf{P} \Omega \mathbf{P}^\top$$

• \mathbf{K} is *internal camera calibration*

$$\mathbf{P} = \mathbf{K} (\mathbf{R} \ | \ \mathbf{t}) \quad \text{where} \quad \mathbf{K} \equiv \begin{pmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

- ω measures 3D angles between — visual planes of image lines
- Can recover \mathbf{K} from ω by *Choleski decomposition*

Absolute Quadric Ω

- A new projective encoding of *Euclidean* and *affine* structure
- Easier to use than traditional absolute conic + plane at infinity

Autocalibration

- New methods for *autocalibration* of moving camera with constant intrinsic parameters
- *Rectify* an initial projective reconstruction by enforcing *absolute quadric projection constraint*

- 1 “*Nonlinear*” constrained numerical optimization method
 - much stabler than existing approaches
 - further constraints on calibration easily incorporated

- 2 “*Quasi-linear*” method from $m \geq 4$ images

General policy

- For precise results, stay close to the image data
- Avoid heavy algebraic elimination, *ad hoc* bases, . . .
- Resolve equations directly by numerical optimization

Absolute Quadric Ω

- A new projective encoding of *metric + affine* structure
- Dual to traditional absolute conic, but simpler to use
 - doesn't require separate encoding of plane at infinity
 - simple image projection
- In Euclidean coordinates $\Omega = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
- In projective coordinates Ω becomes an arbitrary symmetric positive semidefinite contravariant 4×4 rank 3 matrix
- Abstractly, Ω is a degenerate dual 3D quadric ‘squashed’ onto plane at infinity. Its ‘rim’ is the absolute conic \mathcal{C}

Autocalibration

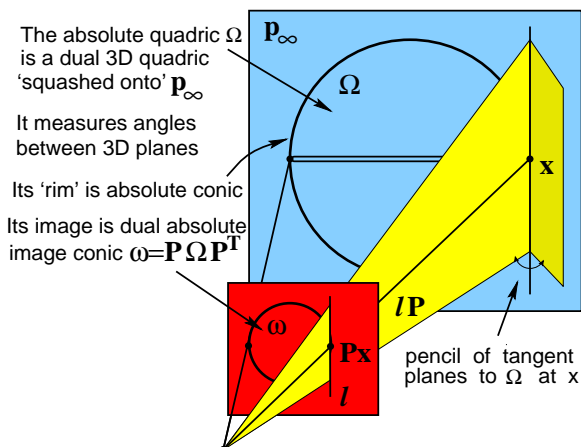
- Calibration and metric 3D structure from a camera with fixed unknown intrinsic parameters moving in an unknown scene
- Requires $m \geq 3$ images and *generic motions*
- Partial information for non-generic motions:

camera preserves a “vertical” \Rightarrow can't recover aspect ratios
 pure translation \Rightarrow affine 3D structure, but no calibration
 pure rotation \Rightarrow calibration, but no 3D structure

- Several existing approaches, but none very stable
 - heavy algebraic elimination \Rightarrow poor numerical conditioning

Basic Principles

- 1 Uncalibrated cameras give only *projective structure*
- 2 Metric structure \iff camera calibration
- 3 Use *motion constancy* of *camera calibration* to recover *projective ‘signature’* of *3D metric structure*
- 4 Use this to *rectify* an initial projective reconstruction



Kruppa Constraints

- Apply general two-image quadric matching constraints to absolute quadric

$$F_{21} \omega F_{21}^T \sim [e_{12}]_{\times} \omega [e_{12}]_{\times}^T$$

- Constraint says that measuring angles between epipolar planes in either image gives the same (3D) answer
- Cross-multiply to eliminate unknown scale factor
 \Rightarrow 2 independent quadratic constraints on ω given F, e
- Solve for ω from $m \geq 3$ images
- ⊕ No explicit 3D reconstruction required
- ⊖ Very sensitive to perturbations in e, F
- ⊖ Recovered Ω could be *any* quadric with constant projection

