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A comparison of Aloha and CSMA in Wireless Ad-Hoc Networks under Different Channel Conditions

Bartłomiej Błaszczyszyn (*INRIA/ENS Paris*), Paul Muhlethaler and Skander Banaouas (*INRIA Rocquencourt*)

Abstract: In this paper we compare Aloha and Carrier Sense Multiple Access (CSMA) in Wireless Ad-Hoc Networks. We use a Signal-to-Interference-and-Noise Ratio (SINR) model where a transmission is assumed to be successful when the SINR is larger than a given threshold. Regarding channel conditions we consider both negligible and standard Rayleigh fading. For slotted and non-slotted Aloha we use analytical models as well as simulations to study the density of successful transmissions in the network. As it is difficult to build precise models for CSMA, we use only simulations to compute the performance of this protocol. We compare the two Aloha versions and CSMA on a fair basis, i.e.; when they are optimized to maximize the density of successful transmissions. For slotted Aloha, the key optimization parameter is the medium access probability, for non-slotted Aloha we tune the mean back-off time, whereas for CSMA it is the carrier sense threshold that is adjusted. Our study shows that CSMA always outperforms slotted Aloha, which in turn outperforms its non-slotted version. The gain in the density of successful transmissions depends however on the model's parameters: the path-loss exponent, SINR threshold, the existence or not of the fading, as well as the transmission range. Our conclusions differ from these of the seminal paper by Nelson and Kleinrock (1983), see [15], where the performance of Aloha is found comparable to this of CSMA in a simple geometric model, the reason being a non-optimal choice of the sensing range in the latter model.

Index Terms—Medium Access Control, MANET, slotted and non-slotted Aloha, CSMA, Poisson point process, shot-noise, SINR, stochastic geometry.

I. INTRODUCTION

Multiple communication protocols are used to organize transmissions of several sources (network nodes) in such a way that scheduled transmissions are likely to be successful. Aloha is one of the most common examples of such a protocol. A major characteristic of Aloha is its great simplicity: the core concept consists in allowing each source to transmit a packet and back-off for some random time before the next transmission, independently of other sources. Slotted Aloha is a variant of this general concept, consisting in synchronizing all sources and making them send packets in some universal time slots. Both slotted and non-slotted Aloha schemes, of course, lead to collisions and some packets have to be retransmitted. Carrier-Sense Multiple Access (CSMA) protocol is perhaps the most simple and popular multiple communication protocol that integrates some collision avoidance mechanism. The main idea of CSMA is to listen before sending a packet; if a node which has a packet to send senses that the channel is busy then the node backs-off for another attempt. The actual transmission will only occur when the channel is sensed idle. This mechanism does not allow for totally collision-free communication protocol, but it is supposed to substantially reduce the fraction of unsuccessful transmissions, and hence retransmissions. Optimization of Aloha and CSMA schemes consists in maximizing the rate of successful transmissions.

This can be done by adjusting the mean back-off time in Aloha and the mechanism CSMA uses to verify if the channel is idle.

Simple classical models allow one to analyze Aloha and CSMA (see [8, 14]). They show that CSMA significantly outperforms Aloha as long as the maximum propagation delays between network nodes remain small compared to the packet transmission delays. However these models are not suitable for wireless network context, as they do not take into account the specificity of the radio propagation of the signal. Consequently, they cannot capture the spatial reuse effect (i.e., the possibility of simultaneous successful wireless transmissions in space) that is a fundamental property of wireless communications.

The aim of this paper is to compare the performance of Aloha and CSMA used in wireless ad-hoc networks. To this regard, we model the geographic locations of network nodes by a planar Poisson point process and use the standard power-law path-loss function of the Euclidean distance to model the mean attenuation of the signal power. Regarding radio channel conditions, we consider both negligible and standard Rayleigh fading. We use a SINR model in which each successful transmission requires that the receiver is covered by the transmitter with a minimum SINR.

In the case of Aloha (both slotted and non-slotted), the above model is amenable to mathematical analysis as shown in [4–6, 9]. We will use (and even slightly develop) this approach as well as simulations (which confirm the analytical results) to evaluate and optimize the performance of Aloha.

The performance of the CSMA in the aforementioned network model with SINR capture condition seems not to be amenable to a similarly simple mathematical analysis, thus we use only simulations to study it.

The main contribution of this paper is the analysis and comparison of the performances of the slotted, non-slotted Aloha and CSMA, all optimized to maximize the rate of successful transmissions, *under various radio propagation assumptions* (path-loss exponent, existence or not of Rayleigh fading). *Our main findings of this analysis are:*

- CSMA always outperforms slotted Aloha, which in turn outperforms the non-slotted Aloha. In a moderate path-loss scenario (path-loss exponent equal to 4), without fading and the SINR level required for capture equal to 10, CSMA offers about 2.4 times larger rate of successful transmissions than slotted Aloha and about 3.2 times larger than non-slotted Aloha.
- The advantage from using CSMA is slightly reduced by increasing path-loss.
- This advantage is significantly reduced by the existence of fading since CSMA is much more sensitive to channel randomness than Aloha. In particular, for Rayleigh fading the

aforementioned comparison of CSMA to slotted and non-slotted Aloha gives the ratios 1.7 and 2.3, respectively.

- The advantage from using CSMA increases with the SINR capture level.
- The above observations are valid when the transmissions are roughly scheduled to nearest neighbors and all the three MAC schemes are optimally tuned. This optimal tuning results in scheduling each node for transmission for about 8%, 6% and 4% of time, for CSMA, slotted and non-slotted Aloha, respectively. These values do not depend on the network density, provided the nearest-neighbor receiver scheduling.
- The optimal tuning of CSMA is obtained by fixing the carrier-sensing power level (used to detect if the channel is idle) to about 8% of the useful signal power received at the nearest neighbor distance. This makes the allowed transmissions successful with high probability (from 0.8 to 0.95). Both smaller and larger values of the carrier-sensing threshold lead to essentially suboptimal performance of CSMA and sometimes even comparable to this of slotted Aloha. This might explain the apparent contradiction of our results to these of [15], which foresee similar performance of Aloha and CSMA.

This paper bring also some *contribution to the development of the mathematical tools for Aloha*. It consists in showing that the so-called *spatial contention factor* (cf [13]), appearing in the Laplace-transform characterization of the interference, is larger in non-slotted Aloha, compared to the slotted one under the same channel assumptions, by some factor that depends in a simple, explicit way only on the path-loss exponent; cf. Fact 3.2. We also suggest the usage of the Bromwich contour inversion integral, developed in [1], to evaluate the coverage probability in the no-fading case; cf. Fact 3.6.

