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Swirling-Sweepers: Constant Volume Modeling

Alexis Angelidis*, Marie-Paule Cani†, Geoff Wyvill‡, Scott King‡
University of Otago, Laboratoire GRAVIR†, University of Otago

Swirling-sweepers is a new method for modelling shapes while preserving volume. The artist describes a deformation by dragging a point along a path. The method is independent of the geometric representation of the shape. It preserves volume and avoids self-intersections, both local and global. It is capable of unlimited stretching and the deformation can be controlled to affect only a part of the model.

1 Motivation

In a virtual modeling context, there is no material. A challenge for computer graphics is to provide a modeling tool that convinces the artist that there is a material. Volume is one of the most important factors influencing the way an artist models with real materials.

The limitation of existing volume-preserving methods is either that they only apply to a specific type of geometric representation, or they only apply to shapes whose volume can be computed, with the exception of [Decaudin 1996]. His technique does not always preserve volume, and is discontinuous at one point.

2 Principle of Swirling-Sweepers

A Swirling-Sweeper is a new space deformation based on our framework called Sweepers [Angelidis et al. 2004b]. It is a blend of simpler deformations that we call swirls. In Figure 1, we show that a swirl is a rotation whose magnitude decreases away from its center, c. We represent the magnitude of rotation by a $C^2$ monotonic scalar function, $\phi$, which vanishes outside a neighborhood of radius $\lambda$ around c. More formally, a swirl is a rotation matrix raised to the power $f(p) = \exp(\phi(||p - c||) \log M) p \tag{1}$

A swirl preserves volume since the determinant of its Jacobian is always equal to 1. There is a convenient way of blending n swirls to produce a more complex deformation:

$f_n(p) = \exp(\sum_{i=1}^{n} \phi(||p - c_i||) \log M_i) p \tag{2}$

See [Angelidis et al. 2004a] for computing exp and log.

From Swirls to Swirling-Sweepers: By specifying a single translation $t$, an artist can input n swirls. As shown in Figure 2, we place n swirl centers, $c_i$, on the circle of center $h$, and radius $r$ lying in a plane perpendicular to $t$. These points correspond to $n$ consistently-oriented unit tangent vectors $\vec{v}_i$. Each pair, ($c_i$, $\vec{v}_i$), together with an angle, $\theta$, define a rotation. Along with radii of neighborhood $\lambda = 2r$, we define $n$ swirls. The radius of the circle, $r$, is left to the user to choose. The following value for $\theta$ will transform $h$ exactly into $h + t$, and preserves volume for sufficiently small $t$:

$\theta = \frac{2||f||}{n} \tag{3}$

*email:alexis@cs.otago.ac.nz
†email:Marie-Paule.Cani@imag.fr
‡GRAVIR is a joint lab of CNRS, INRIA, Institut National Polytechnique de Grenoble and Université Joseph Fourier.
§email:geoff@cs.otago.ac.nz
¶email:scott@cs.otago.ac.nz

Preserving coherency and volume: To preserve coherency and volume, it is necessary to subdivide input vector $t$ into a series of smaller vectors. We use $s = \max(1, \lceil\frac{1}{4}||f||/r\rceil)$ sub-vectors. This decomposition is shown in Figure 3.

Achieving Real-Time: We have a closed-form for the logarithm of a rotation matrix and also for computing $(\exp N)p$, when $N$ is the logarithm of an unknown rotation matrix. These formulas, together with a more complete discussion can be found in [Angelidis et al. 2004a]. They save otherwise expensive numerical approximations.

References

