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Modeling and analysis of slotted and non-slotted Aloha in linear Vehicular Ad-hoc NETWORKS

Bartłomiej Błaszczyszyn*, Paul Mühlethaler† and Yasser Toor‡

Abstract: The aim of this paper is to study slotted and non-slotted Aloha medium access (MAC) scheme in Vehicular Ad-hoc NETWORKS (VANETs). To this regard, we consider a one-dimensional, linear network, which is an appropriate assumption for VANETs and differs from two-dimensional, planar models usually assumed for general Mobile Ad-hoc NETWORKS (MANETs). More precisely, we use a linear version of the Poisson bipolar network model proposed in [2], in which the locations of signal emitting vehicles form a homogeneous Poisson point process on the line, and where the receivers are within a fixed distance from these emitters. We use the well-established signal-over-interference and noise ratio (SINR) capture model assuming power-law mean path-loss and independent Rayleigh fading. First, we consider a capture/outage scenario with fixed bit rate coding, where the SINR must be above a given threshold for a successful packet reception. In this setting we obtain explicit formulas to calculate the probability of capture for both slotted and non-slotted Aloha. From these formulas other characteristics, as the mean density of packet progress, are derived and optimized in MAC parameters. We consider also adaptive coding, where the throughput depends on the SINR. In this scenario we quantify and optimize the mean density of information throughput. Our unified approach to slotted and non-slotted Aloha allows for explicit comparison of both versions of this simple MAC. The obtained results differ quantitatively and even qualitatively from those obtained previously in the analogous analysis of planar MANETs, revealing some specificity of the optimal tuning of the MAC layer in the linear network topology.

Index Terms—VANETs, slotted Aloha, non-slotted Aloha, MAC (Medium Access Control) Layer Optimization, Throughput

I. INTRODUCTION

VANETs are a special case of MANETs where the network is formed between vehicles. VANETs are the most promising civilian applications of MANETs and they are likely to revolutionize our traveling habits by increasing safety on the road while providing value added services.

Most of the academic or industrial proposals for VANETs use some variants of Carrier Sense Multiple Access (CSMA) or another collision avoidance mechanism as their MAC protocols; cf. [9]. However, it is difficult to study these access schemes analytically because they produce complex patterns of nodes transmitting simultaneously. In contrast, the patterns of nodes transmitting simultaneously produced by Aloha is simpler if we assume that the vehicles' positions follow a Poisson point process. For this reason Poisson MANETs with Aloha MAC have received a lot of attention in the literature; see Section II in what follows. Most of this work is done for two-dimensional networks, i.e.; where locations of mobiles are modeled by a planar Poisson point process. This assumption is however not

adequate to VANETs, where locations of vehicles form linear patterns. The first goal of this paper is to adapt some recent stochastic models of MANETs with Aloha, in particular the Poisson bipolar model proposed in [2], and recently extended to non-slotted case in [5], to this specificity of VANETs. Analysis of these linear models reveals some specificity of the optimal tuning of the MAC layer in the linear network topology. *Our main observations to this regard are as follows.*

- The basic expression for the probability of packet capture at the distance R from the receiver, valid for both slotted and non-slotted system provided Rayleigh fading and no external noise, is $P = \exp\{-Kp\lambda RT^{1/\beta}\}$, where λ is the mean number of nodes per unit of network (road) length, p is the Aloha medium access probability, T is the SINR threshold required for the packet capture, β is the path-loss exponent and $K = K(\beta)$ is the so called *spatial contention parameter* that is specific for the Aloha version (slotted or not) and depends only on the path-loss exponent.
- The exponential dependence of P in R (and not in R^2 as for general planar MANETs) makes such “social” network characteristics as *mean total packet progress per unit length of the network and per unit of time* or the *mean number of bit-meters transported by the unit length of the network per unit of time* (in the case of adaptive coding traffic) can be jointly optimized in the medium access probability p and the transmission distance R . This is in contrast to planar MANETs, where such an optimization degenerates in the simple bipolar network model.
- Namely, when the external noise is not negligible the optimal solution is to take $p = 1$ and fix the communication range R to some optimal value (that is characterised in the paper). However, when the external noise is negligible the performance of the network depends only on the product pR and these parameters can be tuned arbitrarily so as to attain some optimal value of pR characterised in the paper.
- Comparison of slotted to non-slotted Aloha also depends on the external noise as well as on the path-loss exponent β . Namely, when the noise is non-negligible then the slotted Aloha essentially outperforms the non-slotted version only for moderate values of $\beta \in [3, 4]$. However, in the case of the absence of the noise, the largest β the more slotted Aloha outperforms its non-slotted version. In both cases (for noise scenario) for small values of β (tending to 1) there is no significant difference between the two versions of Aloha.

The remaining part of this paper is organized as follows: The basic network and interference model is described in

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Section III. In Section IV the network performance is analyzed under a SINR capture (non-outage) condition. In Section V, we assume that the channel throughput is given by the famous Shannon's $\log(1 + \text{SINR})$ law. In both cases, we optimize the global network throughput using the transmission range and Aloha medium access probability. Section VI discusses the numerical examples, followed by the conclusion in Section VII.

II. RELATED WORK

There is a reach literature on the performance evaluation of general MAENTs under CSMA and Aloha. However, only a few papers, as e.g. [7], consider specifically VANETs. To the best of our knowledge all these studies use simulations to evaluate the performances of these networks and/or assume simplified interference models. Aloha can be analyzed much easier as compared to CSMA, even in a quite realistic SINR scenario, as shown recently in [3–5, 8]. However these studies of assume planar (two-dimensional) networks. This approach was partially adapted to linear VANETs in [6], where slotted Aloha is considered only. In this paper we complete this work by considering both slotted and non-slotted Aloha for VANETs in a unified way, inspired by in [5], where planar MANETs are considered.

