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Geometric predicates as arrangements of hypersurfaces: Application to comparison of algebraic numbers

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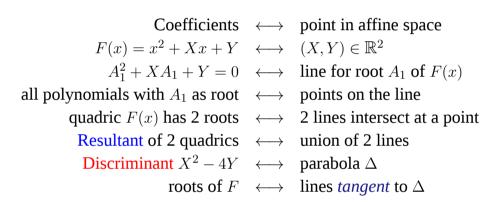
Predicates

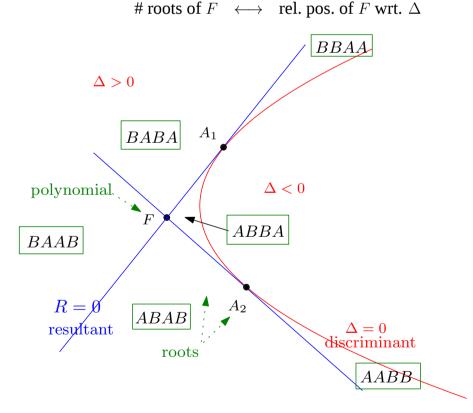
Formulation of a wide range of geometric predicates:

- arrangement of hypersurfaces in \mathbb{R}^d
- cells of dimension $\leq d$ (i.e., vertices, edges, faces)
- point location queries

Geometry in the space of coefficients

Given 2 polynomials of deg. 2, determine their root ordering among the 6 non-degenerate cases *AABB*, *ABAB*, *ABABA*, *BAABA*, *BBAA*.





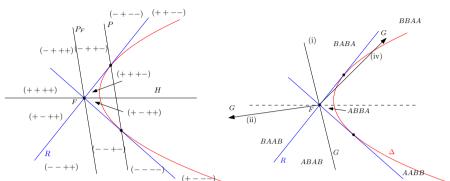
Arrangement consisting of the following hypersurfaces:

- Resultant (pair of lines)
- Discriminant (parabola)

 $\begin{array}{cccc} \text{points in same cell} & \longleftrightarrow & \text{same root ordering} \\ \text{crossing of } & \longleftrightarrow & \text{roots of 2 polys equal} \\ \text{crossing of } & \longleftrightarrow & \text{roots of same poly equal} \\ \end{array}$

Decomposition

ullet exploiting conic properties of pole and polar (duality) or intersecting rays from F with Δ



• When $F(x) = a_2x^2 + a_1x + a_0$ and $G(x) = b_2x^2 + b_1x + b_0$ the tested quanties are:

$$R = b_0^2 a_2^2 - 2 b_0 a_0 b_2 a_2 + a_0^2 b_2^2 - a_1 b_1 b_0 a_2 - a_1 b_1 a_0 b_2 + a_1^2 b_0 b_2 + a_0 b_1^2 a_2$$
(resultant)
$$H = b_1 a_2 - a_1 b_2 \text{ (horizontal line)}$$

$$P = -2 b_0 a_2 + a_1 b_1 - 2 a_0 b_2 \text{ (polar)}$$

 $P_F = a_1b_1a_2 - a_1^2b_2 - 2b_0a_2^2 + 2a_0a_2b_2$ (polar translated)

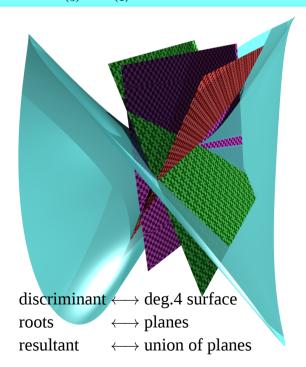
To compare algebraic numbers of degree 2, we have to consider quantities of at most algebraic degree 4 in the worst case. This bound is tight and therefore optimal.

Proof

- walls of arrangement defined by resultant (deg.4) and discriminant (deg.2)
- both irreducible, separating full-dimensional cells
- ⇒ they are *necessary* to define the arrangement
- all subsidiary equations have alg. deg. ≤ 4

Cubics

Given 2 polynomials of deg. 3, determine their root ordering among the $\binom{6}{3} + 2\binom{4}{1} + 2 = 30$ non-degenerate cases.



- quantities of alg. degree at most 6
- optimal, but geometric interpretation not yet known

Higher degree

- difficult to visualise
- *open problem:* optimal decomposition of the arrangement