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On the Stock Estimation for a Harvested Fish Population

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Abstract

We consider a stage-structured model of a harvested fish population and we are interested in the problem of estimating the unknown stock state for each class. The model used in this work to describe the dynamic evolution of the population is a discrete time system including a nonlinear recruitment relationship. To estimate the stock state, we build an observer for the considered fish model. This observer is an auxiliary system that uses the catch data over each time interval and gives a dynamical estimation of the stock state for each stage class. The observer works well even if the recruitment function in the considered model is not well known. The same problem for an age-structured model has been addressed in a previous work.

Keywords: Discrete-time system, Stage structured population models, Estimation, Harvested Fish Population, Observers.

1 Introduction

Overfishing has affected negatively the stock of renewable marine resource. Analyzing the catch data during the last decade, we notice that catch fishing have considerably decreased in many countries and some species are threatened by extinction. The collapse of catch fishing and extinction of species can be explained in most cases by a gap in management policies due in part to the complex problem of controlling the dynamic evolution of marine resource. To improve the efficiency of exploiting stock and preventing the collapse of key fishery, it's essential to understand well the mechanism evolution of the stock and to have a good estimation of the stock fish population. To overcome these types of problems in fishery management, mathematical models are very often used to describe the dynamic evolution of some fish population. With these models it becomes easier to observe and hence to regulate the dynamics of a given fish population.

In this paper, We are interested in the estimation problem of the unknown stock state of a harvested fish population whose dynamics are modeled by a discrete time system including a nonlinear stock recruitment relationship. As already mentioned in the abstract, the estimation problem of the stock state for a harvested fish population was first investigated [18] by considering an age-structured fish-population model. However determining the age for a fish population is a difficult task. In practice, it would be easier and more pertinent to classify a fish population by other quantities that correlate with age such as size or weight. Moreover, most of time the catch data that we have access to, are generally evaluated in term of size or weight. Hence a stage structured fish population model is more realistic and general than an age structured one. The stage-structured fish model we consider here is the following (see for instance Getz's book: page 84 and 35):

$$\left\{ \begin{array}{l} x_1(k+1) = (1-p_1)s_1(k)x_1(k) + s_0f\left(\sum_{i=1}^n b_i x_i(k)\right) \\ x_2(k+1) = p_1s_1(k)x_1(k) + (1-p_2)s_2(k)x_2(k) \\ \quad \quad \quad \vdots \quad \quad \quad \vdots \\ x_{n-1}(k+1) = p_{n-2}s_{n-2}(k)x_{n-2}(k) + (1-p_{n-1})s_{n-1}(k)x_{n-1}(k) \\ x_n(k+1) = p_{n-1}s_{n-1}(k)x_{n-1}(k) + s_n(k)x_n(k) \end{array} \right. \quad (1)$$

where

- n is the number of stage-classes.
- $x_i(k)$ is the number of individuals in the i th class at time k .
- b_i is the fecundity rate of class i .
- s_0 is the individual survival rate of newborn.

- $s_i(k)$ is the individual survival rate in the stage class i at times k :
 $s_i(k) = e^{-M_i - q_i E(k)\tau}$ (see Getz book: pp 221)
- M_i is the natural mortality rate of class i .
- q_i is the catchability coefficient of individuals of the i th class.
- $E(k)$ is the fishing effort at time k .
- p_i is the proportion of individuals in stage class i that move to the next stage class $i + 1$ after the time interval $[k, k + 1[$.
- τ is the length of harvesting season.
- f is the stock-recruitment function.

The widely-used recruitment functions [9, 4, 17, 20, 22] are:

$$\text{Beverton and Holt} \quad f(x_0) = \alpha x_0 / (1 + \beta x_0) ;$$

$$\text{Ricker} \quad f(x_0) = \alpha x_0 e^{-\beta x_0} ;$$

$$\text{Power function} \quad f(x_0) = \alpha x_0^{1-\beta} ;$$

$$\text{Shepherd} \quad f(x_0) = \alpha x_0 / (1 + \beta x_0^c) , \quad (c > 0).$$

$$\text{Deriso - Schnute} \quad f(x_0) = \alpha x_0 (1 - \beta x_0)^{1/\gamma};$$

$$\text{Saila - Lorda} \quad f(x_0) = \alpha x_0^\gamma e^{-\beta x_0}.$$

Where α , β and γ are positive parameters and $x_0 = \sum_{i=1}^n b_i x_i$ is the number of newborns.

The recruitment function is very difficult to model because many factors such as water temperature, air speed and other natural phenomena can often perturb growth of young individuals that are too vulnerable. A weakness of the model (1) is that most of the suggested mathematical expressions for the recruitment function are only valid for some restricted hypotheses. For example Beverton and Holt justified their recruitment function by making hypotheses on food competition and the Ricker recruitment function is based on egg and juveniles predation. Hence the recruitment term in the first equation of model (1) is not well known and we can rewrite the model (1) by considering the recruitment function $s_0 f\left(\sum_{i=1}^n b_i x_i(k)\right)$ as an unknown input $r(x(k))$

to obtain the linear model with unknown single input $r(x(k))$:

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ \vdots \\ x_n(k+1) \end{pmatrix} = A(k) \begin{pmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ \vdots \\ x_n(k) \end{pmatrix} + \begin{pmatrix} r(x(k)) \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix} \quad (2)$$

where

$$A(k) = \begin{pmatrix} (1-p_1)s_1(k) & 0 & \dots & \dots & 0 \\ p_1s_1(k) & \ddots & \ddots & & \vdots \\ 0 & p_2s_2(k) & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & (1-p_{n-1})s_{n-1}(k) & 0 \\ 0 & \dots & 0 & p_{n-1}s_{n-1}(k) & s_n(k) \end{pmatrix}$$

Remark 1 Note that if $p_i = 1$, for $i = 1, \dots, n$ we obtain the age-structured model for which we have build an observer in [18].

While we focus one analyzing a fish-population model subjected to fishing action, quantifying the fishing catch has great importance for the synthesis of the estimator stock states. Regarding the model we address above, the fish number catch over the time period $[k, k+1[$ in each stage class i is given by (see Getz book 's: pp 146):

$$y_i(k) = \int_k^{k+\tau} q_i E(k) x_i(t) dt \quad (3)$$

The total fish number catch is then:

$$y(k) = \sum_{i=1}^n \int_k^{k+\tau} q_i E(k) x_i(t) dt \quad (4)$$

After few computations we can write

$$y(k) = \sum_{i=1}^n \frac{q_i E(k)}{q_i E(k) + M_i} \left(1 - e^{(-M_i - q_i E(k))\tau} \right) x_i(k) \quad (5)$$

Let $C_i(k) = \frac{q_i E(k)}{q_i E(k) + M_i} \left(1 - e^{(-M_i - q_i E(k))\tau} \right)$, then $y(k)$ can be written as:

$$y(k) = \sum_{i=1}^n C_i(k) x_i(k) \quad (6)$$

To summarize the model we consider in this work is:

$$\begin{cases} x(k+1) &= A(k)x(k) + Dr(x) \\ y(k) &= C(k)x(k) \end{cases} \quad (7)$$

where $A(k)$ is defined below, $C(k) = (C_1(k), \dots, C_n(k))$ and $D = (1, 0, \dots, 0)^t$.