III. STOCHASTIC MODELS FOR ALOHA

A. Slotted versus non-slotted Aloha

In slotted Aloha the network nodes are perfectly synchronized to some time slots (each of the length B of the packet). The duration of these slots is the same for the whole network. The network nodes with a pending packet transmit it with the following rule: *each node, at each time slot independently tosses a coin with some bias p which will be referred to as the Aloha medium access probability (Aloha MAP). The packet is transmitted in the current slot if the outcome is heads. Otherwise the packet is not transmitted.*

There is no synchronization in non-slotted Aloha. All the network nodes independently send packets (of the same duration B) and then back off for some exponential random time of rate ε . In non-slotted Aloha the temporal patterns of transmission are independent (across the nodes) renewal processes with the generic inter-arrival time equal to $B + E$ where E is exponential (back-off) with mean ε . This stationary space-time model, called the *Poisson-renewal model* of non-slotted Aloha is studied in [5]. Although the analysis of this Poisson-renewal model for non-slotted Aloha is feasible it does not lead to simple closed formulas. Thus we will use another model of non-slotted Aloha called the *Poisson rain model*, also proposed in [5]. The main difference between the Poisson-renewal model and the Poisson rain model is that the nodes and their receivers are not fixed in time. Rather we may think of these nodes as being “born” at some time, transmitting a packet during the lifetime B and “dying” immediately after.

B. Network and interference model under Aloha

The following two models are linear version of the respective planar models studied jointly in [5].

1) *Slotted Aloha*: To study slotted Aloha we introduce the marked Poisson point $\Phi = \{(X_i, e_i)\}$ with intensity λ on the line \mathbb{R} , where

- $\{X_i\}$ denotes the locations of vehicles,
- $\{e_i\}_i$ is the medium access indicator of station i ; $e_i = 1$ for the station which is allowed to emit and $e_i = 0$ for the station which is not allowed to emit. The random variables e_i are independent, with $\mathbf{P}(e_i = 1) = p$.

Note first that Φ can be represented as a pair of independent Poisson p.p. representing emitters $\Phi^1 = \{X_i : e_i = 1\}$, and nodes $\Phi^0 = \{X_i : e_i = 0\}$ which are not allowed to emit (at a given time slot). These processes have intensities of λp and $\lambda(1 - p)$ respectively.

2) *Non-slotted Aloha*: To study non-slotted Aloha we introduce another Poisson point $\Phi' = \{(X_i, T_i)\}$ with intensity $\lambda_s = \frac{\lambda B}{B+1/\varepsilon}$ on $\mathbb{R} \times \mathbb{R}$

- $\{X_i\}$ denotes the locations of the vehicles,
- $\{T_i\}$ denotes the starting time of the transmission of the vehicle located at $\{X_i\}$.

This model is called a Poisson rain model because the vehicle at location X_i appears at time T_i and disappears at time $T_i + B$.

Remark that the locations of nodes are, as in slotted Aloha, on the line. However in non-slotted Aloha we have to model not only node locations but also their (non-synchronized) packet transmission times. This makes our Poisson model two-dimensional.

Each transmitting vehicle uses the same transmit power S , with a default value of $S = 1$ W. The attenuation due to the distance is modeled by the power-law function $l(r) = (Ar)^{-\beta}$ where r is the distance between the emitter and the receiver. We assume that $A = 1$ without loss of generality. Our mathematical linear model of the network requires $\beta > 1$ (in order for the sum of all powers received at a given location to have a finite mean). Typically beta is larger than 2 and our default value is $\beta = 4$.

In our model $F_{(x,y)}$ denotes the random fading between two vehicles located respectively at x and y . Thus, the actual signal power decay between these two vehicles will be $F_{(x,y)}l(|x - y|)$. Throughout the paper we assume that the values of $F_{(x,y)}$ are independent and exponentially distributed identically with a mean $1/\mu$, which corresponds to the situation of independent Rayleigh fading.

We also consider an independent external noise (i.e., independent of the vehicles' positions e.g., thermal) and denote it at (a given location) by W .

We assume that each vehicle transmits towards its dedicated receiver located within the distance R from it. This is sometimes called the “bipolar network model”. It allows us to study essential network performance characteristics at the medium access level without modeling particular routing schemes.

C. SINR capture

1) *Slotted Aloha*: We now suppose that our network uses slotted Aloha as its access scheme. A vehicle located at x transmits a signal with power S that is received by a vehicle located at y . The Signal over Interference plus Noise Ratio (SINR) of this communication will be:

$$\text{SINR}_{(x,y)} = \frac{SF_{(x,y)}l(|x-y|)}{W + I_{\Phi^1}(y)}, \quad (3.1)$$

where I_{Φ^1} is the shot-noise process of Φ^1 : $I_{\Phi^1}(y) = \sum_{X_i \in \Phi^1} SF_{(y,X_i)}l(|y-X_i|)$.

If we assume a fixed given bit-rate, y successfully receives the signal from x if

$$\text{SINR}_{(x,y)} \geq T, \quad (3.2)$$

where $\text{SINR}_{(x,y)}$ is given by (3.1) and T is the SINR-threshold related to the bit-rate given some particular coding scheme.

If we use an *adaptive coding scheme* in which, for a given SINR level, the appropriate choice of the coding scheme allows a bit-rate close to that given by Shannon's law to be obtained, then the throughput will be:

$$D_{x,y} = \log(1 + \text{SINR}_{(x,y)}). \quad (3.3)$$

2) *Non-slotted Aloha*: We now suppose that our network uses slotted aloha as its access scheme. We still assume that a vehicle located at x transmits a signal with power S that is received by a vehicle located at y .

The Signal over Interference plus Noise Ratio (SINR) of this communication at time $t \in [T_i, T_i + B]$ will be:

$$\text{SINR}_{(x,y)}(t) = \frac{SF_{(x,y)}l(|x-y|)}{W + I_{\Phi'}(y,t)},$$

where $I_{\Phi'}(y,t)$ is the shot-noise process of Φ' at location y and at time t :

$$I_{\Phi'}(y,t) = \sum_{(X_i, T_i) \in \Phi'} SF_{(y,X_i)}l(|y-X_i|)\mathbf{1}_{t \in [T_i, T_i+B]}.$$