Let us briefly say now *what the reader will not find in this paper*. We analyze the rates of successful transmissions offered by the three MAC protocols only in the *saturated traffic model*; i.e. when each node has always a packet to transmit; we do not deal with packet retransmissions due to collisions and with packet queuing. The analysis of such a simplified scenario is usually a first step toward the analysis of the respective queuing models of the MAC schemes, where the MAC parameters are subject to some (e.g. back-pressure) policies supposed to stabilize the protocols in long time horizon. This latter problem is usually addressed in the literature under much simpler network, propagation and capture conditions, and it is still a challenge in our Poisson network with SINR capture.

The remaining part of this paper is organized as follows. In what follows (Section I-A) we recall some previous studies of Aloha and CSMA. Section II introduces the model: distribution of nodes, channel and capture assumptions. It also describes in more details the three MAC protocols studied in this paper. In Section III we presents our analysis tools. Section IV provides our findings regarding the performance of the considered MAC protocols. The conclusions are presented in Section V. A short Appendix provides the proof of our main contribution to development of the mathematical analysis of Aloha.

A. Related Work

Aloha and Time Division Multiple Access (TDMA) are the oldest multiple access protocol. Aloha, which is the “mother” of random protocols, was born in the early seventies, the seminal work describing Aloha [2] being published in 1970. Since

that time it has been widespread in various implementations. The simplicity of the basic idea of Aloha also allows for simple analysis. A first, and now widely taught result regarding the fraction of successful transmissions (cf. e.g [8, 4.2]) was obtained assuming an aggregate, geometry-less process of transmissions following a temporal Poisson process, with some overlapping of two or more packet transmissions necessarily leading to a collision. In this model, the fraction of successful transmissions can attain $1/(2e) \approx 18\%$, when the scheme is optimized by appropriate tuning the of the mean back-off time (intensity of the Poisson process). It was also shown, that this performance can be multiplied by 2 in *slotted-Aloha*, when all the nodes are synchronized and can send packets only at the beginning of some universal time slots.

Although Aloha was primarily designed to manage wireless networks, the lack of geometric (geographic) representation of node locations in the above model makes it more suitable for wired networks. To the authors’ best knowledge, it is in the paper by Nelson and Kleinrock [16] that Aloha was first explicitly studied in a wireless context. The authors showed that in the slotted Aloha, under ideal circumstances, the “expected fraction of terminals in the network that are engaged in successful traffic in any slot does not exceed 21%”. Despite the very simple on-off wireless propagation model used in this paper, this result, as we will show, is surprisingly close to the results that can be obtained using more recent and more sophisticated, physical propagation and interference models (cf. [6, 7, 13]) in the case of the fading-less channel model with the mean path-loss of exponent equal to 3.5. The key element of this latter approach is the explicit formula of the Laplace transform of the interference created by a Poisson pattern of nodes using Aloha. This analysis was recently extended to non-slotted Aloha in [9]. We adopt this approach and slightly extend it further in the present paper.

In the widely referenced paper [10] another simplified propagation model was used to study local interactions of packet transmissions and the stability of spatial Aloha. A more recent study of the stability of spatial Aloha can be found in [10].

In contrast to Aloha, the relatively simple collision avoidance mechanism of CSMA is extremely difficult to analyze. In fact, no satisfactory, analytically tractable model of CSMA is known, despite several proposed approaches, in particular a recent one using Gibbs techniques (cf. [11]) or Matern hard-core point processes; (cf. [5, Ch. 18]). For this reason, in this paper we use simulations to analyze CSMA. However, we want to recall the original geometric approach, again by Nelson and Kleinrock, presented in [15]. This seminal paper presents a comparison of the performance of Aloha and CSMA in the geometric setting with the simple on-off wireless propagation model. This comparison is also the goal of our present study that uses however a more realistic propagation and interference model (see above). Our conclusions apparently *differ* from these of [15], where the performance of CSMA is found comparable to Aloha, as we show that CSMA, with an appropriately tuned sensing threshold, can essentially outperform Aloha. The reason for this difference is presumably not due to the different wireless channel models, but primarily because of a sub-optimal tuning of the CSMA in [15], consisting in too small sensing range (taken to be equal to the carrier range).

II. MODELS

In this section we present the models, which will be used to evaluate and compare the performance of CSMA and Aloha MAC schemes.

A. Distribution of Nodes

The model that we use here was proposed in [6]; see also [5, Chapter 16] for a more detailed presentation, where it is called *the Poisson Bipole model*. It assumes that the nodes of a Mobile Ad hoc NETWORK (MANET) are distributed on the infinite plane according to a homogeneous, planar Poisson point process (p.p.) of intensity λ nodes per unit surface area (say per one square meter). Each node of this network is willing to transmit a packet to its own dedicated receiver located within a distance r meters from it, which is *not* a part of the Poisson point process.

Despite its drawbacks, mostly related to the modeling of the locations of receivers, this model is reasonable, convenient and seems to be widely accepted (cf [12, 17]). The fact that the receivers are not part of the MANET and that they are all at the same distance from their transmitters is a simplification. A natural extension assumes that the transmitters of the MANET choose the nearest neighbours *among* the MANET nodes as their receivers. Such a model is used in [3] to study the local packet transmission delays in MANET (see also [5, Chapter 17]). Its analysis is however more difficult, and in this paper we will use the much simpler, Bipolar model, *choosing the receiver distance r to be $r = a/\sqrt{\lambda}$* , for some constant $a > 0$, i.e.; of the *order of the mean distance to the nearest neighbor* in a Poisson point process of intensity λ . This choice mimics the nearest neighbor scenario.

Using the formalism of the theory of point processes, we will say that a snapshot of the MANET can be represented by an independently marked Poisson point process (P.p.p) $\tilde{\Phi} = \{(X_i, y_i)\}$, where the *locations of nodes* $\Phi = \{X_i\}$ form a homogeneous P.p.p. on the plane, with an intensity of λ , and where the mark y_i denotes the location of the receiver for node X_i . We assume here that one receiver is associated with only one transmitter and that, given Φ , the vectors $\{X_i - y_i\}$ are i.i.d with $|X_i - y_i| = r$.

B. Wireless Channel Model

We assume that whenever node $X_i \in \Phi$ transmits a packet it emits a unit-power signal that is propagated and reaches any given location y on the plane with power equal to $F/l(|X_i - y|)$, where $|\cdot|$ denotes the Euclidean distance on the plane, $l(\cdot)$ is some deterministic path-loss function of the distance and F is a non-negative random factor of unit mean representing the variations of the received power due to channel fading.

An important special case, which is our default assumption in this paper, consists in taking a path-loss function of the form

$$l(u) = (Au)^\beta \quad \text{for } A > 0 \text{ and } \beta > 2. \quad (2.1)$$

Regarding the distribution of the random variable F , called for simplicity fading, we will consider two special cases:

- constant $F \equiv 1$, called *the no fading case*,
- exponential F of parameter 1; this corresponds to the *Rayleigh fading* in the channel. More precisely, we will assume that for any pair: emitter X_i , location y of a potential receiver, and for any packet transmission, an independent copy $F = F_{i,y}$ is

used to model the fading throughout the transmission of this packet.