Now we assume that the linear model with unknown single input (2) which can be written as (7) describes successfully the dynamics of the considered harvested fish population. To compute the state $x(k)$ at time k , we need to know the value of the state $x(k_0)$ at a given time $k_0 < k$. However the model does not give any information about the unknown state value $x(k_0)$. The model provides only information about relation between two consecutive states $x(k)$ and $x(k+1)$, the model gives also measure of captures over time interval $[k, k+1[$. To make the model (2) useful, we have to find how to estimate the unknown value $x(k)$, that is the main concern we will address in this paper. To achieve this goal, we will use a tool from control theory called an observer. We construct an another dynamic system whose state \hat{x} will provide an estimation of the unknown state of the considered model and this will be true regardless of the initial condition of the observer. We briefly recall the definition of an *observer* in control theory. Let us consider the following dynamic system:

$$\begin{cases} x(k+1) = \mathcal{F}(x(k), u(k)) \\ y(k) = \mathcal{H}(x(k), u(k)). \end{cases} \quad (8)$$

where $x(k) \in \mathbb{R}^n$ is the state of the system, $u(k) \in U \subset \mathbb{R}^m$ is the input or the control and $y(k) \in \mathbb{R}^q$ is the measurable output at time k .

Definition 1 *A observer for the dynamical system (8) consists of another dynamical system Σ whose inputs are the inputs and the outputs of system (8) and whose state $\hat{x}(k)$ satisfies $x(k) - \hat{x}(k)$ tends to zero as time k goes to infinity and must remain small if it starts small.*

Σ is an exponential observer of (8) if there exists $\rho < 1$ in such a way that for all $k \geq 0$ and for all initial conditions $(x(0), \hat{x}(0))$, one has

$$|\hat{x}(k) - x(k)| \leq \rho^k |\hat{x}(0) - x(0)|.$$

More details on observer notions can be found in [24, 12]. Observer theory for linear discrete-time system with known inputs is well established. For linear invariant-time systems, the synthesis of the observer is completely given by the Luenberger method [16]. For linear time varying systems, several robust algorithms have been developed, one can cite for instance [6, 3, 1, 14, 15]. In the special case of unknown inputs the synthesis of observers is still a very difficult task. Some algorithms have been developed for some classes of linear systems with unknown inputs [5, 11, 23, 21, 13, 25, 10, 7, 19]. However, most of the available results concern linear time invariant systems. Here we are interested in the design of a global observer for the fishery system (2) which is linear time varying system with known input $E(k)$ and unknown input $r(x(k))$.

2 Observer for linear time varying systems with unknown inputs

Let's consider the following system:

$$\begin{cases} x(k+1) = \Gamma(k)x(k) + \Lambda\zeta, \\ y(k) = \theta(k)x(k), \end{cases} \quad (9)$$

where $x(k)$ and $y(k)$ are respectively the state and the measurable output at time k , $\Gamma(k)$ et $\theta(k)$ are respectively time varying matrix, Λ is an appropriated column vector and ζ is a scalar unknown input. In [10] M. Hou and P.C. Muller have developed a technique to synthesize observers for linear continuous time-invariant systems with unknown inputs. We adapt this technique for linear discrete time-varying systems with unknown inputs of the form (9).

Let $y(k)$ be the measurable output of the system (9), we have

$$\begin{aligned} y(k+1) &= \theta(k+1)x(k+1) \\ &= \theta(k+1)(\Gamma(k)x(k) + \Lambda\zeta) \\ &= \theta(k+1)\Gamma(k)x(k) + \theta(k+1)\Lambda\zeta \end{aligned}$$

Assume that $\theta(k+1)\Lambda$ is invertible $\forall k \geq 0$, we obtain:

$$\zeta = (\theta(k+1)\Lambda)^{-1}(y(k+1) - \theta(k+1)\Gamma(k)x(k))$$

Then we have:

$$\begin{aligned} x(k+1) &= \Gamma(k)x(k) + \Lambda\zeta \\ &= \Gamma(k)x(k) + \Lambda\left((\theta(k+1)\Lambda)^{-1}(y(k+1) - \theta(k+1)\Gamma(k)x(k))\right) \\ &= \Gamma(k)x(k) + \Lambda(\theta(k+1)\Lambda)^{-1}y(k+1) - \Lambda(\theta(k+1)\Lambda)^{-1}\theta(k+1)\Gamma(k)x(k) \\ &= \left(\Gamma(k) - \Lambda(\theta(k+1)\Lambda)^{-1}\theta(k+1)\Gamma(k)\right)x(k) + \Lambda(\theta(k+1)\Lambda)^{-1}y(k+1) \\ &= \left(I_{n \times n} - \Lambda(\theta(k+1)\Lambda)^{-1}\theta(k+1)\right)\Gamma(k)x(k) + \Lambda(\theta(k+1)\Lambda)^{-1}y(k+1). \end{aligned}$$

We define:

$$\bar{\Gamma}(k) = \left(I_{n \times n} - \Lambda(\theta(k+1)\Lambda)^{-1}\theta(k+1)\right)\Gamma(k),$$

and

$$\bar{\theta}(k) = \Lambda(\theta(k+1)\Lambda)^{-1}.$$

Thus we obtain:

$$x(k+1) = \bar{\Gamma}(k)x(k) + \bar{\theta}(k)y(k+1)$$

A candidate observer for system (9) is then given by:

$$\begin{cases} z(k+1) = (\bar{\Gamma}(k) - L(k)\theta(k))z(k) + (L(k) + (\bar{\Gamma}(k) - L(k)\theta(k))\bar{\theta}(k-1))y(k), \\ \hat{x}(k) = z(k) + \bar{\theta}(k-1)y(k). \end{cases} \quad (10)$$

Let $e(k) = x(k) - \hat{x}(k)$ the estimation error, the error dynamics are described by:

$$\begin{aligned}
e(k+1) &= x(k+1) - \hat{x}(k+1) \\
&= \bar{\Gamma}(k)x(k) + \bar{\theta}(k)y(k+1) - z(k+1) - \bar{\theta}(k)y(k+1) \\
&= \bar{\Gamma}(k)x(k) - (\bar{\Gamma}(k) - L(k)\theta(k))z(k) - (L(k) + (\bar{\Gamma}(k) - L(k)\theta(k))\bar{\theta}(k-1))y(k) \\
&= (\bar{\Gamma}(k) - L(k)\theta(k))x(k) - (\bar{\Gamma}(k) - L(k)\theta(k))z(k) \\
&\quad - (\bar{\Gamma}(k) - L(k)\theta(k))\bar{\theta}(k-1)y(k) \\
&= (\bar{\Gamma}(k) - L(k)\theta(k))(x(k) - \hat{x}(k)) \\
e(k+1) &= (\bar{\Gamma}(k) - L(k)\theta(k))e(k)
\end{aligned}$$

Therefore, if the pair of matrices $(\bar{\Gamma}(k), \theta(k))$ is uniformly observable, one can choose the gain matrix $L(k)$ such that the error $e(k)$ tend to zero when k tend to $+\infty$. This means that system (10) is an observer for system (9).

3 Stock estimation via an observer

Our aim is to propose an observer for the considered system (2) with the output given by equation (5). To this end we introduce the following notations:

E_{min} and E_{max} are respectively the minimum and the maximum fishing effort for all time k .

q_{min} and q_{max} are respectively the minimum and the maximum catchability coefficient for all stage class i .

m and M are respectively the minimum and the maximum mortality rate for all stage class i .

We assume that $q_1 \neq 0$.

System (2) can be written in the form (9) with:

$$\Gamma(k) = A(k), \quad \Lambda = D, \quad \zeta = r(x(k)) \text{ and } \theta(k) = C(k)$$

Assume that $q_1 \neq 0$, then $\theta(k+1)\Lambda = C(k+1)D$ is invertible for all $k \geq 0$

since $C(k+1)D = C_1(k+1) = \frac{q_1 E(k+1)}{q_1 E(k+1) + M_1} \left(1 - e^{-(M_1 + q_1 E(k+1))\tau}\right)$ does not vanish if $q_1 \neq 0$.