Note that, unlike in the slotted Aloha, this SINR depends on time and can vary over a given packet reception. It is hence not obvious which value of the SINR should be used to define the successful reception condition analogous to (3.2). A possible choice to this regard, justified by some packet coding with bits interleaving, is to assume that the interference can be actually averaged over the packet duration

$$\bar{I}_{\Phi'}(y) = \frac{1}{B} \int_{T_i}^{T_i+B} I_{\Phi'}(y,t) dt.$$

In what follows we will make this assumption and define the SINR capture condition with respect to this averaged interference

$$\text{SINR}_{(x,y)}^{ns} \geq T, \quad (3.4)$$

where

$$\text{SINR}_{(x,y)}^{ns} = \frac{SF_{(x,y)}l(|x-y|)}{W + \bar{I}_{\Phi'}(y)}.$$

Under this assumption, with an *adaptive coding scheme* the throughput that can be obtained is equal to

$$D_{x,y}^{ns} = \log\left(1 + \text{SINR}_{(x,y)}^{ns}\right). \quad (3.5)$$

In what follows, we will present our analysis of the performance of slotted and non-slotted Aloha, assuming first some particular coding scheme that requires SINR to be larger than a given threshold T (SINR *capture (non-outage) condition*) for the successful transmission at a fixed given bit-rate. Then, in section V, we will assume an *adaptive coding scheme* in which,

for a given SINR level, the appropriate choice of the coding scheme allows us to obtain a bit-rate close to that given by Shannon's law.

IV. CONSTANT BIT RATE CODING

In this section we assume a fixed bit-rate scenario, i.e.; that y successfully receives the signal from x if the SINR is larger than some threshold T related to the bit-rate given some particular coding scheme.

A. Slotted Aloha

1) *Capture probability*: In the case of slotted Aloha the SINR capture condition is given by (3.2). The following explicit formula for the capture probability is fundamental for our analysis of Aloha MAC in linear VANETs.

Proposition 4.1: *Assume $p = 1$. The probability of successful transmission for slotted-Aloha is equal to*

$$P_s(\lambda) = \exp\left\{-K(\beta)\lambda RT^{\frac{1}{\beta}}\right\}\psi_W(\mu T/l(R))$$

where $K(\beta) = 2\pi/(\beta \sin(\pi/\beta))$ is a constant depending on the path-loss exponent and $\psi_W(\xi) = \mathbf{E}[e^{-\xi W}]$ is the Laplace transform of the noise W .

Proof: The proof follows the arguments used originally in [3] for planar network. Namely, using (3.1) we have

$$\begin{aligned} P_s(\lambda) &= \mathbf{P}(FS \geq T(W + I_{\Phi})/l(R)) \\ &= \int_0^\infty e^{-\mu s T/l(R)} d\Pr(W + I_{\Phi} \leq s) \\ &= \psi_{I_{\Phi}}(\mu T/l(R)) \psi_W(\mu T/l(R)), \end{aligned}$$

where $\psi_{I_{\Phi}}$ denotes the Laplace transform of the Poisson shot noise. It is known that (it can be derived from the formula for the Laplace functional of the Poisson p.p. (see e.g. [1])

$$\begin{aligned} \psi_{I_{\Phi}}(\xi) &= \exp\left\{-2\lambda \int_0^\infty \left(1 - \mathbf{E}\left[e^{-\xi SF_{l(|x|)}l(|x|)}\right]\right) dx\right\} \\ &= \exp\left\{-\frac{2\pi\lambda \xi^{1/\beta}}{\mu^{1/\beta} \beta \sin(\pi/\beta)}\right\}. \end{aligned}$$

Consequently,

$$\psi_{I_{\Phi}}(\mu T/l(R)) = \exp\left\{-K_s(\beta)\lambda RT^{\frac{1}{\beta}}\right\}.$$

This concludes the proof. ■

Note that if $W = 0$ then $\psi_W(\xi) \equiv 1$ and the formula for the successful reception probability simplifies to

$$P_s(\lambda) = \exp\left\{-K_s(\beta)\lambda RT^{\frac{1}{\beta}}\right\}.$$

Remark 4.2: Recall (see e.g. [2, Section 16]) that the capture probability in planar (two-dimensional) network model with slotted Aloha is equal to $p_s(\lambda) = \exp(-\kappa_s(\beta)\lambda R^2 T^{2/\beta})$ where $\kappa_s(\beta) = 2\pi^2/(\beta \sin(2\pi/\beta))$. In [8] the name *spatial contention factor* was proposed for this constant. We will use this term in what follows with respect to $K_s(\beta)$ as well as an analogous constant that will appear in the analysis of our linear model of non-slotted Aloha.

We will now consider a general medium access probability $0 \leq p \leq 1$. Recall, in this case the corresponding reception probability is equal to $P_s(p\lambda)$.

Using Campbell's formula (see [11]) we can express the mean total number of successful transmissions per unit length of the network (the *density of successful transmissions*) by $\lambda p P_s(\lambda p)$. Moreover, the *mean progress of the typical transmission* is simply equal to $R P_s(\lambda p)$.

2) *Density of progress*: In the remaining part of this section we will be mainly interested in the *mean density of progress* d_s , defined as the expected total progress of all the transmissions per unit length of the network and per time slot. Again, by Campbell's formula, it can be expressed by $d_s(R, \lambda, p) = \lambda p R P_s(\lambda p)$. This metric directly evaluates the network throughput i.e., number of bit-meters transmitted per unit length of the network and per unit of time.

In the following result we optimize this metric in p . Let us denote a critical communication range by

$$R_s^* = \frac{1}{K_s(\beta) T^{\frac{1}{\beta}} \lambda} = \frac{\beta \sin(\pi/\beta)}{2\pi T^{\frac{1}{\beta}} \lambda}.$$

Proposition 4.3: *If $R \geq R_s^*$ then the value of p that maximizes the mean density of progress $d_s(R, \lambda, p)$ for slotted Aloha is given by*

$$p_s^* = \frac{1}{K_s(\beta) T^{\frac{1}{\beta}} \lambda R} = R^*/R$$

and the maximum value is equal to

$$d_s(R, \lambda, p_s^*) = \frac{1}{K_s(\beta) e T^{\frac{1}{\beta}}} \psi_W(\mu T R^\beta). \quad (4.6)$$

If $R \leq R_s^*$ then $p_s^* = 1$ and

$$d_s(R, \lambda, p_s^*) = \lambda R \exp\left\{-K_s(\beta) \lambda R T^{\frac{1}{\beta}}\right\} \psi_W(\mu T R^\beta). \quad (4.7)$$

Proof: The result follows from Proposition 4.1 by differentiating the explicit formula for mean density of progress with respect to p . ■

We now consider the optimization of the mean density of progress jointly in p and R .