C. Successful Transmission

It is natural to assume that transmitter X_i *successfully transmits* a given packet of length B to its receiver y_i within the time interval $[u, u + B]$ if

$$\text{SIR} = \frac{F/l(|X_i - y_i|)}{\bar{I}} \geq T, \quad (2.2)$$

where T is some signal-to-interference (SIR) threshold and where \bar{I} is the *average interference* suffered by the receiver y_i during this packet transmission interval

$$\bar{I} = \frac{1}{B} \int_u^{u+B} I(t) dt, \quad (2.3)$$

with

$$I(t) = \sum_{X_j \in \Phi, X_j \neq X_i} F_{j,y_i}/l(|X_j - y_i|) \mathbf{I}(X_j \text{ transmits at time } t). \quad (2.4)$$

Note that taking (2.2) as the successful transmission condition, we ignore any external noise. This is a reasonable assumption if the noise is significantly smaller than the interference power \bar{I} , which is the case in our setting. We should remark however, that if necessary, it would not be difficult to extend both the simulation scenario and the analysis to the case with non-null constant or even random noise power.

D. MAC Protocols

We will assume a saturated traffic model, i.e, that each node always has a packet to transmit to its receiver. The times at which any given node can transmit are decided by the Medium Access Protocol (MAC). In this paper we study three MAC protocols: CSMA, slotted Aloha and non-slotted Aloha.

1) *CSMA*: We begin with the most efficient MAC protocol among these studied in this paper. The basic rule of CSMA/CA (called in what follows CSMA for short) is very simple: *each node willing to transmit a packet listens first to the channel and transmits only if it finds the channel idle; otherwise it postpones the transmission for a random "back-off" time.*

Despite the apparent simplicity of this rule, it is not an easy task to model the temporal dynamics of the transmission requests, which arrive and are either immediately accepted or postponed, possibly several times, before being eventually accepted. In our saturated traffic model each node X_i always has a packet that is (re)transmitted an infinite number of times according to the following temporal scheme: before any transmission attempt node X_i takes an exponential back-off time of mean δ , which expires, say, at time a . Then it measures the total power of the received signal I' (the measurement is assumed to take a negligible time)

$$I' = \sum_{X_j \in \Phi, X_j \neq X_i} F_{j,X_i}/l(|X_j - X_i|) \mathbf{I}(X_j \text{ transmits at time } a). \quad (2.5)$$

and starts its packet transmission immediately afterwards *provided* $I' \leq \theta$, where θ is the *carrier-sense threshold*. If, however, $I' > \theta$ then node X_i postpones its transmission for a new copy of the exponential back-off time and reiterates the above procedure. It takes also the exponential back-off time after each transmission of the packet.

Important remarks regarding this scheme are as follows.

- The received power I' , used to verify if the channel is idle, is measured at the transmitter X_i while the interference \bar{I} in the

SIR condition deciding whether this transmission is successful is taken at the receiver y_i . Channels from any given interferer $X_j \neq X_i$ to these two points are assumed to be uncorrelated; i.e., characterized by independent copies of the fading F_{X_j, X_i} and F_{X_j, y_i} .

- The back-off time of exponential length of mean δ is primarily used to avoid collisions due to simultaneous transmissions by several nodes started immediately after completing of some given transmission¹. In this paper we will assume that δ is very small with respect to the packet duration time B , which allows us in the analysis of the channel contention (cf Section II-E) to ignore the fraction of time the channel is idle and not used by any node wishing to transmit.
- The carrier-sense threshold θ is the main, and in our model, the only parameter that will be tuned to maximize the density of successful transmissions and thus optimize the performance of the CSMA.
- The temporal transmission patterns of different nodes are obviously dependent because they depend on the common interference field. We will stipulate in Section II-E the existence of time-stationary models in our infinite planar network. Formally proving the existence of such an infinite space-time model of CSMA is not an easy task² and it is far beyond the scope of the present paper. We will study the performance of this model by a long-time simulation in a large, but finite window.

2) *Slotted Aloha*: Slotted Aloha MAC scheme supposes that all the network nodes are perfectly synchronized to some time slots (each of the length B of the packet, common for the whole network) and transmit packets according to the following rule: *each node, at each time slot independently tosses a coin with some bias p which will be referred to as the Aloha medium access probability (Aloha MAP); it sends the packet in this time slot if the outcome is heads and does not transmit otherwise.* Note that this is a discrete-time incarnation of the generic concept of Aloha consisting in allowing each source to transmit a packet and back-off for some random time independently of other sources.

As in CSMA, we assume a saturated traffic model, i.e., that each node at each time slot has a packet to transmit to its receiver. It is easy to formalise this scenario. A precise description of the stationary space-time model of slotted Aloha can be found in [9]. The Aloha MAP p is the main parameter to be tuned to optimize the slotted Aloha.

3) *Non-Slotted Aloha*: In non-slotted Aloha all the network nodes independently, without synchronization, send packets (of the same duration B) and then back off for some exponential random time of mean ε . In a more formal description of this mechanism one assumes that given a pattern of network nodes, the temporal patterns of their retransmission are independent (across the nodes) renewal processes with the generic inter-arrival time equal to $B + E$ where E is exponential (back-off) with mean ε . A precise description of this stationary space-time model, called the *Poisson-renewal model* of non-slotted Aloha can be found in [9]. Let us however make the following

¹The CSMA back-off time can be non-negligible when it is dynamically tuned by some MAC protocol stabilizing policy (e.g. a back-pressure one). This scenario is however beyond the scope of this paper.

²This is related to the known problems of existence and uniqueness of the infinite Gibbs models.

remarks.

- In contrast to CSMA, the mean back-off time ε is not negligible. In fact, non-slotted Aloha relies on the back-off mechanism to create space-time patterns of active nodes, whose simultaneous transmissions are more likely successful. Thus ε is the main parameter to be tuned in this protocol.
- The analysis of the aforementioned Poisson-renewal model of Aloha is feasible although it does not lead to simple closed formulas. In [9] another model, called the *Poisson rain model*, of non-slotted Aloha has been proposed. The main difference with respect to the scenario considered above is that the nodes X_i and their receivers y_i are not fixed in time. Instead we may think of these nodes as being “born” at some time T_i transmitting a packet during time B and “disappearing” immediately after. The joint space-time distribution of node locations and transmission instances $\Psi = \{(X_i, T_i)\}$ is modeled by a homogeneous Poisson p.p. in $2+1$ dimensions with intensity $\lambda_s = \lambda B / (\varepsilon + B)$. It might be theoretically argued that the Poisson rain model is a good approximation of the Poisson-renewal model when the density of nodes λ is large, and the time instances at which a given node retransmits are very sparse. Indeed, the performance of the Poisson-renewal model is shown in [9] to be very close to this of the Poisson rain model. Thus, in our analytical study of non-slotted Aloha we will use the results regarding the latter one for simplicity, while in our simulations we use the former one.