Applying the construction of Section 2 we have

$$\bar{A}(k) = \left(I_{n \times n} - \frac{1}{C_1(k+1)} D C(k+1) \right) A(k)$$

and

$$\bar{\theta}(k) = \frac{1}{C_1(k+1)} D$$

Therefore a candidate observer for (2) is given by:

$$\begin{cases} z(k+1) &= \bar{A}(k)z(k) + \bar{A}(k)\bar{\theta}(k-1)y(k) \\ \hat{x}(k) &= z(k) + \bar{\theta}(k-1)y(k) \end{cases} \quad (11)$$

This can be written in more detailed form:

$$\left\{ \begin{array}{l}
z_1(k+1) = -\frac{C_2(k+1)p_1s_1(k)}{C_1(k+1)}z_1(k) - \sum_{i=2}^{n-1} \left(p_i s_i(k) \frac{C_{i+1}(k+1)}{C_1(k+1)} + (1-p_i)s_i(k) \frac{C_i(k+1)}{C_1(k+1)} \right) z_i(k) \\
\quad - \frac{C_n(k+1)s_n(k)}{C_1(k+1)}z_n(k) - \frac{C_2(k)p_1s_1}{C_1(k)^2}y(k) \\
z_2(k+1) = p_1s_1(k)z_1(k) + (1-p_2)s_2z_2(k) + \frac{p_1s_1}{C_1(k)}y(k), \\
\vdots \\
z_{n-1}(k+1) = p_{n-2}s_{n-2}(k)z_{n-2}(k) + (1-p_{n-1})s_{n-1}(k)z_{n-1}(k), \\
z_n(k+1) = p_{n-1}s_{n-1}(k)z_{n-1}(k) + s_n(k)z_n(k), \\
\hat{x}_1(k) = z_1(k) + \frac{1}{C_1(k)}y(k), \\
\hat{x}_i(k) = z_i(k) \text{ for } i = 2, \dots, n.
\end{array} \right. \tag{12}$$

The main result can then be stated as follows

Proposition 1 *Assume that $q_1 \neq 0$. There exists $\eta > 0$ such that if $E_{\min} > \eta$, then the system (11) (or system (12)) is a global exponential observer for system (2).*

Proof. Let $e(k) = x(k) - \hat{x}(k)$ be the error state estimation. The error equation is simply $e(k+1) = \bar{A}(k)e(k)$ where $\bar{A}(k) = \left(I_n - \frac{1}{C_1(k+1)}DC(k+1) \right)A(k)$.

We shall use the following matrix norm: $\|\bar{A}(k)\|_1 = \max_j \sum_{i=1}^n |\bar{a}_{ij}(k)|$. To prove that system (11)

is a global exponential observer for system (2), it is sufficient to prove that $\|\bar{A}(k)\|_1 \leq \delta < 1$.

We have $\|\bar{A}(k)\| \leq \|I_{n \times n} - \frac{1}{C_1(k+1)}DC(k+1)\| \|A(k)\|$. It is easy to see that

$$\|I_{n \times n} - \frac{1}{C_1(k+1)}DC(k+1)\| = \max_i \left(\frac{C_i(k+1)}{C_1(k+1)} + 1 \right), \text{ and } \|A(k)\| = \max_i s_i(k).$$

Hence

$$\|\bar{A}(k)\| \leq \max_i \left(\frac{C_i(k+1)}{C_1(k+1)} + 1 \right) \max_i s_i(k).$$

The remaining part of the proof is quite similar to the proof of the main result of [18]. We give it here in order to be self-contained.

For $i = 1 \dots n$, Let: $\gamma_i(k) = \frac{C_i(k+1)}{C_1(k+1)}$. We have:

$$\begin{aligned}\gamma_i(k) &= \frac{q_i E(k+1)(1 - e^{(-M_i - q_i E(k+1))\tau})}{M_i + q_i E(k+1)} \frac{M_1 + q_1 E(k+1)}{q_1 E(k+1)(1 - e^{(-M_1 - q_1 E(k+1))\tau})} \\ \gamma_i(k) &= \frac{M_1 q_i (1 - e^{(-M_i - q_i E(k+1))\tau})}{q_1 (M_i + q_i E(k+1)) (1 - e^{(-M_1 - q_1 E(k+1))\tau})} \\ &\quad + \frac{q_i q_1 E(k+1) (1 - e^{(-M_i - q_i E(k+1))\tau})}{q_1 (M_i + q_i E(k+1)) (1 - e^{(-M_1 - q_1 E(k+1))\tau})}\end{aligned}$$

Since $\frac{q_i q_1 E(k+1)}{q_1 (M_i + q_i E(k+1))} < 1$, we can write

$$\gamma_i(k) \leq \frac{M_1 q_i (1 - e^{(-M_i - q_i E(k+1))\tau})}{q_1 (M_i + q_i E(k+1)) (1 - e^{(-M_1 - q_1 E(k+1))\tau})} + \frac{1 - e^{(-M_i - q_i E(k+1))\tau}}{1 - e^{(-M_1 - q_1 E(k+1))\tau}}$$

The map $x \mapsto \frac{1 - e^{-x\tau}}{x}$ is decreasing, hence

$$\frac{1 - e^{(-M_i - q_i E(k+1))\tau}}{M_i + q_i E(k+1)} \leq \frac{1 - e^{(-m - q_{\min} E_{\min})\tau}}{m + q_{\min} E_{\min}}$$

So we obtain

$$\gamma_i(k) \leq \frac{M q_{\max}}{q_1} \frac{(1 - e^{(-m - q_{\min} E_{\min})\tau})}{m + q_{\min} E_{\min}} \frac{1}{1 - e^{(-M_1 - q_1 E(k+1))\tau}} + \frac{1 - e^{(-M_i - q_i E(k+1))\tau}}{1 - e^{(-M_1 - q_1 E(k+1))\tau}}$$

Since $0 < e^{(-M_i - q_i E(k+1))\tau} \leq 1$, we have $1 - e^{(-M_i - q_i E(k+1))\tau} \leq 1$. Therefore, we have for $i = 1 \dots n - 1$,

$$\gamma_i(k) \leq \frac{M q_{\max}}{q_1} \frac{(1 - e^{(-m - q_{\min} E_{\min})\tau})}{m + q_{\min} E_{\min}} \frac{1}{1 - e^{(-M_1 - q_1 E(k+1))\tau}} + \frac{1}{1 - e^{(-M_1 - q_1 E(k+1))\tau}}.$$

The map $x \mapsto \frac{1}{1 - e^{-x\tau}}$ is decreasing, it follows that:

$$\frac{1}{1 - e^{(-M_1 - q_1 E(k+1))\tau}} \leq \frac{1}{1 - e^{(-m - q_{\min} E_{\min})\tau}}.$$

Therefore,

$$\gamma_i(k) \leq \frac{Mq_{max}}{q_1} \frac{1}{m + q_{min}E_{min}} + \frac{1}{1 - e^{(-m - q_{min}E_{min})\tau}}, \quad \forall i = 1, \dots, n.$$

Otherwise $s_i(k) \leq e^{-m - q_{min}\tau E_{min}} \leq e^{(-m - q_{min}E_{min})\tau}$, $\forall i = 1, \dots, n$.

Hence

$$\bar{A}(k) \leq \left(\frac{Mq_{max}}{q_1} \frac{1}{m + q_{min}E_{min}} + \frac{1}{1 - e^{(-m - q_{min}E_{min})\tau}} + 1 \right) e^{(-m - q_{min}E_{min})\tau}$$

We pose:

$$\delta(E_{min}) = \left(\frac{Mq_{max}}{q_1} \frac{1}{m + q_{min}E_{min}} + \frac{1}{1 - e^{(-m - q_{min}E_{min})\tau}} + 1 \right) e^{(-m - q_{min}E_{min})\tau}$$

We will show now that there exists $\eta > 0$ in such a way that $E_{min} > \eta$ implies $\delta(E_{min}) < 1$.