Proposition 4.4: *If $W > 0$ (with non-null probability) then the maximum (in p and R) of the mean density of progress $d_s^*(R, \lambda, p)$ is equal to*

$$\max_{R \in [0, R_s^*]} \left\{ \lambda R \exp\left\{-K_s(\lambda) \lambda R T^{\frac{1}{\beta}}\right\} \psi_W(\mu T R^\beta) \right\} \quad (4.8)$$

and is attained for $p_s^* = 1$ and an R that maximizes the expression in (4.8). In the absence of noise ($W \equiv 0$) the maximal mean density of progress $d_s(R, \lambda, p)$ is equal to

$$1/(K_s(\beta) e T^{\frac{1}{\beta}})$$

and is attained whenever $p_s R = R_s^*$ with $R \geq R_s^*$.

Proof: The result follows directly from Proposition 4.3. If $W > 0$ then $\psi_W(\mu T R^\beta)$ is a strictly decreasing function of R and the maximum of (4.6) with $R \geq R_s^*$ is attained for $R = R_s^*$. Moreover, the value of (4.7) with $R = R_s^*$ is equal to the value of (4.6) with $R = R_s^*$. Consequently the maximum is attained for some $R \leq R_s^*$ and thus $p_s^* = 1$. If we assume now that $W = 0$, then $\psi_W \equiv 1$. It is then easy to show that the maximum of (4.7) on the interval $R \leq R_s^*$ is attained for $R = R_s^*$ and is equal to the value of (4.6) with $\psi_W \equiv 1$.

Consequently the optimal choice of p and R is $R \geq R_s^*$ and $p_s = R_s^*/R$. This completes the proof. ■

Remark 4.5: The above twofold optimization of the density of progress in the communication range R and the medium access probability p gives a non-degenerate answer. This is in contrast with the planar situation considered in [3]. In this latter context the mean density of progress is of the order $O(1/R)$ and the twofold optimization in p and R leads to $R = 0$ and $p = 1$, which is not an acceptable optimization from a practical point of view. The difference between linear and planar networks comes from the fact that planar networks use an area of the order $O(R^2)$ to transmit a packet whereas the progress is equal to R . This leads to the optimal network consisting of dense small-range communications. In linear networks, the transmission of a packet ‘‘consumes’’ a distance of the order $O(R)$ and the progress is also R and consequently the optimization of the mean density of progress does not degenerate. We will see numerical examples of this optimization in Section VI-A.

Remark 4.6: When the external noise is not negligible ($W > 0$) the optimization of d_s leads to $p^* = 1$ and some optimal communication range $R \leq R_s^*$. We can notice that $R_s^* \leq 1/(2T^{1/\beta} \lambda)$ and thus R_s^* may be smaller than $1/\lambda$ which is the mean distance between two points of the Poisson point process. This is indeed the case when T is not small for example when no sophisticated interference cancellation techniques like spreading or CDMA are used. But a practical choice for R should be at least of the order $1/\lambda$. The network should thus operate as a ‘‘delay-tolerant network’’ and only transmit when neighboring vehicles are close enough to receive the packet.

When the external noise is negligible ($W = 0$) then the VANET network can be optimized with an arbitrary $R \geq R_s^*$ and with the medium access parameter satisfying $p^* = R^*/R$. The numerical examples below in subsection VI-A show that the noise in the order of $W = 10^{-10}$ mW or smaller can be ignored, whereas $W = 10^{-6}$ mW cannot. ¹

B. Non-slotted Aloha

We consider now non-slotted Aloha, with the SINR capture condition given by (3.4).

1) *Capture probability:* The following result gives the basic formula for the capture probability in the linear Poisson rain model of non-slotted Aloha.

Proposition 4.7: *Assume $p = 1$. The probability of successful transmission with non-slotted Aloha is equal to*

$$P_{ns}(\lambda) = \exp\left\{-K_{ns}(\beta) \lambda_s B R T^{\frac{1}{\beta}}\right\} \psi_W(\mu T/l(R))$$

where $K_{ns}(\beta) = 4\pi/((\beta + 1) \sin(\pi/\beta))$.

Proof: Following the same arguments as for slotted Aloha, with the averaged interference $\bar{I}_{\Phi'}$ replacing I_{Φ} we have by (3.4)

$$\begin{aligned} P_{ns}(\lambda_s) &= \mathbf{P}(FS \geq T(W + \bar{I}_{\Phi'})/l(R)) \\ &= \int_0^\infty e^{-\mu s T/l(R)} d\text{Pr}(W + \bar{I}_{\Phi'} \leq s) \\ &= \psi_{\bar{I}_{\Phi'}}(\mu T/l(R)) \psi_W(\mu T/l(R)), \end{aligned}$$

¹A recent study [10] of vehicle-to-vehicle wireless channels suggests the noise order of magnitude $10^{-10.27}$ mW.

where $\psi_{\bar{I}_\phi}$ denotes the Laplace transform of the average interference on the duration B of a transmission. We have the following equation for $I_{\Phi^1}(y, t)$ where the node at location $x = X_i$ transmits:

$$I_{\Phi^1}(y, t) = \sum_{(X_j, T_j) \in \Phi^1, X_j \neq X_i} SF_{(y, X_i)} l(|y - X_i|) \mathbf{1}_{t \in [T_j, T_j + B]}.$$