E. Network Performance under a Given MAC

MAC protocols are supposed to create some space-time patterns of active (transmitting) nodes that increase the chances of successful transmissions. MAC optimization consists in finding the right trade-off between the density of active nodes and the probability that the individual transmissions are successful.

The first step of the analysis of the above trade-off problem consists in evaluating how much a given MAC protocol contends to the channel; i.e., how many packets it attempts to send per one node and per unit of time. In homogeneous models this can be captured on average by the *average fraction of time a typical node is authorized to transmit*. We will denote this metric by τ . By space-time homogeneity $\tau\lambda$ is the spatial density of active nodes at any given time and thus τ can also be interpreted as the probability that a typical node of the MANET is active at a given time. In what follows we will call it *channel occupation* parameter. The way it depends on the basic (tunable) MAC parameters will be explained latter on.

A complete evaluation of the performance of a MAC protocol consists in finding out what fraction of authorized transmissions is successful. We will denote by p_c the *probability that a typical transmission of a typical node is successful* (given this node was authorized by the MAC to transmit). We call it the *coverage probability* for short. By (2.2) we have

$$p_c = \mathbf{P}^0 \{ F \geq l(r)T\bar{I} \}, \quad (2.6)$$

where the probability \mathbf{P}^0 corresponds to the distribution of the random variables “seen by a typical node during its typical transmission” and can be formalized on the ground of the Palm theory for point processes (cf. [5, 16.2.2]). This expression will be the basis of our analytical evaluation of the coverage probability for both slotted and non-slotted Aloha in Section III-C. We can notice that \bar{I} is independent of F in (2.6) because our

MAC schemes do not schedule transmissions according to the channel conditions at the receivers.

We define the optimal performance of a given MAC scheme as the situation where the mean number of successful transmissions per unit of surface and unit of time $\tau\lambda p_c$, called the *density of successful transmissions*, is maximized. For a given MANET density λ , this is equivalent to maximizing τp_c , which can be interpreted as the probability that a typical node of the MANET is transmitting at a given time and this transmission is successful. Following this interpretation we call τp_c *the mean throughput per node*. It will be analytically evaluated for both Aloha schemes and estimated by simulations for Aloha and CSMA MAC.

III. ANALYSIS TOOLS

A. Simulation Scenarios

Our simulations are carried out in a square of 1000 m \times 1000 m in which we generate a Poisson sample of MANET nodes with intensity $\lambda = 0.001$ nodes per square meter. For each MANET node we generate the location of its receiver uniformly on the circle of radius $r = a\sqrt{1000}$ m centered on this node. To avoid side effects, we consider a toroidal metric on this square. (Recall that, roughly speaking, rectangular torus is a rectangle whose opposite sites are “identified”.) Given this metric we consider the distance dependent path-loss model (2.1) with some given path-loss exponent β and $A = 1$. By default $a = 1$, $\beta = 4$, however in some studies we try different values. For each pair of nodes we generate an independent copy of the exponential variable F in the case of Rayleigh fading or take $F \equiv 1$ in no-fading case. Unless explicitly specified, our default value of the SIR threshold is $T = 10$.

For a given distribution of nodes we run the dynamic simulation for each of the three MAC schemes described in Section II-D with some particular choice of their main parameters (the carrier-sense threshold θ for CSMA, MAP p for slotted Aloha and mean back-off time for non-slotted Aloha; the packet transmission duration is always $B = 1$ units of time). We count both the total number of packet transmissions and the number of successful transmissions during the simulation, whose total time is 4000 units of time. Recall that packet transmission takes $B = 1$ unit of time. Thus dividing these numbers of transmissions by the simulation time and by the number of MANET nodes in the square, we obtain one-network-sample estimators of the, respectively, average fraction τ of time a typical node is authorized to transmit and the mean throughput per node τp_c . We repeat the above experiment for 10 random choices of the network and take the empirical means of the above one-sample estimators. The error-bars in all simulation results correspond to a confidence interval of 95%.

B. Analytical Results for Aloha MAC

The analytical results for Aloha are based on the (easy) calculation of the average fraction of time a typical node is authorized to transmit τ and a (more involved) calculation of the Laplace transform of the interference \bar{I} that is the only variable of “unknown” distribution in the expression (2.6) of the coverage probability p_c .

1) *Channel Occupation* τ : It is straightforward to see that in slotted Aloha $\tau = p$. In the Poisson-renewal model of non-slotted Aloha $\tau = B/(B + \varepsilon)$; i.e., the ratio between the packet duration time and the mean inter-transmission time.

2) *Interference Distribution*: The basic observation allowing for an explicit analysis of the coverage probability for all our Poisson models of Aloha is that the distribution of the interference \bar{I} under Palm probability \mathbf{P}^0 in (2.6) corresponds to the distribution of the interference “seen” by an extra receiver added to the original MANET pattern (say at the origin) during an arbitrary period of time of length B (say in $[0, B]$). This is a consequence of the Slivnyak’s theorem; cf. [4, Theorem 1.4.5].

Moreover, note that in the slotted Aloha MAC the interference $I(t) = I$ in (2.3) does not vary during the packet transmission and consequently $\bar{I} = I$. Furthermore, note that the pattern of nodes X_j , which emit at a given time slot and interfere with a given packet transmission (cf. expression (2.4)) is a Poisson p.p. of intensity $p\lambda$. This is a consequence of the independent MAC decisions of Aloha. The general expression of the Laplace transform \mathcal{L}_I of I , which in this case is a Poisson shot-noise variable, is known explicitly; cf. Appendix. Here we recall the expressions for the special cases of interest.

Fact 3.1: For the slotted Aloha model with path-loss function (2.1) and a general distribution of fading F with mean 1 we have:

$$\mathcal{L}_I(\xi) = \exp\{-\tau A^{-2} \xi^{2/\beta} \kappa\}, \quad (3.7)$$

where $\kappa \geq 0$ is some constant depending only on the path-loss exponent and the distribution of the fading F . In particular $-\kappa = \pi\Gamma(1 - 2/\beta)$ in no-fading scenario $F \equiv 1$, $-\kappa = 2\pi\Gamma(2/\beta)\Gamma(1 - 2/\beta)/\beta$ in Rayleigh fading (exponential F).

The constant κ was evaluated in [6] for Rayleigh fading and in [13], for the no-fading scenario, where the name *spatial contention factor* was proposed for this constant.

Regarding the distribution of the averaged interference \bar{I} in non-slotted Aloha, we have the following *new general* result.

Fact 3.2: Assume the Poisson rain model of non-slotted Aloha with space-time intensity of packet transmissions $\lambda_s = \lambda\tau$ and path-loss function (2.1). Assume a general distribution of fading F . Then the Laplace transform $\mathcal{L}_{\bar{I}}(\xi)$ of the averaged interference \bar{I} is given by (3.7) with the spatial contention factor $\kappa = \kappa_{non-slotted}$ equal to

$$\kappa_{non-slotted} = \frac{2\beta}{2 + \beta} \kappa_{slotted},$$

where $\kappa_{slotted}$ is the spatial contention factor evaluated for slotted Aloha under the same channel assumptions.

