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Swirling-Sweepers: Constant Volume Modeling

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Swirling-sweepers is a new method for modeling shapes while preserving volume. The artist describes a deformation by dragging a point along a path. The method is independent of the geometric representation of the shape. It preserves volume and avoids self-intersections, both local and global. It is capable of unlimited stretching and the deformation can be controlled to affect only a part of the model.

1 Motivation

In a virtual modeling context, there is no material. A challenge for computer graphics is to provide a modeling tool that convinces the artist that there is a material. Volume is one of the most important factors influencing the way an artist models with real materials.

The limitation of existing volume-preserving methods is either that they only apply to a specific type of geometric representation, or they only apply to shapes whose volume can be computed, with the exception of [Decaudin 1996]. His technique does not always preserve volume, and is discontinuous at one point.

2 Principle of Swirling-Sweepers

A Swirling-Sweeper is a new space deformation based on our framework called Sweepers [Angelidis et al. 2004b]. It is a blend of simpler deformations that we call swirls. In Figure 1, we show that a swirl is a rotation whose magnitude decreases away from its center, c. We represent the magnitude of rotation by a C2 monotonic scalar function, φ, which vanishes outside a neighborhood of radius λ around c. More formally, a swirl is a rotation matrix raised to the power φ

\[ f(p) = \exp(\phi(||p - c||)) \log(M) p \]  

(1)

A swirl preserves volume since the determinant of its Jacobian is always equal to 1. There is a convenient way of blending n swirls to produce a more complex deformation

\[ f_n(p) = \exp(\sum_{i=1}^{n} \phi(||p - c_i||)) \log(M_i) p \]  

(2)

See [Angelidis et al. 2004a] for computing exp and log.

From Swirls to Swirling-Sweepers: By specifying a single translation \( t \), an artist can input \( n \) swirls. As shown in Figure 2, we place \( n \) swirl centers, \( c_i \), on the circle of center \( h \), and radius \( r \) lying in a plane perpendicular to \( t \). These points correspond to \( n \) consistently-oriented unit tangent vectors \( \tilde{v}_i \). Each pair, \((c_i, \tilde{v}_i)\), together with an angle, \( \theta \), define a rotation. Along with radii of neighborhood \( \lambda = 2r \), we define \( n \) swirls. The radius of the circle, \( r \), is left to the user to choose. The following value for \( \theta \) will transform \( h \) exactly into \( h + t \), and preserves volume for sufficiently small \( t \):

\[ \theta = \frac{2||t||}{n} \]  

(3)

Preserving coherency and volume: To preserve coherency and volume, it is necessary to subdivide input vector \( t \) into a series of smaller vectors. We use \( s = \max(1, \lfloor ||t||/r \rfloor) \) sub-vectors. This decomposition is shown in Figure 3.

Achieving Real-Time: We have a closed-form for the logarithm of a rotation matrix and also for computing \( (\exp(M))p \), when \( N \) is the logarithm of an unknown rotation matrix. These formulas, together with a more complete discussion can be found in [Angelidis et al. 2004a]. They save otherwise expensive numerical approximations.

References