Infinite Homography

- Images of distant points depend only on **camera rotation**

$$|\text{distance}|/|\text{translation}| \gg (\text{focal length in pixels})$$

- **Infinite homography** H_{∞} encodes rotation projectively
 - estimate from distant-point correspondences
 - can also use virtual (e.g. vanishing) points
- **Rigidity constraint:** $\omega \sim H_{\infty} \omega H_{\infty}^T$
 - recover ω linearly from rigidity and ≥ 2 observed H_{∞} 's
- ⊕ Simple, direct and relatively stable
- ⊖ Requires known-distant points, *i.e.* affine structure
 - otherwise H_{∞} can only be recovered indirectly via ω, Ω

- Initial **projective reconstruction** gives P_1, \dots, P_m up to
 - individual rescaling $P_i \rightarrow \lambda_i P_i$
 - overall 3D homography $P_i \rightarrow P_i T$
- In this basis, Ω is unknown, 4×4 rank 3 matrix
- In each image, Ω projects to *same* dual absolute conic ω

$$\omega \sim P_i \Omega P_i^T$$

- Cross-multiply to eliminate scales:

$$\omega^{AB} (P_i \Omega P_i^T)^{CD} - (P_i \Omega P_i^T)^{AB} \omega^{CD} = 0$$

\Rightarrow $15m$ **bilinear equations** in $10+6$ unknowns Ω, ω

- Write constraints symbolically as $\omega \wedge (P \Omega P^T) = 0$
- Solve by **constrained optimization** or **linearization**
 - supplementary quartic constraint $\det(\Omega) = 0$

Solving for Ω, ω

Constrained Numerical Optimization

- Minimize $\sum_i \|\omega \wedge (P_i \Omega P_i^T)\|^2$ given $\det(\Omega) = 0$
- **Sequential quadratic programming (SQP)** based algorithm
- ⊕ Accurate — takes all constraints into account
- ⊕ Additional constraints easily imposed
 - e.g. known skew, aspect ratio, principal point . . .
- ⊖ Initial guess required — but not too critical

“Quasi-Linear” Method

Quadric projection equations $\omega \wedge (P_i \Omega P_i^T) = 0$ give $15m$

linear constraints on $6 \cdot 10 = 60$ unknowns $x = \Omega \otimes \omega$

- 1 Solve linearly for x 's by homogeneous least squares (SVD)
 - 2 Write x 's as 6×10 matrix and project to rank 1 (SVD#2)
 - 3 Extract ω and Ω , project Ω to rank 3 (SVD#3)
- ⊕ No initialization required
 - ⊖ Requires $m \geq 4$ images as nonlinear constraints ignored
 - ⊖ Slower & less stable than SQP — fails for fixating camera

- Iterative Newton-like method for **constrained optimization**

- Minimize cost $f(\mathbf{x})$ given constraints $\mathbf{c}(\mathbf{x}) = \mathbf{0}$

1 Lagrange multipliers \mathbf{z} give implicit solution:

$$\nabla f + \mathbf{z} \cdot \nabla \mathbf{c} = \mathbf{0} \quad \text{with} \quad \mathbf{c}(\mathbf{x}) = \mathbf{0}$$

2 Approximate cost & constraint to 2^{nd} order, to give exactly soluble **constrained quadratic programming** subproblem:

$$\begin{pmatrix} \nabla^2 f & \nabla \mathbf{c}^\top \\ \nabla \mathbf{c} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \delta \mathbf{x} \\ \mathbf{z} \end{pmatrix} = - \begin{pmatrix} \nabla f \\ \mathbf{c} \end{pmatrix}$$

3 Solve, update $\mathbf{x} += \delta \mathbf{x}$, iterate to convergence

- + Quadratic asymptotic convergence

— much better than most *ad hoc* methods

- + Numerically stable & accurate

- + Can handle arbitrary constraints

- Initialization required, may get stuck in local minima

Autocalibration Algorithm

1 Normalize all image coordinates

2 Projective reconstruction \Rightarrow projections \mathbf{P}_i

3 Minimize error $\sum_i \|\omega \wedge (\mathbf{P}_i \Omega \mathbf{P}_i^\top)\|^2$ over ω and Ω
subject to $\det(\Omega) = 0$ (nonlinear & quasi-linear methods)