Let $X = e^{(-m - q_{min}E_{min})\tau}$ and consider

$$P(X) = -\frac{\tau Mq_{max} X}{q_1 \text{Log}(X)} + \frac{X}{1 - X} + X - 1.$$

One can remark that $P(X) = \delta(E_{min}) - 1$.

We have $\lim_{X \rightarrow 0} P(X) = -1$ and $\lim_{X \rightarrow 1} P(X) = +\infty$. So there exists $x_1 \in]0, 1[$ such that $P(x_1) = 0$. Let $x^* = \inf\{x \in]0, 1[/ P(x) = 0\}$ then we have $P(X) < 0$ for all X satisfying $0 < X < x^*$. Now, $0 < X < x^*$ is equivalent to $E_{min} > \frac{-\text{Log}(x^*)}{q_{min}\tau} - \frac{m}{q_{min}}$ since $X = e^{(-m - q_{min}E_{min})\tau}$.

It is then sufficient to choose $\eta = \frac{-\text{Log}(x^*)}{q_{min}\tau} - \frac{m}{q_{min}}$ and this completes the proof of Proposition 1.

■

Remark 2 When we assume that $p_i = 1$ for $i = 1, \dots, n$, the model (2) is equivalent to the age structured fish model (see for instance [9]) for which an observer has been build in [18] using the same techniques.

Remark 3 When the natural mortality rates are all equal, it is possible to give a weaker condition on the minimal value of the fishing effort that ensures the convergence of the observer.

Proposition 2 Assume that $q_1 \leq q_i \forall i = 2, \dots, n$, and moreover that the natural mortality coefficient is the same for all stages, that is, $M_1 = M_2 = \dots = M_n = m$. Then, the system (12) is a global exponential observer for the system (2) if

$$E_{min} > \frac{1}{q_1} \left(\frac{1}{\tau} \text{Log} \left[\frac{q_{max}}{q_1} + 1 \right] - m \right).$$

Remark 4 In general, the fishery's literature provide the fishing mortality rates instead of the catchability parameters. On the other hand the available catch data usually give the total weight of the fishes caught during a season. Therefore we can consider that the measurable output is the seasonal biomass yield, i.e., the total weight of harvested fishes over each period $[k, k + 1)$ instead of their number. If we denote by w_i the mean weight of individuals of class i , then the seasonal biomass yield is

$$Y(k) = \sum_{i=1}^n \frac{q_i E(k)}{M_i + q_i E(k)} (1 - e^{(-M_i - q_i E(k))\tau}) w_i x_i(k) \quad (13)$$

Let $m_i(k) = w_i x_i(k)$ be the biomass in stage class i , then we can write the observer as:

$$\left\{ \begin{array}{l} z_1(k+1) = -\frac{w_2 C_2(k+1) p_1 s_1(k)}{w_1 C_1(k+1)} z_1(k) \\ \quad - \sum_{i=2}^{n-1} \left(p_i s_i(k) \frac{w_{i+1} C_{i+1}(k+1)}{w_1 C_1(k+1)} + (1 - p_i) s_i(k) \frac{w_i C_i(k+1)}{w_1 C_1(k+1)} \right) z_i(k) \\ \quad - \frac{w_n C_n(k+1) s_n(k)}{w_1 C_1(k+1)} z_n(k) - \frac{w_2 C_2(k) p_1 s_1}{w_1 C_1(k)^2} Y(k) \\ z_2(k+1) = p_1 s_1(k) z_1(k) + (1 - p_2) s_2 z_2(k) + \frac{p_1 s_1}{w_1 C_1(k)} Y(k), \\ \vdots \\ z_{n-1}(k+1) = p_{n-2} s_{n-2}(k) z_{n-2}(k) + (1 - p_{n-1}) s_{n-1}(k) z_{n-1}(k), \\ z_n(k+1) = p_{n-1} s_{n-1}(k) z_{n-1}(k) + s_n(k) z_n(k), \\ \hat{m}_1(k) = w_1 z_1(k) + Y(k), \\ \hat{m}_i(k) = w_i z_i(k) \text{ for } i = 2, \dots, n. \end{array} \right. \quad (14)$$

where $\hat{m}_i(k)$ is the estimator of the biomass $m_i(k)$

4 Numerical examples

We now give some simulations to illustrate the efficiency of the observer (12). We consider a three age-classes system with a Ricker stock-recruitment, and the following parameter values:

Parameters of the Ricker function	$\alpha = 1, \beta = 0.0002,$
Fecundity parameters	$b = [2 \ 5 \ 7],$
Catchability coefficients	$q = [0.24 \ 0.36 \ 0.42],$
Natural mortality rates	$M = [0.2 \ 0.2 \ 0.2],$
Length of harvesting season	$\tau = 1/2,$
Fishing effort	$E(k) = 3 + \sin(k)$

The simulations have been done with Scilab. The time evolutions of the state x_i and its corresponding estimator \hat{x}_i for $i = 1, 2, 3$ are drawn in Figures (1), (2) and (3). It can be noticed that the convergence of the error estimation towards zero is quite fast.