Without loss of generality we assume $T_i = 0$ thus $\bar{I}_{\Phi^1}(y)$ satisfies:

$$\bar{I}_{\Phi^1}(y) = \sum_{(X_j, T_j) \in \Phi^1, X_j \neq X_i} SF_{(y, X_i)} l(|y - X_i|) k(T_j)$$

with $k(T_j) = 1/B \int_0^B \mathbf{1}_{t \in [T_j, T_j + B]} dt$ thus $k(t) = \frac{1}{B} \max(0, B - |t|)$. Using now the general expression for the Laplace transform of the Poisson shot-noise, we obtain

$$\mathcal{L}_{\bar{I}_{\Phi^1}}(\xi) = \exp\left\{-2\lambda_s \int_{-\infty}^{\infty} \int_0^{\infty} \left(1 - \mathcal{L}_F(\xi k(t)(r)^{-\beta})\right) dr dt\right\}$$

where \mathcal{L}_F is the Laplace transform of F . Substituting $r := r(\xi k(t))^{-1/\beta}$ for a given fixed t in the inner integral we factorize the two integrals and obtain $\mathcal{L}_{\bar{I}}(\xi) = \exp\{-2\lambda_s \xi^{2/\beta} \zeta \kappa\}$, where $\zeta = \int_{-\infty}^{\infty} (k(t))^{1/\beta} dt$ and $\kappa = \int_0^{\infty} (1 - \mathcal{L}_F(r^\beta)) dr$. A direct calculation yields $\zeta = 2\beta/(1 + \beta)$. Consequently,

$$\psi_{\bar{I}_{\Phi^1}}(\mu T/l(R)) = \exp\left\{-\frac{4\pi\lambda_s R T^{\frac{1}{\beta}}}{(\beta + 1) \sin(\pi/\beta)}\right\}.$$

This concludes the proof. \blacksquare

Remark 4.8: Recall from [5] that the analogous capture probability in planar MANET with non-slotted Aloha is equal to $p_{ns}(\lambda_s) = \exp(-\kappa_{ns}(\beta)\lambda_s R^2 T^{2/\beta})$ with $\kappa_{ns}(\beta) = \frac{4\pi}{\beta} \int_0^{\infty} u^{2/\beta-1} (1 - u \log(1 + u^{-1})) du = 4\pi^2/((\beta + 2) \sin(2\pi/\beta))$.

Remark 4.9: Assuming $p = B/(B + 1/\epsilon) = \tau$, i.e., that slotted and non-slotted Aloha contend to the channel with the same probability τ , and recalling that $\lambda_s = \lambda B/(B + 1/\epsilon)$, one finds that the capture probability in slotted and non-slotted Aloha can be expressed using a general formula

$$P(\lambda) = \exp\left\{-K(\beta)\lambda\tau R T^{\frac{1}{\beta}}\right\} \psi_W(\mu T/l(R)) \quad (4.9)$$

with an appropriate constant $K(\beta)$ corresponding to slotted or non-slotted case. This observation allows for explicit comparisons of many characteristics of the two variants of Aloha, as shown in [5] in the case of planar MANET. We skip here the details and remark only that the ratio of the spatial contention factors of non-slotted and slotted Aloha is equal to $K_{ns}(\beta)/K_s(\beta) = 2\beta/(\beta + 1)$. This mean in particular that when β tends to the critical (for linear model) value 1 then the performance of non-slotted Aloha is close to this of the slotted one.

2) *Density of progress:* The fact that the capture probability in the non-slotted Aloha can be expressed, up to the spatial contention factor $K(\beta)$, via the same formula (4.9) as in the slotted Aloha, allows us to deduce immediately the results regarding the optimization of the density of progress from these obtained in Section IV-A2. Namely, denote the mean density of progress in non-slotted Aloha by $d_{ns} = d_{ns}(R, \lambda, p) = \lambda p R P_{ns}(\lambda p)$

and denote a critical communication range by

$$R_{ns}^* = \frac{1}{K_{ns}(\beta) T^{\frac{1}{\beta}} \lambda} = \frac{(\beta + 1) \sin(\pi/\beta)}{4\pi T^{\frac{1}{\beta}} \lambda_s}.$$

Proposition 4.10: If $R \geq R_{ns}^*$ then the value of p that maximizes the mean density of progress $d_{ns}(R, \lambda, p)$ is given by

$$p_{ns}^* = \frac{1}{K_{ns}(\beta) T^{\frac{1}{\beta}} \lambda R} = R_{ns}^*/R$$

and the maximum value is equal to

$$d_{ns}(R, \lambda, p_{ns}^*) = \frac{1}{K_{ns}(\beta) e T^{\frac{1}{\beta}}} \psi_W(\mu T R^\beta). \quad (4.10)$$

If $R \leq R_{ns}^*$ then $p_{ns}^* = 1$ and

$$d_{ns}(R, \lambda, p_{ns}^*) = \lambda R \exp\left\{-K_{ns}(\beta) \lambda R T^{\frac{1}{\beta}}\right\} \psi_W(\mu T R^\beta). \quad (4.11)$$

Proposition 4.11: If $W > 0$ (with non-null probability) then the maximum (in p and R) of the mean density of progress for non-slotted Aloha $d_{ns}(R, \lambda, p)$ is equal to

$$\max_{R \in [0, R_{ns}^*]} \left\{ \lambda R \exp\left\{-K_{ns}(\beta) \lambda R T^{\frac{1}{\beta}}\right\} \psi_W(\mu T R^\beta) \right\} \quad (4.12)$$

and is attained for $p_{ns}^* = 1$ and an R that maximizes the expression in (4.12). In the absence of noise ($W \equiv 0$) the maximal mean density of progress $d_{ns}(R, \lambda, p)$ is equal to

$$(\beta + 1) \sin(\pi/\beta) / (4e\pi T^{\frac{1}{\beta}})$$

and is attained whenever $p_{ns} R = R_{ns}^*$ with $R \geq R_{ns}^*$.

V. OPTIMAL ADAPTIVE CODING

In Section IV we have assumed that a transmission is successful only if the SINR is above a given threshold T . Here we envisage a situation in which some communication is always feasible with its bit-rate varying with the value of its SINR. This assumption corresponds to an adaptive coding in the channel: if the SINR is high, the coding can be 'loose' and thus the bit-rate is high, whereas with a small SINR the coding must be 'tight' and thus the throughput is low. We start our study by slotted Aloha and in the second sub-section we study non-slotted Aloha using the same methods as for slotted Aloha.

A. Slotted Aloha

We know that Shannon's law $D \log(1 + \text{SINR})$ expresses the well known theoretical maximal-bit rate of the Gaussian channel (AWGN); this limit can be approached using link adaptations and turbo codes.