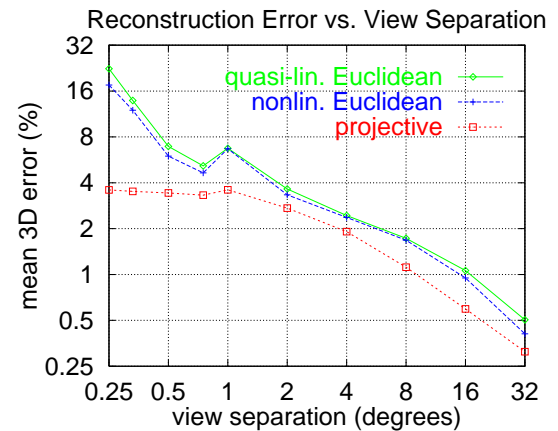
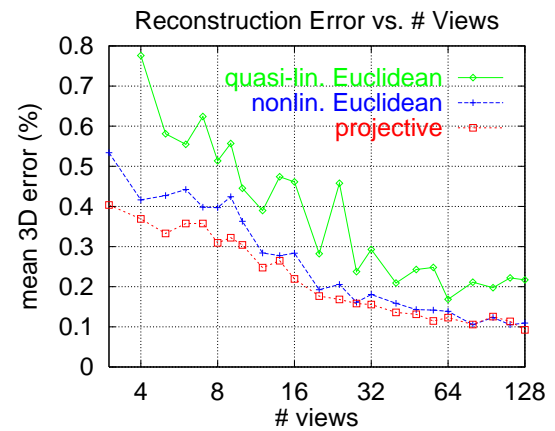
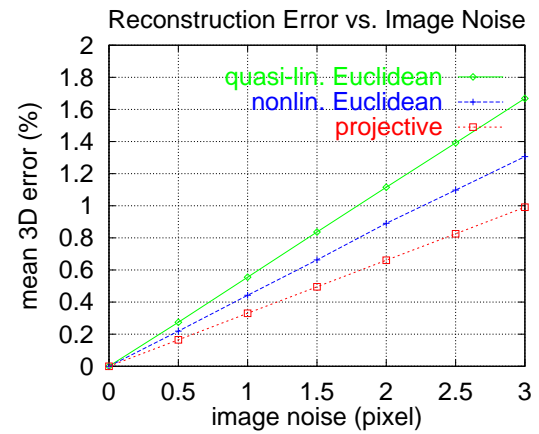
4 Choleski decomposition $\omega = \mathbf{K}\mathbf{K}^\top \Rightarrow$ camera calibration \mathbf{K}

5 SVD Ω & extract rectifying transform $\mathbf{T} \Omega \mathbf{T}^\top = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$

6 $\mathbf{P}_i \rightarrow \mathbf{K}^{-1} \mathbf{P}_i \mathbf{T}^{-1} \sim (\mathbf{R}_i | \mathbf{t}_i)$

7 *Optional:* adjust \mathbf{P}_i to be exactly Euclidean and re-estimate 3D structure with new \mathbf{P}_i 's

8 *Optional:* bundle adjustment . . .



Comparison with Conventional Calibration

- Informal test on precise calibration grid data
- All intrinsic parameters were within $1-2\sigma$ of conventional calibration
- Results may be more accurate than our precision calibration grid . . .

- The *absolute quadric* Ω
 - a simple projective representation of metric structure
- Autocalibration by *absolute quadric projection* constraint
 - gives stable, accurate & practical numerical method

Future Work

- Immediate extensions to
 - varying and/or constrained internal parameters
 - several moving cameras and stereo heads
- Automatic detection of degeneracy & constraint violations
- Specialized methods for small, continuous motions
- General SQP based approach to constrained geometric fitting