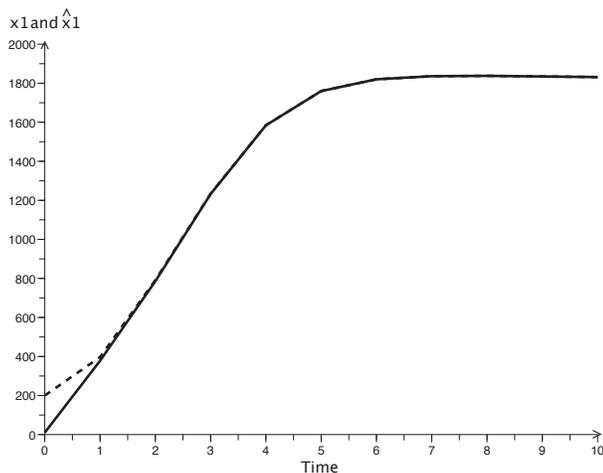


Figure 1: State x_1 and the corresponding estimator \hat{x}_1

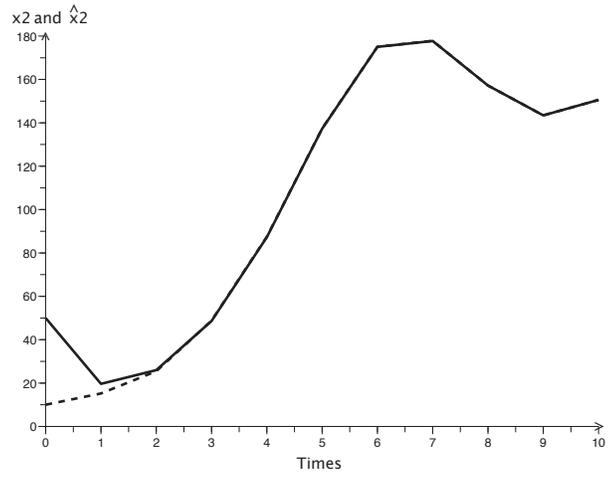


Figure 2: State x_2 and the corresponding estimator \hat{x}_2

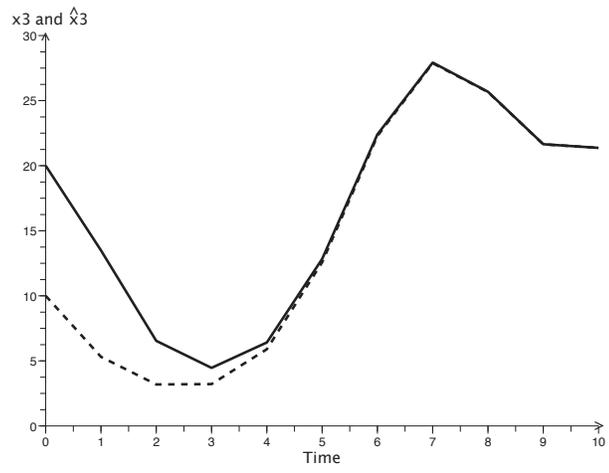


Figure 3: State x_3 and the corresponding estimator \hat{x}_3

5 Applications

5.1 Application using data from ICES

Now we try to illustrate the observer (14) of the fish model with real data. The report of ICES Advisory Committee on fisheries Management [2] (see also [26]) gives catch data for the Arctic cod population in the Atlantic North East for each year between 1946 and 2000. The considered population is structured in mature and immature individuals. The report gives also a biomass estimation for each class. Here using only the total catch for each year, our observer (14) allows to give an estimation of both matures and immatures. The captures, the fishing mortality and the biomass estimation in [2] are reported in table 5.1. To compare our estimations with those of ICES Advisory Committee on fisheries Management, we represent in the same graph the two estimations. The result show that our estimations have the same allure of those of ICES Advisory Committee on fisheries Management.

Table: Biomass and catch in million tonnes of the Arctic cod in the North- Est between year 1946 and 2000. These data are from the "ICES Advisory Committee on Fisheries Management" (Anon, 2001), table 3.7 (catch), table 3.19 (biomass) and tableau 3.23 (mature).

Biomass (million tonnes)			Catch (million tonnes)		Fishing mortality rate	
Year (t)	Immature (x_1)	Mature (x_2)	Immature (C_1)	Mature (C_2)	Immature (f_1)	Mature (f_2)
1946	3.3866	1.1206	0.4222	0.2633	0.1247	0.2350
1947	2.7283	1.1740	0.4959	0.4688	0.1818	0.3993
1948	2.8562	1.0264	0.5176	0.3510	0.1812	0.3420
1949	2.5528	0.7360	0.5503	0.2563	0.2156	0.3482
1950	2.5292	0.6206	0.4284	0.2444	0.1694	0.3938
1951	3.0474	0.5741	0.5263	0.1940	0.1727	0.3379
1952	3.4114	0.5271	0.6951	0.2428	0.2038	0.4606
1953	3.8533	0.4010	0.5209	0.1425	0.1352	0.3554
1954	3.9139	0.4350	0.7196	0.1691	0.1839	0.3887
1955	3.2905	0.3512	0.9268	0.1526	0.2817	0.4345
1956	3.1074	0.3034	1.1425	0.1421	0.3677	0.4684
1957	2.6487	0.2103	0.7023	0.0900	0.2651	0.4280
1958	2.1979	0.1978	0.6019	0.0830	0.2739	0.4196
1959	2.2912	0.4382	0.6197	0.1806	0.2705	0.4121
1960	2.0169	0.3890	0.4343	0.1629	0.2153	0.4188
1961	2.1368	0.4104	0.5113	0.2026	0.2393	0.4937
1962	1.9697	0.3167	0.5785	0.1574	0.2937	0.4970
1963	1.7560	0.2118	0.6335	0.1257	0.3608	0.5935
1964	1.3421	0.1892	0.3270	0.0989	0.2436	0.5227
1965	1.6769	0.1036	0.2991	0.0458	0.1784	0.4421
1966	2.7595	0.1224	0.3425	0.0499	0.1241	0.4077
1967	3.2925	0.1318	0.4620	0.0628	0.1403	0.4765
1968	3.2669	0.2306	0.9045	0.0914	0.2769	0.3964
1969	2.7351	0.1546	1.0495	0.0886	0.3837	0.5731
1970	1.9394	0.2281	0.7180	0.1175	0.3702	0.5151

1971	1.4845	0.3167	0.4008	0.1546	0.2700	0.4882
1972	1.7130	0.3523	0.2886	0.1895	0.1685	0.5379
1973	2.8488	0.3380	0.4363	0.1733	0.1532	0.5127
1974	2.3150	0.1669	0.7231	0.0839	0.3124	0.5027
1975	2.1793	0.1440	0.6522	0.0678	0.2993	0.4708
1976	2.0396	0.1738	0.6082	0.0755	0.2982	0.4344
1977	1.7767	0.3473	0.6734	0.1740	0.3790	0.5010
1978	1.6184	0.2460	0.4947	0.1469	0.3057	0.5972
1979	1.0343	0.1778	0.2649	0.0980	0.2561	0.5512
1980	0.8233	0.1102	0.2400	0.0590	0.2915	0.5354
1981	0.8910	0.1699	0.2437	0.0942	0.2735	0.5544
1982	0.4938	0.3315	0.1283	0.1622	0.2598	0.4893
1983	0.4865	0.3328	0.1323	0.1916	0.2719	0.5757
1984	0.7402	0.2556	0.1484	0.1444	0.2005	0.5649
1985	0.9909	0.1966	0.2047	0.0977	0.2066	0.4969
1986	1.5661	0.1731	0.3319	0.0914	0.2119	0.5280
1987	1.1331	0.1203	0.4392	0.0724	0.3876	0.6018
1988	0.8044	0.2053	0.3171	0.1178	0.3942	0.5738
1989	0.7770	0.1970	0.2461	0.0904	0.3167	0.4589
1990	0.7320	0.3446	0.1112	0.0985	0.1519	0.2858
1991	1.0702	0.6847	0.1425	0.1927	0.1332	0.2814
1992	1.3533	0.8847	0.2037	0.2960	0.1505	0.3346
1993	2.0052	0.7461	0.2974	0.2769	0.1483	0.3711
1994	1.9007	0.6137	0.4394	0.3248	0.2312	0.5292
1995	1.6134	0.5093	0.4738	0.2640	0.2937	0.5184
1996	1.3109	0.5804	0.4360	0.2856	0.3326	0.4921
1997	1.2144	0.5763	0.4373	0.3248	0.3431	0.5636
1998	1.2146	0.3961	0.3558	0.2326	0.2929	0.5872
1999	1.0585	0.2645	0.3388	0.1477	0.3201	0.5584
2000	1.0854	0.2264	0.2836	0.1285	0.2613	0.5676