Using *Shannon's law*, and assuming for simplicity that $D = 1$, we now say in our VANET model that the vehicle at y receives the signal from the vehicle at x with the *throughput* (bit-rate) given by (3.3). By stationarity of Φ^1 the *mean throughput*

$$\tau(R, \lambda p) = \mathbf{E}[D(x, y, \Phi^1)]$$

depends only on the distance $|x - y| = R$ and *not* on the specific locations of (x, y) ; recall that λp is the intensity of the

emitters Φ^1 . We can now prove the following basic result for our VANET model with adaptive coding.

Proposition 5.1: *Assume $p = 1$. The mean throughput for slotted Aloha is equal to*

$$\tau_s(R, \lambda) = \beta \int_0^\infty \exp\left\{K_s(\beta)\lambda Rv\right\} \frac{v^{\beta-1}}{1+v^\beta} \psi_W(\mu R^\beta v) dv.$$

Proof: The proof goes along the same lines as given for the 2D case in [4]. First note that

$$\mathbf{E}[\log(1 + \text{SINR})] = \int_0^\infty \mathbf{P}\{\log(1 + \text{SINR}) > t\} dt.$$

Substituting,

$$\mathbf{P}\{\log(1 + \text{SINR}) > t\} = \mathbf{P}\{\text{SINR} > e^t - 1\} = p_R(\lambda, e^t - 1),$$

where we introduce into the previous notation of p_R the explicit dependence on $T = e^t - 1$, and obtain

$$\tau_s(R, \lambda) = \int_0^\infty P_s(\lambda, e^t - 1) dt.$$

Using Proposition 4.1 and substituting $(e^t - 1)^{1/\beta} = v$ the expected result is obtained. ■

We can now define an important metric; analogous to the mean density of progress considered in the previous section. We will call the *mean density of transport* t_s the expected number of bit-meters transported by the unit length of the network per unit of time. By Campbell's formula it can be expressed in our network as

$$t_s(R, \lambda, p) = R\lambda p \tau_s(R, \lambda p).$$

Recall that this metric is related to the achievable network throughput under the second model (based on Shannon's law).

In what follows, we characterize the choice of the network parameters R and p that maximize d_{trans} . Using the result of Proposition 5.1 it can be shown that $R\tau(R, \lambda)$ converges to 0 when $R \rightarrow 0$, as well as, when $R \rightarrow \infty$. We conjecture that:

(C) $R\tau(R, \lambda)$ with $W \equiv 0$ admits one global maximum for $R = Y^*$ and is strictly increasing for $R < Y^*$.

By Proposition 5.1 this critical (in the absence of noise) communication range Y_s^* can be characterized as the solution of the following equation

$$\begin{aligned} & \int_0^\infty \exp\left\{-K_s(\beta)\lambda Y_s^* v\right\} \frac{v^{\beta-1}}{1+v^\beta} dv \\ &= K_s(\beta)\lambda Y_s^* \int_0^\infty \exp\left\{-K_s(\beta)\lambda Y_s^* v\right\} \frac{v^\beta}{1+v^\beta} dv. \end{aligned}$$

The following result is similar to Proposition 4.4.

Proposition 5.2: *Assume that condition (C) is satisfied. In the absence of noise ($W \equiv 0$) the maximal mean density of transport d_{trans} is attained whenever $pR = Y_s^*$ with $R \geq Y_s^*$. If $W > 0$ (with non-null probability) then the maximum (in p and R) of the mean density of transport d_{trans} is equal to*

$$\max_{R \in [0, Y^*]} \beta \int_0^\infty \exp\left\{-K_s(\beta)\lambda Rv\right\} \frac{v^{\beta-1}}{1+v^\beta} \psi_W(\mu R^\beta v^\beta) dv \quad (5.13)$$

and is attained for $p^* = 1$ and an R that maximizes (5.13).

Proof: Note first by Proposition 5.1 that if $W \equiv 0$ then $d_{\text{trans}}(R, \lambda, p)$ depends on p and R only through the product

pR . This and the definition of Y^* proves the first part of the result. Assume now that $W > 0$. Then $\psi_W(\mu R^\beta v^\beta)$ is strictly decreasing in R and thus the maximum of $d_{\text{trans}}(R, \lambda, p)$ is attained for some $R \leq Y^*$. By assuming that $R\tau(R, \lambda)$ with $W \equiv 0$ is strictly increasing for $R < Y^*$ we conclude that $p^* = 1$. ■

B. Non-slotted Aloha

We assume here that the vehicle at y receives the signal from the vehicle at x with the *throughput* (bit-rate) given by (3.5). With adaptive coding we have the following basic result, which follow by from the corresponding results regarding the slotted Aloha by Remark 4.9.

Proposition 5.3: *Assume $p = 1$. The mean throughput is equal to*

$$\tau_{ns}(R, \lambda) = \beta \int_0^\infty \exp\left\{-K_{ns}(\beta)\lambda Rv\right\} \frac{v^{\beta-1}}{1+v^\beta} \psi_W(\mu R^\beta v) dv.$$

Other results of Section V-A can be also immediately adapted to non-slotted case, replacing $K_s(\beta)$ by $K_{ns}(\beta)$. We skip the details.

VI. NUMERICAL EXAMPLES

A. Slotted Aloha

In this section we first compute the mean density of progress d_s and the mean density of transport t_s taking some particular numerical values for our slotted model parameters. More specifically we will study the impact of noise W . Throughout this section and, if not otherwise specified, we use the following parameters:

- the density of the network is $\lambda = 0.01$ (vehicles per 1 m of the network, i.e., 10 vehicles per 1 km),
- the exponential fading has a mean $1/\mu = 1$
- the path-loss exponent $\beta = 4$

We compute the mean density of progress d_s using the result of Proposition 4.1. We compute d_s for $T = 10$ and different values of noise W , transmission range R and function of the transmission probability p . The results of these computations, carried out with Maple, are given in Figure 1. The optimal density of progress is achieved for $pR = R^* \approx 25.31$; we verify the result of Proposition 4.4 with $W \equiv 0$ mW. We observe that the noise $W = 10^{-10}$ mW does not significantly change the previous result, the density of progress 0.093 is attained at $p = 1$ and $R = 25.6$. Moreover, for $R = 100$ and $p = 0.25$ (yielding $pR = 25 \approx R^*$) the value of the density of progress is 0.085 which is still close to the optimal value. However, with $W = 10^{-6}$ m the network performance must be precisely tuned and a single point $p^* = 1$ and $R = 11.31$ is obtained.