The proof is given in the Appendix.

Remark 3.3: Regarding the ratio of the spatial contention parameters $\zeta = \zeta(\beta) = 2\beta/(2 + \beta)$, that can be seen as the *cost of non-synchronization in Aloha* (cf Remark 3.5 below), note that in the free-space propagation model (where $\beta = 2$) it is equal to 1 (which means that the interference distribution, and so coverage probability, in slotted and non-slotted Aloha are the same). Moreover, $\zeta(\beta)$ increases with the path-loss exponent and asymptotically (for $\beta = \infty$) approaches the value 2. This was only conjectured in [9].

C. Coverage Probability

Evaluation of p_c from (2.6) is straightforward in the case of Rayleigh fading. Indeed, in the case of exponential F , indepen-

dent of \bar{I} one has $\mathbf{P}^0\{F \geq l(r)T\bar{I}\} = \mathbf{E}^0[\exp\{-l(r)T\bar{I}\}] = \mathcal{L}_{\bar{I}}(l(r)T)$. By Facts 3.1 and 3.2 we have the following result.

Fact 3.4: For the Aloha model with path-loss function (2.1) and Rayleigh fading

$$p_c = \exp\left\{-\lambda\tau r^2 T^{2/\beta} \kappa\right\}, \quad (3.8)$$

where

- $\kappa = 2\pi\Gamma(2/\beta)\Gamma(1 - 2/\beta)/\beta$ for slotted Aloha and
- $\kappa = 4\pi\Gamma(2/\beta)\Gamma(1 - 2/\beta)/(2 + \beta)$ for non-slotted Aloha.

Remark 3.5: Note that due to our parametrization $r = a/\sqrt{\lambda}$ (which mimics the nearest-neighbor receiver model), the maximal mean throughput per node τp_c is achieved (in slotted or non-slotted Aloha with Rayleigh fading) for $\tau = \tau^* = \kappa^{-1} a^{-2} T^{-2/\beta}$ and it is equal to τ^*/e . In particular, by Fact 3.2, non-slotted Aloha achieves $\zeta = \zeta(\beta)$ times smaller maximal throughput than slotted Aloha, where ζ is the cost of non-synchronization in Aloha. The dependence of this cost on β is analyzed in Remark 3.3. Here, note only that the well-known result obtained for the simplified collision model with on-off path-loss function, and saying that slotted Aloha offers two times greater throughput than non-slotted Aloha (see [8, Section 4.2]) corresponds in our model to the infinite path-loss exponent; $\zeta(\infty) = 2$.

In the case of a general distribution of fading the evaluation of p_c from the Laplace transform $\mathcal{L}_{\bar{I}}$ is not so straightforward. Some integral formula, based on the Plancherel-Parseval theorem, was suggested in [7] when F has a square integrable density. This approach however does not apply to no-fading case $F \equiv 1$. Here we suggest another, numerical approach, based on the Bromwich contour inversion integral and developed in [1], which is particularly efficient in this case.

Fact 3.6: For Aloha model with constant fading $F \equiv 1$ we have

$$p_c = \frac{2 \exp\{c/(Tl(r))\}}{\pi} \int_0^\infty \mathcal{R}\left(\frac{1 - \mathcal{L}_{\bar{I}}(c + iu)}{c + iu}\right) \cos ut \, du, \quad (3.9)$$

where $c > 0$ is an arbitrary constant and $\mathcal{R}(z)$ denotes the real part of the complex number z .

As suggested in [1], the integral in (3.9) can be numerically evaluated using the trapezoidal rule and the Euler summation rule can be used to truncate the infinite series; the authors also explain how to set c in order to control the approximation error.

D. Carrier-Sense Scaling in CSMA

As mentioned above, a similar analysis of the performance of the CSMA scheme is not possible. In fact, neither the channel contention described by τ nor the distribution of \bar{I} under \mathbf{P}^0 is easy to evaluate for this scheme. Here we want only to comment on some scaling results (with the node density λ) regarding the performance of CSMA.

Note that in the noiseless scenario (cf SIR condition (2.2)), with nearest-neighbor-like distance $r = a/\sqrt{\lambda}$ from transmitter to receiver, and the path-loss function (2.1) the SIR is invariant with respect to a homothetic transformation of the model; i.e., dilating all the distances by some factor, say γ . However, the received powers (as interference I' measured by the transmitters) scale like $\gamma^{-\beta}$. By the well known scaling property of the homogeneous Poisson p.p. ³, this implies that the performance

³The dilation of a planar Poisson p.p. of intensity 1 by a factor $\gamma = \lambda^{-1/2}$ gives a Poisson p.p. of intensity λ .

of the CSMA scheme (values of τ and the distribution of \bar{I}) in our network model is invariant with respect to the MANET density provided the carrier-sense threshold θ varies with λ as $\theta = \theta(\lambda) = \theta(1)\lambda^{\beta/2}$.

In order to present our simulation results for CSMA in a scale-free manner, in Section IV-A we will plot the mean throughput τp_c if function of the modified carrier-sense threshold $\tilde{\theta} := \theta l(r)$ that can be seen as the power normalized by the received signal power (in contrast to θ that is normalized to the emitted signal power). This results in $\tilde{\theta} = \theta(1)\lambda^{\beta/2}(A\lambda^{-1/2})^\beta = \theta(1)A^\beta$ which does not depend on the density of the MANET.

Another way of presenting scale-free results is to express the carrier-sense threshold θ in terms of the equivalent carrier-sense distance R defined as the distance at which a unit of emitted power is attenuated to the value θ , i.e. satisfying $\theta = 1/l(R)$. In our path-loss model this relation makes $R = \theta^{-1/\beta}/A$. We will use this approach when comparing our optimal tuning of CSMA to that proposed in [15]; see Section IV-C.

IV. MAC OPTIMIZATION AND COMPARISON RESULTS

In this section we present our findings regarding analysis and comparison of the performance of Aloha and CSMA.

A. MAC Performance Study

We study the mean throughput per node τp_c achieved by CSMA and Aloha under our default setting ($a = 1$, $\beta = 4$, $T = 10$) with and without fading, depending on the MAC parameters, which are carrier-sense threshold θ , MAP p and mean back-off time ε for, respectively, CSMA, slotted and non-slotted Aloha.

1) CSMA: Figure 1 presents the throughput τp_c achieved by CSMA versus the modified carrier-sense threshold $\tilde{\theta}$. Recall that $\tilde{\theta}$ is the carrier-sense threshold in ratio to the useful power at the received (at the distance $r = a/\sqrt{\lambda}$) ⁴. This makes τp_c and $\tilde{\theta}$ independent of the MANET density; cf. Section III-D. Our first observations are as follows.