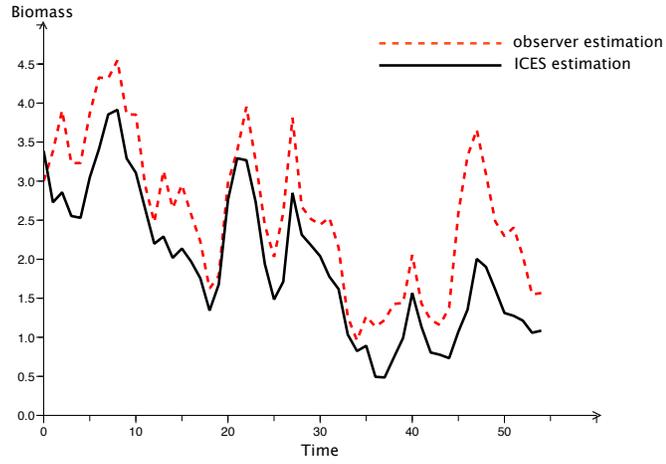


Figure 4: ICES estimation and observer estimation of stage class 1

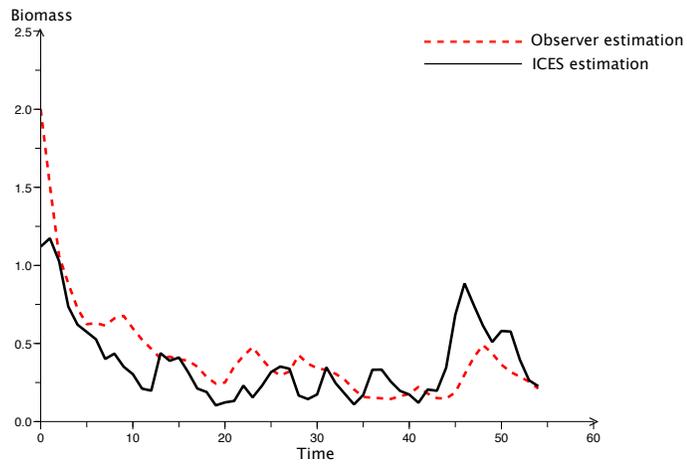


Figure 5: ICES estimation and observer estimation of stage class 2

Table 1: Trachurus spp. : Catch in millions of individuals by age classes (divisions 34.1.3. and 34.3.1) and total catch in thousand of tons.

	AGE								Catch (10 ⁶ individuals)	Catch (1000 t)
	0+	1	2	3	4	5	6	7+		
1972	0,00	1492,00	798,10	403,90	188,00	106,40	52,00	18,50	3058.9	390,123
1973	0,00	2457,00	680,90	250,90	114,50	60,30	43,20	48,50	3625.3	383,052
1974	982,00	3273,20	947,30	240,40	44,90	26,20	12,30	11,80	4562.8	414,632
1975	964,30	2078,10	493,60	152,30	48,90	7,70	4,80	2,60	2803.9	302,317
1976	3153,20	3202,30	685,60	97,40	35,60	16,30	4,60	1,10	4060.3	394,145
1977	36,20	2927,20	1391,20	380,30	52,10	12,00	6,30	1,40	4787.6	392,572
1978	60,00	1650,40	1162,80	338,70	78,20	16,50	3,00	0,50	3268.1	294,151
1979	0,00	1323,30	534,70	231,00	99,00	29,60	10,00	0,80	2246.1	223,165
1980	0,00	150,20	676,60	582,10	799,60	306,60	86,00	11,40	2619.6	503,081
1981	0,00	230,40	415,20	462,20	357,90	207,20	72,60	14,10	1764.0	357,935
1982	0,00	310,50	840,40	542,40	254,10	71,50	8,10	1,10	2045.5	310,464
1983	0,00	591,10	727,30	262,90	135,90	88,40	62,50	31,80	1886.6	280,000*
1984	807,70	424,60	621,40	274,90	167,10	105,70	68,40	41,90	1680.6	300,000*
1985	0,00	145,60	800,10	360,30	109,80	126,40	48,70	2,50	1609.4	320,000**

* : estimated value Table 2: Trachurus spp. : Fishing mortality (divisions 34.1.3 and 34.3.1). (P = mean weight)

	AGE								F PONDERE 1-5
	0+	1	2	3	4	5	6	7+	
1972	0,000000	0,511725	0,788882	0,841723	0,767820	0,616278	0,322343	0,705246	0,705287
1973	0,000000	0,710480	0,691405	0,962460	0,953076	0,933614	0,833445	0,850099	0,850218
1974	0,082420	0,911205	1,462620	0,858620	0,660721	0,921245	0,740458	0,879356	0,879611
1975	0,057783	0,351619	0,472105	0,686220	0,617037	0,312602	0,620453	0,488036	0,487918
1976	0,139428	0,387599	0,261425	0,221046	0,483193	0,632487	0,445506	0,397193	0,397150
1977	0,002260	0,258610	0,413621	0,318220	0,245832	0,424763	0,816728	0,332598	0,332224
1978	0,005763	0,184893	0,215220	0,232136	0,136975	0,167873	0,248101	0,185781	0,185410
1979	0,000000	0,233446	0,115179	0,082385	0,135158	0,096253	0,187148	0,132838	0,132484
1980	0,000000	0,030647	0,250393	0,245816	0,651817	1,229370	0,641243	0,481579	0,481609
1981	0,000000	0,045911	0,152152	0,383773	0,330540	0,502452	2,385422	0,283060	0,282977
1982	0,000000	0,098016	0,327470	0,430687	0,546851	0,139337	0,043822	0,308936	0,308472
1983	0,000000	0,168887	0,496638	0,224000	0,253589	0,540531	0,241525	0,337494	0,336677
1984	0,186031	0,134262	0,380006	0,511330	0,341390	0,457392	0,010983	0,357427	0,357427
1985	0,040390	0,063021	0,577001	0,577000	0,577000	0,577000	0,577000	0,530840	0,530840
M	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50

pig	19,00	65,00	99,00	175,00	274,00	401,00	483,00	563,00	
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Table 3: *Trachurus* spp. :Abundance in millions individuals per age class (divisions 34.1.3 and 34.3.1) and total biomass in thousand tons.

	AGE								BIO- MASS 0-7+
	0+	1	2	3	4	5	6	7+	
1972	9836,60	4635,10	1800,00	871,700	432,000	286,600	236,600	45,1000	1191,90
1973	11053,1	5966,20	1685,30	496,000	227,900	121,600	93,9000	104,000	1066,50
1974	15718,7	674,100	1779,00	512,000	114,900	53,3000	29,0000	24,7000	1080,90
1975	21772,0	8779,60	1634,80	379,000	131,600	36,0000	12,9000	8,40000	1273,90
1976	30610,6	12463,9	3746,50	618,400	115,700	43,1000	16,0000	4,20000	1929,90
1977	20373,3	16140,9	5130,70	1749,60	300,700	43,3000	13,9000	6,20000	2360,90
1978	13264,6	12329,1	7563,30	2057,80	771,900	142,600	17,2000	3,70000	2441,40
1979	10411,7	7999,20	6215,70	3699,10	989,500	408,200	73,9000	8,10000	2455,60
1980	10737,7	6315,00	3841,60	3359,80	2066,20	524,300	224,900	37,2000	2488,70
1981	6939,10	6512,70	3714,70	1813,90	1593,70	653,000	93,0000	71,8000	2024,30
1982	7913,90	4208,80	3772,90	1935,00	749,600	694,500	239,700	5,20000	1738,60
1983	7041,20	4800,00	2314,40	1649,30	763,000	263,100	366,500	139,100	1533,40
1984	6000,00	4270,70	2458,90	854,500	799,600	359,100	93,0000	174,600	1290,90
1985	7300,00	3021,40	2264,90	1019,90	310,800	357,800	137,900	7,50000	1037,30

Although the FAO data provide the catches by age class, we use for our observer only the total catch which is easier to measure in practice. Hence we add in catch table a column which gives the total catch in each time period. Though the FAO data we note that in some time there is no catch in the class 0^+ , for technical reason we will assume that population in class 0^+ is immature and the recruitment class is the class 1, thus population in age class 0^+ will not be considered. We will to use the catch data to estimate the stock fish population via the observer we have build an compare the result with the FAO estimation. To construct the observer (estimator) we need the value of the total catch $y(k)$, the fishing mortality in each class i given by $q_i \tau E(k)$ and the natural mortality rate M_i . In figure (6), (7), (8), (9), (10), (11) and (12) are drown respectively the FAO estimation stock and two corresponding estimation stock of the observer we build for the fish population in age class 1, 2, 3, 4, 5, 6 and 7.

Simulations show that the estimated values given by the observer are close to the ones of the FAO regardless of the initial condition for the observer we consider.