The mean density of transport t_s (in the case of adaptive coding) evaluated using Proposition 5.1 is presented in Figure 2. Similar observations can be made on this Figure. With no external noise the optimum density of transport value 0.53 is reached for $pR = Y^* \approx 21.7$. When $W = 10^{-10}$ mW there is no visible difference with $W = 0$. The maximum 0.53 is reached for $p = 1$ and $R = 21.7$. Moreover for $p = 0.26$ and $R = 100$ the value of the mean density of transport is still 0.5. So noise of order $W = 10^{-10}$ mW can be ignored. However, when $W = 10^{-6}$ mW, the maximum 0.28 is attained when

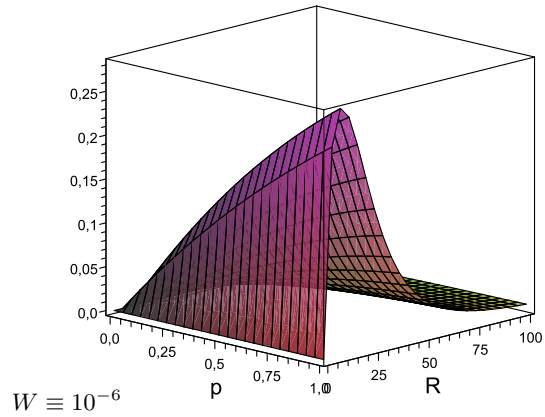
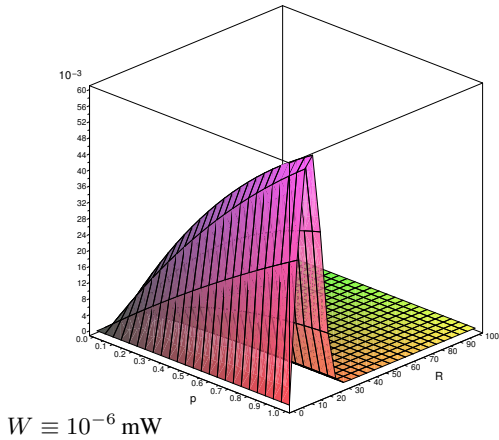
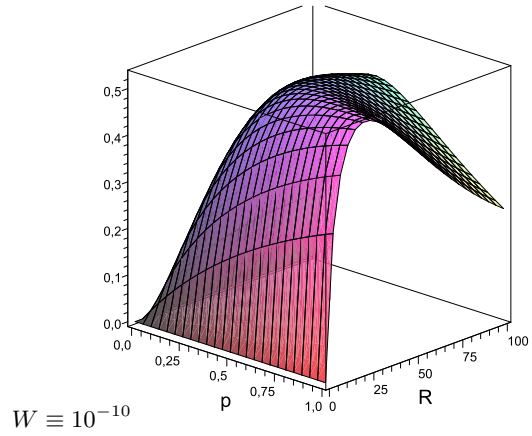
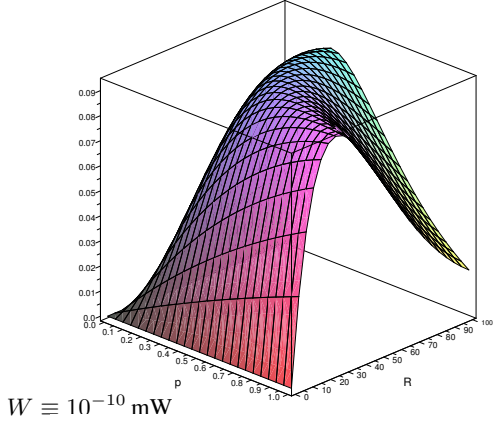
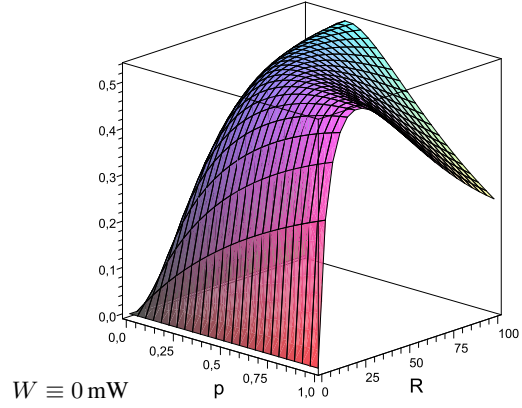
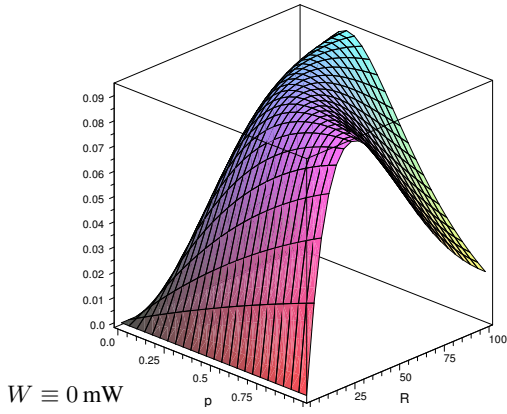


Fig. 1. Mean density of progress versus transmission probability p and transmission range R for three different values of the external noise power W .

$p = 1$ and $R = 8.9$. To maximize the network performance with $W = 10^{-6}$ mW the parameters must be tuned close to this “true” optimum.

B. Comparison of slotted and non-slotted Aloha

In this section, we compare slotted and non slotted Aloha. We begin with no thermal noise $W = 0$. In Figure 3 we plot the optimized mean density of progress for both protocols in function of the path-loss exponent β . We observe that for small values of β slotted and non-slotted Aloha offer similar performances whereas for large values of β slotted Aloha tends to be twice as good as non-slotted Aloha. This observation is confirmed by Figure 4 which plots the ratio of the mean density of progress for slotted and non slotted Aloha.

Fig. 2. Mean density of transport versus transmission probability p and transmission range R for three different values of the external noise power W .