Remark 4.1: In the absence of fading the maximum throughput of 0.068 (unit-size packets per unit of time and per node) is attained by CSMA when the carrier-sense threshold is fixed roughly at the level of $\tilde{\theta} = \tilde{\theta}^* = 0.08$. This optimal tuning of the carrier-sense threshold seems to be quite insensitive to fading. However, the optimal throughput is significantly reduced by fading. Rayleigh fading of mean 1, reduces the CSMA throughput to 63.2% compared with the no fading scenario.

This latter observation is easy to understand as the channel-sensing is done at the emitter and that fading at the receiver is independent of fading at the emitter.

2) Aloha: Figure 2 presents the throughput τp_c with and without fading achieved by slotted Aloha versus the channel occupation time τ , which in this model is equal to the the MAP parameter p . The results of non-slotted Aloha are presented in Figure 3 with $\tau = 1/(1 + \varepsilon)$, where ε is the mean back-off time. Other parameters are as in the default setting. Here are our observations.

⁴In other words, e.g. $\tilde{\theta} = 0.1$ means that the channel is considered by an emitter as idle if the total power sensed by it is at most 10% of the mean useful signal power received by its receiver.

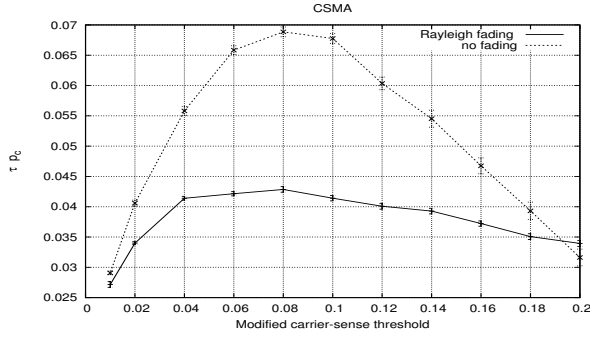


Fig. 1. Mean throughput per node τp_c versus modified carrier-sense threshold θ in CSMA; Rayleigh fading and no-fading scenario.

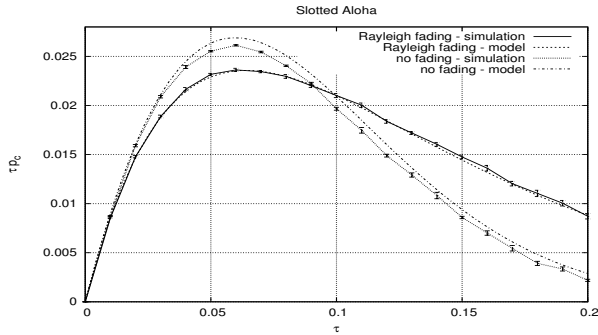


Fig. 2. Mean throughput per node τp_c versus channel occupation $\tau = p$ in slotted Aloha; Rayleigh fading and no fading scenario.

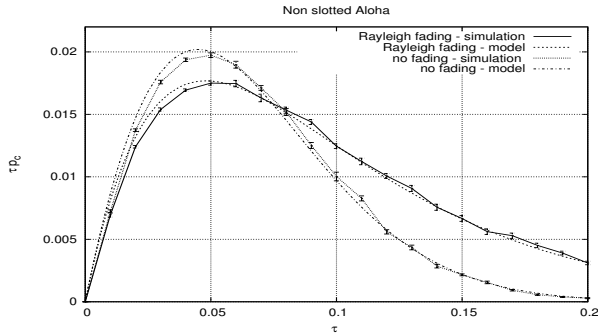


Fig. 3. Mean throughput per node τp_c versus channel occupation time $\tau = 1/(1 + \epsilon)$ for non-slotted Aloha; Rayleigh fading and no fading scenario.

Remark 4.2: In the absence of fading the maximum throughput of 0.028 for the optimal MAP $p = p^* \approx 0.06$. As in CSMA, this optimal tuning seems to be quite insensitive to fading, which in the case of Rayleigh fading can be evaluated explicitly as $p = p^* = \kappa^{-1} T^{-2/\beta}$ (which gives $p^* = 0.064081$ in the default Rayleigh scenario). In contrast to CSMA, Rayleigh fading has a relatively small impact on the slotted Aloha throughput reducing it only to 92% of the throughput achieved in the no-fading scenario (in contrast to 63.2% in CSMA). Similar observations hold for non-slotted Aloha, which in the Rayleigh fading scenario achieves $\zeta = 2\beta/(2 + \beta) = 1.5$ times smaller throughput than the slotted version.

B. Impact of Model Parameters

On Figures 4, 5, 6, 7, 8 and 9 we can study the dependence of the *maximal throughput* achievable by the MAC schemes (at their respective optimal tunings) in function to the path-loss exponent β , SIR threshold T and relative distance to the receiver a (recall that $a = r\sqrt{\lambda}$). It is clear at the that CSMA significantly outperforms both Aloha protocols for all choices of parameters. More fine observations are as follows.

Remark 4.3: The higher path-loss exponent β the smaller

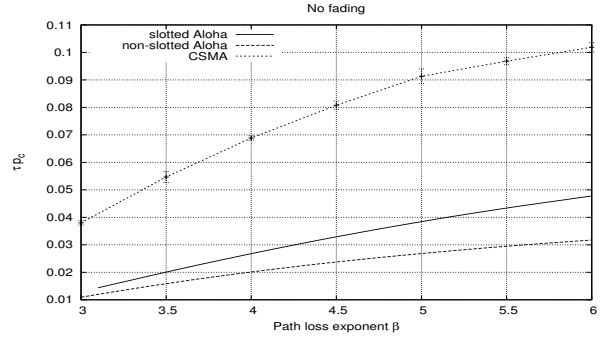


Fig. 4. Maximal achievable mean throughput per node τp_c versus path-loss exponent β in the absence of fading.

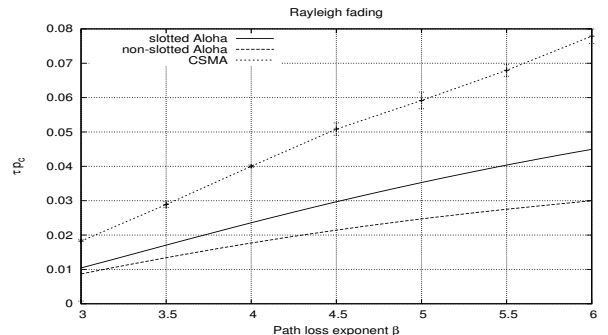


Fig. 5. Maximal achievable mean throughput per node τp_c versus path-loss exponent β with Rayleigh fading.

advantage from using CSMA. When there is no fading, the increase of β from 3 to 6 reduces the gain in throughput of CSMA with respect to slotted Aloha from 2.6 to 2.1 and with respect to non-slotted Aloha from 3.5 to 3.2.