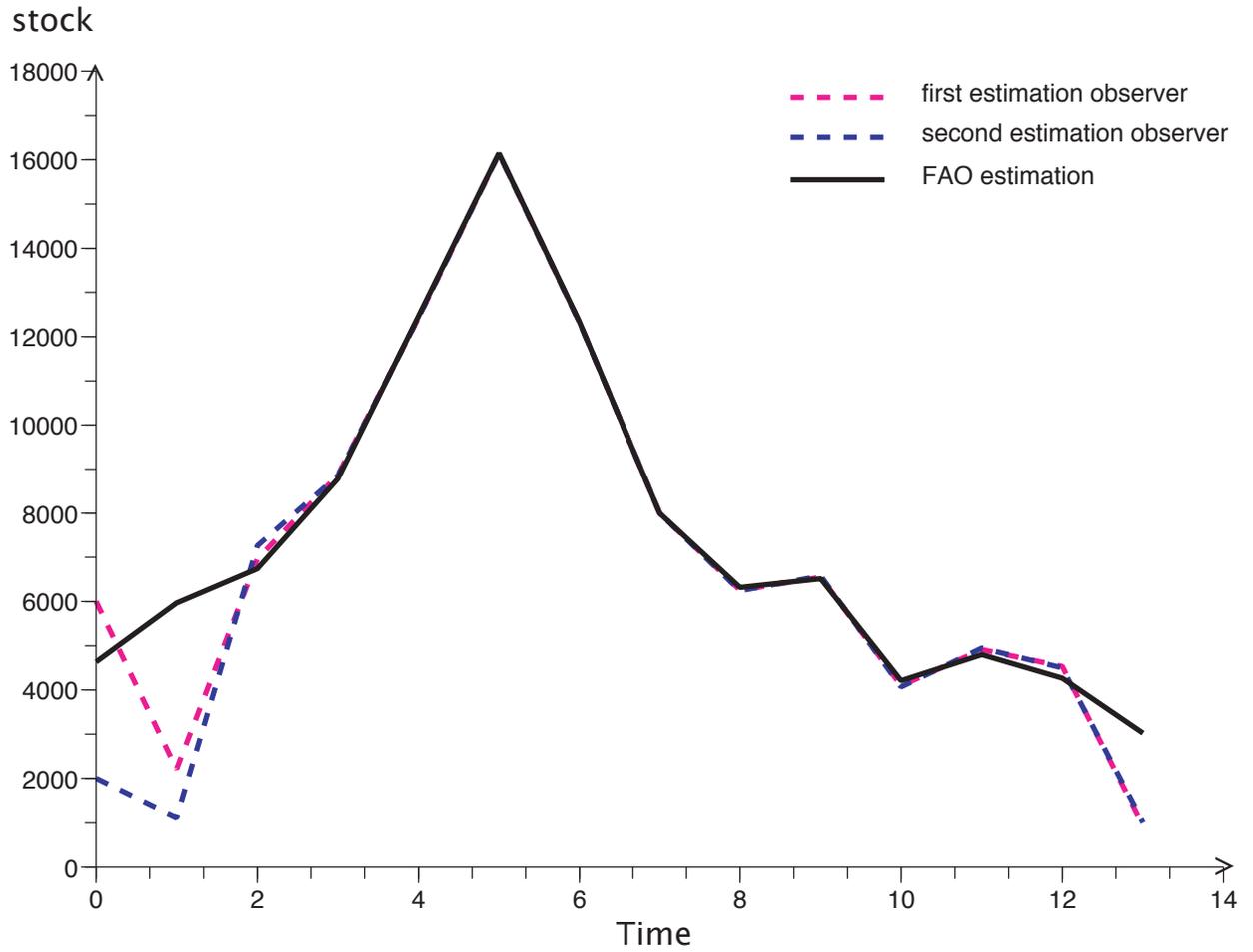


Figure 6: FAO estimation and observer estimation of age class 1

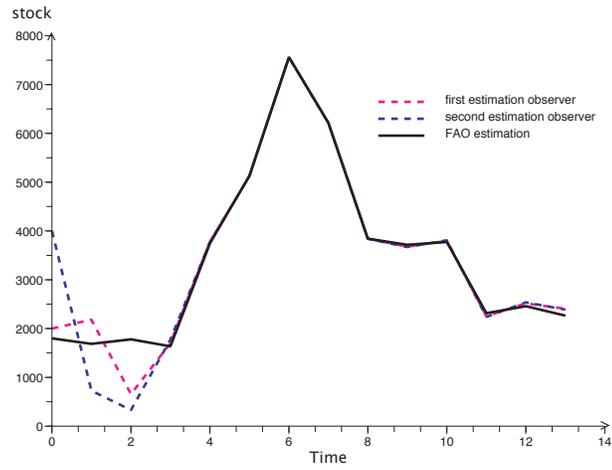


Figure 7: FAO estimation and observer estimation of age class 2

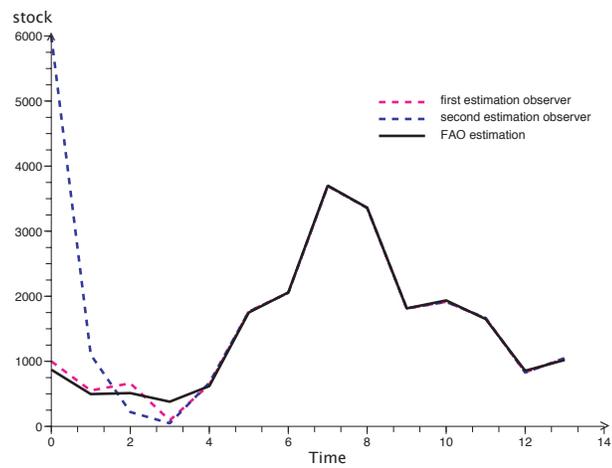


Figure 8: FAO estimation and observer estimation of age class 3

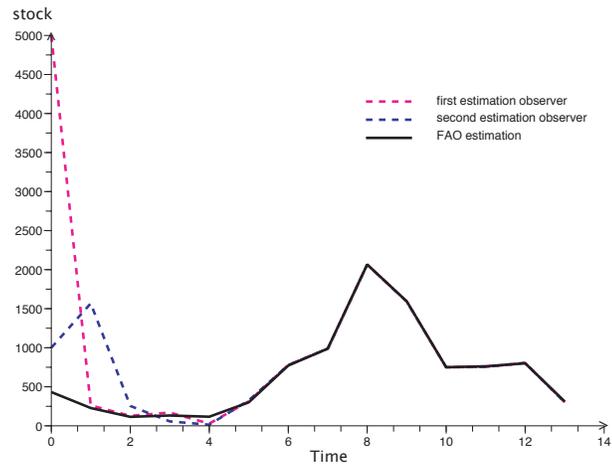


Figure 9: FAO estimation and observer estimation of age class 4

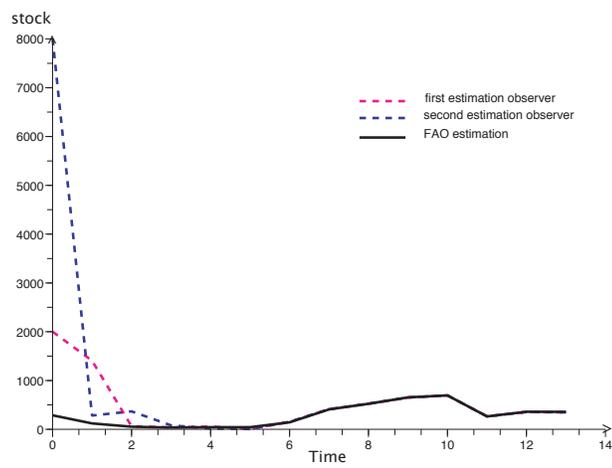


Figure 10: FAO estimation and observer estimation of age class 5

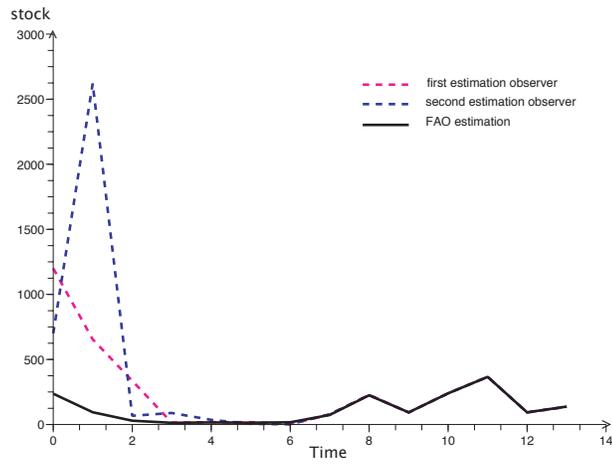


Figure 11: FAO estimation and observer estimation of age class 6

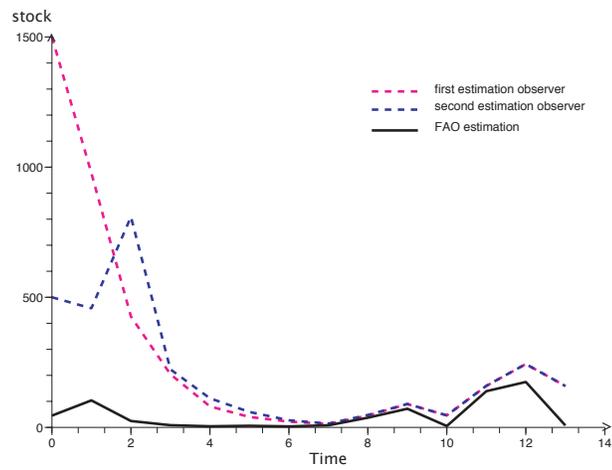


Figure 12: FAO estimation and observer estimation of age class 7

6 Conclusion

We build an observer for a stage-structured model of a harvested fish population. The model we consider is a dynamical discrete-time system including nonlinear relationship. We use the catch data to give an estimation of the stock of each class of the fish population. The observer estimator has the particularity of not using the recruitment function which is very poorly known. We note otherwise that the observer we build is easy to implement and it is successfully test by some real data from FAO and ICES Advisory Committee on fisheries Management. Historically the tools and techniques of automatic control have been developed for artificial engineering systems but nowadays they are more and more applied to "natural systems". The work we address in this paper is an example showing how control theory and computer sciences can be help full for good policies in management fishery.

References

- [1] P. Albertos, Block multirate Input-Output model for sampled-Data control systems. *IEEE Transactions on Automatic Control*, vol 35 :1085–1088, 1990.
- [2] Anon(2001), Advisory Committee on Fisheries Management (ACFM): Report of the Arctic fisheries Working Group. *ICES CM 2001/ACFM:19*.
- [3] J. S. Baras, A. Bensoussan and M. R. James, Dynamic observers as asymptotic limits of recursive filters : special cases. *SIAM Journal on Applied Mathematics*, vol 48, No 5:1147–1158, 1988.
- [4] R.J.H. Beverton, and S. J., Holt 1957 *On the dynamics of exploited fish populations*. Chapman & Hall.
- [5] S.P. Bhattacharyya, Observer design for linear system with unknown inputs. *IEEE Transactions on Automatic Control*, 23:483–484, 1978.
- [6] M. Boutayeb and M. Darouach, Observer design for Linear Time-invariant Systems *Decision and Control, Proceedings of the 39th IEEE Conference on*, vol 4: 3183–3187, 2000.
- [7] M. Darouach, M. Zasadzinski and S.J. Xu, Full-order observer for linear systems with unknown inputs *IEEE Transactions on Automatic Control*, vol 39, No 3:606–609, 1994.
- [8] FAO, *Rapport du deuxième groupe de travail AD HOC sur les Chinchards et les Maquereaux de la zone nord du COPACE*, Organisation des Nations Unies pour l'Alimentation et l'Agriculture. ROME, 1987