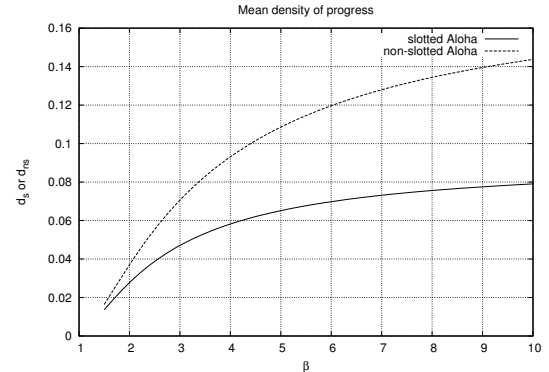


Fig. 3. Mean density of progress for slotted and non slotted Aloha, $W = 0$.

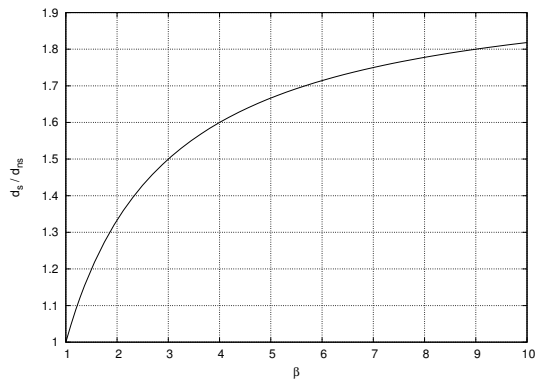


Fig. 4. Ratio of the mean density of progress for slotted and non slotted Aloha, $W = 0$.

In Figure 5 we plot the mean density of transport for slotted and non-slotted Aloha still with $W = 0$. We also observe that for small values of β slotted and non-slotted Aloha offer similar performances whereas for large values of β slotted Aloha tends to be twice as good as non-slotted Aloha.

We then consider the presence of noise $W = 10^{-6}$. Figure 6 presents the optimized mean density of progress for slotted and non-slotted Aloha. We observe that for small or large value of β slotted and non-slotted Aloha provide nearly the same mean density of progress. For intermediate value of β slotted Aloha offers around 40% more density of progress than Aloha. This can be explained because for small value of β it is difficult to control the interference of simultaneous transmissions and for large values of β interference is always very small compared to thermal noise. Thus there is no much difference between slotted and non-slotted Aloha. Only for intermediate values of β , slotted Aloha which has a better control of interference provides significantly better performances. We notice also that for large values of β the mean density of progress vanishes, this can be explained by the proeminence of thermal noise and the use of constant bit rate coding.

Figure 7 presents the mean density of transport for slotted and non-slotted Aloha. We observe that for small values of β slotted and non-slotted Aloha provide nearly the same mean density of transport. For intermediate values of β and large values of β slotted Aloha is between 40% and 30% better than non-slotted Aloha. We observe that the mean density does not vanish when β increase, this is due the adaptive rate coding that we have modeled.

Similar optimizations of the mean density of progress and the mean density of transport with other values of p , R , W , β , μ and S can easily be carried out.

VII. CONCLUSION

In this paper we have adapted existing stochastic models of planar MANETs to the linear scenario of VANETs, allowing for a comprehensive study and comparison of slotted and non-slotted Aloha in these networks under a realistic path-loss and Rayleigh fading scenario. Using these models we show how one can maximize mean packet progress and mean density of information transport by optimizing the Aloha transmission probability and the transmission range. This reveals interesting dependencies between the performance of the network and its

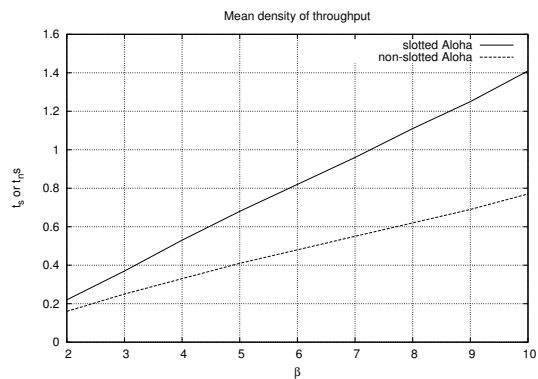


Fig. 5. Mean density of transport for slotted and non slotted Aloha, $W = 0$.

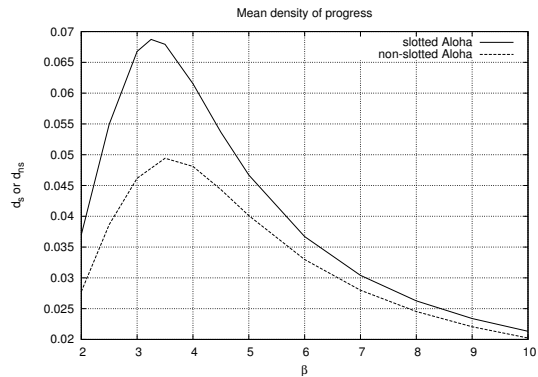


Fig. 6. Mean density of progress for slotted and non slotted Aloha, $W = 10^{-6}$.

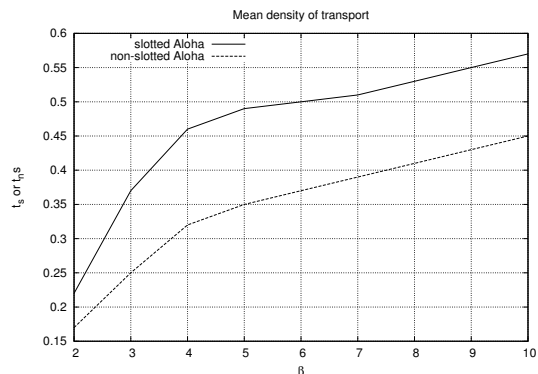


Fig. 7. Mean density of thtransport for slotted and non slotted Aloha, $W = 10^{-6}$.

parameters. These dependencies are intrinsic to linear scenarios usually assumed for VANETs and are different from planar network models typically used for general MANETs.

It is an interesting question for future work to compare the performance of Aloha predicted by our models to this offered by CSMA, which can be simulated.

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