We also note in Figure 4, that in the absence of fading, slotted Aloha attains the expected fraction of 21% of terminals engaged in successful traffic, foreseen in the seminal paper [16], for SINR threshold $T = 10$ and a moderate path loss exponent slightly larger than $\beta = 3.5$.

Remark 4.4: Existence of fading further diminishes the advantage of CSMA. In particular, Rayleigh fading reduces the gain in throughput of CSMA with respect to slotted Aloha to about 1.7 and for non-slotted Aloha to a factor between 2.5 and 2.1 (depending on β).

Studying the impact of the SINR threshold T we observe what follows.

Remark 4.5: The higher T (hence the smaller bit-error rate sustainable in each packet) the more advantage from using CSMA. In particular, when there is no fading and for $\beta = 4$, increasing T from 1 to 11 results in the increase of the gain in throughput of CSMA with respect to slotted Aloha from 2.4 to 3.5 and this latter ratio remains stable for T larger than 11. Regarding similar comparison of CSMA to non-slotted Aloha the gains are from 1.8 to 2.6. In the case of Rayleigh fading the analogous gain factors of CSMA are, respectively, from 1.8 to 2.4 with respect slotted Aloha and from 1.4 to 1.8 with respect to non-slotted Aloha.

Finally we study the impact of the relative distance to the receiver a (in relation to the mean distance to the nearest neighbor in the network). Figures 8, and 9 show clearly that this distance should be kept the smallest possible without disconnecting the network.

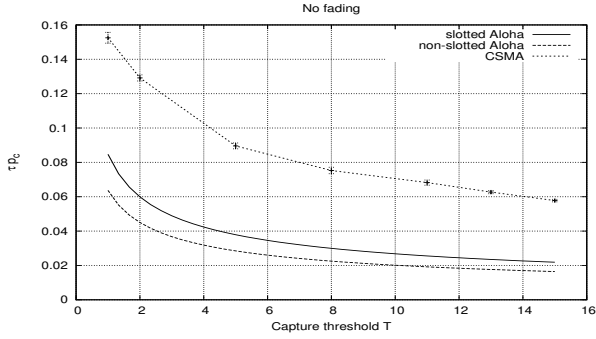


Fig. 6. Maximal achievable mean throughput per node τp_c versus SIR threshold T in the absence of fading.

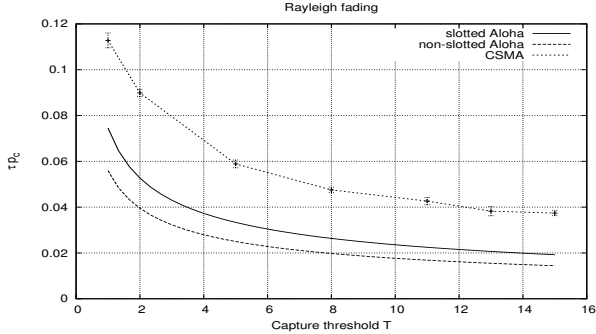


Fig. 7. Maximal achievable mean throughput per node τp_c versus SIR threshold T . Rayleigh fading.

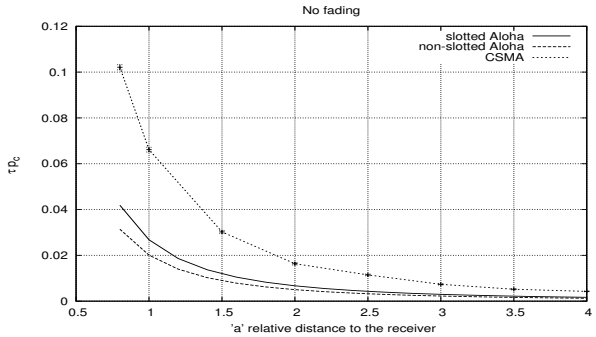


Fig. 8. Maximal achievable mean throughput per node τp_c versus a the distance transmitter receiver. No fading.

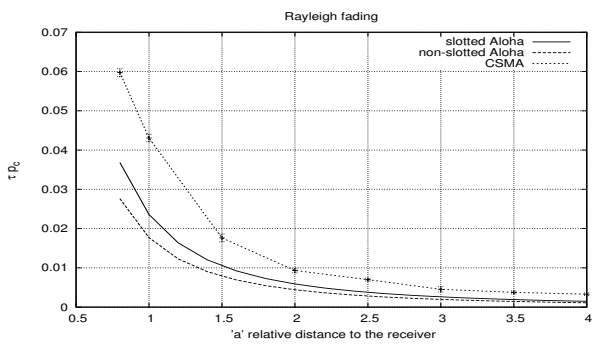


Fig. 9. Maximal achievable mean throughput per node τp_c versus a the distance transmitter receiver. Rayleigh fading.

C. Optimal Tuning of Aloha and CSMA

For Aloha the optimal tuning of τ can be obtained analytically from (3.8) whereas the optimal tuning of CSMA is obtained by simulation. In Figure 10 we present the optimal values of τ versus β with Rayleigh fading for both Aloha and CSMA.

Remark 4.6: We observe that more sophisticated MAC schemes are allowed to contend more to the channel under their optimal tunings (CSMA more than slotted Aloha, which in turn more than the non-slotted one). Obviously, they exhibit

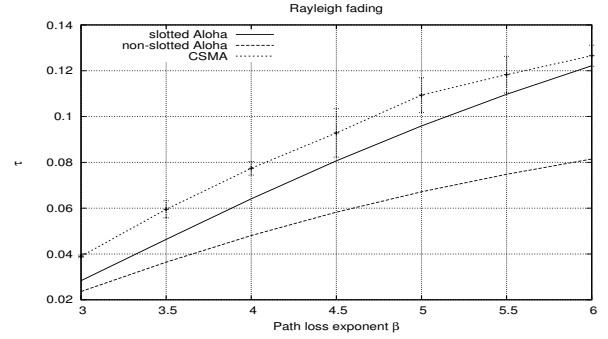


Fig. 10. Optimal value of τ for Aloha and CSMA versus β with Rayleigh fading

also higher capture probabilities. In particular our simulation show the that this probability is close to 1 (between 0.8 and 0.95) for CSMA.

A practical conclusion that can be drawn from the latter observations is that *the carrier-sense threshold in CSMA should be chosen at the largest possible value with which the allowed transmissions are almost always successful.*

D. Nelson&Kleinrock's Model of CSMA Revisited

Remark 4.6 might explain why the significant superiority of CSMA with respect to Aloha was not observed in [15]. Let us be more precise and revisit this latter paper model.

The simple propagation model in [15] assumes a fixed transmission range R and *the same* carrier-sense range. In other words, any two successfully communicating nodes need to be within distance R from each other and no any other transmission should occur in the distance R from the receiver.