- [9] W. M. Getz and R. G. Haight. *Population harvesting. Demographic models of fish, forest, and animal resources*, volume 27 of *Monographs in Population Biology*. Princeton University Press, Princeton, NJ, 1989.

- [10] M. Hou and P.C. Muller, Disturbance decoupled observer design: A unified viewpoint. *IEEE Transactions on Automatic Control*, vol 39, No 6:1338–1341, 1994.
- [11] M. Hou and P.C. Muller, Design of observers for linear systems with unknown inputs. *IEEE Transactions on Automatic Control*, 37:872–875, 1992.
- [12] A. Iggidr, *Controllability, observability and stability of mathematical models*, in Mathematical Models. In Encyclopedia of Life Support Systems (EOLSS). Ed. Jerzy A. Filar. Developed under the auspices of the UNESCO, Eolss Publishers, Oxford,UK, [<http://www.eolss.net>].
- [13] J. Jin, M.J. Tank and C. Park, Time-delayed state and unknown input observation *Int. J. Control*, vol 66 No 5:733–745, 1997.
- [14] E. W. Kamen, Block form observers for linear time-varying discrete-time systems. *Proc. IEEE Conference on Decision and Control*, San Antonio, USA :355–356, 1993.
- [15] E. W. Kamen, Study of linear time-varying discrete-time systems in terms of time-compressed models. *Proc. IEEE Conference on Decision and Control*, USA :3070-3075, 1992.
- [16] D. G. Luenberger. An introduction to observers. *IEEE Transactions on Automatic Control*, AC-16(6):596–602, 1971.
- [17] P. Magal and D. Pelletier. A fixed point theorem with application to a model of population dynamics. *J. Difference Equ. Appl.*, 3(1):65–87, 1997.
- [18] D. Ngom, A. Iggidr, A. Gurio and A. Ouahbi. An Obsever for a nonlinear Age-structured model of a Harvested Fish Population . *Mathematical Biosciences and Engineering*, 5(2):337–354, 2008.
- [19] R. Nikoukhah, S. L. Campbell and F. Delebecque. Observer Design for General Linear Time-invariant Systems. *Automatica*, vol 34, No 3: 575–583, 1998.
- [20] W. E. Ricker. Stock and recruitment. *J. Fish. Res. Board Can.*, 11:559–623, 1954.
- [21] A. Saberi, A.A. Stoorvogel and P. Sunniti, Exact, almost and optimal input decoupled (delayed) observers. *Int. J. Control*, vol 73 No 7:552–581, 2000.
- [22] J. G. Shepherd, A family of general production curves for exploited populations. *Mathematical Biosciences*, 59:77–93, 1982.
- [23] S. Sundaram and C.N. Hadjicostis, Delays observers for linear systems with unknown inputs *IEEE Transactions on Automatic Control*, 52:334–339, 2007.
- [24] E. D. Sontag. *Mathematical control theory. Deterministic finite-dimensional systems*, volume 6 of *Texts in Applied Mathematics*. Springer-Verlag, New York, 1998.

- [25] M.E. Valcher, State observers for discrete-time linear systems with unknown inputs. *IEEE Transactions on Automatic Control*, vol 44, No 2:397–401, 1999.
- [26] A. Wikan and A. Eide, *An Analysis of a Nonlinear Stage-Structured Cannibalism Model with Application to the Northeast Arctic Cod Stock*, *Bulletin of Mathematical Biology*, 66, 2004, pp:1685-1704.