In this model, an *ideal medium access scheme* suggested in [15] should be able to choose from the given pattern of nodes centers for a maximal number of hard (non-intersecting) disks of radius R . The asymptotic analysis of the performance of such an ideal scheme is done in [15] assuming increasing density of nodes λ . Namely, if this density is large, then the optimal scheme should be able to chose the pattern of nodes close to the hexagonal packing, known to obtain the densest packing of hard disks of radius R . Such a packing attains the fraction of 0.90689 of the plane covered by the union of disks. Consequently, since there is no disk overlapping, it would choose the fraction

$$\tau_{ideal} = \frac{0.90689 \times \text{network area}}{\# \text{ of nodes} \times \text{exclusion disk surface area}} = \frac{0.90689}{\lambda \pi R^2}$$

of the nodes of the network, whose density is λ nodes per unit of surface. This expression can be interpreted as the contention parameter of this ideal medium access scheme, which explains our notation. Since all transmissions allowed by this scheme are successful, we have $p_c = 1$ for it and the achieved throughput per node is $0.90689/(\lambda \pi R^2)$.

Regarding CSMA, the simple propagation model with transmission range equal to the carrier-sense range, assumed in [15], corresponds to a choice of nodes such that any selected node is not covered by the transmission range of any other selected node. This task is equivalent to the packing of hard disks of radius $R/2$. For some reason, that is partially explained in that paper, a slightly larger radius $1.2881R/2$ is taken. Similar to the ideal scheme, asymptotic analysis of the hexagonal pattern, gives the contention parameter of this CSMA scheme equal to

$$\tau_{CSMA} = \frac{2.214}{\lambda \pi R^2}.$$

Moreover, assuming that each authorized node chooses its receiver uniformly within the transmission range R , and calculating the fraction of the area within this range that is not covered by any other disk (no collision), the successful transmission probability is calculated as $p_c = 0.2034$. Consequently the throughput achieved by this CSMA is $\tau p_c = 0.4504/(\lambda\pi R^2)$.

Note that apparently the *sub-optimal assumption of the carrier-sense range equal to the transmission range in the above model of CSMA leads to relatively small successful transmission probability* $p_c = 0.2034$, close to that obtained by Aloha, which explains why there is no essential difference between the performance of these two schemes. Our optimally tuned CSMA model seems to be closer to the ideal scheme of [15], at least because the probability of successful transmission is much closer to 1.

Let us now try to compare the performance of our optimal CSMA and the two schemes of [15]. This is not straightforward, since unlike ours, the results of [15] scale in $1/\lambda$ are only valid asymptotically, when $\lambda \rightarrow \infty$ (due to the hexagonal approximation of the perfect packing). However, note that in the model of [15], the expression $N = \lambda\pi R^2$ corresponds to the expected number of nodes within the area contended (blocked) by one given authorized transmission. Consequently the constants $\rho = 0.90689 = \tau p_c N$ and $\rho = 0.4504 = \tau p_c N$ can be interpreted, respectively in the two models, as the expected number of successful transmissions per set of nodes contended (blocked) by one given authorized transmission. This kind of spatial efficiency can be evaluated in our model using the notion of the equivalent carrier-sense distance $R = \theta^{-1/\beta}/A = r\tilde{\theta}^{-1/\beta}$ introduced in Section III-D. Taking $N = \lambda\pi R^2$ with R calculated as such we obtain for our CSMA $\rho = \tau p_c N = \tau p_c \lambda \pi r^2 \tilde{\theta}^{-2/\beta}$. For the optimally tuned CSMA in the standard scenario $a = 1, T = 10, \beta = 4$ without fading we have $\rho = 0.07\pi 0.08^{-1/2} = 0.77750$ of successful transmissions per set of nodes contended (blocked) due to one given authorized transmission. This is a much better performance than $\rho = 0.4504$ for CSMA of [15], and in fact closer to $\rho = 0.90689$ achieved by the ideal scheme of [15].

V. CONCLUSIONS

In this paper we compared slotted and non-slotted Aloha with CSMA in a Poisson ad-hoc network setting with SINR-based capture condition. We assumed the usual power-law path-loss function and both Rayleigh and no-fading scenario. To obtain a fair comparison between these protocols their parameters were tuned to achieve the maximum successful transmission rates. Our analysis shows that CSMA always outperforms both slotted and non-slotted Aloha. However the gain from using CSMA is slightly reduced by increasing path loss and more significantly by the existence of fading. We also showed how to practically tune the carrier-sense threshold in CSMA so as to obtain its optimal tuning for an arbitrary network density. Our models concur with these of [15] even though some results may appear, at first glance, to be somewhat contradictory, because in [15] CSMA is not optimized.

APPENDIX

Proof of Fact 3.2. By (2.3), (2.4) and exchanging the order of integration and summation we express \bar{I} in the following

form

$$\bar{I} = \sum_{X_j \in \Phi, X_j \neq X_i} F_{j,y_i} H_j / l(|X_j - y_i|),$$

where $H_j = \frac{1}{B} \int_u^{u+B} \mathbf{1}(X_j \text{ emits at time } t) dt$. In the Poisson rain model we have $\mathbf{1}(X_j \text{ emits at time } t) = \mathbf{1}(t - B \leq T_j \leq t)$, where T_j is the time at which X_j starts emitting. Integrating the previous function we obtain $H_j = h(T_j)$, where $h(s) = (B - |s|)^+/B$ and $t^+ = \max(0, t)$. Consequently, for the Poisson rain model represented by Poisson p.p. $\Psi = \{X_i, T_i\}$ (cf. Section II-D3) the averaged interference at the typical transmission receiver is equal in distribution to

$$\bar{I} \stackrel{\text{distr.}}{=} \sum_{X_j, T_j \in \Psi} F_j h(T_j) / l(|X_j|),$$

where F_j are i.i.d. copies of the fading. Using the general expression for the Laplace transform of the Poisson shot-noise (see e.g. [4, Prop. 2.2.4]) we obtain for the path-loss function (2.1)

$$\mathcal{L}_{\bar{I}}(\xi) = \exp\left\{-2\pi\lambda_s \int_{-\infty}^{\infty} \int_0^{\infty} r \left(1 - \mathcal{L}_F(\xi h(t)(Ar)^{-\beta})\right) dr dt\right\},$$

where \mathcal{L}_F is the Laplace transform of F . Substituting $r := Ar(\xi h(t))^{-1/\beta}$ for a given fixed t in the inner integral we factorize the two integrals and obtain $\mathcal{L}_{\bar{I}}(\xi) = \exp\{-2\pi\lambda_s A^{-2} \xi^{2/\beta} \zeta \kappa\}$, where $\zeta = \int_{-\infty}^{\infty} (h(t))^{2/\beta} dt$ and $\kappa = \int_0^{\infty} r(1 - \mathcal{L}_F(r^{-\beta})) dr$. A direct calculation yields $\zeta = 2\beta/(2 + \beta)$. This completes the proof.

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